Review Questions

MAT1320, Fall 2015

1. Let $f(x) = \dfrac{3x-4}{x-2}$. What is $f^{-1}(-1)$?

2. Solve equation $2^{2x-1} = 3^{x+1}$.

3. Solve equation $\ln x - \ln(10 - x) = -3$.

4. Evaluate the following expressions:

(a) $\sin(\arcsin(-0.3))$.

(b) $\arccos\left(\cos\left(-\dfrac{\pi}{3}\right)\right)$.

(c) $\arcsin\left(\sin\left(\dfrac{2\pi}{3}\right)\right)$.

(d) $\sin(\arctan a)$.

5. Find limits:

(a) $\lim\limits_{x\to 2} \dfrac{x^2-4}{2+x-x^2}$.

(b) $\lim\limits_{h\to 4} \dfrac{\sqrt{h+5}-3}{h-4}$.

(c) $\lim\limits_{h\to \infty} \dfrac{x^2+\sqrt{x}-1}{\sqrt{2x^4+1}}$.

(d) $\lim\limits_{h\to -\infty} \dfrac{x}{\sqrt{4x^2+1}}$.

6. Suppose a function is defined as

$$f(x) = \begin{cases} ax+3, & x < -2 \\ 2x+b, & -2 \le x \le 3 \\ (a+1)x+2b, & x > 3 \end{cases}.$$

If this function is continuous for all x, what are a and b?

7. Find the derivative of the function $y = e^{\sin(x^2)}$.

8. Some values of a function $y = f(x)$ and it derivative are given in the following table:

x	1	2	3	4	5
$f(x)$	2	4	5	1	3
$f'(x)$	1.2	0.5	0.2	−0.1	2.0

Let $g = f \circ f$. Fill in the following table:

x	1	2	3	4	5
$g(x)$					
$g'(x)$					

9. Find the derivative of **the** function $y = \sqrt{2x-1}$ by the definition.

10. Find the second derivative of $y = \ln(x+\sqrt{x^2+4})$.

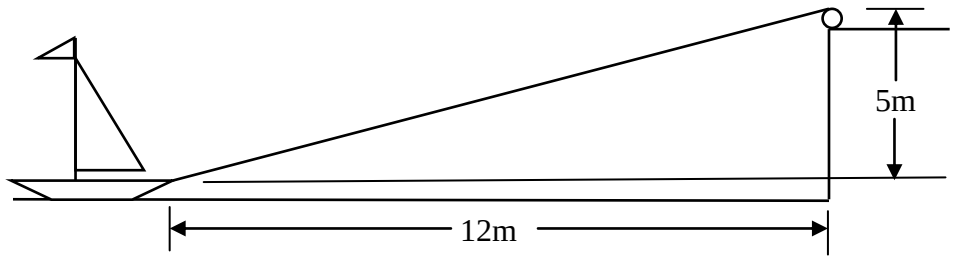
11. Find the 73rd derivative of thee function $y = \cos x$.

12. If a function $y = f(x)$ is defined implicitly by the equation $x^3 - y^3 + x^2y + 3y = 1$. Then the derivative of this function at the point (1, 2) is

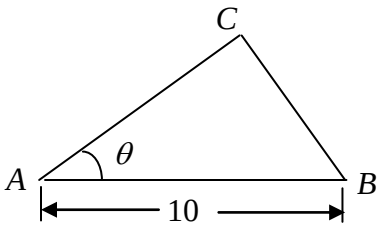
13. Find the derivative of the function $y = \dfrac{(x^4+1)^{2/3}e^{x^2}}{\sqrt{x^2+1}}$. Do not simplify.

14. Find the derivative of $y = (\sin x)^{\sin x}$.

15. The surface of a dock is 5 meters above the deck of a boat. The boat on the water is pulled in by a cable towards the dock. When the boat is 12 meters away horizontally from the dock, it is approaching the dock horizontally at a rate of 0.5 meters per second. How fast is the cable being pulled in?



16. The hypotenuse *AB* of a right triangle is 10 cm. Angle *CAB* is increasing at a rate 0.1 radian per minute. What is the rate of change of the area of the triangle when the length of *BC* is 6 cm?



17. Use the linear approximation of the function $y = \sqrt[3]{5x+7}$ at $x = 4$ to estimate the value of $\sqrt[3]{25}$. Give the estimate as a fraction.

18. If $F(x)$ is an antiderivative of the function $y = \dfrac{1}{1+x^2}$ such that $F(0) = \dfrac{2\pi}{3}$, what is $F(\sqrt{3})$?

19. Suppose $\int_1^3 f(x)dx = 5$, $\int_2^4 f(x)dx = 9$, $\int_1^4 f(x)dx = 11$. Find $\int_2^3 (5f(x) - 2x - 3)dx$.

20. Suppose some values of a function $y = f(x)$ is listed in the following table:

x	1	1.2	1.4	1.6	1.8	2	2.2
y	1.2	1.8	2.1	2.2	1.9	1.7	1.4

Use all the values in the table and the left sum, right sum, trapezoidal rule, and Simpson's rule, respectively, to estimate the definite integral $\int_1^{2.2} f(x)dx$.

21. Use the midpoint rule with $n = 4$ to estimate the definite integral $\int_0^1 e^{-x^2} dx$.

22. Suppose a particle is moving along the x-axis with velocity (in m/sec) $v = 100 - t^2$, $t \geq 0$. Find the total **distance** (not the displacement!) it travels from $t = 0$ to $t = 12$.

23. If $F(x) = \int_{x^2}^{x^3} \sqrt{2t^2 + 1}dt$, find $F'(x)$.

24. Evaluate the definite integral $\int_1^4 \dfrac{x^2 - 1}{\sqrt{x}} dx$.

25. Evaluate the definite integral $\int_0^2 \dfrac{x^3}{\sqrt{2x^2 + 1}} dx$.

26. Calculate the indefinite integral $\int x^2 \arctan x dx$.

27. Calculate definite integral $\int_0^{\pi/3} \sec^4 x dx$.

28. Calculate the definite integral $\int \frac{2x+1}{3x^2-2x-1} dx$.

29. Calculate indefinite integral $\int \frac{x+2}{x^2+2x+5} dx$.

30. Calculate the indefinite integral $\int \frac{x+1}{x(x^2+2x+5)} dx$.

31. Calculate the indefinite integral $\int \frac{1}{(1-x^2)^{3/2}} dx$.

32. Consider function $y = \frac{x^{1/5}}{x+1}$.

(a) Find the first and the second derivatives of this function.

(b) Find critical numbers of this function.

(c) For which values of x is this function increasing / decreasing?

(d) Find all local max / min of this function, if any.

(e) For which values of x is the graph of this function concave up / down?

(f) Find all inflection points, if any.

(g) Find all vertical/ horizontal asymptotes, if any.

(h) Sketch the graph of this function.

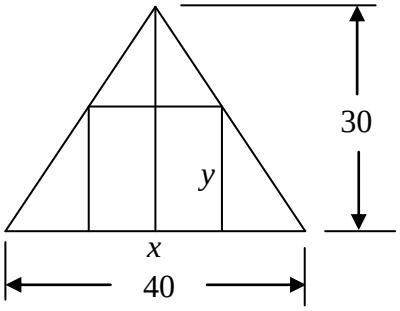
33. Find $\lim_{x\to 0} \frac{x-\sin x}{x-\tan x}$.

34. Find $\lim_{x\to \pi/2} \left(x-\frac{\pi}{2}\right)\tan x$.

35. Find $\lim_{x\to 0} (1-2x)^{1/x}$.

36. Find $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$.

37. Find the maximum area of a rectangle inscribed in a isosceles triangle with base 40 cm and height 30 cm. Justify that what you got is an absolute maximum.



38. A window with perimeter 10 meters has the shape of a rectangle surmounted by an equilateral triangle. Find the dimensions of the window so that the area of the window is maximized.

39. Use Newton's method to find an approximation of a root of the equation $e^x = x + 1$, with $x_1 = 2$. Stop until $| x_{n+1} - x_n | < 0.00001$.