Unit 6: Government policy in competitive markets II – Distribution & incidence

Prof. Antonio Rangel

January 2, 2016

1 Endogenous income and inequality

1.1 Simple model

- There are two markets and two goods:

    - Labor market: consumers sell labor l at wage w to firms
    - Goods market: firms sell good q at price p to consumers

- Consumer side of the model:

    - C consumers
    - no exogenous wealth: $W_i = 0$ for all i
    - $U_i(q, l, m) = q - \frac{l^2}{2\theta_i} + m$
    - $l =$ units of labor provided
    - $\frac{1}{\theta_i} =$ measure of i's disutility of providing labor (i.e., a cost of effort).
    - People with higher θ_i have lower disutility of labor.
    - Consumer's problem:

    $$\max_{q,l \geq 0} q - \frac{l^2}{2\theta_i} + lw - pq$$

    - FOCs for good q: $MB = MC$ with $MB = 1$ and $MC = p$.

– It follows that demand for good q is given by

$$x_i^D(p) = \begin{cases} 0 & \text{if } p > 1 \\ anything & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}$$

– Thus, aggregate demand for the market is given by

$$X_{mkt}^D(p) = \begin{cases} 0 & \text{if } p > 1 \\ anything & \text{if } p = 1 \\ \infty & \text{if } p < 1 \end{cases}$$

– FOCs for labor supplied by consumer i: $MB = MC$ with $MB = w$ and $MC = \frac{l}{\theta_i}$.

– It follows that $l_i^S(w) = \theta_i w$.

– Thus, aggregate labor supply is given by $L_{mkt}^S(w) = w \sum_i \theta_i = wC\bar{\theta}$, where $\bar{\theta}$ denotes average θ.

– Labor income of consumer i: $I_i(w) = \theta_i w^2$

• Firm side of the model:

– F identical firms

– Production function: $F(l) = \gamma l$, $\gamma > 0$

– Firm's problem:
$$\max_{l \geq 0} p\gamma l - wl$$

– FOCs for firm's problem: $MB = MC$ with $MB = p\gamma$ and $MC = w$.

– Thus, we get

$$l_j^D(w, p) = \begin{cases} 0 & \text{if } w > p\gamma \\ anything & \text{if } w = p\gamma \\ \infty & \text{if } w < p\gamma \end{cases}$$

- Aggegate labor demand is then given by:

$$
L_{mkt}^{D}(w, p) = \begin{cases} 0 & \text{if } w > p\gamma \\ anything & \text{if } w = p\gamma \\ \infty & \text{if } w < p\gamma \end{cases}
$$

- Given this, firms' supply at the individual and market level are given by $q_j^{S}(w, p) = \gamma l_j^{D}(w, p)$ and $X_{mkt}^{S}(w, p) = \gamma L_{mkt}^{D}(w, p)$

• Competitive market equilibrium:

- cME given by p^{*}, w^{*}, α^{*} such that :
  1. Consumers optimize over q, l given p^{*}, w^{*}
  2. Firms optimize over q, l given p^{*}, w^{*}
  3. Both markets clear

- The cME in this simple model satisfies the following properties:
$p^{*} = 1$
$w^{*} = \gamma$
$q^{*} = C\bar{\theta}\gamma^{2}$
$l^{*} = C\bar{\theta}\gamma$

- Note that prices and wages are uniquely determined by market parameters, independent of quantity

- Note that there are many possible cMEs, one for each possible quantity q

• Equilibrium level of inequality

- In equilibrium, income of person i is given by $I_i = \theta_i w^2 = \theta_i \gamma^2$

- This implies that the inequaility of income is given by $Var(I_i) = \gamma^4 Var(\theta_i)$

- In equilibrium, the utility of person i is given by:

$$
U_i = -\frac{l^2}{2\theta_i} + lw + q - pq
$$

$$
= -\frac{\theta_i^2 w^2}{2\theta_i} + \theta_i w^2
$$

$$
= \frac{\theta_i \gamma^2}{2}
$$

3

– It follows that the inequality of utility is given by $Var(U_i) = \frac{\gamma^4}{4}Var(\theta_i)$

- Summary: a simple model with individual differences in the disutility of labor generates inequality of income and utility in equilibrium

1.2 Labor income taxes and inequality

- Impact of taxes on market equilibrium

  – Labor income tax $\tau > 0$: for every dollar each consumer earns, she must pay τ to government

  – Revenue returned to consumers using IDENTICAL lump sum transfer $T = \frac{\text{revenue raised}}{C}$

  – Consumer's problem now given by:

  $$\max_{q,l \geq 0} q - \frac{l^2}{2\theta_i} + l(1-\tau)w - pq + T$$

  – Consumer assumes T fixed when maximizing, due to large number of individuals in market

  – No change in q-market $\implies p^* = 1$

  – No change in firms' problem $\implies w^* = \gamma$

  – Labor supply now given by $MB = MC$, with $MB = (1-\tau)w$ and $MC = \frac{l}{\theta_i}$, which implies that

  $$l_i^S(w) = (1-\tau)w\theta_i$$

  and
  $$L_{mkt}^S(w) = (1-\tau)wC\bar{\theta}$$

- Impact of taxes on total tax revenue:

  – $TotalRev(\tau) = \tau w L_{mkt}^S(w^*) = \tau(1-\tau)\gamma^2 C\bar{\theta}$

  – The relationship between total Revenue and the tax rate is often called a Laffer curve

4

- Key property of the Laffer curve: tax revenue increases with tax rate when τ is small, but decreases with the tax rate when τ is sufficiently large

- Impact of taxes on redistribution:

  - Net-Tax = transfers received - taxes paid
  - For individual i, we have that
  $$NetTax(\theta_i) = \tau(1-\tau)\gamma^2\bar{\theta} - \tau(1-\tau)\gamma^2\theta_i$$
  $$= \tau(1-\tau)\gamma^2\left[\bar{\theta} - \theta_i\right]$$
  - $NetTax(\theta_i) > 0$ if and only if $\theta_i < \bar{\theta}$
  - This implies that individuals with an above averege disutility of labor receive a net transfer from the government, and those with below average disutility of labor pay a net tax.

- Impact of taxes on income inequality :

  - $I_i = (1-\tau)^2\gamma^2\theta_i + \tau(1-\tau)\gamma^2\bar{\theta}$
  - Tax reduces income inequality: intercept increasing, slope decreasing in τ
  - $Var(I_i) = (1-\tau)^4\gamma^4 Var(\theta_i)$

- Impact of taxes on utility inequality:

  - Post-tax utility of person i is given by
  $$U_i(\tau) = -\frac{(1-\tau)^2\gamma^2\theta_i^2}{2\theta_i} + (1-\tau)^2\gamma^2\theta_i + \tau(1-\tau)\gamma^2\bar{\theta}$$
  $$= \frac{(1-\tau)^2\gamma^2\theta_i}{2} + \tau(1-\tau)\gamma^2\bar{\theta}$$
  $$= \frac{1}{2}\gamma^2(1-\tau)\left[\theta_i + \tau(2\bar{\theta} - \theta_i)\right]$$
  - $Var(U_i) = \frac{(1-\tau)^4\gamma^4}{4}Var(\theta_i)$

- Lesssons:

  - An income tax can reduce, but not eliminate, inequality
  - There is an efficiency cost of income taxes, since people work less as the tax rate increases

1.3 Optimal labor income tax

- Now let's compute the optimal labor income tax in our model

- Suppose consumers have preferences that depend on the distribution of income, as follows:

$$V_i(q, l, m, \{I_1, \ldots, I_C\}) = \underbrace{\left[q - \frac{l^2}{2\theta_i} + m \right]}_{U_i} - \underbrace{\sigma \sqrt{Var(I)}}_{\text{Inequality pref}}$$

- Optimal tax problem for the government:

$$\max_{\tau \geq 0} \sum_i V_i(\tau)$$

- Notation:

  - $U_i(\tau), V_i(\tau)$: utility as a function of tax rate, as derived above
  - $n(\theta)$: number of consumers of type θ
  - $I_\theta(\tau)$: total income of consumer of type θ as a function of tax rate

- Simplifying objective function we get

$$\sum_i V_i(\tau) = \sum_i U_i(\theta) - C\sigma \sqrt{Var(I_\theta(\tau))}$$

$$= \sum_\theta n(\theta) \left[\frac{w^2(1-\tau)^2\theta}{2} + \tau(1-\tau)w^2\bar{\theta} \right] - C\sigma \sqrt{\frac{\sum n(\theta) \left(I_\theta(\tau) - \overline{I_\theta(\tau)} \right)^2}{C}}$$

- Observe that
$$I_\theta(\tau) - \overline{I_\theta(\tau)} = (1-\tau)^2 w^2 (\theta_i - \bar{\theta})$$

- This implies that

$$\sum n(\theta) \left(I_\theta(\tau) - \overline{I_\theta(\tau)} \right)^2 = (1-\tau)^4 w^4 \sum n(\theta)(\theta_i - \bar{\theta})^2$$
$$= (1-\tau)^4 w^4 Var(\theta)C$$

- So the objective function $\sum_i V_i(\tau)$ can be written as

$$= \frac{w^2(1-\tau)^2}{2} \underbrace{\sum \theta n(\theta)}_{=C\bar{\theta}} + \tau(1-\tau)w^2\bar{\theta} \underbrace{\sum n(\theta)}_{=C} - C\sigma(1-\tau)^2 w^2 SD(\theta)$$

$$\propto w^2 C\bar{\theta} \left[(1-\tau)^2 + 2\tau(1-\tau) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}} \right]$$

$$\propto (1-\tau^2) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}}$$

- As a result, the optimal tax problem can be written as

$$\max_{\tau \geq 0}(1-\tau^2) - 2(1-\tau)^2\sigma\frac{SD(\theta)}{\bar{\theta}}$$

- From the Laffer curve material, we know that this problem has a unique maximum. So the following FOC is necessary and sufficient:

$$-2\tau + 4(1-\tau)\sigma\frac{SD(\theta_i)}{\bar{\theta}} = 0$$

- This implies a precise formula for the optimal tax

$$\tau^{opt} = \frac{2\sigma\frac{SD(\theta_i)}{\bar{\theta}}}{1 + 2\sigma\frac{SD(\theta_i)}{\bar{\theta}}}.$$

- Intuition check:

    - $\tau^{opt} = 0$ when individuals don't care about inequality since $\sigma = 0$
    - $\tau^{opt} = 0$ when there is no inequality since $SD(\theta) = 0$
    - The optimal tax goes to 1 as the distate for inequality increases (i.e., as $\sigma \to \infty$)

- Remark 1: Optimal tax problem induces a fundamental tradeoff: reduce inequality vs. avoid inefficiency

- Remark 2: Is the result robust to alternative model specifications? Basic logic of the problem is robust, although the precise details of the formula depends on the details of the model.

- Remark 3: Solution depends on consumers' objective function. We used an objective function in which consumers care about overall inequality, not about others' utility.

- Remark 4: Key empirical parameters affecting the size of the optimal tax:

    - measure of inequality $\frac{SD(\theta)}{\theta}$
    - strength of social preferences σ
    - general equilibrium effects of taxation

1.4 Second welfare theorem

- Second Welfare Theorem (SWT):

    - Let α be a Pareto optimal allocation
    - Then there is a set of lump-sum $m-$good transfers, with $\sum T_i = 0$, such that α is a cME given transfers $\{T_i\}$

- Intuition: The market for the q-good is not affected by lump-sum transfers.

- Naive interpretation of the SWT:

    - It implies no need to use distortionary taxes to redistribute.
    - It implies usage of lump-sum transfers is sufficient to reach desired P.O. allocation, since they generate no DWL!

- Problem – Lump-sum tax policy in SWT involves unreasonable informational demands:

    - must choose T_i for each consumer
    - therefore must know fundamental parameters (preferences, effort costs) of each individual
    - very unrealistic!

8

2 Price controls

2.1 Simple price controls

- Taxonomy of price control policies

    - Price ceiling: $p \leq p^{max}$
    - Price floor: $p \geq p^{min}$
    - Simple: just price restriction
    - Complex: price restriction plus action necessary to clear market

- When does a simple price ceiling affect equilibrium outcomes?

    - Policy is not binding if $p^{max} \geq p^*$. In this case the market generates the same outcome
    - Policy binds if $p^{max} < p^*$. In this case p^{max} becomes the equilibrium price. But at that price there is excess demand, so a rationing rule is needed (specifying who gets the units that are produced)
    - Efficient rationing rule: units are allocated to highest-value consumers

- Effect of binding price ceiling on social surplus under an efficient rationing rule:

	free mkt	p^{max}	change
CS	A + B + E	A + B + C	C - E
PS	C + D + F	D	-(C+F)
SS	A + … + F	A + B + C + D	-(E+F)

- Note: E+F represents the DWL introduced by the price-ceiling policy

- Remarks:

    - When policy binds, it creates inefficiency
    - Policy can have redistributive effects. For example, in some cases there is a transfer of surplus from firm owners to consumers who are not firm owners.

9

2.2 Complex price controls

- Complex price floor

  - $p \geq p_{min}$
  - Government buys excess supply at equilibrium price p^*
  - Revenue for government purchases financed using an equal lump-sum tax on all consumers
  - Units bought by government are destroyed

- Effects of binding complex price floor on equilibrium outcomes

  - Important quantities: $p^* = p_{min}$, $x^*_{consumed}$, $x^*_{produced}$
  - $x^*_{consumed} < x^*_{produced}$
  - Government buys excess production and destroys it

- Effect of binding complex price floor on social surplus:

	free mkt	p^{min}	change
CS	A+B+E	A-(E+F+G+H+I)	-(B+2E+F+G+H+I)
PS	C+D+F	B+C+D+E+F+I	B+E+I
SS	A+B+C+D+E+F	A+B+C+D-(H+G)	-(E+F+H+G)

- NOTE: E+F+H+G represents the DWL of the policy

- Remarks:

  - DWL bigger than in simple price control
  - Policy also entails a transfer of surplus fron consumers to owners of the firms
  - This policy is particularly inefficient!

3 Economic vs. legal incidence

- Does it matter who pays the tax?

  - Suppose that government needs to raise a tax $\tau > 0$ per unit of good q sold:
  - Tax can be assigned to consumers, producers, or both
  - Class of policies:

    * $0 \leq a \leq 1$: a is fraction of tax paid by consumers
    * tax on consumers per-unit purchased: $\tau^C = a\tau$
    * tax on producers per-unit sold: $\tau^F = (1 - a)\tau$

- Incidence:

  - Legal: who sends a check to the government
  - Economic: who bears the cost of the tax

- RESULT: Equilibrium allocation is independent of a

  - $p = $ mkt price
  - $p + a\tau$: net price paid by consumers
  - $p - (1 - a)\tau$: net price received by firms
  - Consumers treat tax as price increase: $X^D(p|a) = X^D_{no-tax}(p + a\tau)$
  - Likewise for producers: $X^S(p|a) = X^S_{no-tax}(p - (1 - a)\tau)$
  - Market equilibrium $p^*(a)$ solves

    $$X^D_{no-tax}(p^*(a) + a\tau) = X^S_{no-tax}(p^*(a) - (1 - a)\tau)$$

  - Let $p^*_\tau = $ equilibrium price when $a = 0$ (all tax paid by firms)
  - Easy to check that $p^*(a) = p^*_\tau - a\tau$ clears the market for all a
  - But then the net price paid by consumers and received by firms is independent of a
  - This immplies that the equilibrium allocation is also independent of a!

- See graphical intuition provided in video lecture

4 Final remarks

- Key ideas from this unit:

  1. Optimal tax problem involves a tradeoff between redistribution and efficiency
  2. Price controls lead to sizeable deadweight losses, but can improve consumer or producer surplus, through the redistribution of social surplus
  3. Legal incidence $\neq$ economic incidence