Unit 2: Consumer Theory

Prof. Antonio Rangel

January 2, 2016

1 Consumer demand

1.1 The experienced utility function

- Suppose that the consumer only cares about two goods:
  - x, good of interest, measured in units
  - m, a measure of all other consumption in \$. Think of it as amount of \$ spent on all other goods except for x.
  - Assume $x \geq 0$, but can have $m \gtreqless 0$.
- Experienced utility function: $U(x,m)$ = level of experienced utility (= satisfaction or happiness) from consumption bundle x, m.
- Assume U is quasi-linear:
  - $U(x,m) = B(x) + m$, $B' > 0, B'' < 0$.
- Remarks:
  - $B(x)$ = willingness to pay for x units of good x
  - $B(0)$ need not be zero
  - $\frac{\partial U}{\partial m} = 1$, i.e. constant marginal utility; $\frac{\partial U}{\partial x} = B'(x)$, i.e. decreasing marginal utility since $B'' < 0$.
  - $U(x,m)$ measured in dollars

1.2 Utility maximization problem

- Assume consumer is rational; i.e. chooses action to maximize experienced utility

- Consumer's problem, given wealth W and a price p per unit of good x:

$$
\begin{aligned}
& \max_{x \geq 0, m} U(x, W - xp) \\
\sim & \max_{x \geq 0} B(x) + W - xp \qquad \text{by QL assumption} \\
\sim & \max_{x \geq 0} B(x) - xp \qquad \text{by dropping constant } W
\end{aligned}
$$

- Familiar structure: benefit minus cost

- Notation: $x^*(p)$ = optimal choice = demand for x at p.

- It follows that $m^*(p) = W - px^*(p)$

- Solution:

  - Concavity conditions satisfied
  - Case 1: Interior ($x^*(p) > 0$): when $B'(0) > p$ and B' crosses p .
  - Case 2: Corner solution at $x^*(p) = 0$: when $B'(0) < p$.
  - Case 3: $x^* = \infty$: when $B'(x) > p$ for all x.
    Not very interesting, since crossing conditions usually satisfied via $B'(\infty) = 0$
  - Economic intuition: starting at zero, keep buying as long as MB > MC

1.3 The demand function

Demand curve for individual consumer

- Demand function: $x^*(p)$, defined for all $p > 0$

- In economics, we always graph price on the vertical axis, and quantity on the horizontal axis

- Key result: Demand function $\sim$ MB curve

- With x as independent variable, graph B'
- Taking p as independent variable, same locus of points gives the demand function.

1.4 Example

- $U(x, m) = A \ln x + m$
- Observe that all solutions must be interior, since $B'(0) = \infty > p$.
- Solving $B'(x) = p$ yields $x^*(p) = \frac{A}{p}$
- A is a taste parameter. As A increases the demand curve shifts right (i.e., the amound demanded increases at any p).

1.5 Example

- $U(x, m) = A \ln x + m$
- What is the demand function if there is a mandatory minimum purchase of at least 1 unit of x ?
- For $p \leq A$, demand curve the same as in previous example, $x^*_{bef}(p) = x^*_{after}(p)$
- For $p > A$, demand curve now is $x^*_{after}(p) = 1 > x^*_{bef}(p)$. In this case, since the consumers are forced to buy more than they want, they are made worse off by the policy.

1.6 Properties of the demand function

- RESULT: Law of Demand: $\frac{dx^*}{dp} \leq 0$, w/ inequality if $x^* > 0$.
  Outline of proof:
  - For prices in the range of corner solutions: $x^*(p) = 0 \implies \frac{dx^*}{dp} = 0$
  - $x^*(p) > 0 \implies$ interior solution $\implies B'(x^*(p)) = p$
    Taking derivatives of both sides w.r.t. p yields $B'' \frac{dx^*}{dp} = 1$
    $\implies \frac{dx^*}{dp} = \frac{1}{B''}$
    Therefore $B'' < 0 \implies \frac{dx^*}{dp} < 0$

- RESULT: No income effects: $\frac{dx^*}{dW} = 0$
  - Why? With quasilinear preferences, the FOC that characterizes x^* does not depend on how much of the good m is consumed
- Both results approximately hold for goods which account for a small fraction of consumer's overall spending
- Both can fail when U is non-quasilinear

2 Consumer surplus

2.1 Consumer Surplus: Basics

- How do we measure well-being of consumer?
- Consumer surplus = NET benefit of buying optimally at price p, measured in dollars
- Mathematically, it is defined as follows:

$$\begin{aligned} CS(p) &= U(x^*(p), W - px^*(p)) - U(0, W) \\ &= B(x^*(p)) - B(0) - px^*(p) && \text{by quasilinearity} \\ &= \int_0^{x^*(p)} B'(x)dx - \int_0^{x^*(p)} p \;\; dx && \text{the Fundamental Theorem of Calculus} \\ &= \int_0^{x^*(p)} (B'(x) - p)\, dx && \text{by linearity of the integral} \end{aligned}$$

- Definition in the first line says $CS(p)$ equals experienced utility of buying $x^*(p)$ minus experienced utility of buying zero.
- Graphical description: CS = area between B' and p, from $x = 0$ to $x = x^*$
- Properties of CS:
  - $CS \geq 0$
  - $CS > 0$ iff $x^*(p) > 0$

- Can see both from the graphical description
- Can also see it from a *revealed preference* argument: $(0, W)$ is always feasible, i.e. the consumer can always buy nothing. Therefore CS can't be negative: if the consumer buys anything, it must make her better off.

2.2 Example

- Suppose $U(x, m) = 2A\sqrt{x} + m$
  - $B' = \frac{A}{\sqrt{x}}$
  - $x^*(p) = \frac{A^2}{p^2}$
- Consider two different ways of computing $CS(p)$.
- Direct method:

$$\begin{aligned} CS(p) &= U(x^*(p)) - U_{no\_trade} \\ &= \left[2A\sqrt{x^*(p)} + W - px^*(p)\right] - [0 + W] \\ &= \frac{2A^2}{p} - \frac{A^2}{p^2}p \\ &= \frac{A^2}{p} \end{aligned}$$

- Graphical/integral method:

$$\begin{aligned} CS(p) &= \int_0^{x^*(p)} (B'(x) - p)\, dx \\ &= \int_0^{\frac{A^2}{p^2}} \left(\frac{A}{\sqrt{x}} - p\right) dx \\ &= \left[2A\sqrt{x} - px\right]_0^{\frac{A^2}{p^2}} \\ &= \frac{A^2}{p} \end{aligned}$$

2.3 Effect of price changes on consumer surplus

- Consider effect on CS of price decrease from p_0 to p_1
- It is given by:

$$\begin{aligned}\Delta CS_{p_0 \to p_1} &= CS(p_1) - CS(p_0)\\ &= \text{ Change in experienced utility following price change from } p_0 \text{to } p_1\\ &= x_0^* \Delta p + \int_{x_0^*}^{x_1^*} (B' - p_1)\, dx\end{aligned}$$

where $\triangle p = p_0 - p_1$

- Fist term = value of buying the old x_0^* units at lower price p_1
- Second term = value of buying additional units at price p_1

2.4 More on consumer surplus

- Let's extend the notion of consumer surplus to more general situations
- Let θ = complete description of consumer's problem
- $x^*(\theta)$ = optimal choice at situation θ
- Now, $CS(\theta) = U(x^*(\theta)) - U_{no\_trade}$
- Example:
  - $U(x, m) = 2A\sqrt{x} + m$
  - θ : price = p, mandatory minimum purchase = 1 unit
  - Now, we have

$$x^*(\theta) = \begin{cases} 1 & \text{if } p \geq A \\ \frac{A^2}{p^2} & \text{if } p < A \end{cases}$$

and

$$CS(\theta) = \begin{cases} 2A - p & \text{if } p \geq A \\ \frac{A^2}{p} & \text{if } p < A \end{cases}$$

- Important lesson from example: There are situations in which CS can be negative. They often involve situations in which consumers are forced to make purchases that they would not make volutarily.

3 Recovering preferences from data

3.1 What if we don't know the utility function?

- In reality, we never know consumers' experienced utility functions. However, we can infer changes in consumer surplus from behavior alone!

3.2 How to compute CS using only behavioral data

- Given observations of several price/quantity pairs, can estimate the function $x^*(p)$ using statistics
- Then, construct the inverse demand function $p^*(x) = x^{*-1}(p)$
- But we know that at an interior allocation, $p^*(x) = B'(x)$, as long as the consumer makes decisions by maximizing her experienced utility function
- It follows that
$$CS(p) = \int_0^{x^*(p)} (p^*(x) - p)\, dx$$
- This gives CS as a function of $x^*(p), p^*(\cdot)$, and p, all of which are observable
- Can use this to estimate the change in consumer surplus that would follow from an unobserved change in prices, the introduction of a new tax, etc.

3.3 Recovering the utility function from observed behavior

- Suppose $x^*(\cdot)$ observed.
- We recover U using the following steps:
  - Assume U is quasi-linear, i.e. $U(x, m) = B(x) + m$
  - Then $p^*(x) = B'(x)$
  - So $B(x) = B(0) + \int_0^x p^*(u)du$

- $U(x,m) = \int_0^x p^*(u)du + m + constant$

- So we can recover $U(x.m)$ up to a constant, which is equal to $B(0)$.
- Constant typically unimportant in most applications.
- What if U is not quasilinear?
  - OK if $U \approx$ quasi-linear, even if not exactly so
  - Method generalizes (taught in advanced courses, requires substantially more math)

3.4 Example

- Consider an example of how to recover $U(x,m)$ from $x^*(p)$
- Suppose we observe that $x^*(p) = \frac{10}{p} - 1$
- Then $p^* = \frac{10}{x+1}$
- So $B(x) = 10\ln(x+1)$, and $U(x,m) = 10\ln(x+1) + m + constant$

3.5 Example

- θ_0 = initial situation at which consumer buys freely at price p
- At initial situation observe $x^*(p) = \frac{9}{p} - 1$.
- θ_1 = new situation at which individuals buy freely, but also get 2 free units (regardless of how much they buy)
- Question: Predict whatt is $CS(\theta_1)$?
- Solve in three steps.
- Step 1: Recover $U(x,m)$ from the initial observed demand.
  - Get $U(x,m) = 9ln(x+1) + m$ (similar to previous example)
- Step 2: Predict $x_1^*(p)$ by maximizing the recovered prefences

  - Important: $x_1^*(p)$ denotes the amount bought, not the amount consumed which is equal to $x_1^*(p) + 2$.
  - Get

$$x_1^*(p) = \begin{cases} 0 & \text{if } p \geq 3 \\ \frac{9}{p} - 3 & \text{if } p < 3 \end{cases}$$

- Step 3: Predict $CS(\theta_1)$
  - Get

$$CS(\theta_1) = \begin{cases} 9ln3 & \text{if } p \geq 3 \\ 9ln\frac{9}{p} + 9 - 3p & \text{if } p < 3 \end{cases}$$

3.6 Application: Valuing new products

- Hypothetical example: compute the value of introducing household robots for a typical US consumer
- Data:

	p	x
2020	9000	1
2025	1000	9

- Graph the data, and fit a line through them (trivial in this example, since only two points)
- Get $x^*(p) = max\{0, 10 - \frac{p}{1000}\}$
- Value of introducing robots $= CS(p) - CS(p = \infty) = CS(p)$.
- Using graphical method:
  - $CS_{2020} = \frac{1}{2} \cdot 1 \cdot 1000 = \$500$
  - $CS_{2025} = \frac{1}{2} \cdot 9 \cdot 9000 = \$40,500$
- Lesson: Value of a new product depends strongly on the price at which it's sold.

4 Consumer mistakes

4.1 Decision mistakes

- A simple model of decision mistakes:

  1. Decision Utility vs. Experienced Utility
     - Experienced utility: describes well-being/hedonics
     - Decision utility: describes objective that is maximized at decision time
  2. $x^*(\theta) = \max_x U^{DU}(x)$ s.t. feasibility constraints in θ
     $x^{opt}(\theta) = \max_x U^{EU}(x)$ s.t. feasibility constraints in θ
  3. Rational behavior: $U^{DU} = U^{EU} \implies x^* = x^{opt}$
  4. Mistakes: $U^{DU} \neq U^{EU} \implies x^* \neq x^{opt}$

- Example:

  – $U^{EU} = 2A\sqrt{x} + m$
  – $U^{DU} = 4A\sqrt{x} + m$
  – So $x^{opt}(p) = \frac{A^2}{p^2}, x^*(p) = \frac{4A^2}{p^2}$. Note the gap between the consumer choice and the optimal choice.

- Remarks:

  1. $B^{DU}(x) \neq B^{EU}(x) + constant \implies$ mistakes arise
  2. If mistakes, then:
     – Utility function recovered from behavior $\neq U^{EU}$
     – CS calculted with the recovered utility function is incorrect
  3. Thus, critical to know if mistakes likely in a given context

- Example (continued):

  – The CS estimated under the assumption that the consumer is rational is $CS^{est}(p) = \frac{4A^2}{p}$.
  – But the true CS, given the consumer's true EU function, is $CS^{true}(p) = 0$.

4.2 Application: Addiction

- Here is a simple model to think about the consumption of addictive substances:
  - $U^{EU}(x,m) = B(x) + m$
  - $U^{DU}(x,m) = \theta B(x) + m$, $\theta \gg 1$
  - Can have $CS^{true}(p) < 0$, in which case prohibition can be welfare improving!

5 Final remarks

- Key ideas:
  1. Modeling behavior: $x^*(\theta)$ given by $max_{x\geq 0} U^{DU}(x)$ s.t. constraints in θ
  2. Measuring well-being: Consumer surplus, $CS(\theta) = U^{EU}(x^*(\theta)) - U^{EU}_{no\_trade}$
  3. Given rationality, can recover $B(x)$ from $x^*(p)$ and measure CS from $p^*(x)$

- Tips on problem solving:
  - Must specificy maximization problem correctly
  - Consumer behavior ($x^*(\theta)$) follows from maximizing U^{DU}given the constraints in situation θ
  - Optimal level of consumption ($x^{opt}(\theta)$) follows from maximizing U^{EU} given the constraints in situation θ
  - Don't mix up demand $x^*(p)$ and inverse demand $p^*(x)$
  - Careful when computing CS using the notion of 'area under the curve'