MAT 1348 Review Sheet

Some of the Most Important Concepts, Methods, Theorems, and Questions

NOTE: You must be able to give precise definitions of important concepts, as well as state important theorems.

- **Logic and Proofs**

  Concepts

  1. propositions, logical connectives, equivalence
  2. tautology, contradiction, contingency
  3. disjunctive normal form
  4. rules of inference, arguments; validity of an argument
  5. counter-examples
  6. types of proof: direct, indirect, by contradiction, proof of equivalence, proof by cases

  Methods

  1. truth tables
  2. truth trees
  3. proofs

  Questions

  1. Translate English sentences into compound propositions.
  2. Is a given proposition a tautology/contradiction?
  3. Are given propositions equivalent?
  4. Find a disjunctive normal form of a given proposition.
  5. Is a given argument valid?
  6. Prove a given theorem using a specified method of proof.

- **Sets**

  Concepts

  1. set, cardinality, subset, equality of sets
  2. important number sets: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{Z}^+$, $\mathbb{Z}^-$, $\mathbb{Q}^+$, $\mathbb{Q}^-$, etc. Here:

$$\mathbb{N} = \{0, 1, 2, 3, \cdots\} \quad \text{The Natural Numbers}$$

$$\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, \cdots\} \quad \text{The Integers}$$

$$\mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x > 0\} \quad \text{The Strictly Positive Integers}$$

$$\mathbb{Z}^- = \{x \in \mathbb{Z} \mid x < 0\} \quad \text{The Strictly Negative Integers}$$
$$\mathbb{Q} = \{\frac{m}{n} \mid m, n \in \mathbb{Z}, n \neq 0\} \quad \text{The Rational Numbers}$$
$$\mathbb{R} = \quad \text{The Real Numbers}$$

Similarly, $\mathbb{Q}^+, \mathbb{Q}^-, \mathbb{R}^+, \mathbb{R}^-$ are the strictly positive and strictly negative rationals and reals, as above.

3. set operations: union, intersection, complement, difference, symmetric difference; power set, cartesian product of sets

Questions

1. Find the cardinality of a power set, cartesian product etc.
2. Prove a set identity (using a rigorous proof, known set identities, or membership tables).

- **Functions**

Concepts

1. function
2. injective (one-to-one) and surjective (onto) functions; bijections
3. invertible functions, inverse
4. composition of functions

Questions

1. Determine whether a given function is injective, surjective, invertible, or a bijection.
2. Find the inverse of an invertible function.

- **Relations**

Concepts

1. relation
2. reflexive, symmetric, antisymmetric, transitive relations
3. equivalence relations, equivalence classes, partitions
4. congruences

Questions

1. Determine whether a given relation is reflexive, symmetric, antisymmetric, or transitive.
2. Prove that a given relation is an equivalence relation.
3. Find the equivalence classes of a given equivalence relation (or the corresponding partition of the set).

- **Counting**

  Concepts

  1. Product Rule, Sum Rule, Principle of Inclusion – Exclusion
  2. Pigeonhole Principle
  3. combinations, permutations
  4. binomial coefficients, Binomial Theorem

  Questions

  1. How many...?
  2. What is the smallest number of ... to guarantee...? or Show that in any set of... we can find... such that....
  3. Find the coefficient of ... in the expansion of...

- **Mathematical Induction**

  Concepts

  1. Mathematical Induction

  Questions

  1. Prove... using Mathematical Induction.

- **Graphs**

  Concepts

  1. graph, simple graph, directed graph
  2. complete graphs, cycles, paths, complete bipartite graphs
  3. bipartite graphs
  4. subgraphs, isomorphism

  Theorems

  1. Handshaking Theorem
  2. Characterization of bipartite graphs

  Questions

  1. How many edges in a graph with a given degree sequence?
  2. Does there exist a graph/simple graph with the given degree sequence?
  3. Is a given graph bipartite? (If so, give a proper 2-vertex colouring. If not, find an odd cycle.)
  4. Are two given graphs isomorphic? (If so, give an isomorphism and check it with the adjacency matrix method given in class. If not, find an invariant in which they disagree.)