Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

SUPPLEMENTAL/DEFERRED EXAMINATION

MAT 1320C, February 2011

Professor: Weixuan Li

Duration: 3 hours

Last name _____ First name _____ Student number _____

Instructions:

1. The exam is closed book. Only non-programmable and non-graphing calculators (TI-30 or equivalent) are allowed.

2. The exam has three parts in 14 pages (including this cover page) with a total of 50 marks.

3. Write your solution in the space provided for each question. If you need extra space to write your solution, use the back of the pages. Please clearly indicate where your answer is whenever you do this.

4. Check your answer carefully to avoid arithmetic calculation mistakes.

5. Do not separate the pages.

Part 1. Multiple-choice Questions

Write your answers to questions in this part in the following boxes:

1. ☐ 2. ☐ 3. ☐ 4. ☐ 5. ☐ 6. ☐ 7. ☐ 8. ☐

1. Evaluate $\int_{0}^{\pi/3} 2\cos^3\theta \sin\theta\, d\theta$.

(A) $1/2$;   (B) $1/4$;   (C) $1/\sqrt{2}$;   (D) $1/16$;   (E) $1/8$;   (F) $1/32$.

2. Consider the function $f(x) = 2x^3 - 6x^2 + 15x - 10$. Which of the following statements is true?

(A) $f(x)$ attains a local (i.e., relative) maximum at $x = 1$.

(B) $f(x)$ attains a local (i.e., relative) minimum at $x = 1$.

(C) $(1, 1)$ is a critical point of $f(x)$, but it is not a local maximum or a local minimum.

(D) $(1, 1)$ is an inflection point.

(E) $(1, 0)$ is not a critical point, nor an inflection point.

3. If $f(x) = \arctan(2x)$, then $f'(1) =$

(A) 2 / 3;   (B) 2;   (C) 2 / 5;   (D) 1 / 2;   (E) 1 / 3;   (F) 1 / 5.

4. Which one of the following values is closest to $\int_0^3 2^x \, dx$?

(A) 4.85;   (B) 7;   (C) 8;   (D) 8.79;   (E) 9.89;   (F) 10.1.

5. On what interval(s) is the function $f(x) = (x^2 - 4x + 6)e^x$ concave up?

(A) It is never concave up;   (B) $x > 1$ only;   (C) $x < -1$ and $x > 1$ only;
(D) $-\infty < x < \infty$;   (E) $-1 < x < 1$ only;   (F) $x < -1$ only.

6. Where are the critical points of the function $f(x) = x^3 + 3x + 17$?

(A) It has none;   (B) $x = \pm 1$ only;   (C) $x = 0$ only;
(D) $x = 0$ and $x = \pm 1$ only;   (E) $x = 1$ only;   (F) $x = 0$ and $x = 1$ only.

7. What is the slope of the tangent line to the curve $xe^x + x^2y^2 + 2y = 4$ at the point $(x, y) = (0, 2)$?

(A) undefined;   (B) 1;   (C) $1/2$;   (D) $-1/2$;   (E) -1;   (F) 0.

8. At what value(s) of x does the graph of $y = 2 \ln |x + 1|$ have a horizontal tangent line?

(A) 1 and -1 only;   (B) never;   (C) 0, 1, and -1 only;
(D) 0 only;   (E) 2 and -2 only;   (F) 0, 2, and -2 only.

Section B: Short Answer Questions (2 marks each)

9. If $g(x) = \int_{1}^{2x} \ln(2t^2 - t)dt$, what is $\dfrac{d}{dx}g(x)$?

Answer.

10. If $f(x)$ is a continuous function such that $\int_{0}^{10} f(x)dx = 12$ and $\int_{5}^{10} f(x)dx = 15$, what is $\int_{0}^{5} 2f(x)dx$?

Answer.

11. Newton's method is being used to find an approximation of the root of $f(x) = x^3 + 7x - 7$ with $x_1 = 1$. What is the value of x_2?

Answer.

12. Some values of an increasing function $y = f(x)$ are given by the following table:

x	0	1	2	3	4	5
$f(x)$	4	10	16	24	33	40

Use the right sum and the left sum to find an overestimate and an underestimate of the area between the graph of this function and the x-axis with $0 \le x \le 5$.

13. Find the linear approximation to $f(x) = \sqrt{2x+4}$ near $x = 0$, and use it to estimate $\sqrt{6}$ to three decimal places.

14. The top of a ladder 3 meters long is sliding down a wall at a speed 0.5 m / s. How fast is the bottom of the ladder sliding away from the wall when the bottom of the ladder is 1 meter away from the wall?

15. Evaluate $\lim_{x \to 0^+} (\csc x - \cot x)$.

16. What is the largest rectangular area that can be enclosed with 1000 m of fence?

Section C. Long Answer Questions

17. (4 marks) Find the derivatives of the following functions:

(a) $f(t) = \left(\dfrac{t^2 + 3t}{4t + 1} \right)^5$.

(b) $y = \sin(x^2 e^{3x} - \ln x)$.

18. (2 marks) Sketch the graph of a continuous function satisfying all the following properties:

(i) $f(x) = 0$ if $x = -2$ and $x = 2$;
(ii) $f'(x) < 0$ if $-2 < x < 0$;
(iii) $f'(x) > 0$ if $x < -2$ or $x > 0$;
(iv) $f''(x) < 0$ if $x < -1$;
(v) $f''(x) > 0$ if $x > -1$;
(vi) $f(0) = -4$.

Label the local maxima, local minima, and inflection points (if any).

19. (8 marks) Find the following integrals:

(a) $\int \arcsin x\, dx$.

(b) $\int_{0}^{1} \frac{x}{(x+1)(x+2)}\, dt$.

(Continue on next page)

(c) $\int \frac{e^x}{1+e^{2x}}\,dx$.

(d) $\int 3x(2x^2+3)^{2/3}\,dx$.

20. (4 marks) (a) Approximate the integral $\int_0^4 \frac{1}{x+1}\, dx$ numerically to 4 decimal places with $n = 4$ subdivisions with the Midpoint Rule and Simpson's Rule.

(b) Evaluate this definite integral to find its true value.