Semi-Relativistic Coulomb Collision Operators for Current Drive Applications

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Introduction and Motivation

- A semi-relativistic Fokker-Planck operator for electron-electron collision was first used by Karney and Fisch to calculate lower-hybrid and electron cyclotron current drive efficiencies.
- There is evidence from our recent numerical work that this collision operator should be appropriate for current drive applications under ITER conditions.
- We extended Karney and Fisch's work by expressing the semi-relativistic Fokker-Planck coefficients in terms of a pair of Rosenbluth-Trubnikov potential functions, and worked out the general Legendre expansion of these two potentials.
- This general Legendre expansion can be used to implement the semi-relativistic collision term efficiently in Fokker-Planck codes, such as in CQL3D.

Whether the semi-relativistic collision operator is precise enough under ITER conditions?

- To justify the use of semi-relativistic collision operator under ITER conditions, here we give some numerical examples about the precision of the semi-relativistic collision operator.
- These numerical results indicate that the semi-relativistic collision operator should be precise enough for current drive applications under ITER conditions.

Whether the semi-relativistic collision operator is precise enough under ITER conditions?

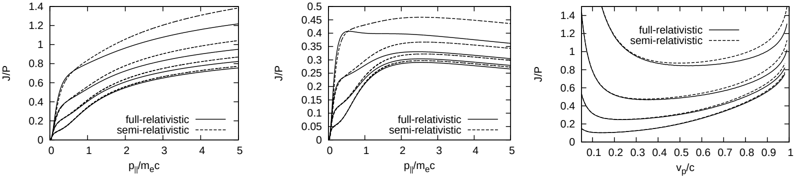

Figure 1: Efficiencies for localized excitation of Landau-damped waves (left figure), cyclotron-damped waves (middle figure) and narrow Landau-damped waves (right figure). The curves from bottom to top show the efficiencies for different temperature $\Theta = T_e/m_e c^2 = 0.02, 0.05, 0.1, 0.2$. The solid lines correspond to the full-relativistic model, while the dashed lines correspond to the semi-relativistic model.

Collision operator in Fokker-Planck form

The collision term for a relativistic plasma of species a colliding off species b may be written in the Fokker-Planck form as[1, 4, 5]

$$C(f_a, f_b) = -\frac{\partial}{\partial \boldsymbol{u}} \cdot \left[-\boldsymbol{D}^{a/b} \cdot \frac{\partial f_a}{\partial \boldsymbol{u}} + \boldsymbol{F}^{a/b} f_a \right],$$

in which the coefficient $\boldsymbol{D}^{a/b}$ and $\boldsymbol{F}^{a/b}$ are defined by

$$\boldsymbol{D}^{a/b}(\boldsymbol{u}) = \frac{q_a^2 q_b^2}{8\pi\epsilon_0^2 m_a^2} \ln \Lambda^{a/b} \int \boldsymbol{U}(\boldsymbol{u}, \boldsymbol{u}') f_b(\boldsymbol{u}') d\boldsymbol{u}', \tag{1}$$

$$\boldsymbol{F}^{a/b}(\boldsymbol{u}) = -\frac{q_a^2 q_b^2}{8\pi\epsilon_0^2 m_a m_b} \ln \Lambda^{a/b} \int \frac{\partial}{\partial \boldsymbol{u}'} \cdot \boldsymbol{U}(\boldsymbol{u}, \boldsymbol{u}') f_b(\boldsymbol{u}') d\boldsymbol{u}', \tag{2}$$

Here f_a and f_b are the distribution functions for the two species, $\boldsymbol{u}$ is the ratio of momentum to species mass, and $\ln \Lambda^{a/b}$ is the Coulomb logarithm.

Extend Karney and Fisch's work

- If either the test or the background species is weakly relativistic, the full relativistic collision kernel can be approximated by the simple Landau semi-relativistic kernel[2, 4],

$$\boldsymbol{U}(\boldsymbol{v},\boldsymbol{v}')=\frac{\boldsymbol{I}}{|\boldsymbol{v}-\boldsymbol{v}'|}-\frac{(\boldsymbol{v}-\boldsymbol{v}')(\boldsymbol{v}-\boldsymbol{v}')}{|\boldsymbol{v}-\boldsymbol{v}'|^3}.$$

- In this work, we extend Karney's work by expressing the semi-relativistic Fokker-Planck coefficients in terms of two potential functions, and worked out the general Legendre decomposition of these two potential functions.

Potential form of the semi-relativistic Fokker-Planck coefficients

Define two potential functions,

$$h_b(\boldsymbol{v}) = -\frac{1}{8\pi}\int |\boldsymbol{v}-\boldsymbol{v}'|\gamma'^5 f_b(\gamma'\boldsymbol{v}')d^3\boldsymbol{v}',$$

$$g_b(\boldsymbol{v}) \equiv -\frac{1}{4\pi}\int \left[\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|}\left(\frac{1}{\gamma'}+\frac{1}{\gamma'^3}\right)+\frac{(\boldsymbol{v}\cdot\boldsymbol{v}'-v'^2)^2}{|\boldsymbol{v}-\boldsymbol{v}'|^3\gamma'}\right]\gamma'^5 f_b(\gamma'\boldsymbol{v}')d^3\boldsymbol{v}'.$$

In terms of these two potential functions, the diffusion and friction coefficients can be written as,

$$\boldsymbol{D}^{a/b}(\boldsymbol{u}) = -\frac{4\pi c_{ab}}{m_a^2}\frac{\partial^2 h_b(\boldsymbol{v})}{\partial\boldsymbol{v}\partial\boldsymbol{v}},$$

$$\boldsymbol{F}^{a/b}(\boldsymbol{u}) = -\frac{4\pi c_{ab}}{m_a m_b}\frac{\partial}{\partial\boldsymbol{v}}g_b(\boldsymbol{v}).$$

Legendre expansion of Fokker-Planck coefficients I

If the background distribution f_b and the potential functions are expanded in terms of Legendre polynomials (assuming axial symmetry),

$$f_b(u,\theta) = \sum_{l=0}^{\infty} f_b^l(u) P_l(\cos\theta),$$

$$h_b(v,\theta) = \sum_{l=0}^{\infty} h_b^l(v) P_l(\cos\theta),$$

$$g_b(v,\theta) = \sum_{l=0}^{\infty} g_b^l(v) P_l(\cos\theta),$$

one can get a relation between h_b^l and f_b^l,

$$\begin{aligned} h_b^l(v) \quad = \quad & \frac{1}{2(4l^2-1)} \left[\int_0^v \frac{(v')^{l+2}}{v^{l-1}} \left(1 - \frac{2l-1}{2l+3} \times \frac{(v')^2}{v^2} \right) \gamma'^5 f_b^l(\gamma' v') dv' \right. \\ + \quad & \left. \int_v^c \frac{v^l}{(v')^{l-3}} \left(1 - \frac{2l-1}{2l+3} \times \frac{v^2}{(v')^2} \right) \gamma'^5 f_b^l(\gamma' v') dv' \right]. \end{aligned}$$

Legendre expansion of Fokker-Planck coefficients II

And a relation between g_b^l and f_b^l,

$$\begin{aligned} g_b^l(v) &= -\frac{1}{2l+1}\int_0^v \left[\frac{1}{c^2}\frac{v'^{l+2}}{v^{l-1}}\gamma'^4\frac{l(l-1)}{2l-1} - \frac{1}{c^2}\frac{v'^{l+4}}{v^{l+1}}\gamma'^4\left(\frac{l^2+l-1}{2l+3}\right)\right. \\ &+ \left.\frac{(v')^{l+2}}{v^{l+1}}\left(\gamma'^2+1\right)\gamma'^2\right] f_b^l(\gamma' v')dv' \\ &- \frac{1}{2l+1}\int_v^c \left[-\frac{1}{c^2}\frac{v^{l+2}}{v'^{l-1}}\gamma'^4\frac{(l+1)(l+2)}{2l+3} + \frac{1}{c^2}\frac{v^l}{v'^{l-3}}\gamma'^4\left(\frac{l^2+l-1}{2l-1}\right)\right. \\ &+ \left.\frac{v^l}{(v')^{l-1}}\left(\gamma'^2+1\right)\gamma'^2\right] f_b^l(\gamma' v')dv'. \end{aligned}$$

Summary

- We have derived a general Legendre expansion for the Fokker-Planck coefficients of the semi-relativistic collision operator.
- This general Legendre decomposition can be used to implement the semi-relativistic collision term efficiently in Fokker-Planck codes, such as in CQL3D[9].

References

[1] S. T. Beliaev and G. I. Budker. *Sov. Phys. Dokl*, 1:218, 1956.

[2] Charles F. F. Karney and Nathaniel J. Fisch. *Physics of Fluids*, 28(1):116–126, 1985.

[3] Charles F. F. Karney. *Comp. Phys. Rep.*, 4:183–244, 1986.

[4] L. D. Landau. *Phys.Z.Sowjetunion*, 10:154, 1936.

[5] Bastiaan J. Braams and Charles F. F. Karney. *Phys. Fluids*, 1B(7):1355–1368, July 1989.

[6] Y. M. Hu, Y. J. Hu, and Y. R. Lin-Liu. In *Poster at 9th Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Heating*, Sanya, China, 2010.

[7] Marshall N. Rosenbluth, William M. MacDonald, and David L. Judd. *Phys. Rev.*, 107(1):1–6, Jul 1957.

[8] B. A. Trubnikov. Consultants Bureau, New York, 1965.

[9] R.W. Harvey and M.G. McCoy. In *IAEA Conf. Proc.,Technical Committee Meeting on Advances in Simulation and Modeling of Thermonuclear Plasmas*, page 527, Montreal, Canada, 1992. International Atomic Energy Agency.

[10] Bastiaan J. Braams and Charles F. F. Karney. *Phys. Rev. Lett*, 59(16):1817, 1987.

[11] Jr. T.M. Antonsen and K.R. Chu. *Physics of Fluids*, 25(8):1295–1296, 1982.

[12] Ronald H. Cohen. *Physics of Fluids*, 30(8):2442–2449, 1987.

[13] Y. R. Lin-Liu, V. S. Chan, and R. Prater. *Physics of Plasmas*, 10(10):4064–4071, 2003.