Notes on misc terminologies in tokamak physics

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Abstract

These are my notes when reading Wesson's book on tokamak[1].

1 Global energy confinement time

The global energy confinement time τ_E is defined by

$$\tau_E = \frac{W}{P_{\text{loss}}}, \tag{1}$$

where $W = \int \frac{3}{2} n(T_i + T_e) dV$ is the total plasma stored energy, P_{loss} is total loss power from the plasma. The global energy confinement time τ_E characterizes the time needed for the plasma to loss all its stored energy. The energy conservation requires that

$$\frac{dW}{dt} = P_{\text{absorb}} - P_{\text{loss}}, \tag{2}$$

where P_{absorb} is the absorbed power of the plasma. Using Eq. (2), Eq. (1) is written as

$$\tau_E = \frac{W}{P_{\text{absorb}} - \frac{dW}{dt}}. \tag{3}$$

The stored energy W, and its time derivative dW/dt can be measured in experiments. However, neither P_{absorb} and P_{loss} can be directly measured in experiments. In practice, the absorbed power can be estimated via discounting the source power of the auxiliary heating system by an emperical factor. This factor is usuall different for different heating schemes. For example, P_{absorb} on EAST tokamak is estimated as

$$P_{\text{absorb}} = \alpha_1 P_{\text{NBI}} + \alpha_2 P_{\text{LHW}} + \alpha_3 P_{\text{ICRH}} + \alpha_4 P_{\text{ECRH}}, \tag{4}$$

where P_{NBI}, P_{LHW}, P_{ICRH}, and P_{ECRH} are the source power of the NBI, LHW, ICRH, and ECRH heating system used in a specified discharge, respectively. The values of the coefficients α_1, α_2, α_3, and α_4 are chosen emperically by the person who estimate P_{absorb} (for example the value of α_1 is chosen to be 0.5).

Thus equation (3) along with Eq. (4) provides a means of determining τ_E form experimentally known quantities. For steady state, equation (3) reduces to

$$\tau_E = \frac{W}{P_{\text{absorb}}}. \tag{5}$$

1.1 H factor

To characterize the degree of improvement/degradation of the global energy confinement time in a given plasma relative to a given scaling law, we define the H factor (confinement improvement factor)

$$H \equiv \frac{\tau_E}{\tau_{E\,\text{scaling}}}, \tag{6}$$

where τ_E is the meassured (as above) global energy confinement time for a given plasma, $\tau_{E\,\mathrm{scaling}}$ can be one of the many empirical scaling laws for the energy confinement time. One of the widely used empirical scaling law for ELMy plasmas is the PB98(y,2) scaling law, which is given by

$$\tau_{E98(y,2)} = 0.0562 I^{0.93} B^{0.15} n^{0.41} P^{-0.69} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}. \tag{7}$$

When this scaling law is used in Eq. (6) to calculate H factor, this factor is called $H_{98(y,2)}$, i.e.,

$$H_{98(y,2)} = \frac{\tau_E}{\tau_{E98(y,2)}}. \tag{8}$$

(The PB98(y,2) scaling law is used to predict the performance of the ITER ELMy plasmas.) The value of $H_{98(y,2)}$ can be less than 1, which means the confinement is not as good as the PB98(y,2) scaling law predicts. One of the L-mode scaling law can also be used in Eq. (6) to calculate H factor, i.e.,

$$H = \frac{\tau_E}{\tau_E^L}, \tag{9}$$

which is the usual definition of H factor. For a high confinement (H-mode) plasma, the H factor given by Eq. (9) is usually about 2, i.e. the energy confinement time of a H-mode plasma is usually twice as long as that of the L-mode plasma.

1.2 Plasma stored energy

Using the force balance equation, the pressure can be expressed in terms of the magnetic field.

The force balance equation

$$\nabla p = \mathbf{j} \times \mathbf{B}, \tag{10}$$

can be written as

$$\nabla\left(p + \frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0}(\mathbf{B} \cdot \nabla \mathbf{B}). \tag{11}$$

Neglecting the curvature term, the above equation is written

$$\nabla\left(p + \frac{B^2}{2\mu_0}\right) = 0. \tag{12}$$

$$p = \frac{B_a^2}{2\mu_0} - \frac{B^2}{2\mu_0} \tag{13}$$

$$\begin{aligned}
\Psi_t &= \int B_\phi \, d^2 S \\
&\approx \int B d^2 S \\
&\approx \int \sqrt{B_a^2 - 2\mu_0 p}\, d^2 S \\
&= B_a \int \sqrt{1 - \frac{p}{B_a^2 / 2\mu_0}}\, d^2 S
\end{aligned}$$

Assuming low-beta, i.e., $p/(B_a^2/2\mu_0) \ll 1$, then the above equation can be approximated by

$$\Psi_t = B_a \int \left(1 - \frac{p}{B_a^2/\mu_0}\right) d^2 S$$

$$\Psi_t = B_a S - \frac{\mu_0}{B_a} \int p d^2 S$$

$$\Psi_t = B_a S - \frac{\mu_0}{B_a} \int p d^2 S \tag{14}$$

The plasma stored energy E_p is given by

$$E_p = \int p d^3 V \approx 2\pi R_0 \int p d^2 S$$

Using Eq. (14), E_p is written

$$E_p = \frac{B_a}{\mu_0}(B_a S - \Psi_t) 2\pi R_0$$

The plasma stored energy can be obtained by using the toroidal magnetic flux.

2 Pfirsch-Schluter current

3 Density limit

Refer to my notes on tokamak equilibrium.

4 Beta limit

Refer to my notes on tokamak equilibrium.

5 Magnetic measurements

5.1 Magnetic probe: measurement of magnetic field and its temporal variation

Faraday's law gives

$$\varepsilon = -\frac{d\Phi}{dt}, \tag{15}$$

where Φ is the magnetic flux through a loop, ε is the emf. If the loop (magnetic probe) is a coil with N turns, the induced voltage of the coil is N times the emf ε, i.e.,

$$V = N\varepsilon. \tag{16}$$

Using this, Eq. (15) is written as

$$V = -N\frac{d\Phi}{dt}. \tag{17}$$

The magnetic flux through a coil is given by

$$\Phi = \mathbf{B} \cdot \mathbf{n} A = B_n A,$$

where $\mathbf{n}$ is the unit normal vector of the loop, B_n is the normal component of the magnetic field, A is the area of the loop. Using this, Eq. (17) is written

$$V = -NA\frac{dB_n}{dt}. \tag{18}$$

The voltage V can be directly measured and thus provides the temporal variation of B_n. The temporal variation of B_n may be due to internal plasama fluctuations. Therefore the magnetic probe measurement provides information of the temporal evolution of these fluctuations.

Equation (18) can be integrated over time, yielding

$$B_n(t) - B_n(0) = -\frac{1}{NA}\int_0^t V dt. \tag{19}$$

The induced voltage V can be integrated over time, which, according to Eq. (19), gives the total change of the strength of the normal magnetic field. To obtain the value of B_n at t, we need to determe the inital value $B_n(0)$, which can be obtained in some simple way, for example, starting the measurement before any magnetic field is present so that $B(0)=0$.

For divertor manetic configuration (i.e., a magnetic configuration with at least one X point, where the poloidal magnetic field is zero), the magnetic probe can be used to determine the location where the poloidal magnetic field is zero, thus determine the location of X point.

5.2 Poloidal flux loop: measuring the poloidal magnetic flux and toroidal voltage

The poloidal flux loop (often called "flux loop" by tokamak operators) is a simple toroidal loop. Since it is in the toroidal direction, it measures the poloidal magnetic flux and its time change rate (toroidal voltage). There are usually many (e.g. 37 on EAST[2], 41 on DIII-D) flux loops at different locations on the poloidal plane. The measured value of poloidal flux can be used to obtain a contour of the poloidal flux and thus obtain a flux surface. With the feedback control (by adjusting the current in the poloidal field coils), these measurement can be used to control the shape of the last-closed-flux-surface. This is the so-called iso-flux control (also called gap control).

5.3 Measurement of toroidal current

To measure the total toroidal current of tokamak plasmas, the Rogowski coil is usually used. The structure of the Rogowski coil is shown in Fig. (). Next, we derive the relation between the voltage V in the Rogowski coil and the total toroidal current I. The magnetic filed in a tokamak has two components, one is the toroidal component and the other is the poloidal component. The unique structure of the Rogowski coil makes the change of toroidal magnetic flux not contribute to the voltage in the coil (since the voltage in the two big loops due to the change of toroidal magnetic flux cancel each other). Thus only the poloidal flux contribute to the voltage in the coil, which can be calculated as follows. The number of turns for the arch-length dl along the big coil is given by

$$dN = n\,dl, \tag{20}$$

where n is the number of turns per unit length. The resulting voltage is written as

$$dV = \varepsilon\,dN = \varepsilon n\,dl,$$

where ε is the emf in each small coil (only the change of the poloidal flux conntribute to ε). Thus the total voltage in the coil is written

$$V = \oint \varepsilon n\,dl, \tag{21}$$

The emf ε can be written as

$$\varepsilon = -\frac{d\Phi}{dt} = -A\frac{dB_n}{dt}, \tag{22}$$

Using this in Eq. (21) yeilds

$$\begin{aligned} V &= -An\oint \frac{dB_n}{dt}dl \\ &= -An\frac{d}{dt}\oint B_n dl \\ &= -An\frac{d}{dt}\oint \mathbf{B}_p\cdot d\mathbf{l}. \end{aligned} \tag{23}$$

Integrating Eq. (23) over time, we obtain

$$\oint \mathbf{B}_p(t)\cdot d\mathbf{l} - \oint \mathbf{B}_p(t=0)\cdot d\mathbf{l} = -\frac{1}{An}\int_0^t V(t')dt' \tag{24}$$

On the other hand, neglecting the displacement current, Ampere's law is written as

$$\oint \mathbf{B}_p \cdot d\mathbf{l} = \mu_0 I. \tag{25}$$

Using this, Eq. (24) is written

$$I(t) - I(t=0) = -\frac{1}{\mu_0 A n}\int_0^t V(t')dt'. \tag{26}$$

Equation (26) gives the relation between the voltage V in the Rogowski coil and the change of the toroidal current I. If we start the measurement before the plasma is setup, then we know $I(t=0)=0$ and Eq. (26) can be used to calculate the toroidal current $I(t)$ at any later time.

5.4 Internal inductance

Bibliography

[1] John Wesson. *Tokamaks*. Oxford University Press, 2004.
[2] B. J. Xiao. The first diverted plasma on east tokamak. *34th EPS Conference on Plasma Phys*, 2007.