Influence of energetic particles on MHD modes —application of NOVA-K code

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Background

- In addition to thermal ions and electrons, tokamak plasmas usually contain super-thermal species, which are called fast or energetic particles
- These energetic particles (EP) can be created from various sources, such as ion/electron cyclotron heating, neutral beam injection, and fusion reaction
- The energetic particles can destabilize various MHD waves
- The destabilizing effects (drive effects) of EP can be calculated by the NOVA-K code

NOVA-K code

NOVA-K is a 2D Kinetic-MHD linear stability code for tokamaks with energetic particles.

- Authors: C. Z. Cheng, N. Gorelenkov, G. Y. Fu (PPPL)
- NOVA-K is written in Fortran Language
- Approximate number of code lines: 10,000 lines
- Code was first used in 1988 after the discovery of TAE

Physical model of NOVA-K code

- NOVA-K uses the perturbative method to calculate the destabilizing effects of EP
- NOVA-K are composed of two codes, one of which, NOVA, calculates the ideal MHD modes without the EP
- Then NOVA-K post-processor uses the calculated MHD modes structure to calculate the orbit integration of EP to obtain the contribution of EP to the growth rate of the modes
- Besides the EP drive effects, NOVA-K also includes various damping effects:
  (1) Thermal electron/ion Landau damping
  (2) Continuum damping
  (3) Collisional damping of trapped electrons
  (4) Radiative damping

Ideal MHD equations

Momentum equation

$$\rho_m\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-\nabla p+(\nabla\times\boldsymbol{B})/\mu_0\times\boldsymbol{B}$$

Faraday's Law

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{B})$$

Equation of state

$$\frac{\partial p}{\partial t}+\boldsymbol{u}\cdot\nabla p=-\gamma p\nabla\cdot\boldsymbol{u}$$

Linearized ideal MHD equations

- $\boldsymbol{\xi}$, $\boldsymbol{B}_1$, and p_1 are perturbed plasma displacement, magnetic field and plasma pressure, respectively.
- Time dependence of perturbation: $A(\boldsymbol{r},t) = A(\boldsymbol{r})\exp(-i\omega t)$
- ω^2 is a real number
- Linearized ideal MHD equations:

$$-\omega^2 \rho_{m0} \boldsymbol{\xi} = -\nabla p_1 + (\nabla \times \boldsymbol{B}_1)/\mu_0 \times \boldsymbol{B}_0 + (\nabla \times \boldsymbol{B}_0)/\mu_0 \times \boldsymbol{B}_1$$

$$\boldsymbol{B}_1 = \nabla \times (\boldsymbol{\xi} \times \boldsymbol{B}_0)$$

$$p_1 + \boldsymbol{\xi} \cdot \nabla p_0 + \gamma p_0 \nabla \cdot \boldsymbol{\xi} = 0$$

Kinetic effect of EP on linear MHD stability

- NOVA-K includes kinetic effect through perturbed pressure of energetic/thermal particles in the momentum equation

$$
\begin{aligned}
-\omega^2 \rho_{m0} \boldsymbol{\xi} &= -\nabla p_1 - \nabla \cdot \delta \boldsymbol{p}_h \\
&+ (\nabla \times \boldsymbol{B}_1)/\mu_0 \times \boldsymbol{B}_0 + (\nabla \times \boldsymbol{B}_0)/\mu_0 \times \boldsymbol{B}_1
\end{aligned}
$$

Quadratic form and perturbative calculation of kinetic effects

$$\delta W_f + \delta W_k - \omega^2 \delta K = 0$$

Zeroth-order equation

$$\delta W_f(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0) - \omega_0^2 \delta K(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0) = 0$$

The first-order equation

$$\delta W_f(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_1) - \omega_0^2 \delta K(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_1) - 2\omega_0\omega_1 \delta K(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0) + \delta W_k(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0, \omega_0) = 0$$

$$\Longrightarrow \frac{\omega_1}{\omega_0} = \frac{\delta W_k(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0, \omega_0)}{2\omega_0^2 \delta K(\boldsymbol{\xi}_0^\star, \boldsymbol{\xi}_0)}$$

where

$$\delta W_k = \int d^3\boldsymbol{x} \boldsymbol{\xi}^\star \cdot \nabla \cdot \delta P_k$$

Expression of perturbation

- Due to toroidal symmetry, perturbation can be represented by a single toroidal mode, $\exp(-in\zeta)$.
- However, due to the poloidal nonuniform, different poloidal harmonics are coupled, perturbation should include all Fourier series in θ

$$\boldsymbol{\xi}(\psi,\theta,\zeta,t)=\sum_{m=-\infty}^{m=+\infty}\boldsymbol{\xi}_m(\psi)\exp[i(m\theta-n\zeta-\omega t)]$$

- ψ dependence of the perturbation is treated in NOVA-K using finite-element expansion

Decomposition of ideal MHD perturbation

NOVA-K decomposes the displacement vector and perturbed magnetic field as

$$\boldsymbol{\xi} = \frac{\xi_\psi}{|\nabla\psi|^2}\nabla\psi + \frac{\xi_s}{B^2}(\boldsymbol{B}_0 \times \nabla\psi) + \frac{\xi_b}{B^2}\boldsymbol{B}_0,$$

$$\boldsymbol{B}^{(1)} = \frac{Q_\psi}{|\nabla\psi|^2}\nabla\psi + \frac{Q_s}{|\nabla\psi|^2}(\boldsymbol{B}_0 \times \nabla\psi) + \frac{Q_b}{B_0^2}\boldsymbol{B}_0.$$

Ideal MHD eigen-mode equations

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \begin{pmatrix} \xi_\psi \\ \delta p_1 \\ \xi_s \\ \nabla \cdot \boldsymbol{\xi} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where A_{ij} can be

$$(\nabla\psi) \cdot \nabla,$$

$$\boldsymbol{B} \cdot \nabla,$$

$$(\boldsymbol{B} \times \nabla\psi) \cdot \nabla,$$

Perturbed EP distribution due to ideal MHD perturbation

- Equilibrium distribution: $F = F(P_{\varphi 0}, \varepsilon_0, \mu_0)$
- Ideal MHD perturbation: $\boldsymbol{B}_1 = \nabla \times (\boldsymbol{\xi} \times \boldsymbol{B}_0)$
- Perturbed distribution:

$$\delta f = -\frac{Ze}{c}\boldsymbol{\xi}_\perp \nabla\psi \frac{\partial F}{\partial P_{\varphi 0}} - \mu_0 \frac{B_{1\parallel}}{B_0}\frac{\partial F}{\partial \mu_0} - \delta g,$$

where

$$\delta g = -i(\omega - n\omega_\star)\frac{\partial F}{\partial \varepsilon_0}\int_0^t \mathcal{L}^{(1)} d\tau,$$

$$\mathcal{L}^{(1)} = -\left(m v_\parallel^2 - \mu_0 B_0\right)\boldsymbol{\xi}\cdot\boldsymbol{\kappa} + \mu_0 B_0 \nabla\cdot\boldsymbol{\xi}_\perp,$$

$$\omega_\star = \frac{\partial F}{\partial P_{\varphi 0}}\bigg/\frac{\partial F}{\partial \varepsilon_0}.$$

Unperturbed guiding-center orbits

- Unperturbed guiding-center orbits

$$\dot{\boldsymbol{R}} = v_{\|}\boldsymbol{b} + \frac{1}{m\Omega}\boldsymbol{b}\times\Big(\mu\nabla B + m v_{\|}^{2}\boldsymbol{\kappa}\Big),$$

$$\dot{v}_{\|} = \frac{1}{m}\Big(- \mu\boldsymbol{b}\cdot\nabla B + m v_{\|}\boldsymbol{\kappa}\cdot\dot{\boldsymbol{R}}\Big).$$

- Finite-Orbit-Width (FOW) effect is included in NOVA-K.

Toroidicity-Induced Alfvén Eigen-mode

- Toroidicity-Induced Alfvén Eigen-mode (TAE) is a type of global shear Alfvén eigenmode.
- Exists only in toroidal geometry: created due to toroidal coupling.
- Frequency is discrete, lies within the "gaps" in the shear Alfvén continuum.
- Can be strongly destabilized by α-particles in burning tokamak plasmas.

NOVA-K input

- The input file for the equilibrium is “inequ”, which is a file formatted as a table. The entries of the table set various equilibrium parameters
- Most important are (1) pressure and safety factor profiles. (2) shape of LCFS
- The energetic particles equilibrium distribution can be set in NOVA_param. For example, the argument tip is related to the distribution function type, (refer to taem.f)
  - "s" for slowing down distribution, $F_h(\varepsilon, \Lambda, \psi) = n_h(\psi)\delta(\Lambda - \Lambda_0)/\varepsilon^{3/2}$
  - "m" for Maxwellian with standard input

NOVA-K output

- Shear Alfvén continuum.
- Frequency and mode structure of ideal MHD modes
    - External/Internal kink modes
    - Toroidal Alfvén Eigenmodes(TAE)
- Drive/damping rate
    - Energetic particles drive/damping
    - Thermal electron/ion Landau damping
    - Continuum damping
    - Collisional damping of trapped electrons
    - Radiative damping

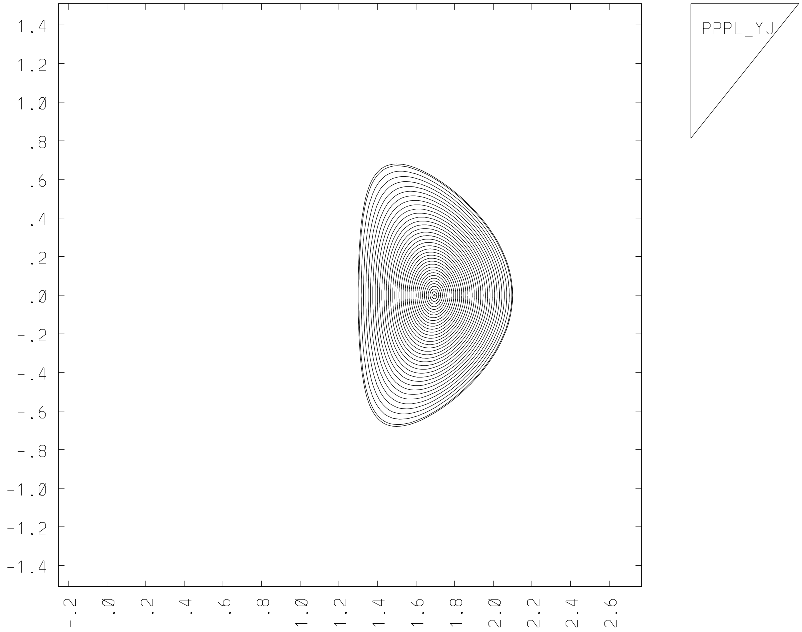

Figure 1. EAST shape parameter: $R_0 = 1.7m$, $a = 0.4m$, elongation $\kappa = 1.7$, triangularity $\delta = 0.5$.

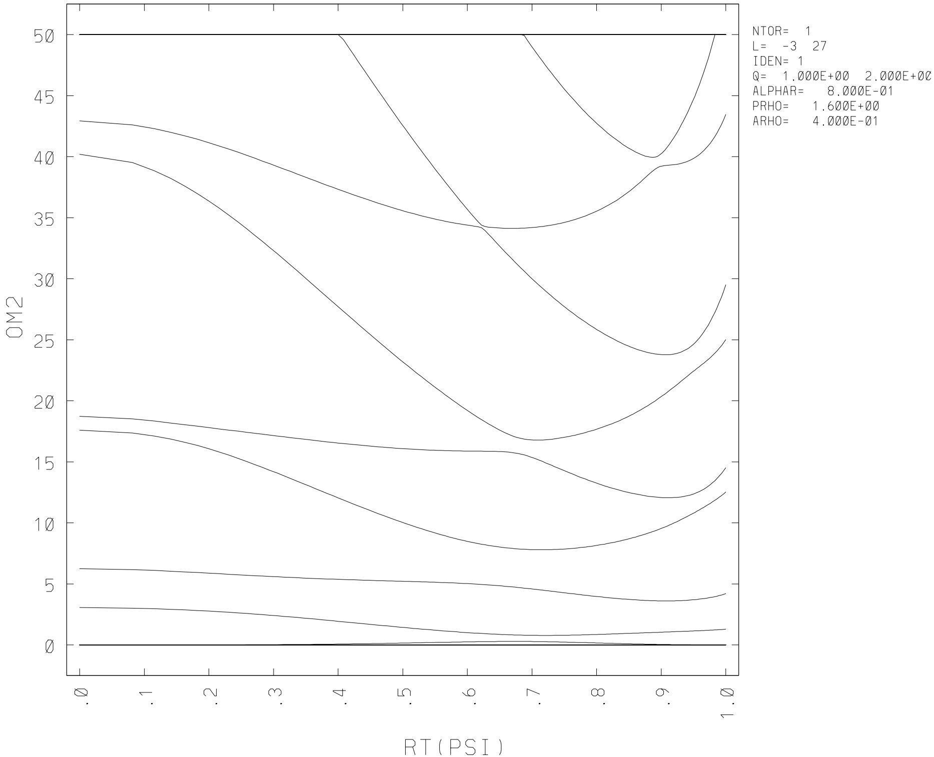

Figure 2. n=1 continuum spectrum

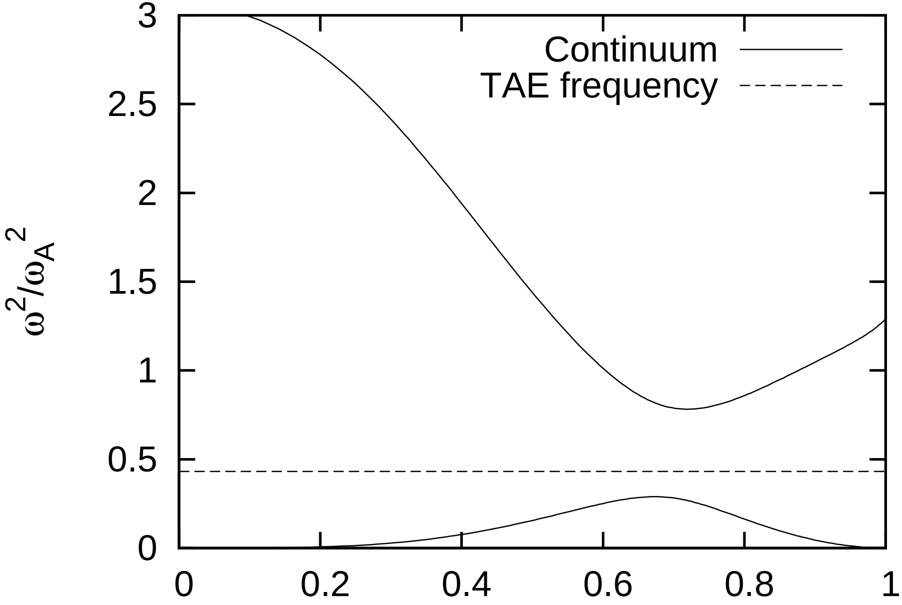

Figure 3. n=1 continuum spectrum and TAE frequency

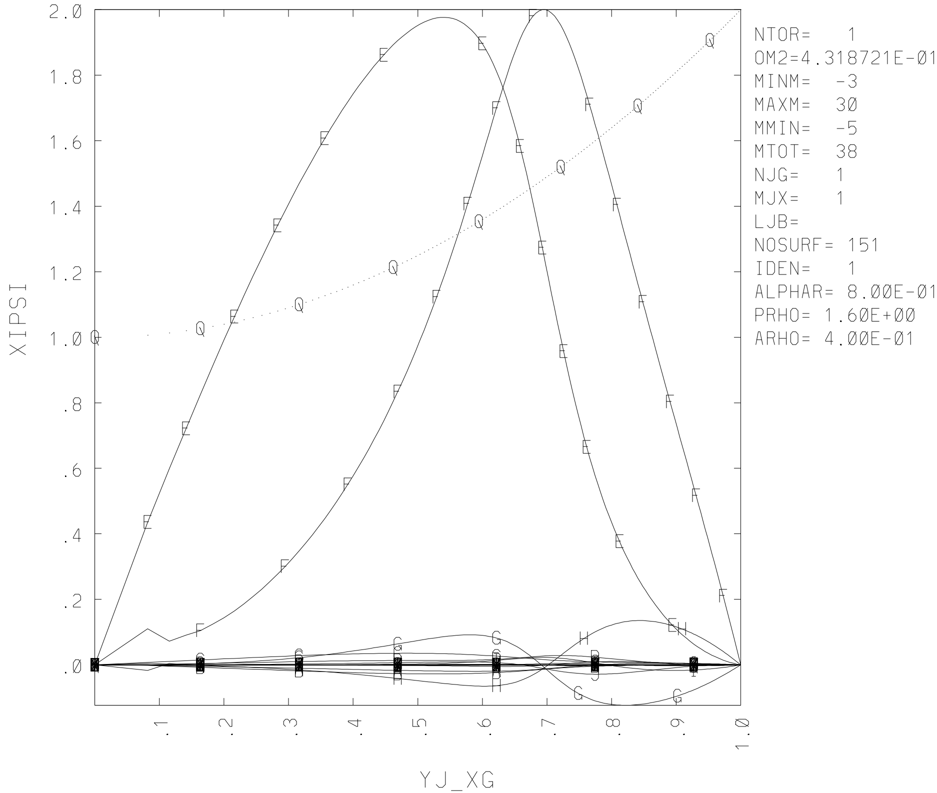

Figure 4. Mode structure of n=1 TAE. Dominant poloidal modes are $m = 1$ and $m = 2$

Example of NOVA-K output

gam_ecoll -9.784951544844977E-005

gam_eLandau -2.373366177356202E-003

gam_DLandau -8.925676315388741E-004

gam_TLandau -1.885105163939624E-010

gam_HLandau -5.120318917926147E-009

gam_CLandau 0.000000000000000E+000

radiative damping 0.000000000000000E+000

Fast ion beta, beta_h 8.000000000000000E-002

Fast ion growth rate without FLR, gam_h -1.319028439767093E-002

Fast ion growth rate with FLR, gam_hFLR -1.950236033379125E-003

beta_hcrit=-beta_h*(all dampings above)/gam_h

or beta_hcrit=-beta_h*(all dampings above)/gam_hFLR

Thanks!