Preliminary concepts

Sets

Sets are well-defined collections of objects. Such Objects are called **elements if members** of the set. *Vectors* are sets of points, and *matrices* are sets of vectors.

Belonging and inclusion

Belonging:

$$a \in A$$

A is a *subset* of B, or B includes A:

$$A \subset B$$

Two sets are equal if and only if they have the *same elements*.

Set specification

D: the set of all dogs
d: dog
The set of all black dogs:

$$B = \{d \in D : d \; is \; black\}$$

or

$$B = \{d \in D | \; d \; is \; black\}$$

The colon $:$ or vertical bar $|$ read as "such that".

Ordered pairs

An unordered pair is a set whose elements are x, y, and $x, y = y, x$. Therefore, presentation order does not matter, the set is the same.

In machine learning, we usually **do** care about the presentation order. So we need to define an ordered pair.
An **ordered pair** is denoted as (x, y), with x as the *the first coordinate* and y as the *second coordinate*. $(x, y) \neq (y, x)$.

Relations

We can derive the idea of **relations** among sets or between elements and sets. In set theory, **relations** are defined as *sets of ordered pairs*, and denoted as R. We can express the relation between x and y as:

$$x \; R \; y$$

Further, for any $z \in R$, we can obtain the notions of **domain** and **range**. The domian is a set defined as:

$$domR = \{x: \; for \; some \; y \; \; (x \; R \; y)\}$$

This reads as: the values of x such that for at leat one element of y, x has a relation with y.
The **range** is a set defined as:

$$ranR = \{y: \; for \; some \; x \; \; (x \; R \; y)\}$$

This reads as: the set formed by the values of y such that at least one element of x, x has a relation with y.

Fuctions

A **function** from X to Y is a relation such that:

> $dom f = X$
> such that for each $x \in X$ there is a unique element of $y \in Y$ with $(x, y) \in f$

We can say that a function "transform" or "map" or "sent" x onto y, and for each "argument" x there is a unique value y that f "assumes" or "takes".

We typically denote a relation a relation or function or transformation or mapping from X onto Y as:

$$f : X \ \rightarrow \ Y$$

or

$$f(x) \ = \ y$$

Each value $f\left(x\right)$ maps uniquely onto one value of y.
For $f : X \ \rightarrow \ Y$, the domain of f equals to X, but the range does not necessarily equals to Y. The range includes only the elements for which Y has a relation with X.

The ultimate goal of machien learning is learning functions from data.
The domain X is usually a vector (or set) of *variables* or *features* mapping onto a vector of *target values*.