A RELATIVISTIC THEORY OF ELECTRON CYCLOTRON CURRENT DRIVE EFFICIENCY

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A fully relativistic theory of electron cyclotron current drive (ECCD) efficiency based on Green's function techniques is considered. Numerical calculations of the current drive efficiency in a uniform magnetic field are performed. The numerical results with parameter regimes relevant to ITER operation are compared with those of two simplified models in which the electron-electron Coulomb collision operator is respectively approximated by its high-velocity limit and a semirelativistic form. Our results indicate that the semirelativistic approximation of the collision operator should be appropriate for modeling the ECCD efficiency under ITER conditions.

KEYWORDS: *ECCD efficiency, Coulomb collision operator, fully relativistic*

I. INTRODUCTION

Owing to its ability to drive local current on- and off-axis, electron cyclotron current drive (ECCD) has been considered the best tool for current profile control to achieve high beta and high confinement in tokamak operation. The EC heating and current drive system is planned for ITER. Although relativistic effects have been known to play important roles in the wave absorption and current drive, existing ECCD models rely on either a semirelativistic Coulomb collision operator or the high-velocity model collision operator to describe the electron-electron interaction.[1–3] The predictions of the ECCD efficiency by these models were confirmed by the experiments in the present-day tokamaks. Their validity for the ITER plasmas remains to be established. In this work, we present a fully relativistic theory of the ECCD efficiency. The relativistic collision operator of Beliaev and Budker[4] is used to model the Coulomb collision among electrons. The relativistic generalization of the local Kennel-Engelmann wave-induced diffusion operator[5] in velocity space is used to describe the wave-particle interaction. The ECCD efficiency is calculated using the Green's function techniques. An equivalent relativistic formulation of the problem has been given previously by Kalisov and Kernbichler.[6] In the parameter regime relevant to ITER operation, the current drive efficiencies without considering trapped electron effects will be numerically evaluated and compared with the predictions of the existing models. Our results can be used as a guide for upgrading Fokker-Planck codes and approximate current drive packages for ITER applications.

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II. ECCD EFFICIENCY

Following Cohen,[2] or equivalently Lin-Liu, Chan, and Prater,[3] we use Green's function techniques to calculate ECCD efficiency. The perturbed electron distribution f_1 and the corresponding current drive response function f_r satisfy, respectively, the linearized Fokker-Planck equation and the adjoint equation, given as

$$v_\parallel \mathbf{b}\cdot\nabla f_1 - C_e^l f_1 = S_w(f_M) \tag{1}$$

and

$$v_\parallel \mathbf{b}\cdot\nabla f_r - C_e^{l+} f_r = \frac{v_\parallel B}{\langle B^2\rangle} \ , \tag{2}$$

where

$$\mathbf{b} = \mathbf{B}/B$$

$v_\parallel$ = velocity component along the magnetic field

C_e^l = linearized Coulomb collision operator

C_e^{l+} = adjoint collision operator

$$\int d\Gamma f C_e^l g = \int d\Gamma\, g C_e^{l+} f \ . \tag{3}$$

For the case of a homogeneous magnetic field $B = B_0$, from Eqs. (1) and (2) we obtain, respectively,

$$j_\parallel = -B_0 q_e \int d\Gamma f_r S_w(f_M) \tag{4}$$

and

$$C_e^{l+} f_r = -\frac{v_\parallel}{B_0} \ . \tag{5}$$

From Eq. (5) it is easy to see that f_r has the form of $f_r^{(1)}(u)\cos\theta$, where θ is the angle between the velocity and the magnetic field. So the equation can be reduced to a one-dimensional integro-differential equation.

The wave-particle interaction is described by the relativistic generalization of the local Kennel-Engelmann wave-induced diffusion operator[5] in velocity space, $S_w(f)$, whose explicit expression is given in Ref. 1. The dimensionless ECCD efficiency defined in Ref. 3 can be expressed as

$$\zeta^* = \frac{e^3 n_e}{2\pi\varepsilon_0^2 T_e}\frac{\langle j_\parallel\rangle}{Q} = -\frac{1}{\Theta}\frac{2}{\log\Lambda^{e/e}}\left\langle\frac{B}{B_{\max}}\right\rangle \times \frac{\displaystyle\int \gamma^2\, d\gamma D_l f_M \tilde{L}\tilde{f}_r}{\displaystyle\int \gamma^2\, d\gamma D_l f_M} \ , \tag{6}$$

where

$$j_\parallel = -q_e B\left\langle \int d\Gamma f_r S_w(f_M)\right\rangle \ , \tag{7}$$

$$Q = \left\langle \int d\Gamma\, w S_w(f_M)\right\rangle \ , \tag{8}$$

$$\langle(\cdots)\rangle = \frac{\displaystyle\oint \frac{dl_p}{B_p}(\cdots)}{\displaystyle\oint \frac{dl_p}{B_p}} \ , \tag{9}$$

$$w = (\gamma - 1)mc^2 \ ,$$

$$\Theta = \frac{T_e}{mc^2} \ ,$$

$$\tilde{L} \equiv \frac{\partial}{\partial\gamma} + n_\parallel\frac{\partial}{\partial u_\parallel} \ ,$$

$$\gamma = \sqrt{1+u^2} \ ,$$

$$u = \gamma v \ ,$$

$$\tilde{f}_r = \frac{B_{\max}\nu_c}{c} f_r \ ,$$

$$\nu_c = \frac{\Gamma^{e/e}}{c^3} \ ,$$

$$\Gamma^{e/e} = \frac{n_e q_e^4 \log\Lambda^{e/e}}{4\pi\epsilon_0^2 m^2} \ ,$$

$$f_M = \frac{n_e}{4\pi m^3 c^3 \Theta K_2(\Theta^{-1})}\exp\left(-\frac{\gamma}{\Theta}\right) \ ,$$

$$D_l = |\psi_l|^2\left(\frac{u_\perp}{\gamma}\right)^2 \ ,$$

$$\psi_l = J_{l+1}(x)\frac{(E_1 - iE_2)}{2} + J_{l-1}(x)\frac{(E_1 + iE_2)}{2} + \frac{u_\parallel}{u_\perp}J_l(x)E_\parallel \ ,$$

$$x = l\frac{n_\perp u_\perp}{y} \ ,$$

$$y = \frac{l\omega_c}{\omega} \ ,$$

$$\omega_c = \frac{q_e B}{m} \ ,$$

$$u_\parallel = \frac{\gamma - y}{n_\parallel} \ ,$$

$$n_\parallel = \frac{k_\parallel c}{\omega} \ ,$$

and

$$n_\perp = \frac{k_\perp c}{\omega} \ .$$

Here $j_\parallel$ is the wave-driven current, Q is the absorbed wave power density, w is the particle energy, dl_p is the line element along the poloidal circumference, B_p is the poloidal magnetic field, $d\Gamma$ is the volume element in velocity space; T_e, m, c, u, n_e, q_e, $\log\Lambda^{e/e}$, B, $B_{\max}$, K_2, J_l, l, E, k, ω_c, and ω are, respectively, electron temperature, electron mass, light speed, momentum per unit mass normalized to c, electron density, electron charge, the

Coulomb logarithm, magnetic field, the maximum of B on the flux surface, the second-order modified Bessel function of the second kind, the Bessel function of order l, cyclotron harmonic number, wave electric field, wave number, electron cyclotron frequency, and wave frequency. The subscript $\parallel$ ($\perp$) denotes the parallel (perpendicular) component with respect to the magnetic field. E_1 and E_2 are the perpendicular components of the wave electric field, which are given by using hot plasma dispersion relations,[7] and f_M is the electron's Maxwellian distribution.

In the widely used TORAY-GA ray-tracing code[7,8] Cohen's definition of dimensionless ECCD efficiency is used:

$$\zeta_c = \frac{\langle j_\parallel \rangle}{e n_e v_t} \left(\frac{Q}{\nu_t n_e m v_t^2} \right)^{-1} = -(2\Theta)^{-1} \frac{\int \gamma^2 \, d\gamma D_l f_M \tilde{L} \tilde{f}_r}{\int \gamma^2 \, d\gamma D_l f_M} , \tag{10}$$

where

$$\nu_t = \frac{\Gamma^{e/e}}{v_t^3} ,$$

$$\Gamma^{e/e} = \frac{n_e e^4 \ln \Lambda^{e/e}}{4\pi \epsilon_0^2 m^2} ,$$

and

$$v_t = \sqrt{\frac{2T_e}{m}} . \tag{11}$$

The relation between Cohen's ECCD efficiency ζ_c and ζ^* defined in Eq. (6) is given as

$$\frac{\zeta_c}{\zeta^*} = \frac{\ln \Lambda^{e/e}}{4} . \tag{12}$$

In the following, we use ζ_c to present the results of our calculations.

III. COLLISION MODELS

We consider three collision operators for modeling the electron-electron collisions. The first is the fully relativistic Coulomb collision operator of Beliaev and Budker.[4] A differential form of this operator was constructed by Braams and Karney.[9] Their formulation will be extensively used here. The second operator to be considered is a semirelativistic form of the collision operator first used by Karney and Fisch to model radio frequency and ECCD (Ref. 10). The third is Fisch's relativistic high-velocity model.[3,11]

The linearized adjoint collision operator is given as $C_e^{l+} f_r = f_M^{-1} C_e^l (f_r f_M)$. By substituting $f_r = f_r^{(1)}(u) \cos\theta$ into Eq. (5) and dividing both sides of the equation by $\cos\theta$, the right side of the equation can be expressed as

$$\frac{1}{\cos\theta} C_e^{l+} (f_r^{(1)}(u) \cos\theta) = \frac{1}{f_M \cos\theta} [C^{e/e}(f_M f_r^{(1)}(u)\cos\theta, f_M) + C^{e/e}(f_M, f_M f_r^{(1)}(u) \cos\theta) + C^{e/i}(f_M f_r^{(1)}(u) \cos\theta, f_{iM})] , \tag{13}$$

where f_{iM} is the ion's Maxwellian distribution. Assuming that the ions are infinitely massive relative to electrons, the electron-ion collision term becomes relatively simple,

$$\frac{C^{e/i}(f_M f_r^{(1)}(u)\cos\theta, f_{iM})}{f_M \cos\theta} \frac{B_0}{c} \approx -\frac{Z\gamma\chi}{u^3} , \tag{14}$$

where Z is the effective ion charge and $\chi = (B_0 \nu_c / c) f_r^{(1)}(u)$. The test particle part of the electron-electron collision term can be written as

$$\frac{C^{e/e}(f_M f_r^{(1)}(u)\cos\theta, f_M)}{f_M \cos\theta} \frac{B_0}{c} = D_{fuu}^{e/e} \frac{\partial^2 \chi}{\partial u^2} + \left(\frac{2D_{fuu}^{e/e}}{u} + \frac{\partial D_{fuu}^{e/e}}{\partial u} + F_{fu}^{e/e} \right) \frac{\partial \chi}{\partial u} - \frac{2}{u^2} D_{f\theta\theta}^{e/e} \chi , \tag{15}$$

and the field particle contribution gives

$$\frac{C^{e/e}(f_M, f_M f_r^{(1)}(u)\cos\theta)}{f_M \cos\theta} \frac{B_0}{c} \equiv I_f^{e/e} = \frac{1}{\gamma} f_m(u) \chi + \frac{1}{\Theta} \int_0^u D_1 \frac{u'^2}{\gamma\gamma'} f_m(u') \chi \, du' + \frac{1}{\Theta} \int_u^\infty D_2 \frac{u'^2}{\gamma\gamma'} f_m(u') \chi \, du' . \tag{16}$$

Here $\gamma' = \sqrt{1 + u'^2}$ and $f_m = (4\pi m^3 c^3 / n_e) f_M$; $D_{fuu}^{e/e}$, $D_{f\theta\theta}^{e/e}, F_{fu}^{e/e}, D_1$, and D_2 are as given in the Appendix of Ref. 9. So the normalized one-dimensional integro-differential equation for the fully relativistic model is written as

$$D_{fuu}^{e/e} \frac{\partial^2 \chi}{\partial u^2} + \left(\frac{2D_{fuu}^{e/e}}{u} + \frac{\partial D_{fuu}^{e/e}}{\partial u} + F_{fu}^{e/e} \right) \frac{\partial \chi}{\partial u} - \frac{2D_{f\theta\theta}^{e/e} + Z\gamma/u}{u^2} \chi + I_f^{e/e} + \frac{u}{\gamma} = 0 . \tag{17}$$

The semirelativistic collision operator used by Karney and Fisch[10] can be obtained from that of Beliaev and Budker[4] by assuming that one of the colliding particles is nonrelativistic. The normalized adjoint equation is given as

$$\frac{1}{u^2}\frac{\partial}{\partial u}u^2 D_{suu}^{e/e}\frac{\partial \chi}{\partial u} - \frac{u}{\gamma\Theta}D_{suu}^{e/e}\frac{\partial \chi}{\partial u} - \frac{2D_{s\theta\theta}^{e/e} + 3Z\gamma/4\pi u}{u^2}\chi + 3I_s^{e/e}(\chi) + \frac{3u}{4\pi\gamma} = 0 \ , \tag{18}$$

where

$$D_{suu}^{e/e} = \int_0^u \frac{\gamma^3 u'^4}{\gamma'^2 u^3} f_m(u')\,du' + \int_u^\infty \gamma' u' f_m(u')\,du' \ ,$$

$$D_{s\theta\theta}^{e/e} = \int_0^u \frac{u'^2}{2u^3}(3\gamma\gamma'^2 u^2 - \gamma^3 u'^2) f_m(u')\,du' + \int_u^\infty \gamma' u' f_m(u')\,du' \ ,$$

$$\begin{aligned} I_s^{e/e} = {} & \frac{\chi(u) f_m(u)}{\gamma} + \frac{1}{5}\int_0^u u'^2 f_m(u')\chi(u') \\ & \times \frac{1}{\Theta}\left\{\frac{\gamma}{u^2}\frac{u'}{\gamma'^4}\left[\Theta(4\gamma'^2+6) - \frac{1}{3}(4\gamma'^3 - 9\gamma')\right] + \frac{\gamma^2}{u^2}\frac{u'}{\gamma'^4}\left[\frac{u'^2}{\Theta}\gamma' - \frac{1}{3}(4\gamma'^2+6)\right]\right\} du' \\ & + \frac{1}{5}\int_u^\infty u'^2 f_m(u')\chi(u') \\ & \times \frac{1}{\Theta}\left\{\frac{\gamma'}{u'^2}\frac{u}{\gamma^4}\left[\Theta(4\gamma^2+6) - \frac{1}{3}(4\gamma^3 - 9\gamma')\right] + \frac{\gamma'^2}{u'^2}\frac{u}{\gamma^4}\left[\frac{u^2}{\Theta}\gamma - \frac{1}{3}(4\gamma^2+6)\right]\right\} du' \ . \end{aligned}$$

For the relativistic high-velocity model, the corresponding equation is given as[3]

$$\frac{\gamma^2}{u^2}\frac{\partial \chi}{\partial u} + (Z+1)\frac{\gamma}{u^3}\chi - \frac{u}{\gamma} = 0 \ . \tag{19}$$

We note that Eq. (19) is a simple differential equation; there is no field particle contribution and it does not conserve momentum.

We have numerically solved Eqs. (17), (18), and (19) for χ for various collision models. Their predictions of the current drive efficiency are easily obtained by using Eq. (6).

IV. COMPARISON OF ECCD EFFICIENCIES BETWEEN COLLISION MODELS

We focus on the calculation of ECCD efficiency for the first-harmonic O-mode, because it will be most relevant to the ITER ECCD operation. For a given EC frequency normalized to the cyclotron frequency ω/ω_c and a parallel index of refraction $n_\parallel$, polarization of the wave electric field is calculated by the dispersion relation given in Ref. 7. Equations (17), (18), and (19) are solved to determine χ for various values of ion charge Z and electron temperature T_e. Figure 1 shows the dimensionless efficiency ζ_c as a function of ω_c/ω with $n_\parallel = 0.4$ for $Z = 1.6$ and 5 at three different electron temperatures ($T_e = 1.0, 10, 25$ keV). At low temperature ($T_e \le 1.0$ keV), the results of the three collision models agree well with each other. Under ITER conditions ($T_e \approx 25$ keV), the efficiency given by the semirelativistic model still agrees well with that given by the fully relativistic model. On the other hand, the efficiency given by the high-velocity model is appreciably smaller even at $T_e = 10$ keV in the case of $Z = 1.6$. For the case of $Z = 5$, we find that the discrepancies between the results of the high-velocity model and the two more complete models are much more

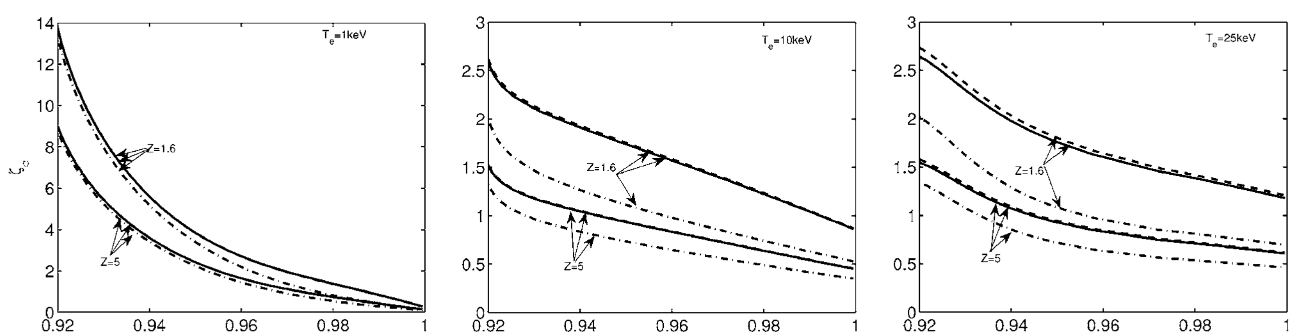

Fig. 1. Dimensionless efficiency ζ_c as a function of ω_c/ω, evaluated for the fundamental O-mode with $n_\parallel = 0.4$ and $Z = 1.6$ and 5 at different electron temperatures T_e. The dashed-dotted, dashed, and solid lines represent the results from the high-velocity, semirelativistic, and fully relativistic collision models, respectively.

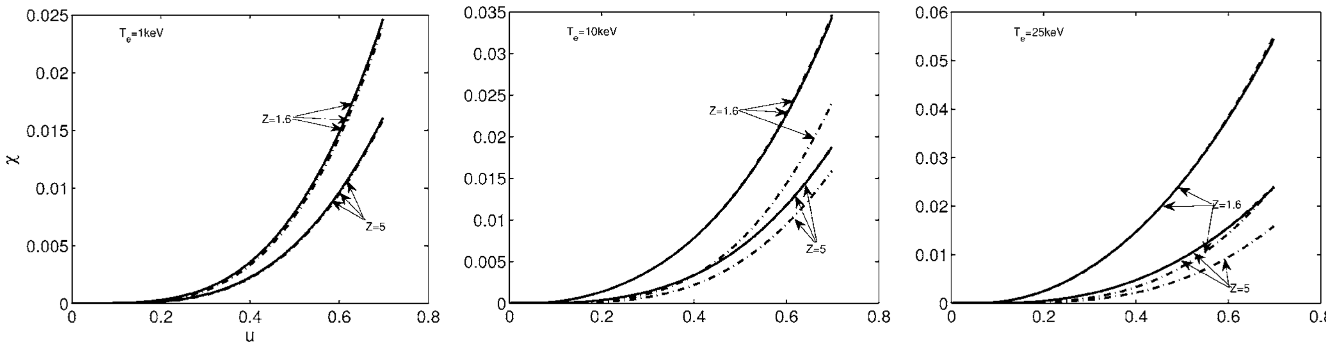

Fig. 2. Normalized response function χ as a function of u, evaluated for $Z = 1.6$ and 5 with different electron temperatures T_e. The dashed-dotted, dashed, and solid lines represent the results from the high-velocity, semirelativistic, and fully relativistic collision models, respectively.

reduced because the electron-ion collision term tends to be dominant as Z increases. We have done similar calculations with different values of $n_{\parallel}$ and reached similar conclusions, as discussed previously. Figure 2 plots the normalized response function χ as a function of u for various values of Z and T_e. No appreciable difference for the functions of semirelativistic and fully relativistic models is observed. The discrepancies between the results of the high-velocity model and those two other models are quite apparent. As expected, the discrepancy increases as T_e increases, and at a fixed temperature, it decreases as Z increases. Figure 3 compares the predictions of the current drive efficiency by the fully relativistic collision model with and without the field particle contribution term $C^{e/e}(f_M, f_M f_r^{(1)}(u)\cos\theta)$. When this term is discarded, the momentum-conserving property of the like-particle collision operator is lost. The results shown in Fig. 3 indicate that the efficiencies given by the momentum nonconserving model are generally smaller than those given by the complete model, especially for the cases of high electron temperatures and low values of Z. This is the fact that was first used by Prater et al. in the code benchmark work of Ref. 12 to explain why the high-velocity model underestimates the current drive efficiency.

V. CONCLUSION

In summary, the ECCD efficiencies in a uniform magnetic field are calculated using the fully relativistic, semirelativistic, and high-velocity collision models with plasma and wave parameters relevant to ITER operation. At low electron temperatures ($T_e \leq 1.0$ keV), the three models agree well with each other in predicating the ECCD efficiency. On the other hand, at the temperature regime expected in the core region of the ITER plasma ($T_e \approx 25$ keV), it is found that the high-velocity model tends to underestimate the efficiency notably, but the predictions of the semirelativistic model still agree well with those of the fully relativistic model. These results demonstrate that the semirelativistic collision model should

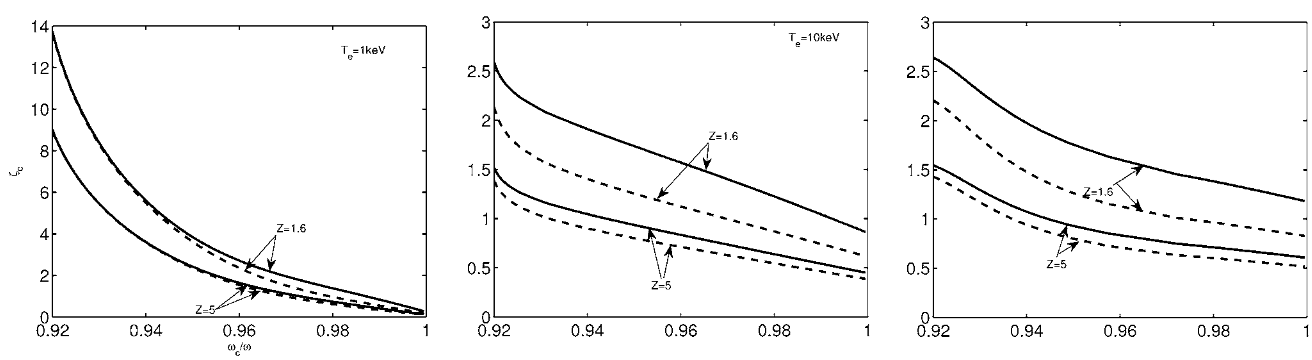

Fig. 3. Dimensionless efficiency ζ_c as a function of ω_c/ω, evaluated for the fundamental O-mode with $n_{\parallel} = 0.4$ and $Z = 1.6$ and 5 at different electron temperatures T_e by using the fully relativistic collision operator with (solid line) and without (dashed line) momentum conservation.

be appropriate for modeling ECCD operation in ITER. Although we do not consider the trapped electron effect in this work, we believe that its inclusion will not alter the major conclusion reached here.

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