Problem 2 [8.5 points]

At the beginning of each semester, university bookstores around the country face high demand for course textbooks during the short period of time consisting of the first few days of the semester. Hundreds of textbook titles are sold. Consider one textbook title sold by a given bookstore. The bookstore needs to determine the selling price of a new copy and the selling price of a used copy.

We consider the joint pricing problem of the new and used formats of a given textbook. To simplify, we assume that demand is deterministic; that is, once we set prices we know exactly what demand would be for each format of the textbook. Ignore integrality constraints in this problem.

The Multinomial Logit (MNL) demand model is a renowned and widely used tool in both marketing and operations research, primarily for its effectiveness in analyzing consumer choice data. Currently, MNL demand models remain essential for companies, serving as a fundamental instrument in comprehending and forecasting product demand.

Let us assume that the demand for the new and used copies of the textbook under consideration follows this model. In particular, the demands for the new and used copies of the textbook are given by the following expressions:

$$d_{new} = 250 \times \frac{e^{(2 - 0.01 \times p_{new})}}{1 + e^{(2 - 0.01 \times p_{new})} + e^{(1 - 0.01 \times p_{used})}}$$

$$d_{used} = 250 \times \frac{e^{(1 - 0.01 \times p_{used})}}{1 + e^{(2 - 0.01 \times p_{new})} + e^{(1 - 0.01 \times p_{used})}}$$

where p_{new} is the price set for the new book and p_{used} is the price set for the used book. The model features the exponential function where e = 2.7183 (in particular, $e^0 = 1.0000$, $e^{0.5} = 1.6487$, $e^1 = 2.7183$, $e^2 = 7.3891$, etc.).

For example, if the price of the new book is set to $200 and the price of the used book is set to $50 then the corresponding demands are:

$$d_{new} = 250 \times \frac{e^{(2 - 0.01 \times 200)}}{1 + e^{(2 - 0.01 \times 200)} + e^{(1 - 0.01 \times 50)}} = 250 \times \frac{e^0}{1 + e^0 + e^{0.5}} = 69$$

$$d_{used} = 250 \times \frac{e^{(1 - 0.01 \times 50)}}{1 + e^{(2 - 0.01 \times 200)} + e^{(1 - 0.01 \times 50)}} = 250 \times \frac{e^{0.5}}{1 + e^0 + e^{0.5}} = 113$$

In this case, the "price ratio" of the used book price to the new book price is $50/$200 = 0.25, and the "average price" per book sold is (69 x 200 + 113 x 50) / (69 + 113) = $107.

The cost of procuring one copy of the new book is $75 and the cost of procuring one copy of the used book is $25. The contribution margin from this textbook title is ($200 - $75) x 69 + ($50 - $25) x 113 = $11,450.

a) **[7 points]** Formulate an optimization problem to maximize the bookstore's contribution margin from this textbook. The university requires that the "price ratio" (the price of the used book divided by the price of the new book) is at most 0.5. Solve the problem using SciPy. What are the optimal prices and what is the corresponding optimal contribution margin?

b) **[1.5 points]** What is the corresponding "average price" in part a)? The university is concerned that the prices of textbooks are a financial burden on students and has instructed the bookstore to limit the "average price" of this textbook to no more than $180. Do you think the "average price" constraint imposed by the university necessarily leads to lower prices for the new <u>and</u> used books? Explain.