Introduction to Coordinate Frames and Satellite Orbits: John Draim's constellation

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The guided project within the AMVO curriculum's Orbital Mechanics component contains a series of linked questions. Each group must produce a comprehensive report, due for submission to *Atenea* one week before the final examination. It is imperative to note that late submissions will not undergo evaluation. The report's content must be concise and confined to a maximum of four pages, excluding supplementary sections for references, visual aids, programming codes, or tables. The groups should provide precise answers to the questions posed, while also providing a brief but clear overview of the methodology employed in arriving at the solutions. Additionally, any software functions developed as part of the project must be compiled and submitted in a ZIP format via the ATENEA platform, jointly with the PDF report.

Introduction

In the 80's, John Draim published a series of articles presenting his research work on demonstrating that only using four satellites it was able to provide full coverage over the Earth's surface continuously. The benefits of such a constellation setup are evident, mainly in the economical, and the number of applications is widely extensive, from communications, Earth observation, up to meteorological purposes.

Objective

This practical session is devoted to analysing a John Draim constellation of four satellites over two days. Students will be asked to compute the orbital parameters during the exercise time window, the definition and translation between reference frames and the visual representation of the obtained data and results.

Input data

The four-satellite constellation is defined using as a foundation a modified (flattened) geometrical figure known as a tetrahedron. The satellite orbits are placed parallel to the planes of this perturbed tetrahedron, passing through the Earth's centre. Each of the satellite orbits is then made elliptic, with the following orbital parameters in the Terrestrial Reference Frame (TRF):

Table 1 John Draim's constellation orbital parameters.

Satellite	**Inclination**	**Orbital period**	**Eccentricity** e	**Argument of Perigee** ω	**RAAN** Ω	**Mean Anomaly** M_0
S_1	31.3º	26.49 h	0.263	-90º	0º	0
S_2	31.3º	26.49 h	0.263	90º	90º	270
S_3	31.3º	26.49 h	0.263	-90º	180º	180
S_4	31.3º	26.49 h	0.263	90º	270º	90

From these parameters, it is then generated the file "***SATPT.txt***", which contains the satellite position and velocity data for one year at a rate of 600 seconds. The following table describes the contained information:

Table 2 Information contained in the input file "SATPT.txt"

Column	**Field**	**Description**	**Units**
1	Sat. Id.	Identification of the satellite	---
2	Epoch	Time since January 1 2000 11:00:00, in UT1	Seconds
3	Satellite Position	X coordinate in Terrestrial Reference Frame (TRF).	metres
4	Satellite Position	Y coordinate in Terrestrial Reference Frame (TRF).	metres
5	Satellite Position	Z coordinate in Terrestrial Reference Frame (TRF).	metres
6	Satellite Velocity	X component of satellite velocity in Terrestrial Reference Frame (TRF).	metres/second
7	Satellite Velocity	Y component of satellite velocity in Terrestrial Reference Frame (TRF).	metres/second
8	Satellite Velocity	Z component of satellite velocity in Terrestrial Reference Frame (TRF).	metres/second

1. Geocentric Satellite coordinates

a. Using the data on file "***SATPT.txt***", compute the estimated position vector and plot the geometric distance with respect to the Earth's centre as a time function, during a period of three days (72 hours), assuming a two-body modelling. Every work group shall take a different time window (the day of birth of the youngest member in the group).
   i. Transform the satellite coordinates (X, Y, Z) given in an Earth-Centered Earth-Fixed (ECEF) coordinate system to spherical geocentric coordinates (r, λ, ϕ) using the algorithms presented in the theory lectures.
   ii. Plot the geocentric distance, separating the satellites. Comment on the resulting figure.

2. Satellite ground tracks in the geocentric Terrestrial Reference Frame (TRF)

a. Using the spherical coordinates obtained in the previous step, plot the tracks over the Earth surface for all satellites. Comment on the figure, highlighting the link orbital parameters of Table 1 and the shape of the satellite ground tracks.

b. Based on the previous answer, what is the number of orbital periods (or analogously, time) required for the satellites to approximately repeat its ground track. Take the necessary extra days and redo the computations and its associated plots.

3. Assessing the constellation covering in the terrestrial reference frame

c. Using the spherical coordinates obtained in the previous exercise, plot the ground tracks of all the satellites.
d. Using a global grid, defined by fixed steps of latitudes and longitudes over the Earth (every 5 degrees), compute the number of epochs in which every cell of such a grid have:
    i. 1 satellite in view (Satellite elevation is greater than 5 degrees).
    ii. 2 satellites in view
    iii. 3 satellites in view
    iv. 4 satellites in view
e. According the previous analysis, compute the time interval in which the different cells are covered for 1, 2, 3 and 4 satellites, respectively. Comment on the results, and emphasizes on the relation of time with respect to number of satellites that predominates.
c. For an observer located in the ESEIAAT campus, compute the number of satellites visible and its interval during the 72 hours period.

4. Satellite ground tracks in the geocentric Celestial Reference Frame (CRF)

Transform the TRF coordinates of the satellites to the inertial CRF, i.e. not-tied to Earth rotation. This can be approximated by performing a rotation around the Z-axis with a magnitude of Greenwich Mean Sidereal Time (GMST)(i.e. $R_3(-\theta_{GMST})$) on the TRF coordinates (X,Y,Z) to roughly align with the CRF (neglecting the precession, nutation and pole movement).

a. Compute the GMST at the corresponding observation time.

b. Transform the state vector (r,v) from the terrestrial to the celestial refence frame.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{CRF} = \boldsymbol{R_3}(-\theta_{GMST}) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{TRF}$$

$$\begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix}_{CRF} = \boldsymbol{R_3}(-\theta_{GMST}) \begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix}_{TRF}$$

c. Plot the obtained orbits in the inertial space using spherical geocentric coordinates (r, λ, ϕ) as in the previous exercise.

5. Displaying the tetrahedron and optimizing the constellation
    a. Given the fact that John Draim's constellation is based on the tetrahedron, a geometrical shape composed of 4 identical triangular faces, plot for a full constellation period the resulting modified triangles. Take, for instance, equally separated spaces of time, or more suitable, equally angular spaces of mean anomaly.

Additional Information

References

[1] Vallado, D. A. (2001). Fundamentals of astrodynamics and applications (Vol. 12). Springer Science & Business Media.

[2] Draim, J. E. (1987). A common-period four-satellite continuous global coverage constellation. Journal of Guidance, Control, and Dynamics, 10(5), 492-499.

[3] Draim, J. E. (1985). Three-and four-satellite continuous-coverage constellations. Journal of Guidance, Control, and Dynamics, 8(6), 725-730.

[4] Draim, J. E. (1991). Continuous global N-tuple coverage with (2N+2) satellites. Journal of Guidance, Control, and Dynamics, 14(1), 17-23.

Some useful parameters

Earth:

Mass (1024 kg)	5.9722
Equatorial radius (km)	6378.137
Polar radius (km)	6356.752
Volumetric mean radius (km)	6371.000
Core radius (km)	3485
Ellipticity (Flattening)	0.003353
GM (x 106 km3/s2)	0.39860
J2 (x 10-6)	1082.63
Semimajor axis (106 km)	149.598
Sidereal orbit period (days)	365.256
Mean orbital velocity (km/s)	29.78
Max. orbital velocity (km/s)	30.29
Min. orbital velocity (km/s)	29.29
Orbit inclination (deg)	0.000
Orbit eccentricity	0.0167
Sidereal rotation period (hrs)	23.9345
Inclination of equator (deg)	23.44