



## Module 5: Mixed-Integer Optimization Assignment

This is a group assignment

### Instructions

Read the mini-case **Hornby Products Company** (reproduced from Baker's book and attached below; the table exhibits are available electronically on Canvas).

Construct an (mixed-)integer linear programming optimization model in Python using PuLP to examine the problem of determining the optimal number and locations of warehouses to lease. **Produce a recommendation of the optimal number and locations of warehouses to lease.**

Each team needs to submit the following files through Canvas via a link that will be made available under the Assignment section (it is sufficient for one member of the team to submit it on behalf of the entire team):

- **Python models as an ipynb file, with code and corresponding output clearly displayed.** The file name should be your group name and assignment name, e.g. Section\_21\_Team\_1 Assignment\_5.ipynb
- **A pdf file (or html) file exported from your ipynb file, with code and corresponding output clearly displayed.** The file name should be your group name and assignment name, e.g. Section\_21\_Team\_1 Assignment\_5.pdf (Section\_21\_Team\_1 Assignment\_5.html)
- **An electronic copy of a short report.** In the report, you should:
  - The **mathematical formulation** of your optimization model (variables, objective function, constraints), any assumptions you made (Legible, hand-written math formulation is acceptable).
    - Provide clear definition of the decision variables
    - Briefly state the purpose of each constraint
    - Legible, hand-written math formulations are acceptable.
  - State any additional assumptions you made if the problem statement is not 100% clear about certain aspects of the problem
  - Present the optimal solution in an easy-to-communicate format

**Grading points will be allocated to the clarity and professional quality of the report.**

Reminder: This is a group deliverable. **Work with your designated team members only.**

### *Case: Hornby Products Company\**

The Hornby Products Company, headquartered in Denver, Colorado, markets a broad line of handcrafted home furnishings that are produced either in its own plants or by local artisans working under contract with the company. Hornby Products has established markets throughout most of the area west of the Mississippi. Its products are distributed to these markets mainly through a series of specialized manufacturer's representatives. In a few areas, the company utilizes architectural firms and interior decorators as distributors.

The company has been so successful in western markets that management has decided to expand its market area to the east. The most recent expansion has been into a region east of the Mississippi from Illinois to western New York and as far south as Alabama and South Carolina. The company is currently serving this new region from its warehouse in Denver and a regional warehouse in St Paul. Sales in the eastern region have grown to such a level that management has decided to establish a system of distribution warehouses to serve this market. The company is now asking, how many additional warehouses are needed, and where should they be located?

## **History**

The Hornby product line contains the full gamut of home furnishings, from heavy pieces of handcrafted furniture to delicate pottery and statues. The company's management has always insisted that the workmanship in its products meet the highest standard. Because of this insistence, the company has attained an industry-wide reputation for outstanding quality. In addition to product quality, management has also concentrated on the quality of its customer service. As its reputation for quality and service has spread, the company has begun to experience very rapid growth. With a vigorous management team, it appears possible to sustain this rapid growth rate without much difficulty.

The company has several policies that have permitted this growth without major capital additions. Subcontracting of production has reduced the necessary investment in plant and equipment. Leasing rather than buying warehouses has practically eliminated investment in this area. The company has, however, deviated from its minimum investment policy in order to maintain a high level of customer service. A full line of products is stocked at every distribution warehouse and every effort is made to provide delivery within 24 hours. The company considers this policy critical to expanding the market for its products.

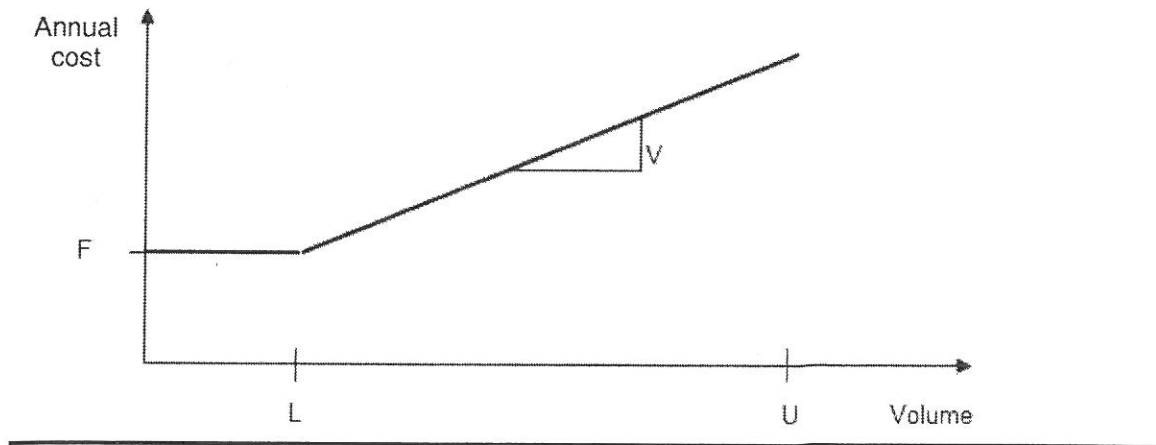
Hornby Products has contracted with 22 different manufacturer's representatives and architectural firms in the expansion east of the Mississippi. As the company's product line has become established in that area, the representatives have been placing larger orders, necessitating more and more shipments from Denver rather than from St Paul. Denver shipments have been required in order not to drastically deplete the stock at St Paul. This has made it difficult to adhere to the 24-hour delivery policy. The situation has worsened to the point where management has concluded that the sales growth will be stifled unless distribution warehouses

\*Adapted from Berry, W.L. and D. Clay Whybark, *Computer Augmented Cases in Operations and Logistics Management* (1972) South-Western Publishing. An edited version was graciously provided by Alan Neebe.

are set up to serve the eastern region. The final impetus to establish these warehouses was provided by the representative in Indianapolis, who complained that late delivery of samples had caused the loss of a sizable contract for the furnishings of a major hotel chain. Hornby management recognized that this would be an increasing problem if better delivery service couldn't be arranged. They went to work on establishing a new distribution system for the eastern region.

**EXHIBIT 7.1** *Potential Warehouse Locations for the New Region*



**EXHIBIT 7.2** *Cost Structure at a Typical Warehouse***Alternatives**

As a first step in establishing the system, company officers visited each of the cities in which they had manufacturing representatives. These 22 cities are shown on the map in Exhibit 7.1. (After initially drawing the map, the company officers decided that Denver should continue to serve Little Rock and Willow Springs.) Eight of these cities were judged to have sufficient transportation services and warehouse facilities for distribution warehouses. Preliminary work was done on estimating the costs and determining the availability of specific facilities. Since company policy was to lease distribution warehouses, little difficulty was encountered in finding at least some warehouse space in each of the eight possible cities.

The typical warehouse facility offers a leasing arrangement that follows the cost structure diagrammed in Exhibit 7.2. The cost structure consists of a fixed cost ( $F$ ) and a variable cost ( $V$ ). In addition, there is a lower limit ( $L$ ) and an upper limit ( $U$ ) associated with annual volume. The

**EXHIBIT 7.3** *Warehouse Cost Data*

W/H	Location	Fixed cost	Variable cost	Min. (L)	Max. (U)
1	Atlanta	2700	6	200	1750
2	Buffalo	2900	8	150	1250
3	Chicago	3500	9	250	2000
4	Cincinnati	2200	7	200	1500
5	Detroit	3300	8	200	1750
6	Pittsburgh	3000	8	200	1500
7	Richmond	2000	6	150	1000
8	St. Louis	1800	5	200	1500



upper limit is a volume ceiling dictated by material handling equipment: no more than this amount of product can be moved through the warehouse. The lower limit functions as a guarantee of sorts to the leasing company. In each city, lease arrangements require a certain minimum capacity to be leased if a warehouse is established. Specifically, Hornby can choose to operate at a volume lower than  $L$ , but the fixed cost  $F$  would be charged in any event. Only volumes in excess of  $L$  incur additional costs, and here the charge is equal to the variable cost  $V$  multiplied by the excess volume. Exhibit 7.3 presents the detailed cost and capacity estimates.

For the purposes of evaluating expansion alternatives, sales levels have been forecast two years into the future. Detailed sales forecasts for each representative have been converted to hundredweights (cwt) sold annually at each location. (The term *hundredweight* refers to 100 pounds of product.) At Hornby Products Company, the use of hundredweights as a unit of measure turns out to be a relatively accurate means of aggregating demand, and it is convenient in working out transportation costs. The sales forecasts are given in Exhibit 7.4.

**EXHIBIT 7.4** *Forecast for Annual Demands*

Rep. no.	Location	Volume
1	Atlanta	275
2	Birmingham	160
3	Buffalo	240
4	Charleston	260
5	Charlotte	135
6	Chattanooga	160
7	Chicago	400
8	Cincinnati	200
9	Cleveland	320
10	Columbus	220
11	Detroit	190
12	Evansville	100
13	Ft Wayne	140
14	Indianapolis	310
15	Knoxville	125
16	Louisville	340
17	Memphis	240
18	Nashville	210
19	Peoria	150
20	Pittsburgh	340
21	Richmond	300
22	St Louis	260

# **EXHIBIT 7.5** *Unit Costs for Distribution*

	Atlanta	Buffalo	Chicago	Cincinnati	Detroit	Pittsburgh	Richmond	St Louis
Atlanta	1	13	16	11	15	11	8	12
Birmingham	4	15	15	12	16	13	11	10
Buffalo	13	1	10	8	4	4	7	13
Charleston	7	6	11	6	8	6	4	10
Charlotte	4	10	15	10	13	8	4	14
Chattanooga	2	11	14	9	13	9	7	11
Chicago	16	10	1	9	6	9	14	7
Cincinnati	9	6	7	3	6	6	7	7
Cleveland	13	3	7	6	3	5	8	10
Columbus	10	4	8	4	5	4	7	9
Detroit	15	4	6	8	1	7	11	11
Evansville	9	10	7	6	9	8	10	4
Ft. Wayne	14	8	2	7	4	9	12	6
Indianapolis	12	9	4	6	7	8	11	4
KNOXville	4	9	12	7	11	9	6	9
Louisville	8	8	8	4	8	8	8	6
Memphis	8	14	12	10	14	12	12	6
Nashville	6	12	11	8	12	10	9	7
Peoria	16	11	2	10	8	11	15	5
Pittsburgh	11	2	9	6	5	3	5	11
Richmond	8	7	14	9	11	7	1	14
St Louis	12	13	7	9	11	11	14	1

The cost of transportation from each possible warehouse to each representative has also been determined. Exhibit 7.5 presents the warehouse-to-representative transportation costs.

Fortunately, there is no question of meeting the customer service requirements policy in the new region, because any of the warehouses being considered could deliver to any of the eastern representatives within 24 hours.