Linear Regression

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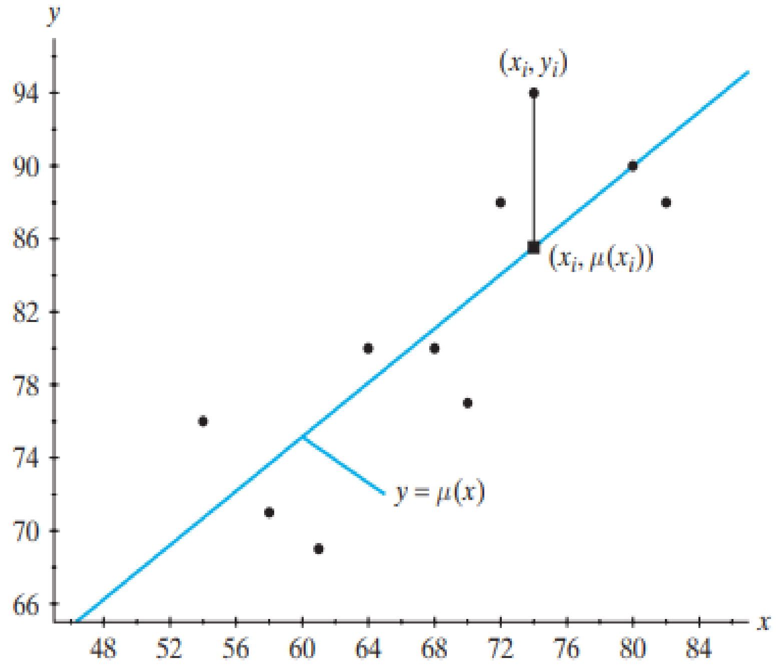
- Department of Digital Analytics

Simple regression problem

- $E(Y|x) = \mu(x)$ is a linear function of x.
- The data points are $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$
- We have linear model,

$$Y_i = \alpha_1 + \beta x_i + \varepsilon_i$$

- (it could be assumed to be equal to forms of $\alpha + \beta x,\ \alpha + \beta x + \gamma x^2, \alpha e^{\beta x}$,..)



```python
import pandas as pd
import statsmodels.api as sm

iris_df = pd.DataFrame(iris.data, columns=['y','b','c','x'])
iris_df = iris_df.iloc[:30]
model = sm.OLS.from_formula('y~x', iris_df)
result = model.fit()

print(result.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.181
Model:                            OLS   Adj. R-squared:                  0.152
Method:                 Least Squares   F-statistic:                     6.178
Date:                Wed, 07 Jun 2023   Prob (F-statistic):             0.0192
Time:                        01:35:32   Log-Likelihood:                -9.4236
No. Observations:                  30   AIC:                             22.85
Df Residuals:                      28   BIC:                             25.65
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      4.6394      0.168     27.628      0.000       4.295       4.983
x              1.5701      0.632      2.486      0.019       0.276       2.864
==============================================================================
Omnibus:                        1.473   Durbin-Watson:                   2.015
Prob(Omnibus):                  0.479   Jarque-Bera (JB):                1.068
Skew:                           0.458   Prob(JB):                        0.586
Kurtosis:                       2.872   Cond. No.                         10.7
==============================================================================
```

- $Y_i = \alpha_1 + \beta x_i + \varepsilon_i, \quad \varepsilon \sim N(0, \sigma^2)$
- Let $\alpha_1 = \alpha - \beta\bar{x}$

$$\rightarrow Y_i = \alpha + \beta(x_i - \bar{x}) + \varepsilon_i \ ,$$
$$where\ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- In order to find proper α, β, σ^2, since $\boldsymbol{Y_i \sim N(\alpha + \beta(x_i - \bar{x}), \sigma^2)}$

$$L(\alpha, \beta, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{[y_i - \alpha - \beta(x_i - \bar{x})]^2}{2\sigma^2})$$
$$= (\frac{1}{\sqrt{2\pi\sigma^2}})^{n/2} \exp\{-\frac{\sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})]^2}{2\sigma^2}\}$$

- maximize $L(\alpha, \beta, \sigma^2)$

= minimize $-\ln L(\alpha, \beta, \sigma^2) = \frac{n}{2}\ln(2\pi\sigma^2) + \frac{\sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})]^2}{2\sigma^2}$

- Select α and β to minimize, so we find the two first-order partial derivatives
- $H(\alpha,\beta) = \sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})]^2$
- $\frac{\partial H(\alpha,\beta)}{\partial \alpha} = 2\sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})](-1) = 0 \rightarrow \boldsymbol{\hat{\alpha} = \bar{Y}}$
- $\frac{\partial H(\alpha,\beta)}{\partial \beta} = 2\sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})][-(x_i - \bar{x})] = 0 \rightarrow \boldsymbol{\hat{\beta} = \frac{\sum_{i=1}^{n} Y_i(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$
- To find the maximum likelihood estimator of σ^2,

$$\frac{\partial[-\ln L(\alpha,\beta,\sigma^2)]}{\partial(\sigma^2)} = \frac{n}{2\sigma^2} - \frac{\sum_{i=1}^{n}[y_i - \alpha - \beta(x_i - \bar{x})]^2}{2(\sigma^2)^2} = 0 \rightarrow \boldsymbol{\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{\alpha} - \hat{\beta}(x_i - \bar{x})]^2}$$

- $\Leftrightarrow$ $\boldsymbol{\mu_{Y|X} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(X - \mu_X)}$
- $\hat{\alpha}, \hat{\beta}, \widehat{\sigma^2}$ are **mutually independent**.
- Maximum likelihood estimates of α, β are also called **ordinary least squares estimates (OLS).**

참고)

- $\frac{\partial H(\alpha,\beta)}{\partial \alpha} = 0 \rightarrow \sum_{i=1}^{n} y_i - n\alpha - \beta\sum_{i=1}^{n}(x_i - \bar{x}) = 0$
  since $\sum_{i=1}^{n}(x_i - \bar{x}) = 0 \rightarrow \sum_{i=1}^{n} y_i - n\alpha = 0$, thus $\hat{\alpha} = \bar{Y}$
- $\frac{\partial H(\alpha,\beta)}{\partial \beta} = 0$ (With α replaced by $\bar{y}$) $\rightarrow$
  $\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) - \beta\sum_{i=1}^{n}(x_i - \bar{x})^2 = 0$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n} y_i(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \ (\leftarrow \sum_{i=1}^{n}\bar{y}(x_i - \bar{x}) = 0)$$

Calculation

```
                            OLS Regression Results
==============================================================================
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Model:                            OLS   Adj. R-squared:                  0.152
Method:                 Least Squares   F-statistic:                     6.178
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Skew:                           0.458   Prob(JB):                        0.586
Kurtosis:                       2.872   Cond. No.                         10.7
==============================================================================
```

```python
n = 30
p = 1
df = n-p-1

# beta
upper = 0
lower = 0
for i in range(n):
    iris_df_i = iris_df.iloc[i,:]
    upper += (iris_df_i['x']-iris_df['x'].mean())*iris_df_i['y']
    lower += (iris_df_i['x']-iris_df['x'].mean())**2
beta = upper/lower
print('beta : ',beta)

alpha = iris_df['y'].mean()
alpha_1 = alpha-beta*iris_df['x'].mean()
print('intercept : ' ,alpha_1)
```

```python
sigma_square = 0
for i in range(30):
    iris_df_i = iris_df.iloc[i,:]
    sigma_square_i = (iris_df_i['y'] - alpha - \
                      beta*(iris_df_i['x']-iris_df['x'].mean()))
    sigma_square += sigma_square_i**2
sigma_square /= 30
print('sigma_square : ' ,sigma_square)

summ = 0
for i in range(n):
    iris_df_i = iris_df.iloc[i,:]
    summ+=(iris_df_i['x']-iris_df['x'].mean())**2
std_beta = (sigma_square / summ) **(1/2)
std_beta_amend = std_beta * ((n/df)**(1/2))
print('std_beta : ', std_beta_amend)
```

```
beta :  1.5701357466606322
intercept :  4.639366515837106
sigma_square :  0.10974057315233794
std_beta :  0.6316837699752129
```

Details

```
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- P-value

  $H_0: \beta_j = 0 \quad \text{vs} \quad H_1: \beta_j \neq 0$

  (귀무가설 : Y에 끼치는 변수 X_j의 영향이 없다. (↔ 대립가설 : 영향이 있다.))
- 95% confidence interval 확인

$$\hat{\boldsymbol{\alpha}} = \overline{\boldsymbol{Y}},\ \hat{\boldsymbol{\beta}} = \frac{\sum_{i=1}^{n} Y_i(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

- $\hat{\alpha}, \hat{\beta}$: normal distribution
  (linear function of independent and normally distributed random variables $Y_1, Y_2, \ldots, Y_n$)
- $E(\hat{\alpha}) = E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(Y_i) = \frac{1}{n}\sum_{i=1}^{n}[\alpha + \beta(x_i - \bar{x})] = \alpha$
- $Var(\hat{\alpha}) = (\frac{1}{n})^2 \sum_{i=1}^{n} Var(Y_i) = \frac{\sigma^2}{n}$
- $E(\hat{\beta}) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})E(Y_i)}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})[\alpha+\beta(x_i-\bar{x})]}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\cancel{\alpha \sum_{i=1}^{n}(x_i-\bar{x})}+\beta \sum_{i=1}^{n}(x_i-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \beta$
- $Var(\hat{\beta}) = \sum_{i=1}^{n}\left[\frac{x_i-\bar{x}}{\sum_{j=1}^{n}(x_j-\bar{x})}\right]^2 Var(Y_i) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{[\sum_{i=1}^{n}(x_i-\bar{x})^2]^2}\sigma^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

$\hat{\alpha}, \hat{\beta}$는 α, β의 unbiased estimators (maximum likelihood estimator)

- Let $\hat{Y}_i = \hat{\alpha} + \hat{\beta}(x_i - \bar{x})$

→ $Y_i - \hat{Y}_i = Y_i - \hat{\alpha} - \hat{\beta}(x_i - \bar{x})$: i-th residual (=ε_i)

- Square of normal distribution = chi-square with degree of freedom 1

→ $\sum_{i=1}^{n} \left\{\frac{\varepsilon_i}{\sigma}\right\}^2 = \sum_{i=1}^{n} \left\{\frac{Y_i - \alpha - \beta(x_i - \bar{x})}{\sigma}\right\}^2$ $(\sim\chi^2(n)$: $\chi^2(1)$의 sum)

$= \frac{1}{\sigma^2}\sum_{i=1}^{n}[(\hat{\alpha} - \alpha) + (\hat{\beta} - \beta)(x_i - \bar{x}) + \{Y_i - \hat{\alpha} - \hat{\beta}(x_i - \bar{x})\}]^2$

$= \frac{n(\hat{\alpha} - \alpha)^2}{\sigma^2} + \frac{(\hat{\beta} - \beta)^2 \sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2} + \sum_{i=1}^{n} \frac{\{Y_i - \hat{\alpha} - \hat{\beta}(x_i - \bar{x})\}^2}{\sigma^2}$ (나머지 항들은 독립)

Since $\frac{n(\hat{\alpha} - \alpha)^2}{\sigma^2}$ and $\frac{(\hat{\beta} - \beta)^2 \sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2}$ have $\chi^2(1)$ (표준정규분포의 제곱)

→ $\sum_{i=1}^{n} \frac{\{Y_i - \hat{\alpha} - \hat{\beta}(x_i - \bar{x})\}^2}{\sigma^2} = \frac{n\widehat{\sigma^2}}{\sigma^2} \sim \chi^2(n-2)$

Variance 의 정의

- $E(\hat{\beta}) = \beta$
- $\mathrm{Var}(\hat{\beta}) = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$
- $\frac{n\widehat{\sigma^2}}{\sigma^2} \sim \chi^2(n-2)$

$\rightarrow \frac{(\widehat{\beta} - \mathrm{E}(\widehat{\beta}))/\mathrm{s.e}(\widehat{\beta})}{\sqrt{\frac{n\widehat{\sigma^2}}{\sigma^2} \times \frac{1}{(n-2)}}} = \frac{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\left(\frac{\widehat{\beta} - \beta}{\cancel{\sigma}}\right)}{\sqrt{\frac{n\widehat{\sigma^2}}{\cancel{\sigma^2}(n-2)}}} \sim T$ with n-2 degrees of freedom

→ p-value 계산하고 각 변수에 대한 귀무가설 검증 가능

(Z : 표준정규분포, V : 자유도가 ν인 chi-square 분포 → $\frac{Z}{\sqrt{V/\nu}}$ ~자유도가 ν인 T 분포)

$\rightarrow [\hat{\beta} - t_{\gamma/2}\sqrt{\frac{n\widehat{\sigma^2}}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}, \hat{\beta} + t_{\gamma/2}\sqrt{\frac{n\widehat{\sigma^2}}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}]$

($\hat{\beta} \pm t_{\gamma/2}\, s.e(\hat{\beta})\sqrt{\frac{n}{(n-2)}}$ 로 대체 가능)

: 100(1- γ)% confidence interval for β.

- $E(\hat{\alpha}) = \alpha$ • $\mathrm{Var}(\hat{\alpha}) = \frac{\sigma^2}{n}$

- $\frac{(\hat{\alpha} - \mathrm{E}(\hat{\alpha}))/\mathrm{s.e}(\hat{\alpha})}{\sqrt{\frac{n\widehat{\sigma^2}}{\sigma^2} \times \frac{1}{(n-2)}}} = \frac{\frac{\cancel{\sqrt{n}}(\hat{\alpha}-\alpha)}{\cancel{\sigma}}}{\sqrt{\frac{\cancel{n}\widehat{\sigma^2}}{\cancel{\sigma^2}(n-2)}}} = \frac{\hat{\alpha}-\alpha}{\sqrt{\frac{\widehat{\sigma^2}}{(n-2)}}} \sim T$ with n-2 degrees of freedom

$\rightarrow [\hat{\alpha} - t_{\frac{\theta}{2}}\sqrt{\frac{\widehat{\sigma^2}}{(n-2)}}, \hat{\alpha} + t_{\frac{\theta}{2}}\sqrt{\frac{\widehat{\sigma^2}}{(n-2)}}]$ ($\hat{\alpha} \pm t_{\gamma/2}\, s.e(\hat{\alpha})\sqrt{\frac{n}{(n-2)}}$ 로 대체 가능)

: 100(1-θ)% confidence interval for α

- $\frac{n\widehat{\sigma^2}}{\sigma^2} \sim \chi^2(n-2) \rightarrow [\frac{n\widehat{\sigma^2}}{\chi^2_{\frac{\theta}{2}}(n-2)}, \frac{n\widehat{\sigma^2}}{\chi^2_{1-\frac{\theta}{2}}(n-2)}]$: 100(1-θ)% confidence interval for σ^2

- 단순회귀이기 때문에 자유도가 n-2이지만 변수가 p개인 다항회귀라면 각 계수들은 자유도가 n-p-1인 T 분포를 따르게 됨
- T값에 각 $\hat{\alpha}$와 $\hat{\beta}$의 표준편차(*n/(n-p-1))를 곱하여 confidence interval 계산

Calculation

```
                            OLS Regression Results
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Method:                 Least Squares   F-statistic:                     6.178
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Df Model:                           1
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x              1.5701      0.632      2.486      0.019       0.276       2.864
==============================================================================
Omnibus:                        1.473   Durbin-Watson:                   2.015
Prob(Omnibus):                  0.479   Jarque-Bera (JB):                1.068
Skew:                           0.458   Prob(JB):                        0.586
Kurtosis:                       2.872   Cond. No.                         10.7
==============================================================================
```

```python
from scipy.stats import t
n = 30
p = 1
df = n-p-1
mean = beta
rv = t(df)

t_val = (beta / std_beta) / (n/df)**(1/2)
print('t값 : ',t_val)

p_value = (1-rv.cdf(t_val))*2
print('p-value : ',p_value)

interval_length = rv.ppf(0.975) * (std_beta * (n/df)**(1/2))

print(f'confidence interval : [{mean-interval_length},{mean+interval_length}]')
```

```
t값 :  2.4856357266673696
p-value :  0.019172036972222236
confidence interval : [0.2761902008329515 4,2.8640812923796926]
```

Prediction interval

- $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n) \rightarrow$ estimate α, β / $Y_{n+1} = \alpha + \beta(x_{n+1} - \bar{x}) + \varepsilon_{n+1}$
- Let $W = Y_{n+1} - \hat{\alpha} - \hat{\beta}(x_{n+1} - \bar{x}) \rightarrow E[W] = E[Y_{n+1}] - \alpha - \beta(x_{n+1} - \bar{x}) = 0$
- $Var(W) = \sigma^2 + \frac{\sigma^2}{n} + \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}(x_{n+1} - \bar{x})^2 = \sigma^2[1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}]$
  ($Y_{n+1}, \hat{\alpha}, \hat{\beta}$ are independent)

- $T = \dfrac{Y_{n+1} - \hat{\alpha} - \hat{\beta}(x_{n+1} - \bar{x}) \Big/ \cancel{\sigma} \sqrt{[1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}]}}{\sqrt{\frac{n\widehat{\sigma^2}}{(n-2)\cancel{\sigma^2}}}}$ with n-2 degrees of freedom,

$(d = \sqrt{\frac{n\widehat{\sigma^2}}{(n-2)}} \sqrt{[1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}]})$

- $P\left[\hat{\alpha} + \hat{\beta}(x_{n+1} - \bar{x}) - dt_{\gamma/2} \le Y_{n+1} \le \hat{\alpha} + \hat{\beta}(x_{n+1} - \bar{x}) + dt_{\gamma/2}\right] = 1 - \gamma$

: prediction interval for Y_{n+1}

Polynomial regression problem

Boston 집값 예측 문제
\# of observations : 506
\# of variables : 13

INDUS 변수
$E(\hat{\beta})$: 0.0206, $S.E(\hat{\beta})$: 0.061

```python
from scipy.stats import t
n = 506
p = 13
df = n-p-1
mean = 0.0206
std = 0.061
t_val = 0.334

rv = t(df)
interval_length = rv.ppf(0.975) * std # * (n/df)**(1/2)

print('p-value :',(1-rv.cdf(t_val))*2)
print(f'confidence interval :[{mean-interval_length},{mean+interval_length}]')
```

```
p-value : 0.7385218727795606
confidence interval :[-0.09925263869488882,0.14045263869488888]
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   MEDV   R-squared:                       0.741
Model:                            OLS   Adj. R-squared:                  0.734
Method:                 Least Squares   F-statistic:                     108.1
Date:                Tue, 06 Jun 2023   Prob (F-statistic):          6.72e-135
Time:                        08:46:57   Log-Likelihood:                -1498.8
No. Observations:                 506   AIC:                             3026.
Df Residuals:                     492   BIC:                             3085.
Df Model:                          13
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     36.4595      5.103      7.144      0.000      26.432      46.487
CRIM          -0.1080      0.033     -3.287      0.001      -0.173      -0.043
ZN             0.0464      0.014      3.382      0.001       0.019       0.073
INDUS          0.0206      0.061      0.334      0.738      -0.100       0.141
CHAS           2.6867      0.862      3.118      0.002       0.994       4.380
NOX          -17.7666      3.820     -4.651      0.000     -25.272     -10.262
RM             3.8099      0.418      9.116      0.000       2.989       4.631
AGE            0.0007      0.013      0.052      0.958      -0.025       0.027
DIS           -1.4756      0.199     -7.398      0.000      -1.867      -1.084
RAD            0.3060      0.066      4.613      0.000       0.176       0.436
TAX           -0.0123      0.004     -3.280      0.001      -0.020      -0.005
PTRATIO       -0.9527      0.131     -7.283      0.000      -1.210      -0.696
B              0.0093      0.003      3.467      0.001       0.004       0.015
LSTAT         -0.5248      0.051    -10.347      0.000      -0.624      -0.425
==============================================================================
Omnibus:                      178.041   Durbin-Watson:                   1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              783.126
Skew:                           1.521   Prob(JB):                    8.84e-171
Kurtosis:                       8.281   Cond. No.                     1.51e+04
==============================================================================
```

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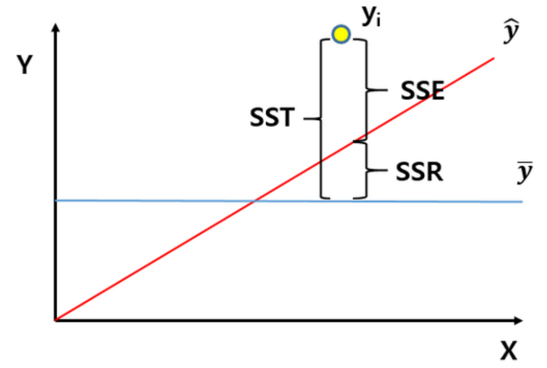
NOX          -17.7666      3.820     -4.651      0.000     -25.272     -10.262
RM             3.8099      0.418      9.116      0.000       2.989       4.631
AGE            0.0007      0.013      0.052      0.958      -0.025       0.027
DIS           -1.4756      0.199     -7.398      0.000      -1.867      -1.084
RAD            0.3060      0.066      4.613      0.000       0.176       0.436
TAX           -0.0123      0.004     -3.280      0.001      -0.020      -0.005
PTRATIO       -0.9527      0.131     -7.283      0.000      -1.210      -0.696
B              0.0093      0.003      3.467      0.001       0.004       0.015
LSTAT         -0.5248      0.051    -10.347      0.000      -0.624      -0.425
==============================================================================
Omnibus:                      178.041   Durbin-Watson:                   1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              783.126
Skew:                           1.521   Prob(JB):                    8.84e-171
Kurtosis:                       8.281   Cond. No.                     1.51e+04
==============================================================================
```



SST(Y의 전체 변동) : $\sum(y_i - \bar{y})^2$
SSR(모형에 의해 설명되는 변동) : $\sum(\hat{y}_i - \bar{y})^2$
SSE(모형에 의해 설명이 되지 않는 변동) : $\sum(y_i - \hat{y}_i)^2$

- $R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$
- $R^2_{adj} = 1 - \frac{SSE/(n-p-1)}{SST/(n-1)}$
- F-statistics : 모델 유의성 검정

Polynomial regression problem

- F-statistics : 모델 유의성 검정
  $H_0: \beta_1 = \cdots = \beta_p = 0 \ (\Leftrightarrow Y = \alpha + \varepsilon)$ vs $H_1: not\ H_0$
  귀무가설 : 독립변수들이 종속변수를 설명하는데 효과가 없다.
  대립가설 : 독립변수들 중 적어도 하나 이상은 종속변수를 설명하는데 효과가 있다.

- 에러에 의해 설명되는 부분과 모델에 의해 설명되는 비율을 비교할 수 있음

$$F = \frac{SSR/p}{SSE/(n-p-1)} = \frac{\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2/\cancel{\sigma^2}p}{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2/\cancel{\sigma^2}(n-p-1)}$$

with degree of freedom (p, n-p-1)

$\sum_{i=1}^{n}\frac{(y_i-\hat{y}_i)^2}{\sigma^2} \sim \chi^2(n-p-1)$ (앞에서 이미 보임)

$\frac{SST}{\sigma^2} = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{\sigma^2}$ $is\ the\ form\ of$ $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

(S는 표본표준편차)

$\rightarrow \frac{SSR}{\sigma^2} \sim \chi^2(p)$

```
from scipy.stats import f
n,p = df.drop('MEDV',axis=1).shape

y_pred = result.predict(df.iloc[:,:-1]).values
y_real = df['MEDV'].values
y_mean = df['MEDV'].mean()

upper = np.sum([(y_pred[i]-y_mean)**2 for i in range(n)])
lower = np.sum([(y_real[i]-y_pred[i])**2 for i in range(n)])

F = (upper/p)/(lower/(n-p-1))
print('F값 : ',F)
```

```
F값 :  108.0766661743256
```