1번 C289002 이영훈

$$(2)\ \int_0^1\int_1^2 (4x^3 - 9x^2y^2)\,dy\,dx$$

$$4x^3y - 3x^2y^3\Big]_1^2 = (8x^3 - 24x^2) - (4x^3 - 3x^2)$$

$$= 4x^3 - 21x^2$$

$$\int_0^1 4x^3 - 21x^2\,dx = [x^4 - 7x^3]_0^1 = (1-7) - 0 = -6$$

$$(4)\ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_{-1}^{5} \cos y\,dx\,dy$$

$$\cos y \cdot x\Big]_{-1}^{5} = 6\cos y$$

$$6\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos y\,dy = 6[\sin y]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 6\left(1 - \frac{1}{2}\right) = 3$$

$$(6)\ \int_0^1\int_1^2 \frac{x \cdot e^x}{y}\,dy\,dx$$

$$x \cdot e^x \cdot \ln y\Big]_1^2 = x \cdot e^x \cdot \ln 2$$

$$\ln 2\int_0^1 x \cdot e^x dx$$

$$\ln 2\left\{[x \cdot e^x]_0^1 - \int_0^1 e^x dx\right\}$$

$$\ln 2\left\{[x \cdot e^x]_0^1 - [e^x]_0^1\right\}$$

$$\ln 2\,(e - e + 1) = \ln 2$$

$$(8)\ \int_0^1\int_0^3 e^{x+3y}\,dy\,dx$$

$$\int_0^3 e^x \cdot e^{3y}\,dy$$

$$e^x \cdot \left[\frac{e^{3y}}{3}\right]_0^3 = \frac{e^x}{3}(e^9 - 1)$$

$$\frac{e^9-1}{3}\int_0^1 e^x dx$$

$$\boxed{\frac{e^9-1}{3}(e-1)}$$

$$(10)\ \int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$$

$$\int_0^1 y\int_0^1 x \cdot \sqrt{x^2+y^2}\,dx\,dy$$

$$x \cdot \frac{2}{3}(x^2+y^2)^{\frac{3}{2}} \cdot \frac{1}{\cancel{2x}}$$

$$\frac{1}{3}(x^2+y^2)^{\frac{3}{2}}\Big]_0^1$$

$$\frac{1}{3}\left((y^2+1)^{\frac{3}{2}} - y^3\right)$$

$$\frac{1}{3}\left[\int_0^1 y(y^2+1)^{\frac{3}{2}}dy - \int_0^1 y^4 dy\right]$$

$$y \cdot \frac{2}{5}(y^2+1)^{\frac{5}{2}} \cdot \frac{1}{2y}$$

$$\frac{1}{5}(y^2+1)^{\frac{5}{2}}\Big]_0^1 - \frac{1}{5}[y^5]_0^1$$

$$\frac{1}{15}\left(2^{\frac{5}{2}} - 1 - 1\right)$$

$$\frac{4\sqrt{2}}{15} - \frac{2}{15}$$

$$(12)\ \int_0^1\int_0^1 \sqrt{s+t}\,ds\,dt$$

$$\int_0^1\int_0^1 (s+t)^{\frac{1}{2}}\,ds\,dt$$

$$\int_0^1 (s+t)^{\frac{1}{2}}ds$$

$$\frac{2}{3}(s+t)^{\frac{3}{2}}\Big]_0^1 = \frac{2}{3}(1+t)^{\frac{3}{2}} - \frac{2}{3}t^{\frac{3}{2}}$$

$$\frac{2}{3}\int_0^1 (1+t)^{\frac{3}{2}} - t^{\frac{3}{2}}\,dt$$

$$\frac{2}{5}(1+t)^{\frac{5}{2}} - \frac{2}{5}t^{\frac{5}{2}}\Big]_0^1$$

$$\frac{2}{5}\left[(1+t)^{\frac{5}{2}} - t^{\frac{5}{2}}\right]_0^1$$

$$\frac{2}{5}(4\sqrt{2} - 1 - 1)$$

$$\frac{2}{3}\left(\frac{8\sqrt{2}}{5} - \frac{4}{5}\right)$$

$$\frac{16\sqrt{2}}{15} - \frac{8}{15}$$

2번

(2) $\iint_R \cos(x+2y)\,dA$, $R=[\overset{x}{0,\ \pi}]\times[\overset{y}{0,\ \frac{\pi}{2}}]$

$$\int_0^{\pi} \cos(x+2y)\,dx$$

$$\sin(x+2y)\Big]_0^{\pi} = \sin(\pi+2y) - \sin 2y$$

$$\int_0^{\frac{\pi}{2}} \sin(\pi+2y) - \sin 2y\,dy$$

$$-\frac{1}{2}\cos(\pi+2y) + \frac{1}{2}\cos 2y\Big]_0^{\frac{\pi}{2}}$$

$$\frac{1}{2}\left[\left[-\cos(\pi+2y) + \cos 2y\right]_0^{\frac{\pi}{2}}\right]$$

$$(-\cos 2\pi + \cos\pi) - (-\cos\pi + \cos 0)$$

$$-1 - 1 \qquad 1 + 1$$

$$-2 - 2 = -4 \qquad -2$$

(6) $\iint_R \frac{x}{1+xy}\,dA$, $R=[0,\ 1]\times[0,\ 1]$

$$\int_0^1\int_0^1 \frac{x}{1+xy}\,dy\,dx$$

$$(\ln(1+xy))_y = \frac{x}{1+xy}$$

$$\ln(1+xy)\Big]_0^1$$

$$\int_0^1 \ln(1+x)\,dx$$

$$x\cdot\ln(1+x)\Big]_0^1 - \int_0^1 \frac{x}{1+x}\,dx$$

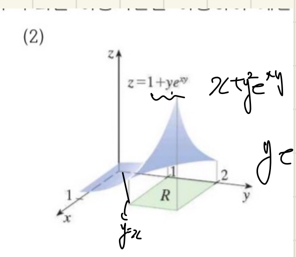
$$-\left[[x]_0^1 - [\ln(1+x)]_0^1\right]$$

$$\ln 2 \qquad -1 + \ln 2$$

$$2\ln 2 - 1$$

3번

3. 다음 그림은 xy평면 영역 R에서 높이 z인 입체의 부피를 이중적분을 이용하여 계산하시오.



$x+y^2e^{xy}$

ye^{xy}

$y=x$

$1\le y\le 2$

$0\le x\le 1$

$$\int_1^2\int_0^1 1+ye^{xy}\,dx\,dy$$

$$\left[x + e^{xy}\right]_0^1$$

$$\cancel{1} + e^y \cancel{-1}$$

$$\int_1^2 e^y\,dy$$

$$e^y\Big]_1^2 \quad e^2 - e$$

(4) $\iint_R \frac{1+x^2}{1+y^2}\,dA$, $R=[0,\ 1]\times[0,\ 1]$

$$\int_0^1 \frac{1+x^2}{1+y^2}\,dx$$

$$\frac{1}{1+y^2}\left[x + \frac{1}{3}x^3\right]_0^1$$

$$\frac{1}{1+y^2}\cdot\frac{4}{3}$$

$$\frac{4}{3}\int_0^1 \frac{1}{1+y^2}\,dy$$

$$\frac{4}{3}\cdot\frac{1}{2y}\cdot\ln(1+y^2)\Big]_0^1$$

$$\frac{2}{3}\ln 2$$

(8) $\iint_R \frac{x}{x^2+y^2}\,dA$, $R=[1,\ 2]\times[0,\ 1]$

$$\int_0^1\int_1^2 \frac{x}{x^2+y^2}\,dx\,dy$$

$$\frac{1}{2}\left[\ln(x^2+y^2)\right]_1^2$$

$$\frac{1}{2}\left(\ln(4+y^2) - \ln(1+y^2)\right)$$

$$\frac{1}{2}\left[\int_0^1 \ln(4+y^2)\,dy - \int_0^1 \ln(1+y^2)\,dy\right]$$

$$y\cdot\ln(4+y^2) - \int_0^1 y\cdot\frac{2y}{4+y^2}\,dy$$

$$\frac{2y^2+8}{4+y^2} - \frac{8}{4+y^2}$$

$$2 - \frac{8}{4+y^2}$$

$$2y - \frac{1}{2}\cdot 8\cdot\tan^{-1}\frac{y}{2}$$

$$\left[y\cdot\ln(4+y^2) - 2y + 4\tan^{-1}\frac{y}{2}\right]_0^1$$

$$-\left[y\ln(1+y^2) - 2y + 2\tan^{-1}y\right]_0^1$$

$$\ln 5 - \cancel{2} + 4\tan^{-1}\frac{1}{2}$$

$$-\ln 2 + \cancel{2} - 2\cdot\frac{\pi}{4}$$

$$\frac{1}{2}\left(\ln\frac{5}{2} - \frac{\pi}{2} + 4\tan^{-1}\frac{1}{2}\right)$$

4번

(2) $\int_0^1 \int_{2x}^2 (x-y)\,dy\,dx$

$xy - \frac{1}{2}y^2 \Big]_{2x}^{2} = (2x - 2) - (\cancel{2x^2} - \cancel{2x^2})$

$= 2x - 2$

$\int_0^1 2x - 2\,dx = x^2 - 2x \Big]_0^1 = (1-2) - 0 = -1$

(4) $\int_0^2 \int_y^{2y} xy\,dx\,dy$

$\left[\frac{y}{2}x^2\right]_y^{2y} = 2y^3 - \frac{1}{2}y^3$

$= \frac{3}{2}y^3$

$\frac{3}{2}\int_0^2 y^3 dy = \frac{3}{2}\left[\frac{1}{4}y^4\right]_0^2$

$= \frac{3}{2}(4-0) = 6$

(6) $\int_0^1 \int_0^v \sqrt{1-v^2}\,du\,dv$

$\sqrt{1-v^2}\cdot u \Big]_0^v$

$\int_0^1 v \cdot \sqrt{1-v^2}\,dv$

$\cancel{v}\cdot \frac{2}{3}(1-v^2)^{\frac{3}{2}} \cdot \frac{1}{(-2v)}$

$-\frac{1}{3}(1-v^2)^{\frac{3}{2}}\Big]_0^1$

$\frac{1}{3}$

5번

(2) $\iint_D \frac{4y}{x^3+2}\,dA,\ D = \{(x,\ y) \mid 1 \le x \le 2,\ 0 \le y \le 2x\}$

$\int_1^2 \int_0^{2x} \frac{4y}{x^3+2}\,dy\,dx$

$\frac{2}{x^3+2}y^2\Big]_0^{2x}$

$\frac{2}{x^3+2}\cdot 4x^2 = \frac{8}{x^3+2}x^2$

$\frac{8}{3}\int_1^2 \frac{3x^2}{x^3+2}dx$

$\frac{8}{3}\left[\ln(x^3+2)\right]_1^2 = \frac{8}{3}(\ln 10 - \ln 3) = \frac{8}{3}\left(\ln\frac{10}{3}\right)$

(4) $\iint_D x^2\,dA,\ D = \{(x,\ y) \mid 1 \le x \le e,\ 0 \le y \le \ln x\}$

$\int_1^e \int_0^{\ln x} x^2\,dy\,dx$

$x^2 y\Big]_0^{\ln x}$

$\int_1^e x^2 \ln x\,dx$

$\ln x \cdot \frac{1}{3}x^3\Big]_1^e - \int_1^e \frac{1}{3}x^2\,dx$

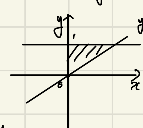
$\frac{1}{3}e^3 - \left[\frac{1}{9}x^3\right]_1^e$

$\frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9}$

$\frac{2}{9}e^3 + \frac{1}{9}$

(6) $\iint_D x\sqrt{y^2 - x^2}\,dA$, D는 $x=0,\ y=1,\ y=x$로 둘러싸인 영역

$0 \le y \le 1 \qquad 0 \le x \le y$



$\int_0^1 \int_0^y x\sqrt{y^2-x^2}\,dx\,dy$

$(y^2-x^2)^{\frac{3}{2}}$

$\frac{2}{3}(y^2-x^2)^{\frac{1}{2}}\cdot(-2x)$

$\left[-\frac{1}{3}\times(y^2-x^2)^{\frac{3}{2}}\right]_0^y$

$\int_0^1 \frac{1}{3}y^3\,dy$

$\frac{1}{12}y^4\Big]_0^1 = \frac{1}{12}$

(8) $\iint_D xy^2\,dA$, D는 $x=0,\ x=\sqrt{1-y^2}$으로 둘러싸인 영역

$0 \le x \le 1$

$0 \le y \le \sqrt{1-x^2} \quad x^2+y^2=1$

$\int_0^1 \int_0^{\sqrt{1-x^2}} xy^2\,dy\,dx$

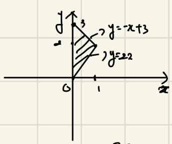
$\frac{1}{3}xy^3\Big]_0^{\sqrt{1-x^2}}$

$\int_0^1 \frac{1}{3}x(1-x^2)^{\frac{3}{2}}dx$

$\frac{1}{3}\times\left(-\frac{1}{5}\right)(1-x^2)^{\frac{5}{2}}$

$-\frac{1}{15}(-1) = \frac{1}{15}$

(10) $\iint_D 2xy\,dA$, D는 $(0,\ 0),\ (1,\ 2),\ (0,\ 3)$을 꼭지점으로 갖는 삼각형 영역



$0 \le x \le 1 \qquad 2x \le y \le -x+3$

$\int_0^1 \int_{2x}^{-x+3} 2xy\,dy\,dx$

$xy^2\Big]_{2x}^{-x+3} = x(-x+3)^2 - 4x^3$

$x(x^2 - 6x + 9) - 4x^3$

$-3x^3 - 6x^2 + 9x$

$-\frac{3}{4}x^4 - 2x^3 + \frac{9}{2}x^2 \qquad -\frac{3}{4} - 2 + \frac{9}{2}$

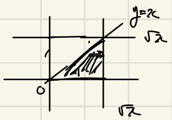
$-\frac{3}{4} - \frac{8}{4} + \frac{18}{4}$

$\frac{n}{4}$

G 6-4

(2) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy$

$y \le x \le \sqrt{\pi}$, $0 \le y \le \sqrt{\pi}$



$0 \le x \le \sqrt{\pi}$, $0 \le y \le x$

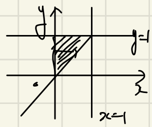
$\int_0^{\sqrt{\pi}} \int_0^{x} \cos(x^2)\,dy\,dx$

$\int_0^{\sqrt{\pi}} x \cdot \cos(x^2)\,dx$

$\frac{1}{2} \sin x^2 \Big]_0^{\sqrt{\pi}} = \frac{1}{2}(0-0) = 0$

(4) $\int_0^1 \int_x^1 e^{\frac{x}{y}}\,dy\,dx$

$0 \le x \le 1$ $x \le y \le 1$



$0 \le y \le 1$ $0 \le x \le y$

$\int_0^1 \int_0^y e^{\frac{x}{y}}\,dx\,dy$

$y \cdot e^{\frac{x}{y}} \Big]_0^y$

$ye - y$

$= y(e-1)$

$e-1 \left[\frac{1}{2}y^2\right]_0^1$

$(e-1)\frac{1}{2}$

(6) $\int_0^8 \int_{\cancel{\sqrt[3]{y}}\ \sqrt[3]{y}}^{2} \cancel{e}\ e^{x^4}\,dx\,dy$

$\sqrt[3]{y} \le x \le 2$ $0 \le y \le 8$

$y = x^3$

$0 \le x \le 2$ $0 \le y \le x^3$

$\int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx$

$\int_0^2 e^{x^4} \cdot x^3\,dx$

$\frac{1}{4} e^{x^4} \Big]_0^2$

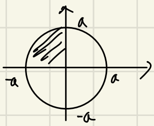
$\frac{1}{4}(e^{16}-1)$

12단

(2) $\int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y \, dx \, dy$

$x^2+y^2=a^2$

$0 \le r \le a$ $\quad \frac{\pi}{2} \le \theta \le \pi$



$J(r,\theta)=r$

$\int_{\frac{\pi}{2}}^{\pi} \int_0^a r^2 \cdot r \cdot r \cos^2\theta \cdot \sin\theta \, dr \, d\theta$

$\cos^2\theta \sin\theta \, \frac{1}{5} r^5 \Big]_0^a$

$\frac{a^5}{5} \int_{\frac{\pi}{2}}^{\pi} \cos^2\theta \cdot \sin\theta \, d\theta$

$\cos\theta = t$

$-\sin\theta \cdot d\theta = dt$

$-\frac{a^5}{5} \left[\frac{1}{3} t^3 \right]_0^{-1}$

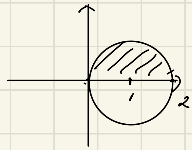
$\frac{a^5}{15}$

(4) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2} \, dy \, dx$

$x^2-2x+1+y^2=1$

$(x-1)^2+y^2=1$ $\quad 0 \le r \le 1$

$0 \le \theta \le \pi$



$\int_0^{\pi} \int_0^1 r \, dr \, d\theta$

$\frac{1}{2} r^2 \Big]_0^1 = \frac{1}{2}$

$\frac{1}{2} [\theta]_0^{\pi} = \frac{\pi}{2}$