Electron shielding current in neutral beam current drive in general tokamak equilibria and arbitrary collisionality regime

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Neutral beam injection (NBI) and current drive

- NBI produces a current of fast ions circulating around the torus.

- The momentum trasfer from fast ions to electrons by collisions causes the electrons to drift toroidally in the same direction as the fast ions.

- The electron current due to this drift is in the reverse direction to the fast ions current, so there is some cancellation between these two components, and the name ‘‘electron shielding current’’.

- The degree of cancellation depends on the beam velocity v_f, charge number of fast ions Z_f, effective charge number of plasma ions $Z_{\rm eff}$, and the fraction of trapped electrons f_t.

- Since there can be near cancellation of the fast ion and electron currents, correct calculation of the net current requires accurate evaluation of the electron shielding current.

Previous works

- In the usual regeme of neutral beam current drive, the ratio of the net current to the fast ion current can be written in the form

$$F \equiv \frac{\langle j_{\parallel} B \rangle}{\langle j_{f\,\parallel} B \rangle} = 1 - \frac{Z_f}{Z_{\rm eff}}(1 - G),$$

- The G factor represents the trapped electrons correction to the electron shielding current. In uniform plasmas the G factor is zero.

- Previous theoretical calculation of the G factor is limited to the case in which either the inverse aspect ratio or the electron collisionality is small. In Wesson's book: $G = 1.46\sqrt{\varepsilon}A(Z_{\rm eff})$.

- In the banana regime, Lin-Liu and Hinton found that $G = \mathcal{L}_{31}$, where $\mathcal{L}_{31}$ is one of the bootstrap current coefficients and employ a formula for $\mathcal{L}_{31}$ that is valid in general tokamak equilibria to express the G factor.

Our contribution

- In this work, we extend the work of Lin-Liu and Hinton to arbitrary collisionality regime.
- We show that the G factor can still be expressed in terms of the bootstrap current coefficient $\mathcal{L}_{3\,1}$.
- We further make use of the bootstrap current coefficient formula that is valid for arbitrary collisionality regime, to provide a convenient formula for calculating the G factor.
- The formulas presented can be easily applied to numerical codes modeling neutral beam current drive to improve the calculation capabilities of these codes.

Basic equation

- In the presence of the fast ions generated by NBI, the perturbed electron distribution satisfies the Fokker-Planck Eq.

$$v_\parallel \hat{\boldsymbol{b}} \cdot \nabla f_{e1} - C_l(f_{e1}) = C^{e/f}(f_{e\,m}),$$

where $C_l(f_{e1}) = C(f_{e1},\ f_{e\,m}) + C(f_{e\,m},\ f_{e1}) + C^{e/i}(f_{e1})$, $C^{e/f}(f_{e\,m})$ is the collision term of electrons with fast ions, which is assumed to be known and acts as an inhomogeneous term.

- The operators C_l and $v_\parallel \hat{b} \cdot \nabla$ have the self-adjoint property:

$$\int d\boldsymbol{v} g C_l(f_{e\,m} h) = \int d\boldsymbol{v} h C_l(f_{e\,m} g).$$

$$\left\langle \int d\boldsymbol{v} g v_\parallel \hat{\boldsymbol{b}} \cdot \nabla h \right\rangle = -\left\langle \int d\boldsymbol{v} h v_\parallel \hat{\boldsymbol{b}} \cdot \nabla g \right\rangle.$$

Green's function formulation for $j_{e\parallel}$

Using the self-adjoint property, the parallel current density $j_{e\parallel}$ contributed by f_{e1}. can be obtained through the following way:
First solve the adjoint equation

$$-v_{\parallel}\hat{\boldsymbol{b}}\cdot\nabla\chi_e - C_l(\chi_e) = q_e v_{\parallel} B f_{em},$$

to obtain the current response function χ_e, then $j_{e\parallel}$ can be expressed as

$$\langle j_{e\parallel} B\rangle = \left\langle \int d\boldsymbol{v}\frac{\chi_e}{f_{em}} C^{e/f}(f_{em})\right\rangle.$$

Collision of electrons with fast ions

- In the usual situation of NBI, the fast ions beam velocity is much less than the electron thermal velocity, i.e., $v_f \ll v_{te}$.
- In this case, the collision term of electrons with fast ions can be approximated by

$$C^{e/f}(f_{em}) = \frac{m_e}{T_e} \nu_{ef} v_\parallel u_{f\parallel} f_{em},$$

where $u_{f\parallel}$ is the average parallel velocity of fast ions,

$$\nu_{ef} = \left(\frac{n_f Z_f^2}{n_e} \right) \frac{\Gamma^{e/e}}{v^3}$$

$$\Gamma^{e/e} = n_e e^4 \ln\Lambda^{e/e} / (4\pi\epsilon_0^2 m_e^2).$$

Electron shieding current $j_{e\,\|}$ (1/2)

$$\langle j_{e\,\|}B\rangle = -\frac{Z_f}{Z_{\rm eff}}\frac{1}{Ip_e}\left\langle j_{f\,\|}B\int d\boldsymbol{v}\chi_e\nu_{e\,i}\frac{Iv_\|}{\Omega_e}\right\rangle$$

- where $j_{f\,\|} = Z_f e n_f u_{f\,\|}$ is the fast ion parallel current, $p_e = n_e T_e$, $\Omega_e = -Be/m_e$, $I = B_\phi R$
- To evaluate $\langle j_{e\,\|}B\rangle$, one needs to know the response function χ_e
- In both banana and Pfirsch-Schlüter regimes, the term

$$D \equiv \int d\boldsymbol{v}\chi_e\nu_{e\,i}Iv_\|/\Omega_e$$

  can be shown to be a flux surface function. In these cases, D can be taken out of the flux surface average operator.

- $\langle j_{e\,\|}B\rangle = -\frac{Z_f}{Z_{\rm eff}}\frac{1}{Ip_e}\langle j_{f\,\|}B\rangle D.$

Electron shieding current $j_{e\,\|}$ (2/2)

- In the case of the intermediate collisionality regime D is not exactly a flux surface function and has poloidal angle dependent collisionality corrections.
- The numerical results of Sauter's neoclassical bootstrap current theory indicates that these finite collisionality corrections in D are weakly dependent on the poloidal angle.
- We write Eq. (6) approximately, by taking D out of the flux surface average operator and replacing it by $\langle D \rangle$, as

$$\begin{aligned}
\left\langle j_{e\,\|} B \right\rangle &= -\frac{Z_f}{Z_{\rm eff}} \frac{1}{I p_e} \left\langle j_{f\,\|} B \int d\boldsymbol{v} \chi_e \nu_{e\,i} \frac{I v_{\|}}{\Omega_e} \right\rangle \\
&\approx -\frac{Z_f}{Z_{\rm eff}} \frac{1}{I p_e} \langle j_{f\,\|} B \rangle \left\langle \int d\boldsymbol{v} \chi_e \nu_{e\,i} \frac{I v_{\|}}{\Omega_e} \right\rangle
\end{aligned}$$

Relationship with bootstrap current coefficient

According to the neoclassical bootstrap current theory of Sauter [PoP1999, cited $\geqslant 300$], we have

$$\frac{1}{Ip_e}\left\langle \int d\boldsymbol{v}\chi_e\nu_{ei}\frac{Iv_{\parallel}}{\Omega_e}\right\rangle = 1-\mathcal{L}_{31}$$

where $\mathcal{L}_{31}$ is the dimensionless electron density gradient bootstrap current coefficient. Thus electron shielding current is written as

$$\langle j_{e\parallel}B\rangle = -\langle j_{f\parallel}B\rangle\frac{Z_f}{Z_{\mathrm{eff}}}(1-\mathcal{L}_{31})$$

The total current is the sum of the beam current carried by the fast ions and the electron shielding current, i.e., $j_{\parallel}=j_{f\parallel}+j_{e\parallel}$. Then

$$F\equiv\frac{\langle j_{\parallel}B\rangle}{\langle j_{f\parallel}B\rangle}=1-\frac{Z_f}{Z_{\mathrm{eff}}}(1-\mathcal{L}_{31})\Longrightarrow G=\mathcal{L}_{31}$$

Bootstrap current coefficient $\mathcal{L}_{31}$

The formula of $\mathcal{L}_{31}$ given by Sauter is

$$\mathcal{L}_{31} = \left(1 + \frac{1.4}{Z_{\mathrm{eff}}+1}\right)X - \frac{1.9}{Z_{\mathrm{eff}}+1}X^2 + \frac{0.3}{Z_{\mathrm{eff}}+1}X^3 + \frac{0.2}{Z_{\mathrm{eff}}+1}X^4$$

with

$$X = \frac{f_t}{1 + (1 - 0.1\, f_t)\sqrt{\nu_{e\star}} + 0.5(1 - f_t)\nu_{e\star}/Z_{\mathrm{eff}}},$$

where $\nu_{e\star} = qR\nu_{ee}/(\varepsilon^{3/2}v_{te})$ is the ratio of the electron collision frequency to the bounce frequency, q and ε are the safety factor and inverse aspect ratio of a toroidal magnetic surface, respectively; $\nu_{ee} = \sqrt{2}n_e\, e^4 \ln\Lambda_e/(12\pi^{3/2}\,\epsilon_0^2\sqrt{m_e}T_e^{3/2})$; f_t is the effective trapped fraction

$$f_t = 1 - \frac{3}{4}\left\langle\frac{B^2}{B_{\max}^2}\right\rangle \int_0^1 \frac{\lambda d\lambda}{\left\langle\sqrt{1 - \lambda B/B_{\max}}\right\rangle}.$$

The formulas are valid for general tokamak equilibria and arbitrary collisionality regime.

Results

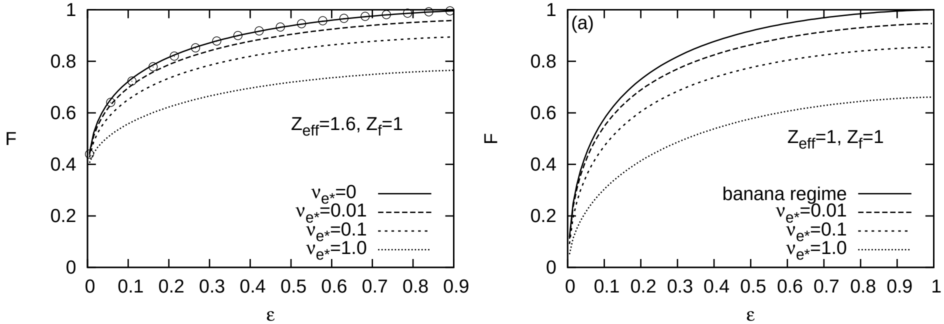

Figure 1. The ratio F of the total neutral beam driven current to the fast ion current, as a function of the inverse aspect ratio ε.

- The results for small values of $\nu_{e\star}$ are close to those in the banana regime as expected
- The ratio of the total current to the fast ion current is reduced when the value of $\nu_{e\star}$ is increased. This means the electron shielding current is actually increased when $\nu_{e\star}$ is increased.
- The explanation for this is that the collision increases the fraction of circulating electrons so that more electrons can contribute to the shielding current.

Compared with Hirshman's formula

- Up to now, there is only one collisional model for the electron shielding current in NBCD available in the literature, which is based on a formula given by Hirshman. This model has been implemented in two widely used transport codes, TRANSP and ONETWO, to model NBCD.

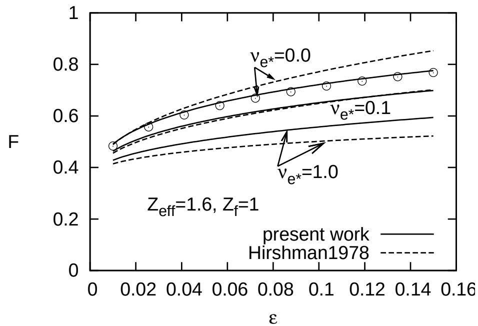

Figure 2. Comparison of the predictions for the F factor from the present work and the Hirshman's model.

Summary

- We show that for arbitrary aspect ratio and arbitrary collisionality regime the electron shielding current in NBCD can be approximately expressed in terms of the electron density gradient coefficient of the bootstrap current $\mathcal{L}_{31}$.
- We employ the existing formula for $\mathcal{L}_{31}$ valid for general tokamak equilibria and arbitrary collisionality regime to provide a general formula for calculating the electron shielding current.
- These formulas for the electron shielding current can be easily included in transport codes to model neutral beam current drive.
- For details refer to: Y. J. Hu, Y. M. Hu, and Y.R. Lin-Liu, Phys. Plasmas 19, 034505 (2012)

Thank you for your attention!