Rf current drive in general tokamak geometry

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Outline

- Drift kinetic equation in toroidal geometry
- Adjoint method for current drive problem
- Current drive in banana regime
- A example of electron cyclotron current drive(ECCD)

Drift kinetic equation in toroidal geometry

Neglecting cross-field drift

$$\frac{\partial f}{\partial t} + v_{\parallel} \hat{\boldsymbol{b}} \cdot \nabla f - C(f) = - \nabla_v \cdot \boldsymbol{S}_w,$$

where
f is guiding center distribution function, $f = f(w, \mu, \sigma, \boldsymbol{r})$;
w is kinetic energy;
μ is magnetic moment, $\mu = m v_\perp^2 / 2B$; $\sigma = \mathrm{sgn}(v_\parallel) = \pm 1$;
$\hat{\boldsymbol{b}}$ is magnetic field unit vector;
$C(f)$ is collision term;
$\boldsymbol{S}_w$ is wave-induced flux in velocity space;

Rf Current drive

- Generation of toroidal current in tokamaks by injecting electromagnetic waves
- Time-independent, linear current drive

$$v_{\|}\hat{\boldsymbol{b}} \cdot \nabla f - C^l(f) = -\nabla_v \cdot \boldsymbol{S}_w,$$

- Linearized collision operator:

$$C^l(f_e) = C(f_e, f_{em}) + C(f_{em}, f_e) + C^{e/i}(f_e)$$

- Effect of wave on plasma is modeled by wave diffusion term $\boldsymbol{S}_w = \boldsymbol{D}_w \cdot \nabla_v f$
- Trapped particles effect: $v_{\|} = \sigma\sqrt{\frac{2}{m}(w - \mu B)}$.

Adjoint method

Solve DKE to determine parallel current: $j_{\|} = q_e \int \; f v_{\|} d\Gamma$.

$$v_{\|}\hat{\boldsymbol{b}} \cdot \nabla f - C^l(f) = -\nabla_v \cdot \boldsymbol{S}_w$$

Adjoint equation

$$-v_{\|}\hat{b} \cdot \nabla \chi - C^{l+}(\chi) = \frac{v_{\|}B}{\langle B^2 \rangle}$$

$$\Longrightarrow \frac{j_{\|}}{B} = -q_e \left\langle \int d\Gamma \chi \nabla_v \cdot \boldsymbol{S}_w \right\rangle$$

where C^{l+} is adjoint operator of C^l,

$$\int d\Gamma g C_e^{l+}(h) = \int d\Gamma h \, C_e^l(g)$$

Banana regime

$$\frac{\partial\chi}{\partial t} - v_{\parallel}\hat{b}\cdot\nabla\chi - C^{l+}(\chi) = D(f)$$

$$v_{\parallel}\hat{b}\cdot\nabla\chi \approx \omega_h\chi$$

$$C^{l+}(\chi) \approx \omega_l\chi$$

$$D(f) \approx \omega_l\chi$$

Provided $\omega_l \ll \omega_h$ and considering a phenomena at frequency ω_l

$$\frac{\partial\chi}{\partial t} \approx \omega_l\chi$$

then we are working in banana regime.

Expand DKE in banana regime.

$$\frac{\partial\chi}{\partial t} - v_\parallel \hat{b}\cdot\nabla\chi - C^{l+}(\chi) = \frac{v_\parallel B}{\langle B^2\rangle}$$

Expand $\chi = \chi_0 + \chi_1$, where $\chi_i = (\omega_l/\omega_h)^i$, then

$$v_\parallel \hat{b}\cdot\nabla\chi_0 = 0 \tag{1}$$

$$\frac{\partial\chi_0}{\partial t} - v_\parallel \hat{b}\cdot\nabla\chi_1 - C_e^{l+}(\chi_0) = \frac{v_\parallel B}{\langle B^2\rangle} \tag{2}$$

For passing electrons, Eq.(2) is bounce averaged to give

$$\Longrightarrow \tau_b\frac{\partial\chi_0}{\partial t} - \oint \frac{dl}{v_\parallel} C_e^{l+}(\chi_0) = \oint \frac{B}{\langle B^2\rangle} dl$$

$$\Longrightarrow \frac{1}{v}\left\langle \frac{B}{\xi} C_e^{l+}(\chi_0) \right\rangle = -1$$

Current drive efficiency

$$j_{\|} = -Bq_e \left\langle \int d\Gamma \chi_0 \nabla_v \cdot \boldsymbol{S}_w \right\rangle$$

Averaged energy absorbed per unit volume per unit time by electrons from the wave

$$P = -\left\langle \int d\Gamma w \nabla_v \cdot \boldsymbol{S}_w \right\rangle$$

Current drive efficiency at a flux surface

$$\text{CD efficiency} = \frac{\langle j_{\|} \rangle}{P}$$

Collision operator

- Nonrelativistic collision operator
- Weakly relativistic collision operator
- Fully relativistic collision operator
- Relativistic high-velocity-limit collision operator

$$C^l(f) = [\nu_{ei}(u) + \nu_D(u)]L(f) + \frac{1}{u^2}\frac{\partial}{\partial u}u^2\lambda_s(u)f$$

Pitch angle scattering operator

$$L(f) = \frac{1}{2}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial}{\partial \xi}f$$

Wave diffusion operator

- Lower-hybrid wave:

$$\boldsymbol{D} = \delta(\boldsymbol{x} - \boldsymbol{x}_R)\delta(\omega - k_\parallel v_\parallel)\hat{\boldsymbol{p}}_\parallel \hat{\boldsymbol{p}}_\parallel$$

- Electron cyclotron wave:

$$\nabla \cdot \boldsymbol{S}_w = \delta(\boldsymbol{x} - \boldsymbol{x}_R)\tilde{\Lambda} D_0 \delta(\omega - k_\parallel v_\parallel - l\omega_c/\gamma)\tilde{\Lambda} f,$$

$$D_0 \propto \left(\frac{k_\perp u_\perp}{2\omega_c}\right)^{2l-2} \frac{u_\perp^2}{\gamma^2},$$

$\tilde{\Lambda}$ is a differential operator in velocity space,

$$\tilde{\Lambda} = \frac{\partial}{\partial w} + \frac{k_\parallel}{\omega}\frac{1}{m_e}\frac{\partial}{\partial u_\parallel}.$$

Electron cyclotron current drive

$$B = B_0/(1+\varepsilon\cos\theta_p),\ B_p = B_{p0}/(1+\varepsilon\cos\theta_p)$$

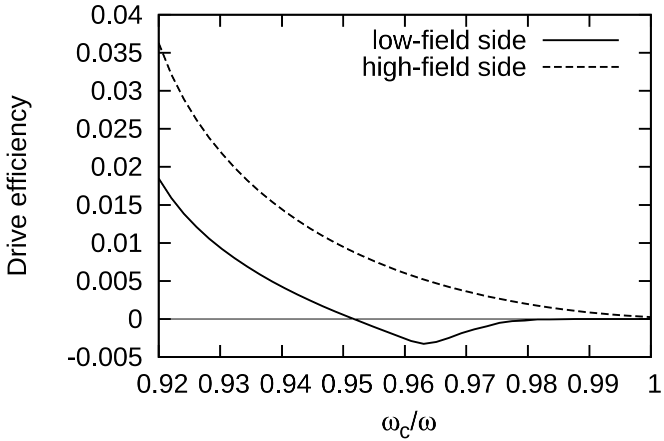
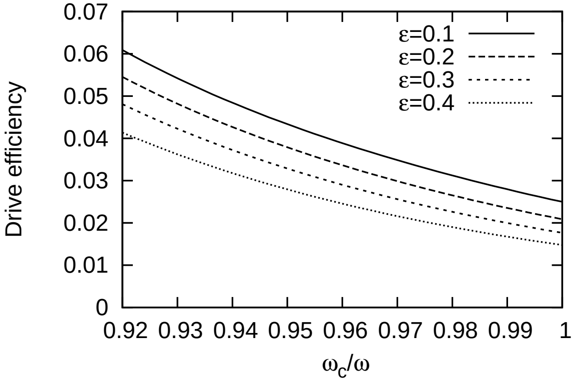


Figure 1. Low-field-side($\theta_p = 0°$) and high-field-side($\theta_p = 180°$) absorption. $\varepsilon = 0.3$, $T_e = 2\mathrm{keV}$, $n_{\parallel} = 0.4$, $Z_{\mathrm{eff}} = 1.67$, $l = 1$.

- Near high-field-side absorption ($\theta_p = 165°$, Left Fig.)
- Near low-field-side absorption ($\theta_p = 15°$, Right Fig.)



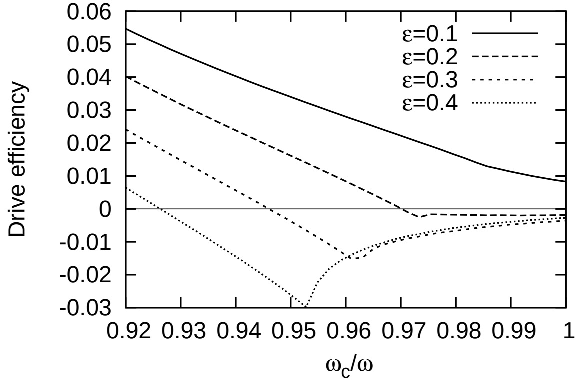


Figure 2. Drive efficiency as a function of ω_c/ω at different flux surface. $T_e = 25\mathrm{keV}$, $n_{||} = 0.4$, $l = 1$.

Trapped particles effect: Ohkawa effect

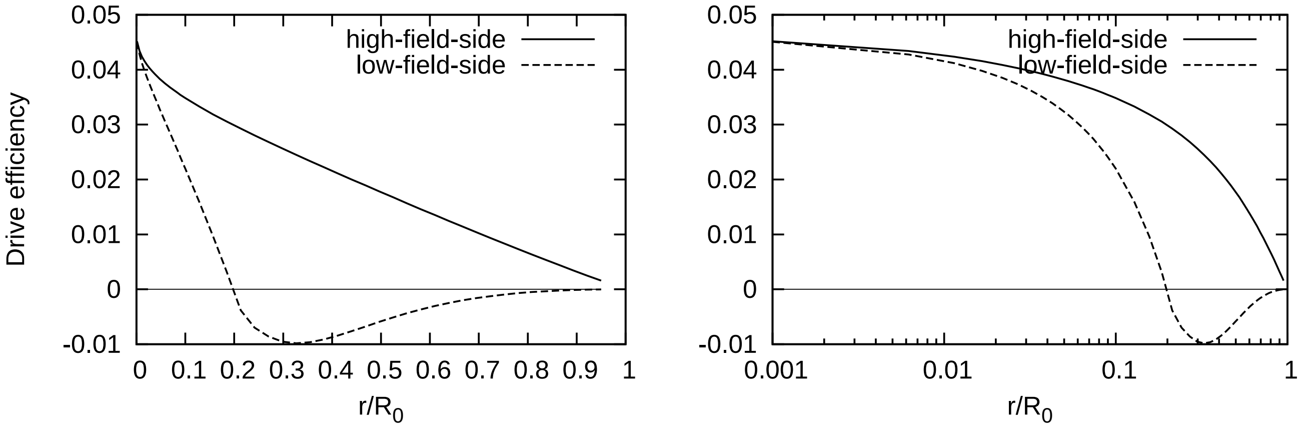


Figure 3. $T_e = 25\mathrm{keV}$, $n_{||} = 0.4$, $Z_{\mathrm{eff}} = 1.67$, $l = 1$, $\omega_c/\omega = 0.97$.

Thanks!