Numerical simulation of beam-driven Alfvén eigenmodes on EAST tokamak

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1 EAST neutral beam injection geometry

In the EAST experiment campaign during 2014/06-2014/10, the NBI is in the co-current direction. The toroidal magnetic field is in the opposite direction of the current. The pitch angle of the fast ions generated by the NBI is defined by the included angle between the beam velocity and the magnetic field. Next, we estimate the pitch angle of the EAST NBI. Since the poloidal magnetic field is much smaller than the toroidal field, the pitch angle of the beam velocity with respect to the local magnetic field can be approximated as the included angle θ between the beam velocity and the toroidal direction. Using the definition of the tangential radius of the beams, the value of θ at at R can be calculated as

$$\theta(R) = \pi - \left[\frac{\pi}{2} - \text{Arcsin}(R_{\tan}/R)\right]. \tag{1}$$

Using this formula, we obtain, at $R = 1.8$ (correspond approximately to the location of the magnetic axis), the pitch angle of the beam with $R_{\tan} = 1.26m$ is $\theta = 134.43°$; the pitch angle of the beam with $R_{\tan} = 0.73$ is $\theta = 114.93°$. The averaged pitch angle of the two beams is approximated as $(134.43° + 114.93°)/2 = 124.68°$.

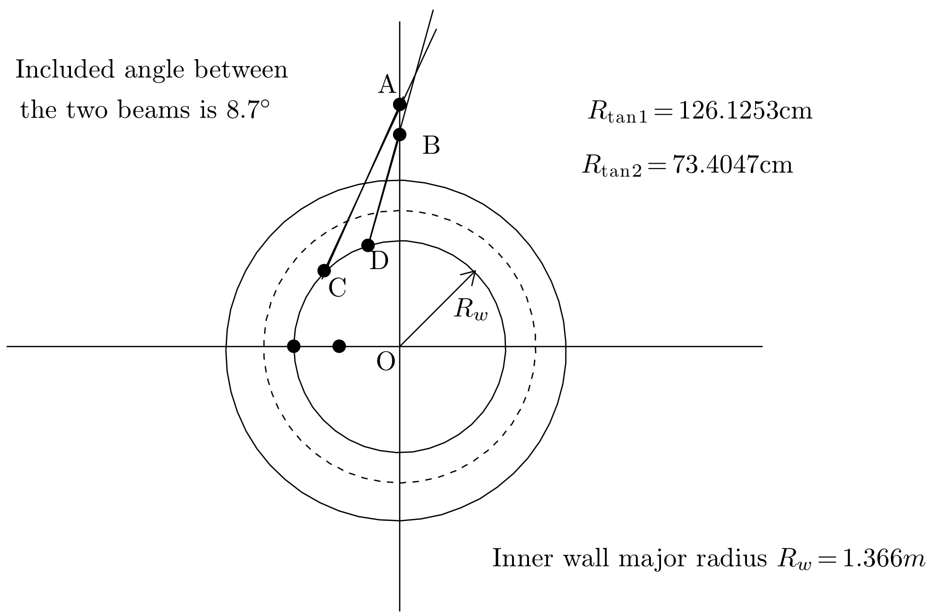

Figure 1. Top view of the neutral beam injection geometry on EAST tokamak. The two beams both lie in the midplane of the device, with the included angle between them being 8.7°. The tangential radius (the perpendicular distance from the axsymmetrical axis of the device to the beam line) of the two beams are 73.4047cm and 126.1253cm, respectively. Because the major radius of the inner wall of EAST ($R_w = 1.366m$) is larger than the tangential radius of the two beams, both the beam lines insect the inner wall.

The current is in the count clockwise direction (top view) and the magnetic field is the clockwise direction(top view).

2 NBI fast ions distribution

The isotropic “slowing down” distribution of fast ions is given by

$$f(v) = \begin{cases} \dfrac{n(\psi)}{v^3 + v_{\text{crit}}^3} \ \text{for}\ 0 < v \leqslant v_{\text{birth}} \\ 0 \qquad\qquad \text{for}\ v > v_{\text{birth}} \end{cases}, \tag{2}$$

where v_{birth} is the birth velocity of the fast ions (for the 80keV Deuterium NBI on EAST

$$E_h = 80\text{keV} \tag{3}$$

,

$$v_{\text{birth}} = \sqrt{2E_b/m_D} = 2.76 \times 10^6 m/s$$

),

v_{crit} is the critical velocity for the collisional friction of fast ions with electrons and ions being equal, which is given by

$$v_{\text{crit}} = \left(\frac{m_e}{m_i}\frac{3\sqrt{\pi}}{4}\right)^{1/3} v_{te}, \tag{4}$$

(The derivation of Eq. (4) is given in Sec. 3.1.) For $T_e = 2\text{keV}$, $v_{\text{crit}} = 2.38 \times 10^6 m/s$.

[In the numerical implementation, the jump of the distrbution function at $v = v_b$ is usually mimicked by the error function erfc, i.e., equation (2) is approximated by

$$f(v) = \frac{n}{v^3 + v_{\text{crit}}^3}\frac{1}{2}\text{erfc}\left(\frac{v - v_{\text{birth}}}{\Delta v}\right), \tag{5}$$

where erfc is the error function and Δv is set to a small value (in my simulation Δv chosen so that $\Delta v / V_A = 0.05$, where V_A is the Alfven speed at the magnetic axis).]

Define the normalized magnetic moment

$$\lambda = \frac{\mu B_0}{\varepsilon}, \tag{6}$$

where B_0 is the strength of the equilibrium magnetic field at the magnetic axis. Equation (6) can be further written

$$\lambda = \frac{\frac{m v_\perp^2}{2B} B_0}{\varepsilon} = \frac{B_0}{B}\frac{m v_\perp^2}{m v^2} = B_0 \frac{\sin^2\theta}{B}. \tag{7}$$

In terms of the midplane coordinates (v_0, θ_0), the above equation is written

$$\lambda = B_0 \frac{\sin^2\theta_0}{B_{\min}}, \tag{8}$$

where $B_{\min}$ the strength of the magnetic field at the low-field-side of the midplane. Using $B_0 \approx B_{\min}$ and $\theta_0 \approx 124°$ for EAST, we obtain $\lambda \approx 0.68$. [In the numerical implementation, the dependence of f on λ is assumed of the form

$$\exp\left(-\frac{(\lambda - \lambda_0)^2}{\Delta\lambda}\right), \tag{9}$$

where λ_0 and $\Delta\lambda$ are the centeal value of λ and the width of Gaussian function, respectively. In the simulation, I choose $\lambda = 0.68$ and $\Delta\lambda = 0.1$.]

The radial profile of the density of the fast ions is assumed of the form

$$\exp\left(-\frac{\psi^2}{\psi_{\text{scale}}}\right),$$

where ψ is the normalized poloidal flux and ψ_{scale} is the radial scale length. In the simulation, I choose $\psi_{\text{scale}}=0.4$.

$$f(\psi,v,\lambda)=n(\psi)\frac{1}{v^3+v_{\text{crit}}^3}\frac{1}{2}\text{erfc}\left(\frac{v-v_{\text{birth}}}{\Delta v}\right)\exp\left(-\frac{(\lambda-\lambda_0)^2}{\Delta\lambda}\right). \tag{10}$$

$$n(\psi)=n_0\exp\left(-\frac{\psi}{\psi_{\text{scale}}}\right) \tag{11}$$

Use the EAST discharge #38300 at 3.9s as a reference equilibrium. The magnetic field strength at magentic axis is 1.63Tesla. The Alfvén velocity at the magnetic axis is $V_A=3.835\times 10^6 m/s$. For the 80keV Deuterium NBI on EAST, $v_{\text{birth}}=\sqrt{2E_b/m_D}=2.76\times 10^6 m/s$. The ratio of the beam velocity to the Alfvén velocity is

$$\frac{v_{\text{birth}}}{V_A}=\frac{2.76\times 10^6}{3.835\times 10^6}=0.72 \tag{12}$$

EAST NBI: 4MW $\times\, 10s$ @ 80keV

2.1 Normaliztion used in MEGA simulation

The time unit used in Mega simulation is $1/\Omega_h$, where $\Omega_h=B_0|q_h|/m_h$ is the cyclotron angular frequency of the energetic particles (EPs), B_0 is the strength of the magnetic field at the magnetic axis, q_h and m_h are the charge and the mass of the EPs. The velocity unit in Mega is the Alfvén speed at the magnetic axis, $V_{A0}=B_0/\sqrt{\mu_0\rho_{m0}}$, where ρ_{m0} is the mass density at the magnetic axis. Since both time unit and velocity unit has been chosen, the length unit used in Mega is a derived unit, which is defined by V_{A0}/Ω_f.

The magnetic field unit used in Mega is the strength of the magnetic field at the magnetic axis.

The Alfven time is defined by $t_A=R_0/V_{A0}$, where R_0 is the major radius of magnetic axis.

MEGA code (MHD+EPs hybrid codes)

$$f(\theta,t)=\sum_{m=0}^{\infty}A_m(t)\cos(m\theta)+\sum_{m=0}^{\infty}B_m(t)\sin(m\theta)$$

$$A_m\cos(m\theta)+B_m\sin(m\theta)$$

$$m=1$$

$$m=2$$

$$m=3$$

2.2 Magnetic flux coordinates used in analyzing the MEGA simulation results

Simulation performed in the cylindrical coordinates (R, ϕ, Z).

Flux coordinates (ψ, θ, ϕ) used in analyzing MEGA simulation results

Here ψ is the normalized poloidal flux
ϕ is the usual toroidal angle
θ is chosen to make magnetic field lines straight on (θ, ϕ) plane

proportional to ϕ along the magnetic field line so that the magnetic field lines are straight lines (with slope q) on (θ, ϕ) plane.

3 Simulation results

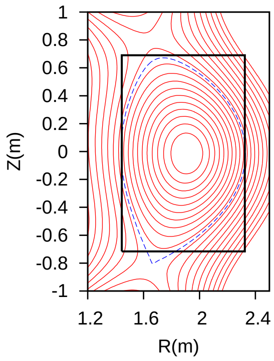

Figure 2. Flux surfaces shape and the compuational region of Mega simulation for EAST discharge #38300 at $3.9s$. The center of the compuational region is chosen to be at $(R = R_0, Z = Z_{\rm axis})$, where R_0 is major radius of LCFS, $Z_{\rm axis}$ is the Z coordinate of the magnetic axis.

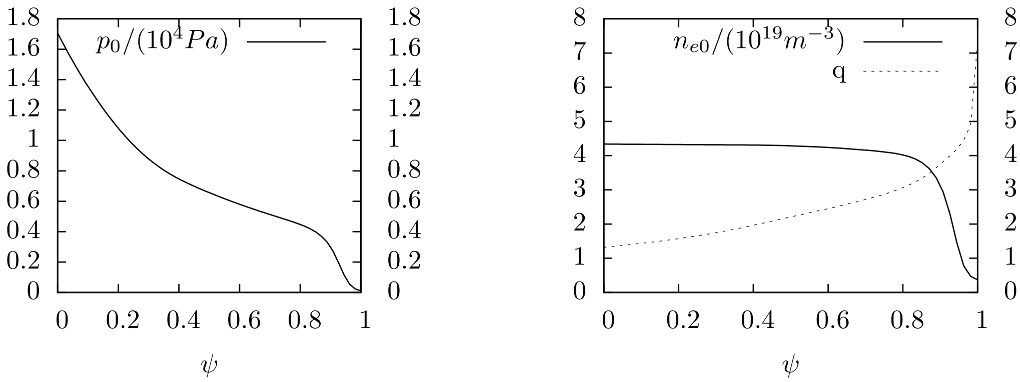

Figure 3. Profiles of the pressure (a), electron number density and safety factor (b) for EAST discharge #38300 at 3.9s.

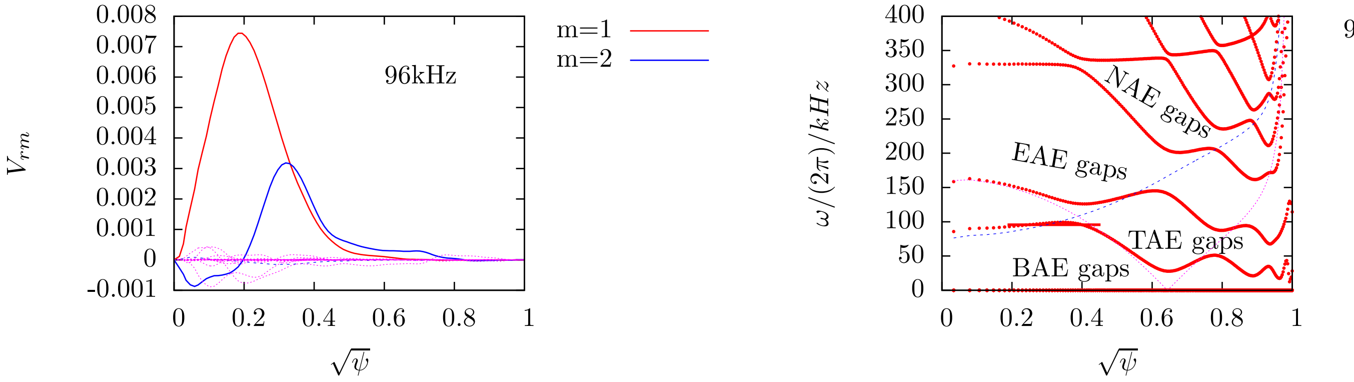

Figure 4. (a) $n=1$ TAE mode structure and (b) $n=1$ Alfven continua.

$$\frac{V_A}{R_0}$$

$$t=\bar{t}\;/\Omega_h=\bar{\tau}39.1/\Omega_h$$

$$0.0476243\times7.85009\times10^7/39.1=95.6151\times10^3$$

3.1 Derivation of the critical velocity

The velocity of the fast ions generated in NBI is much larger than the thermal velocity of ions but smaller than the electron thermal velocity, i.e,

$$v_{ti}\ll v_f<v_{te}. \tag{13}$$

For the α particles created in D-T reaction, the relation in Eq. (13) also applies. [The ratio of α particle's velocity to v_{te} is given by

$$\frac{v_\alpha}{v_{te}}=\sqrt{\frac{T_\alpha}{T_e}\frac{m_e}{m_\alpha}}. \tag{14}$$

For an electron temperature $T_e=20$keV, the above equation gives

$$\frac{v_\alpha}{v_e}=\sqrt{\frac{3.5\times10^6\text{eV}}{20\times10^3\text{eV}}\frac{1}{1836}}=0.304, \tag{15}$$

which indicates that the velocity of α particles are still smaller than the electron thermal velocity.] Next, we derive the criticle velocity of fast ions for which the collision friction of the fast ions with electrons and ions is equal. The collision friction coefficient due to an isotropic field-particles is given by[2]

$$F^{a/b}(v)=-\frac{4\pi\Gamma^{a/b}}{3n_b}\frac{m_a}{m_b}\frac{1}{v^2}\int_0^v3(v')^2f_b(v')dv', \tag{16}$$

where $\Gamma^{a/b}=\frac{n_bq_a^2q_b^2}{4\pi\epsilon_0^2m_a^2}\ln\Lambda^{a/b}$. Consider the collision friction of fast ions with thermal ions. Assume the distribution of thermal ions are Maxwellian, then Eq. (16) is written

$$F^{f/i}(v)=-\frac{4\pi\Gamma^{f/i}}{3n_i}\frac{m_f}{m_i}\frac{1}{v^2}\int_0^v3(v')^2f_{Mi}(v')dv'. \tag{17}$$

Since v_f is much larger than the thermal velcity of ions, the collision friction coefficients can be approximated by the high-velcity-limit (i.e. to set the upper limit of the integration to be $+\infty$), which gives

$$F^{f/i} = -4\pi\Gamma^{a/i}\frac{m_a}{m_i}\frac{1}{v^2}\left(\frac{m_i}{2\pi T_i}\right)^{3/2} v_{ti}^3\frac{\sqrt{\pi}}{4} \tag{18}$$

For the collision friction of fast ions with thermal electrons, since $v_f < v_{te}$, Eq. (16) is written

$$\begin{aligned}F^{f/e} &= -\frac{4\pi\Gamma^{a/e}}{n_e}\frac{m_a}{m_e}\frac{1}{v^2}\int_0^v (v')^2 f_{Me}(v')dv' \\ &= -\frac{4\pi\Gamma^{a/e}}{n_e}\frac{m_a}{m_e}\frac{1}{v^2}\int_0^v (v')^2 n_e\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{v^2}{v_t^2}\right)dv' \\ &= -4\pi\Gamma^{a/e}\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\frac{m_a}{m_e}\frac{v_t^3}{v^2}\int_0^x x^2\exp(-x^2)dx, \end{aligned} \tag{19}$$

where $x = v/v_{te}$. Sinc $x < 1$, we expand $e^{-x^2} \approx 1$. Using this, Eq. (19) is written

$$\begin{aligned}F^{f/e} &= -4\pi\Gamma^{a/e}\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\frac{m_a}{m_e}\frac{v_{te}^3}{v^2}\int_0^x (x^2)dx \\ &= -4\pi\Gamma^{a/e}\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\frac{m_a}{m_e}\frac{v_{te}^3}{v^2}\left(\frac{x^3}{3}\right)\end{aligned} \tag{20}$$

The critical velocity v_{crit} is given by the balance of $F^{f/i}$ and $F^{f/e}$, i.e. $F^{f/i} = F^{f/e}$. Using this, along with Eqs. (18) and (20), we obtain

$$-4\pi\Gamma^{a/i}\frac{m_a}{m_i}\frac{1}{v_{\text{crit}}^2}\left(\frac{m_i}{2\pi T_i}\right)^{3/2} v_{ti}^3\frac{\sqrt{\pi}}{4} = -4\pi\Gamma^{a/e}\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\frac{m_a}{m_e}\frac{v_{te}^3}{v_{\text{crit}}^2}\left(\frac{v_{\text{crit}}^3}{3v_{te}^3}\right),$$

Using $\Gamma^{a/b} = \frac{n_b q_a^2 q_b^2}{4\pi\epsilon_0^2 m_a^2}\ln\Lambda^{a/b}$ and $\ln\Lambda^{f/i} \approx \ln\Lambda^{f/e}$, the above equation is written

$$v_{\text{crit}} = \left(\frac{n_i Z^2}{n_e}\frac{m_e}{m_i}\frac{3\sqrt{\pi}}{4}\right)^{1/3} v_{te},$$

which corresponds to the fast ions kinetic energy

$$E_{\text{crit}} \equiv \frac{1}{2}m_f v_{\text{crit}}^2 = \frac{m_f}{m_i}\left(\frac{m_i}{m_e}\right)^{1/3}\left(\frac{n_i Z^2}{n_e}\frac{3\sqrt{\pi}}{4}\right)^{2/3} T_e, \tag{21}$$

which agrees with the equation given in Ref. [1].

4 Coupling of EPs and MHD

The MHD momentum equation is given by

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = Q_{\text{MHD}}\boldsymbol{E} - \nabla p + \boldsymbol{J}_{\text{MHD}}\times\boldsymbol{B}, \tag{22}$$

where Q_{MHD} and $\boldsymbol{J}_{\text{MHD}}$ are the charge density and current density of the MHD plasma, respectively. For a system consisting of MHD plasma and energetic partilces, Q_{MHD} and $\boldsymbol{J}_{\text{MHD}}$ are written as

$$Q_{\text{MHD}} = Q - Q_h, \tag{23}$$

$$\boldsymbol{J}_{\text{MHD}} = \boldsymbol{J} - \boldsymbol{J}_h, \tag{24}$$

where Q and Q_h are the total charge density and the charge density of EPs, respectively, $\boldsymbol{J}$ and $\boldsymbol{J}_h$ are the total current density and the current density of EPs, respectively. Using Eqs. (23) and (24), equation (22) is written

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = (Q - Q_h)\boldsymbol{E} - \nabla p + (\boldsymbol{J} - \boldsymbol{J}_h)\times\boldsymbol{B}. \tag{25}$$

Thus, the charge density and current density of EPs are coupled to the MHD momentum equation.

Note that the MHD+EPs plasma system in tokamak device is created by three means, i.e., electromagnetic wave heating, neutral beam injection, and the fusion reactions. None of the three means introduce net charges into the plasma. Thus the total charge neutrality can be assumed. Further, we assume the local charge neutrality is valid, i.e., the total charge density of the MHD+EPs system is assumed to be zero, i.e., $Q=0$. Using these, and noting that the total current density is given by $\boldsymbol{J}=(\nabla\times\boldsymbol{B})/\mu_0$, the momentum equation is written

$$\rho\left(\frac{\partial\boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-Q_h\boldsymbol{E}-\nabla p+[(\nabla\times\boldsymbol{B})/\mu_0-\boldsymbol{J}_h]\times\boldsymbol{B}. \tag{26}$$

The current density of energetic particles is given by

$$\boldsymbol{J}_h=\int(\boldsymbol{v}_\parallel^\star+\boldsymbol{v}_B+\boldsymbol{v}_E)Z_hefd^3v-\nabla\times\int\mu\boldsymbol{b}fd^3v, \tag{27}$$

where Z_he the charge of the energetic particles, the last term is the magnetization current (which seems to be independent of the charge of the EPs, why?). Consider the current density contributed by the $\boldsymbol{E}\times\boldsymbol{B}$ drift, i.e., $\int\boldsymbol{v}_Efd^3v$, which can be written as

$$\int\left(\frac{1}{B_\parallel^\star}\boldsymbol{E}\times\boldsymbol{b}\right)Z_hefd^3v. \tag{28}$$

Then the Lorentz force of this current is written

$$-\boldsymbol{B}\times\int\left(\frac{1}{B_\parallel^\star}\boldsymbol{E}\times\boldsymbol{b}\right)Z_hefd^3v, \tag{29}$$

which can be further written as

$$\int\left[\frac{1}{B_\parallel^\star}(\boldsymbol{B}\cdot\boldsymbol{E})\boldsymbol{b}-\frac{1}{B_\parallel^\star}B\boldsymbol{E}\right]Z_hefd^3v, \tag{30}$$

Using that $B_\parallel^\star\approx B$ and $\boldsymbol{E}$ is approximately perpendicular to $\boldsymbol{B}$ (for ideal MHD, this is exact), the above expression is written as

$$-\boldsymbol{E}\int Z_hefd^3v, \tag{31}$$

i.e., $-Q_h\boldsymbol{E}$. Using this in Eq. (26), we find that the electric force $Q_h\boldsymbol{E}$ happens to be canceled out, which yields

$$\rho\left(\frac{\partial\boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-\nabla p+\left[(\nabla\times\boldsymbol{B})/\mu_0-\boldsymbol{J}_h'\right]\times\boldsymbol{B}, \tag{32}$$

where

$$\boldsymbol{J}_h'=\int(\boldsymbol{v}_\parallel^\star+\boldsymbol{v}_B)Z_hefd^3v-\nabla\times\int\mu\boldsymbol{b}fd^3v, \tag{33}$$

is the current density of EPs with the contribution of $\boldsymbol{E}\times\boldsymbol{B}$ drifts removed. Equations (32) and (33) are the MHD+EPs coupling model used in Mega simulation[3].

5 misc

For 3.5MeV α particles created in D-T reaction, $v_{\text{birth}}=1.295\times10^7m/s$;

$$a_m\cos(m\theta)+b_m\sin(m\theta) \tag{34}$$

$$a_m \cos(m\theta) + b_m \sin(m\theta) = A_m \cos(m\theta + \alpha) = A_m \cos(\alpha)\cos(m\theta) - A_m \sin(\alpha)\sin(m\theta) \tag{35}$$

$$\cos(\alpha) = \frac{a_m}{A_m} \tag{36}$$

$$\sin(\alpha) = -\frac{b_m}{A_m} \tag{37}$$

$$1 = \left(\frac{a_m}{A_m}\right)^2 + \left(\frac{b_m}{A_m}\right)^2 \tag{38}$$

$$A_m^2 = a_m^2 + b_m^2 \tag{39}$$

$$a_m \cos(m\theta) + b_m \sin(m\theta) = A_m \sin(m\theta + \alpha) = A_m \sin(\alpha)\cos(m\theta) + A_m \cos(\alpha)\sin(m\theta) \tag{40}$$

$$\sin(\alpha) = \frac{a_m}{A_m}$$

$$\cos(\alpha) = \frac{b_m}{A_m}$$

$$1 = \left(\frac{a_m}{A_m}\right)^2 + \left(\frac{b_m}{A_m}\right)^2 \tag{41}$$

$$A_m^2 = a_m^2 + b_m^2 \tag{42}$$

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