Linear programming sensitivity analysis and the sensitivity report.

LP models are imperfect

Learn: change your problem. Solve it again.

- **Sensitivity analysis**
- **Parametric analysis**
- **Scenario analysis**

Sensitivity Analysis

Sensitivity analysis:

- Measure how sensitive the optimum solution is to small changes in the data.
- Methods
  - Perturb the data, reoptimize, and analyze
  - LP and NLP Sensitivity Reports (none for IPs)

3

Parametric Analysis

- **Parametric analysis**: Change one parameter (perhaps from 0 to ∞) and see what happens to the optimum solution and the optimum objective value.
  - e.g. , if the price of oil is part of the LP, see what happens as the price of oil changes from \$25 per barrel to \$200 per barrel.
  - for small problems, any coefficient can be a parameter.
  - For large problems, parameters chosen more carefully

4

Scenario analysis

- **Run different scenarios in which lots of data change (possibly a few scenarios, possibly many.)**
  - **e.g., great economic environment, average environment, poor environment.**
- **This can reveal if a solution is robust,**
  i.e., works well in lots of different situations,

Overview

- **2-Variable example**
  - **Provides intuition on the SR report**
  - **shadow prices and reduced costs**
- **Example from Applied Math Programming**
  - **http://web.mit.edu/15.053/www**
  - **Illustrates how the sensitivity report (SR) is used**

Graphical sensitivity analysis for a 2-variable example

A 2-variable LP example

MIT Corp is making widgets. The widgets come in two types: red widgets, and gray widgets. Red widgets sell for $4. Gray widgets for $3. They both cost $1 to produce. MIT Corp needs to produce at least 1,000 of each type. It can produce at most 5,000 red widgets and at most 6,000 gray widgets, and at most 8,000 widgets in total.

- **R = number of red widgets (in 1000s)**
- **G = number of gray widgets (in 1000s)**
- **z = profit (in $1000s)**

**Suggestion: pause the video,
and write the LP yourself.
Then restart the video.**

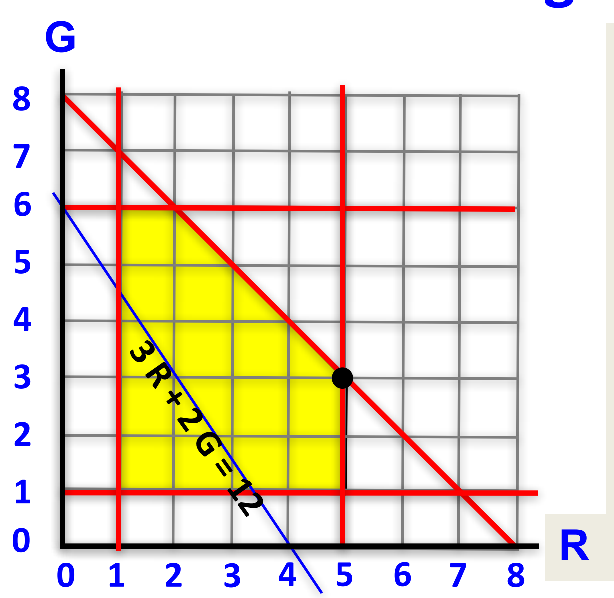
LP Formulation

$$
\begin{array}{llllll}
\textbf{maximize} & 3R + 2G & & & & \\
\textbf{subject to} & R + G & \le & 8 & & (1) \\
 & R & \ge & 1 & & (2) \\
 & G & \ge & 1 & & (3) \\
 & R & \le & 5 & & (4) \\
 & G & \le & 6 & & (5) \\
 & R \ge 0, \quad G \ge 0 & & & &
\end{array}
$$

Suggestion: pause the video, and draw the feasible region yourself. Then restart the video.

11

LP Feasible region



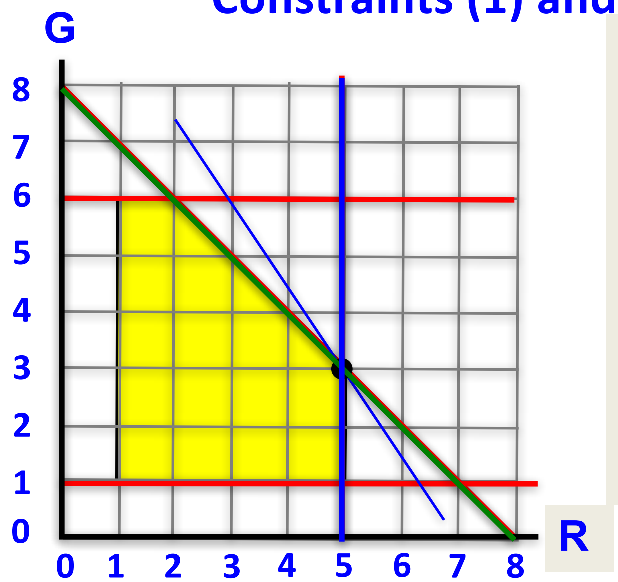
$$
\begin{array}{lrcrcrl}
\text{max} & 3R & + & 2G & & & \\
\text{s.t} & R & + & G & \le & 8 & (1) \\
 & R & & & \ge & 1 & (2) \\
 & & & G & \ge & 1 & (3) \\
 & R & & & \le & 5 & (4) \\
 & & & G & \le & 6 & (5) \\
 & \multicolumn{6}{l}{R \ge 0, \quad G \ge 0}
\end{array}
$$

Optimum: R = 5, G = 3, z* = 21.

12

The structure of the optimal solution:

Constraints (1) and (4) hold with equality.

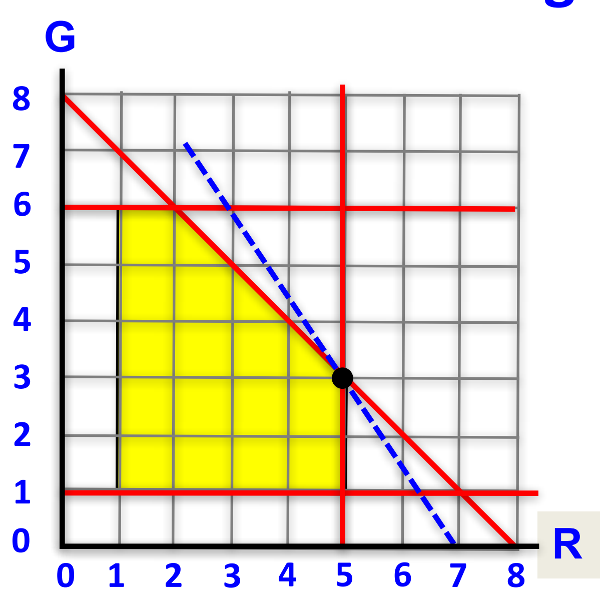


max	3 R	+	2 G			
s.t	R	+	G	≤	8	(1)
	R			≥	1	(2)
			G	≥	1	(3)
	R			≤	5	(4)
			G	≤	6	(5)
	$R \geq 0$,		$G \geq 0$			

Optimum: R = 5, G = 3, z* = 21.

13

LP Feasible region



If the cost coefficients change a little, the optimal extreme point will stay optimal.

If RHS coefficients change a little, the optimum extreme point will change, but the optimum structure will stay the same. Constraints (1) and (4) will both be satisfied with equality.

14

Example 1. Changing a cost coefficient.

Suppose that objective function changes to
$z = (3+p) R + 2 G$.

- The solution (5, 3) remains optimal if p is small
- The solution (5, 3) is optimal if $-1 \le p \le \infty$.

Changing a RHS coefficient

Suppose that RHS of constraint (5) changes
from 5 to $5 + q$ for a small values of q.

- Then the optimum objective value becomes $21 + q$ if q is very small.
- The optimal objective is $21 + q$ if $-3 \le q \le 2$.

LP sensitivity analysis overview

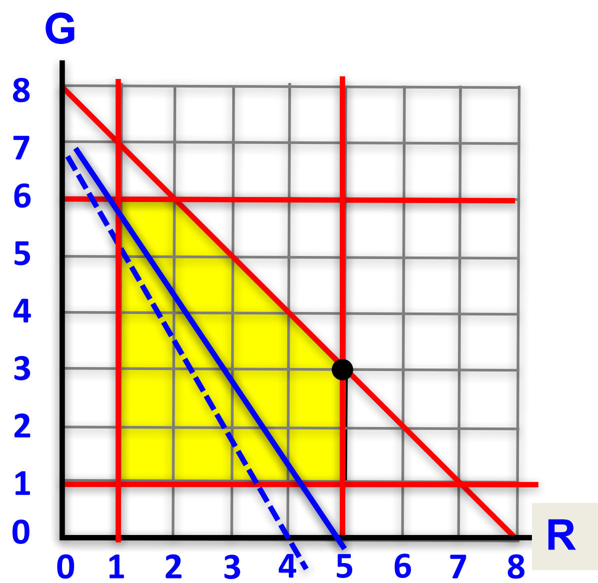
- Geometric intuition of elements of LP sensitivity analysis
  - shadow prices
  - allowable increases and decreases
  - reduced costs
- Using sensitivity analysis data in "pricing out"
  - A method for answering many questions on sensitivity analysis

17

Changing the cost coefficients

18

Changing cost coefficients: the slope changes



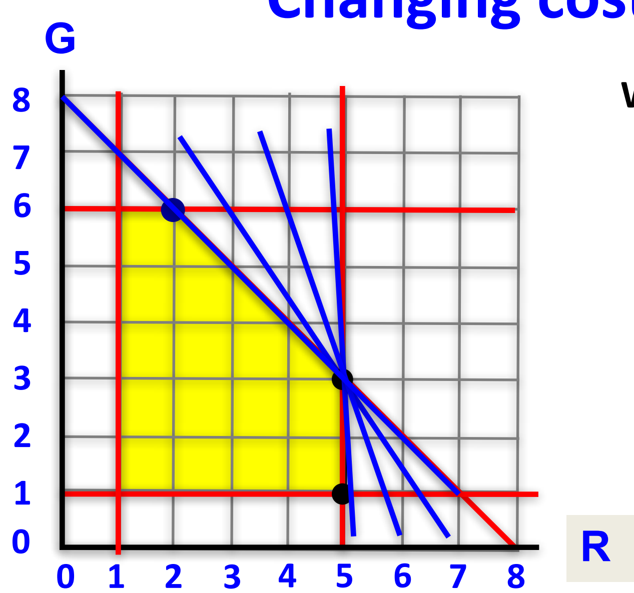
What happens if objective is
(3 + p) R + 2 G?

If p is small,

- The slope of the isoprofit lines changes a little.
- The same extreme point remains optimal.

19

Changing cost coefficients



What happens if objective is
(3 + p) R + 2 G?

How much can "p" increase or decrease and still have (5, 3) optimal?

"p" can increase to ∞

"p" can decrease to -1.

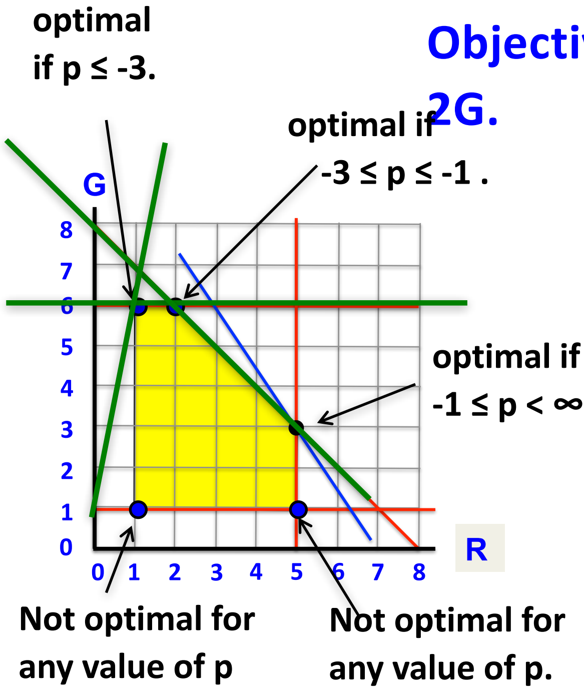
(2, 6) optimizes 1.99 R + 2 G

20

The sensitivity report: cost coefficients

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
R	5	0	$\boxed{3}$	1E+30	1
G	3	0	2	1	2

Changing a cost coefficient parametrically



Objective function is (3+p)R + 2G.

Let z(p) be the optimal objective value as a function of p.

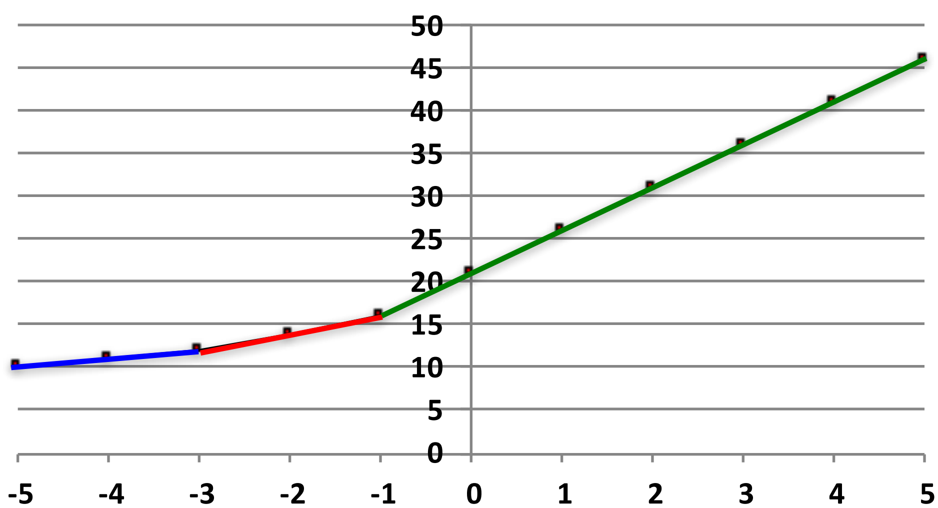
If $-1 \le p < \infty$, then opt = (5, 3), and
$z(p) = (3+p)\times 5 + (2\times 3) = 21 + 5p$.

If $-3 \le p \le -1$, then opt = (2, 6), and
$z(p) = (3+p)\times 2 + (2\times 6) = 18 + 2p$.

If $p \le -3$, then opt = (1, 6), and
$z(p) = (3+p)\times 1 + (2\times 6) = 15 + p$.

23

The optimal value as a function of p



24

The shadow price and its connection to changing a RHS coefficient

Changing RHS coefficients

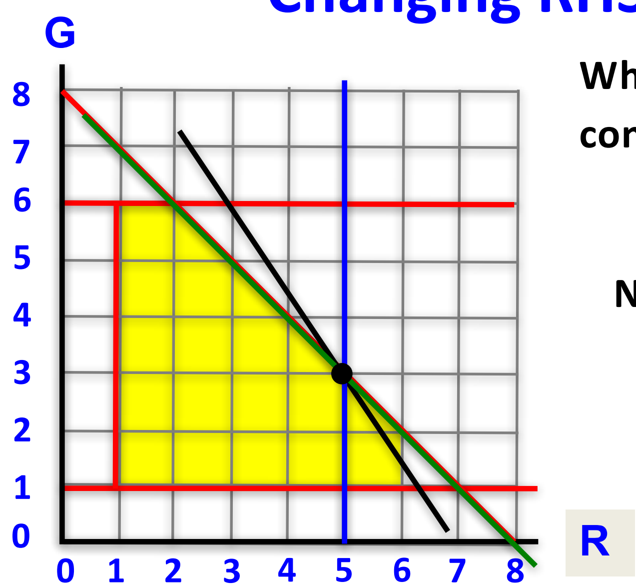
What happens if the fourth constraint is changed to:

$R \leq 6$?

What happens to the optimal solution?

What happens to the optimal objective value?

Changing RHS coefficients



What happens if the fourth constraint is changed to:

R ≤ 6?

New solution:

R = 6

G = 2

$z^* = 3R + 2G = 22.$

z^* has increased by 1.

27

Shadow price: first definition

The **shadow price** of a constraint of a linear program is the increase in the optimal objective value if the RHS of the constraint increases by 1. (Caveat to follow).

e.g., The constraint becomes R ≤ 6,

z^* increases from 21 to 22.

The shadow price of the constraint is 22 – 21 = 1.

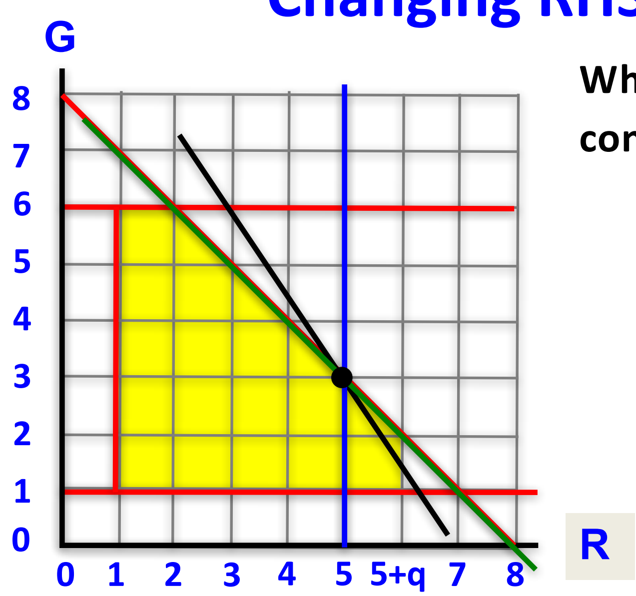
28

The sensitivity report: RHS coefficients

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Sum	8	2	8	3	2
LB-R	5	0	1	4	1E+30
LB-G	3	0	1	2	1E+30
UB-R	5	1	5	2	3
UB-G	3	0	6	1E+30	3

29

Changing RHS coefficients



What happens if the fourth constraint is changed to:

$R \leq 5 + q$?

$R + G = 8$ (1)

$R = 5 + q$ (4)

$\Rightarrow$ $G = 3 - q$

$z^* = 3R + 2G =$

$= (15 + 3q) + (6 - 2q)$

$= 21 + 1q.$

30

Shadow price: a more precise definition

The shadow price of a constraint of a linear program is the increase in z^* per unit increase in the RHS coefficient.

e.g., The constraint becomes $R \le 5 + q$,
z^* increases from 21 to $21 + (1 \times q)$.
The shadow price of the constraint is 1.

z^* increases linearly in q. $z^*(q) = 21 + (1 \times q)$.
The shadow price is the slope of the line.
The shadow price is 1.

Allowable changes in RHS coefficient

The shadow price of a constraint is ONLY valid within a range for each constraint.

Suppose that the RHS is 5 + q and all other data is unchanged.
Then $z^* = 21 + (1 \times q)$ so long as $-3 \le q \le 2$

The allowable increase and decrease for a RHS coefficient

Allowable changes in RHS coefficient

The shadow price of a constraint is ONLY valid within a range for each constraint.

Suppose that the RHS is 5 + q and all other data is unchanged.
Then $z^* = 21 + (1 \times q)$ so long as $-3 \le q \le 2$

35

The sensitivity report: RHS coefficients

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Sum	8	2	8	3	2
LB-R	5	0	1	4	1E+30
LB-G	3	0	1	2	1E+30
UB-R	5	1	5	2	3
UB-G	3	0	6	1E+30	3

36

A question and answer

Suppose that the number of red widgets that can be made is increased to 6,500. By how much will our profit increase?

RHS increases from 5 to 6.5.

z* increases by 1 × 1.5 = 1.5.

Profit increases by $1,500.

37

Another question and answer

Suppose that the number of red widgets that can be made is increased to 7,500. By how much will our profit increase?

RHS increases from 5 to 7.5.

This is outside of the allowable increase.

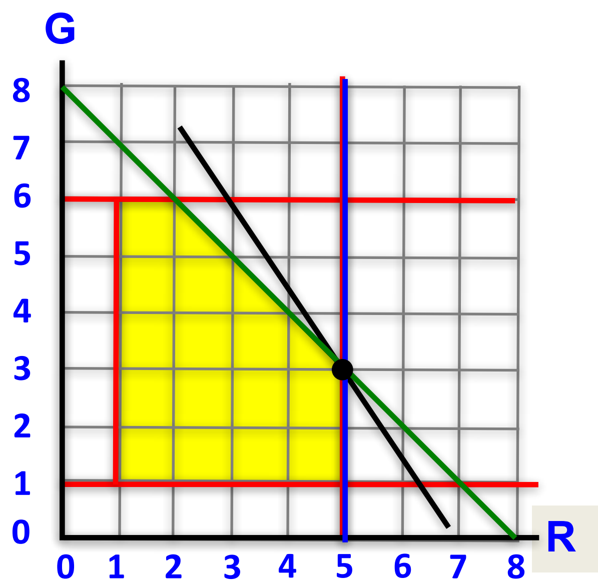
If the RHS increased from 5 to 7, then z* increases by 1 × 2 = 2.

Profit increased by $2,000.

If the RHS increases to more than 7, profit increases by ≥ $2,000.

38

On the allowable increases and decreases

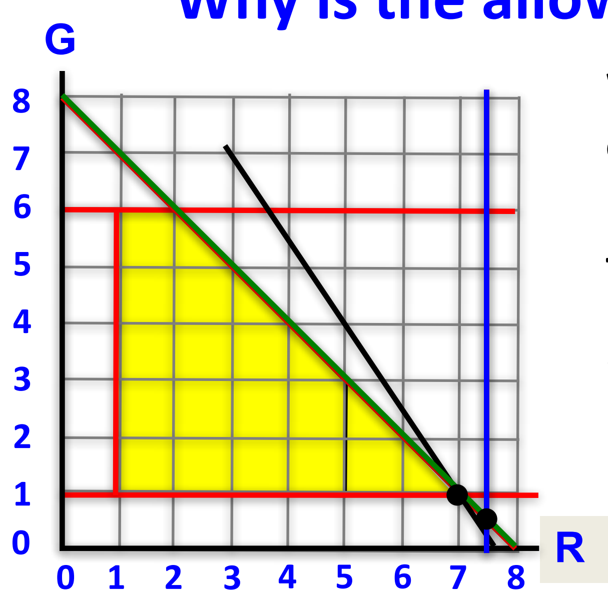


The optimum solution is at the intersection of the green and blue lines.

If we change the RHS of the blue line to 5 + q, the shadow price is 1, so long as the intersection of the blue and green lines is feasible.

39

Why is the allowable increase 2?



What happens if the fourth constraint is changed to:

$R \leq 7.5$?

The structure with (1) and (4) holding with equality is infeasible.

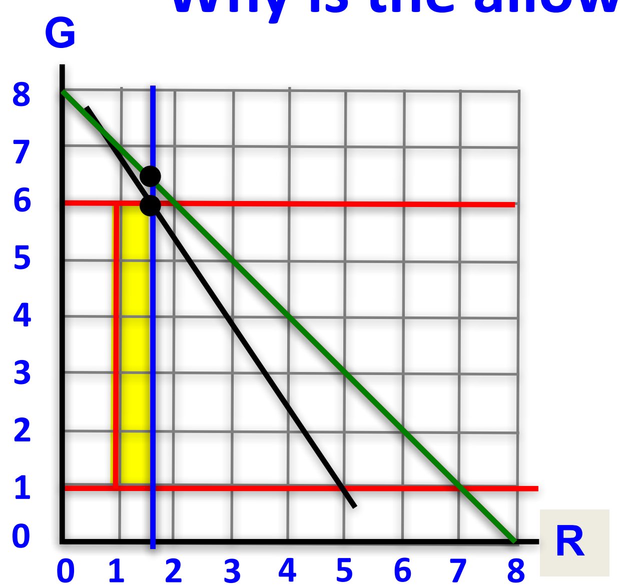
New opt solution:

R = 7

G = 1

40

Why is the allowable decrease 3?



What happens if the fourth constraint is changed to:

R ≤ 1.5?

The structure with (1) and (4) holding with equality is infeasible.

New opt solution:

R = 1.5
G = 6

41

42

A 3-variable example from the book Applied Mathematical Programming

book available for free at http://web.mit.edu/15.053/www

43

Example from AMP

Manufacturer of glassware

- Produces 6-oz glasses, 10-oz glasses, and champagne glasses.
- Limited production capacity
- Limited storage space in their warehouse
- Limit on production of 6-oz glasses (due to lack of demand)

x_1 = No. of cases of 6-oz glasses in 100s.

x_2 = No. of cases of 10-oz glasses in 100s.

x_3 = No. of cases of champagne glasses in 100s.

44

Formulation of glassware example
(page 77 of AMP)

$$\begin{aligned} \max \quad & 5x_1 + 4.5x_2 + 6x_3 \\ \text{s.t.} \quad & 6x_1 + 5x_2 + 8x_3 \le 60 && \text{production capacity} \\ & 10x_1 + 20x_2 + 10x_3 \le 150 && \text{storage capacity} \\ & x_1 \le 8 && \text{demand limit on 6-oz glasses} \\ & x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0 \end{aligned}$$

45

On the structure of the optimal solution

- When a linear program has 3 variables, each extreme point of the feasible region can be obtained as the solution of three equality constraints.
- If the problem has n variables, each extreme point is obtained by solving n equality constraints.

46

The solution is obtained by solving three equations:

$$\boxed{\begin{array}{l} 6x_1 + 5x_2 + 8x_3 = 60 \\ 10x_1 + 20x_2 + 10x_3 = 150 \end{array}}$$

$$x_1 \leq 8$$

$$x_1 \geq 0,\ x_2 \geq 0,\ \boxed{x_3 = 0}$$

x_1 = 6 3/7

x_2 = 4 2/7

x_3 = 0

z^* = 51 3/7

47

The SR Report

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
6-oz glasses	6 3/7	0	5	0.4	4/11
10-oz glasses	4 2/7	0	4.5	2	1/3
champagne glasses	0	- 4/7	6	4/7	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	11/14	60	5.5	22.5
warehouse capacity	150	1/35	150	90	22
6-oz glass demand	6 3/7	0	8	1E+30	1 4/7

48

Changing a single coefficient

Changing a single cost coefficient

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
6-oz glasses	6 3/7	0	$\boxed{5}$	0.4	4/11
10-oz glasses	4 2/7	0	4.5	2	1/3
champagne glasses	0	- 4/7	6	4/7	1E+30

Suppose that the revenue from 6-oz glasses increased from 5 to 5.3. Will the optimum solution stay the same?

What if it increased to 5.5? What if it is decreased to 4.5?

Changing a RHS coefficient

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	$\boxed{11/14}$	60	5.5	22.5
warehouse capacity	150	1/35	150	90	22
6-oz glass demand	6 3/7	0	8	1E+30	1 4/7

Suppose additional hours of production capacity can be purchased. What is the most the glass manufacturer would pay for an additional hour of capacity?

How many hours would they purchase at this price?

51

Shadow Price

Shadow price for production capacity: 11/14 ≈ .786

It would be worth a little less than $.79 per hour.

They would be willing to purchase 5.5 hours at about $.79 per hour.

52

Why not just reoptimize?

- If we want to change a single coefficient, why not just make the change, and use Solver or OpenSolver again?
  - Reoptimizing is a good idea for small linear programs.
  - For integer programs, reoptimizing is the only option.
    - There is no sensitivity report for IPs and MIPs
  - For large LPs (100s, 1000s or more), the sensitivity report is really useful.
  - And by "pricing out" we can learn even more information.

53

Changing two or more RHS coefficients

54

Using the SR report for multiple changes

The shadow price information is valid for sufficiently small changes, even if there are multiple changes in the RHS.

The allowable increases and decreases in the SR report are only valid if exactly one cost coefficient changes or if one RHS coefficient changes.

What if we want to change more than one coefficient?

- e.g., production capacity increases by 2 and warehouse capacity decreases by 10.

55

Using the SR report for multiple changes

If you want to change exactly two RHS coefficients, then divide all allowable increases and decreases in the SR report by 2.

If you want to change exactly two cost coefficients, then divide all allowable increases and decreases in that SR report by 2.

This is a special case of the 100% rule. (Section 3.7 AMP)

56

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	11/14	60	5.5	22.5
warehouse capacity	150	1/35	150	90	22
6-oz glass demand	6 3/7	0	8	1E+30	1 4/7

To change two RHS coefficients, divide allowable changes by 2.

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	11/14	60	2.75	11.25
warehouse capacity	150	1/35	150	45	11
6-oz glass demand	6 3/7	0	8	1E+30	11/14

57

Changing two RHS coefficients

Suppose that production capacity increases by 2 and warehouse capacity decreases by 10. What is the total change in the profit?

Part of the SR report, for two changes in RHS.

Name	Shadow Price	Allowable Increase	Allowable Decrease	Impact
production capacity	11/14	$\boxed{2.75}$		2 × 11/14 = 11/7
warehouse capacity	1/35		11	-10 × 1/35 = -2/7

z increases by 11/7 – 2/7 = 9/7 ≈ 1.29

58

Changing 3 RHS coefficients

- If you want to change 3 (or K) RHS coefficients, then divide all allowable increases and decreases in the SR report by 3 (or K).

- The same approach works for changing cost coefficients.

“Pricing out” : using shadow prices.

Analyzing a new variable using shadow price information.

MIT Corp is considering making a third type of widget: blue widgets. (Decision variable B.)

They sell for \$4.60 and cost \$1 to make. (Profit = \$3.60)

MIT Corp wants the constraint that

$$R + G + 2B \leq 8$$

Will it be profitable to make any blue widgets?

61

The revised model

$$\begin{aligned} \max\ z = 3R + 2G + 3.6B \\ R + G + 2B &\leq 8 \\ R &\geq 1 \\ G &\geq 1 \\ R &= 5 \\ G &\leq 6 \\ R \geq 0,\ G \geq 0 \end{aligned}$$

Figure out the optimal objective value if B = 1.

Check to see if the objective is higher than the original objective.

This is called "pricing out" variable B.

62

The revised model with B = 1

$$\max\ z = 3R + 2G + 3.6$$
$$R + G + 2 \le 8$$
$$R \ge 1$$
$$G \ge 1$$
$$R = 5$$
$$G \le 6$$
$$R \ge 0,\ G \ge 0$$

Same as original model except that we have an extra 3.6 of profit and we have

$R + G \le 6$

63

Change in profitability if B = 1.

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Sum	8	2	8	3	2

- Increase in profit if B = 1.

  \+ 3.6 (profitability from B)

  \- 2 × 2 (decrease in profitability because RHS of the constraint " R + G ≤ 8"decreased by 2)

  Net impact : 3.6 – 4 = -0.4.

64

Pricing out x_3 of the glass problem

$$\begin{array}{ll} \max & 5x_1 + 4.5x_2 + 6x_3 \\ \text{s.t.} & 6x_1 + 5x_2 + 8x_3 \le 60 \\ & 10x_1 + 20x_2 + 10x_3 \le 150 \\ & x_1 \le 8 \\ & x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0 \end{array}$$

In the optimal solution, $x_3 = 0$. What would be the impact on the optimum solution value of requiring that $x_3 = 1$?

65

Change in formulation if $x_3 = 1$.

$$\begin{array}{ll} \max & 5x_1 + 4.5x_2 + 6 \\ \text{s.t.} & 6x_1 + 5x_2 + 8 \le 60 \\ & 10x_1 + 20x_2 + 10 \le 150 \\ & x_1 \le 8 \\ & x_1 \ge 0,\ x_2 \ge 0 \end{array}$$

$$\begin{array}{ll} \max & 5x_1 + 4.5x_2 + 6 \\ \text{s.t.} & 6x_1 + 5x_2 \le 60 - 8 \\ & 10x_1 + 20x_2 \le 150 - 10 \\ & x_1 \le 8 \\ & x_1 \ge 0,\ x_2 \ge 0 \end{array}$$

66

Pricing out x_3.

	Shadow Price	Impact on opt. obj value
max $5x_1 + 4.5x_2 \quad + 6$		6
s.t. $6x_1 + 5x_2 \le 60 - 8$	11/14	- 8 × 11/14
$10x_1 + 20x_2 \le 150 - 10$	1/35	- 10 × 1/35
$x_1 \quad \le 8$	0	- 0 × 0
$x_1 \ge 0,\ x_2 \ge 0$		= 6 – 88/14 – 10/35
		= -4/7.

67

Computing an allowable increase when pricing out

68

Allowable increases when pricing out

We can give an allowable increase for pricing out

- For new variables.
- For existing variables whose optimal value is 0.

For variables that are non-zero in an optimal solution, the reduced cost is 0, and the technique in this video cannot be used to create bounds on the allowable increase.

69

Change in formulation if x_3 = p.

$$\begin{array}{lll} \max & 5x_1 + 4.5x_2 + 6p & \\ \text{s.t.} & 6x_1 + 5x_2 + 8p & \leq 60 \\ & 10x_1 + 20x_2 + 10p & \leq 150 \\ & x_1 & \leq 8 \\ & x_1 \geq 0,\ x_2 \geq 0 & \end{array}$$

$$\begin{array}{lll} \max & 5x_1 + 4.5x_2 & + 6p \\ \text{s.t.} & 6x_1 + 5x_2 & \leq 60 - 8p \\ & 10x_1 + 20x_2 & \leq 150 - 10p \\ & x_1 & \leq 8 \\ & x_1 \geq 0,\ x_2 \geq 0 & \end{array}$$

If p is small, then the opt objective value "increases" by -4/7 × p.

We are changing the RHS for two constraints.

70

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	11/14	60	5.5	22.5
warehouse capacity	150	1/35	150	90	22
6-oz glass demand	6 3/7	0	8	1E+30	1 4/7

To change two RHS coefficients, divide allowable changes by 2.

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
production capacity	60	11/14	60	2.75	11.25
warehouse capacity	150	1/35	150	45	11
6-oz glass demand	6 3/7	0	8	1E+30	11/14

$$\begin{array}{llll}
\max & 5x_1 + 4.5x_2 & & + 6p \\
\text{s.t.} & 6x_1 + 5x_2 & \le & 60 - 8p \\
& 10x_1 + 20x_2 & \le & 150 - 10p \\
& x_1 & \le & 8 \\
& x_1 \ge 0, \ x_2 \ge 0 & &
\end{array}$$

Shadow Price	Allowable Decrease
11/14	11.25
1/35	11

$8p \le 11.25$

$10p \le 11$

Therefore, $p \le 11/10$.

The allowable increase is possibly larger than 11/10

If we require that "$x_3 = p$" and if $p \le 11/10$, then the optimal objective value "increases" by $-4/7 \times p$.

Reduced costs and pricing out

Reduced costs

Name	Final Value	Reduced Cost
6-oz glasses	6 3/7	0
10-oz glasses	4 2/7	0
champagne glasses	0	$\boxed{-4/7}$

The reduced cost of a variable (say x_3) is the shadow price of its nonnegativity constraint, i.e., the constraint "$x_3 \geq 0$."

The reduced cost of x_3 is the change in the optimal objective value if we change the constraint to "$x_3 \geq 1$."

Reduced costs

The reduced cost of x_3 is the shadow price of its nonnegativity constraint.

The reduced cost for x_3 is $-4/7$.

On the sign of the reduced cost

Let $\bar{c}_j$ denote the reduced cost of x_j for a maximization problem.

- **$\bar{c}_j$ is the shadow price of "$x_j \geq 0$."**
- **$\bar{c}_j \leq 0$. Can you see why?**

Let z^* denote the max objective value.

Suppose we require $x_j \geq p$ for some $p > 0$.

Then the feasible region gets smaller.

Let z' be the max objective value for the modified problem.

Then $z' \leq z^*$, and thus $\bar{c}_j \leq 0$.

On the sign of the reduced cost

The shadow of a "≥" for a maximization problem problem is non-positive.

The shadow price of a "≤" for a maximization problem problem is non-negative.

Let z^* denote the max objective value.

Suppose we require $x_j \geq p$ for some $p > 0$.

Then the feasible region gets smaller.

Let z' be the max objective value for the modified problem.

Then $z' \leq z^*$, and thus $\bar{c}_j \leq 0$.

77

Pricing out computes the reduced cost

Let $\bar{c}_j$ denote the reduced cost of x_j for a maximization problem.

Let x^* denote the optimum solution for the original problem.

Suppose we require $x_j \geq p$ for some sufficiently small $p > 0$.

Let x' be the optimum solution for the modified problem.

Then

- $\bar{c}_j$ can be computed by pricing out.
- If $x^*_j = 0$, then $x'_j = p$. We can choose $p < x^*_j$
- If $x^*_j > 0$, then $\bar{c}_j = 0$, because $x^*_j > p$, and $x' = x^*$ and $z' = z^*$.

No feasible solution is better than x*.

78

Pricing out, again,
and some final remarks on the SR

Pricing out x_2 of the glass problem

$$\begin{aligned}
\text{max} \quad & 5x_1 + 4.5x_2 + 6x_3 \\
\text{s.t.} \quad & 6x_1 + 5x_2 + 8x_3 \le 60 \\
& 10x_1 + 20x_2 + 10x_3 \le 150 \\
& x_1 \le 8 \\
& x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0
\end{aligned}$$

In the optimal solution, x_2 = 4 2/7.

Its reduced cost is 0.

Take into account the resources used in making exactly one unit.

Pricing out x_2.

		Shadow Price	Impact on opt. obj value
max	$5x_1 + 4.5x_2 + 6x_3 \quad + 4.5$		4.5
s.t.	$6x_1 + 5x_2 + 8x_3 \le 60 - 5$	11/14	- 5 × 11/14
	$10x_1 + 20x_2 + 10x_3 \le 150 - 20$	1/35	- 20 × 1/35
	$x_1 \le 8$	0	- 0 × 0
	$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$		= 4.5 –55/14 – 20/35 = 0

81

Pricing out using constraint coefficients

$$
\begin{aligned}
\max \quad & 5x_1 + 4.5x_2 + 6x_3 \\
\text{s.t.} \quad & 6x_1 + 5x_2 + px_3 \le 60 \\
& 10x_1 + 20x_2 + 10x_3 \le 150 \\
& x_1 \le 8 \\
& x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0
\end{aligned}
$$

For what values of p is it profitable to produce champagne glasses?

82

Pricing out x_3.

	Shadow Price	Impact on opt. obj value
max $5x_1 + 4.5x_2 \quad + 6$		6
s.t. $6x_1 + 5x_2 \le 60 - p$	11/14	- p × 11/14
$10x_1 + 20x_2 \le 150 - 10$	1/35	- 10 × 1/35
$x_1 \le 8$	0	- 0 × 0
$x_1 \ge 0,\ x_2 \ge 0$		

$= 6 - 11p/14 - 10/35 \ge 0$

$\Rightarrow p \le 80/11 \approx 7.27$

83

Final remarks

- Sensitivity analysis is broadly useful.
- The LP Sensitivity Report gives lots of sensitivity analysis information for free.
- Pricing out can be used to "explain" why a product is not produced, or what changes to make in order to make the product profitable.

84