Print your name ____

Problems [2 points each for a question. Total about 172 points (include 32 points of ICP)]

Directions: Show your work step by step to receive full credit. Box your final answer and record its appropriate unit.

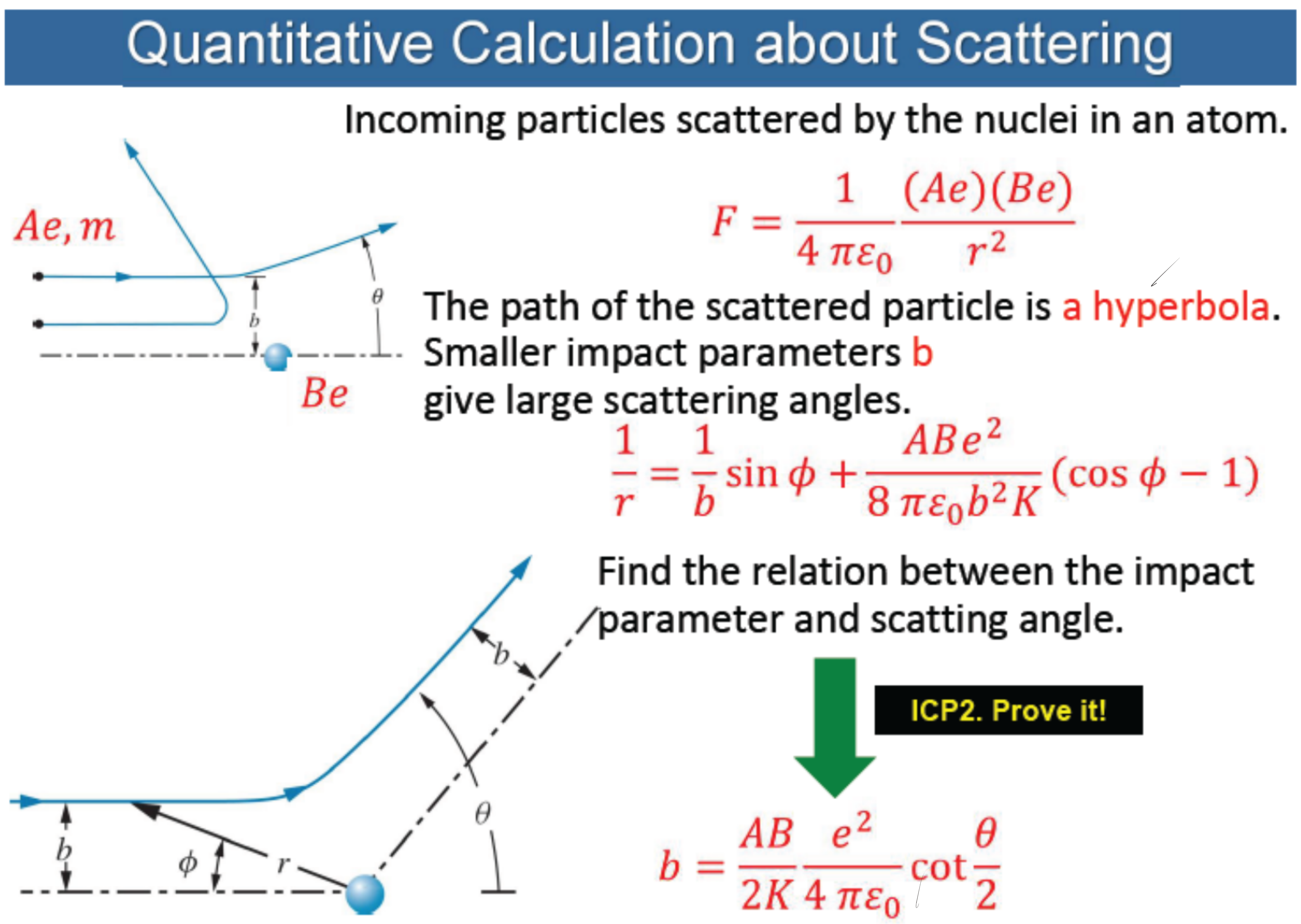
Some Commonly Used Constants and Conversion Factors

(see Appendix A for a more complete list)

Speed of light	$c = 2.998 \times 10^{8}$ m/s
Electronic charge	$e = 1.602 \times 10^{-19}$ C
Boltzmann constant	$k = 1.381 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J$\cdot$s $= 4.136 \times 10^{-15}$ eV$\cdot$s
Avogadro's constant	$N_A = 6.022 \times 10^{23}$ mole $^{-1}$
Electron mass	$m_e = 5.49 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$
Proton mass	$m_p = 1.007276$ u $= 938.3$ MeV$/c^2$
Neutron mass	$m_n = 1.008665$ u $= 939.6$ MeV$/c^2$
Bohr radius	$a_0 = 0.0529$ nm
Hydrogen ionization energy	13.6 eV
Thermal energy	$kT = 0.02525$ eV $\cong \frac{1}{40}$ eV ($T = 293$ K)
$hc = 1240$ eV$\cdot$nm (MeV$\cdot$fm)	$\hbar c = 197$ eV$\cdot$nm (MeV$\cdot$fm)
$\dfrac{e^2}{4\pi\varepsilon_0} = 1.440$ eV$\cdot$nm (MeV$\cdot$fm)	1 u $= 931.5$ MeV$/c^2$ 1 eV $= 1.602 \times 10^{-19}$ J

In Class Problem 1-16, total 32 points.

1) Rutherford scattering with negative charge beam: We have discussed Rutherford scattering problem with positive charge beam in the lecture, see the following.



(a) Assuming that the input beam is negative charged （入射粒子束是负电）, such as -Ae (e is the positive charge unit, and both A and B are positive number). Please re-derive the above two formula (1/r=, b=) in this case.

(b) Will the negative charge fall into the nuclei? Try to explain it according to the formula that you derive.

(c) The track of the negative charge scattering beam （负电散射粒子束的轨迹） will be significantly different with the positive charge case. Please draw several typical tracks for the negative charge beam and explain them.

(d) Assume the nuclei is Lithium nuclei, and the input beam is 1 KeV Beta ray. Calculate the impact parameters b for various scattering angle θ = 0, 30, 45, 90, 180, and plot the function b(θ).

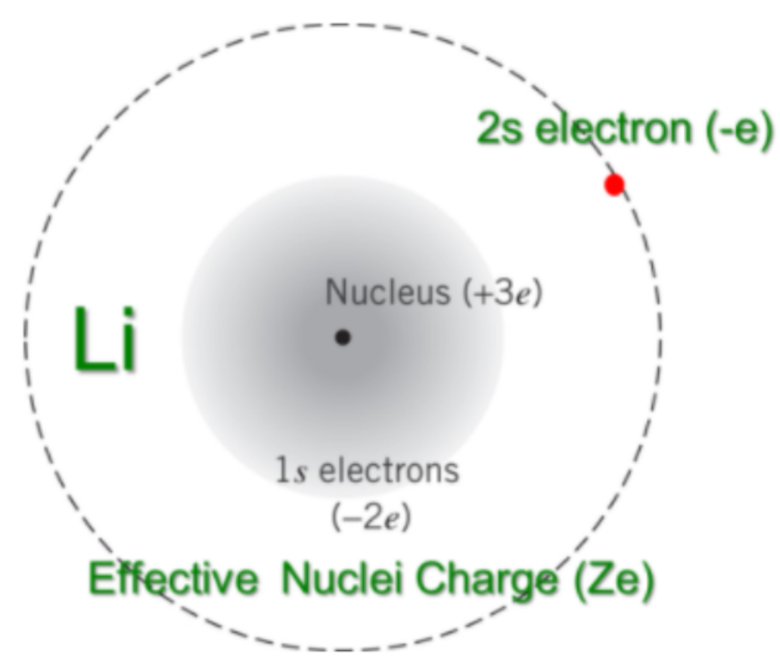
2) Compton Scattering: X-ray photons of the wavelength of 0.01535 nm are incident on free electrons at rest. After the interaction, some photons with the wavelength of 0.01644 nm are observed.

(a) Relative to the direction of the original X-rays, at what angle would we observe the photons with the wavelength of 0.01644 nm?

(b) What is the kinetic energy given to the electrons by these scatted photons with the wavelength of 0.01644 nm?

(c) What is the maximum wavelength can be observed for the scattered photons in this experiment, and what is the direction of these photons relative to the direction of the original X-rays.

3) Bohr Model of Hydrogen-like Atoms: Lithium atoms can be treated as hydrogen-like atoms with an effective nuclei charge **Z**e and a single 2s electron rotating around the effective nuclei. Apply the method that we have used to build the Bohr model of hydrogen to solve the following problems:



(a) Prove that the radius of the 2s electron orbits is given by

$$r_n = \frac{4\pi\varepsilon_0}{m}\frac{\hbar^2}{\boldsymbol{Z}e^2}n^2$$

where *n=1,2,3...* All the characters represent the same physics constants as the formulas of the Bohr model of Hydrogen.

(b) Prove that the energy level of the 2s electron orbits is given by

$$E_n = -\frac{m\boldsymbol{Z}^2e^4}{32\,(\pi\varepsilon_0\hbar)^2}\frac{1}{n^2}$$

All the characters represent the same physics constants as the formulas of the Bohr model of Hydrogen.

(c) If the effective nuclear charge for lithium 2s orbit electron is 1.26e, then what is the ground state energy, the first excited state energy, and the ionization energy for lithium atom 2s electron. (Require to use eV as units in your results)

4) Hydrogen Atom: Give expressions for the following quantities in terms of e, h, c, ε_0, m_e and m_p.

(a) The energy needed to ionize a hydrogen atom.

(b) The difference in frequency of the Lyman alpha line in hydrogen and deuterium atoms.

(c) The magnetic moment of the electron.

5)

Bohr theory and the correspondence principle
This exercise gives an alternative approach to the theory of the hydrogen atom presented in Section 1.3 that is close to the spirit of Bohr's original papers. It is somewhat more subtle than that usually given in elementary textbooks and illustrates Bohr's great intuition. Rather than the *ad hoc* assumption that angular momentum is an integral multiple of $\hbar$ (in eqn 1.7), Bohr used the correspondence principle. This principle relates the behaviour of a system according to the known laws of classical mechanics and its quantum properties.

Assumption II The correspondence principle states that in the limit of large quantum numbers a quantum system tends to the same limit as the corresponding classical system.

Bohr formulated this principle in the early days of quantum theory. To apply this principle to hydrogen we first calculate the energy gap between adjacent electron orbits of radii r and r'. For large radii, the change $\Delta r = r' - r \ll r$.

(a) Show that the angular frequency $\omega = \Delta E/\hbar$ of radiation emitted when an electron makes a quantum jump between these levels is

$$\omega \simeq \frac{e^2/4\pi\epsilon_0}{2\hbar}\frac{\Delta r}{r^2}\,.$$

(b) An electron moving in a circle of radius r acts as an electric dipole radiating energy at the

orbital frequency ω given by eqn 1.4. Verify that this equation follows from eqn 1.3.

(c) In the limit of large quantum numbers, the quantum mechanical and classical expressions give the same frequency ω. Show that equating the expressions in the previous parts yields $\Delta r = 2\,(a_0 r)^{1/2}$.

(d) The difference in the radii between two adjacent orbits can be expressed as a difference equation.[44] In this case $\Delta n = 1$ and

$$\frac{\Delta r}{\Delta n} \propto r^{1/2}\,. \qquad (1.45)$$

This equation can be solved by assuming that the radius varies as some power x of the quantum number n, e.g. if one orbit is labelled by an integer n and the next by $n+1$, then $r = an^x$ and $r' = a\,(n+1)^x$. Show that $\Delta r = axn^{x-1} \propto n^{x/2}$. Determine the power x and the constant a.

6)

Rydberg atoms

(a) Show that the energy of the transitions between two shells with principal quantum numbers n and $n' = n + 1$ is proportional to $1/n^3$ for large n.

(b) Calculate the frequency of the transition between the $n' = 51$ and $n = 50$ shells of a neutral atom.

(c) What is the size of an atom in these *Rydberg states*? Express your answer both in atomic units and in metres.

7)

Incident photons of energy 10.39 keV are Compton scattered, and the scattered beam is observed at 45.00° relative to the incident beam. (*a*) What is the energy of the scattered photons at that angle? (*b*) What is the kinetic energy of the scattered electrons?

8)

X-ray photons of wavelength 0.02480 nm are incident on a target and the Compton-scattered photons are observed at 90.0°. (*a*) What is the wavelength of the scattered photons? (*b*) What is the momentum of the incident photons? Of the scattered photons? (*c*) What is the kinetic energy of the scattered electrons? (*d*) What is the momentum (magnitude and direction) of the scattered electrons?

9)

High-energy gamma rays can reach a radiation detector by Compton scattering from the surroundings, as shown in Figure 3.26. This effect is known as *back-scattering*. Show that, when $E \gg m_e c^2$, the back-scattered photon has an energy of approximately 0.25 MeV, independent of the energy of the original photon, when the scattering angle is nearly 180°.

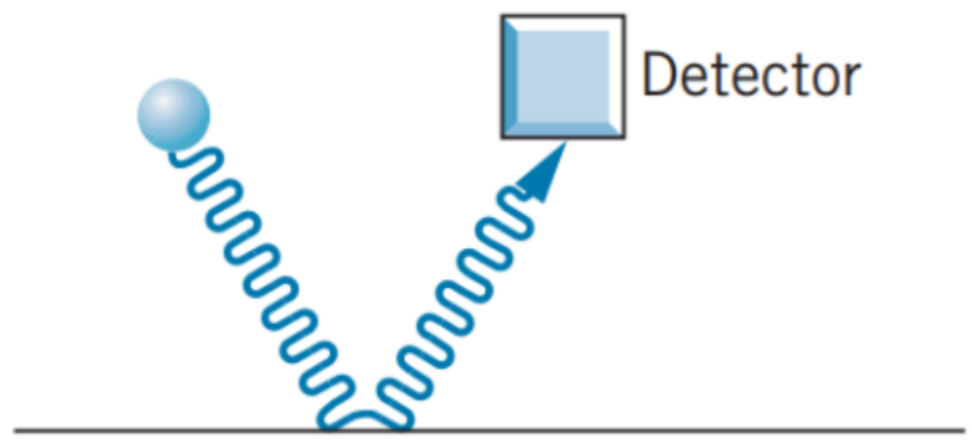


FIGURE 3.26

10)

Gamma rays of energy 0.662 MeV are Compton scattered. (*a*) What is the energy of the scattered photon observed at a scattering angle of 60.0°? (*b*) What is the kinetic energy of the scattered electrons?

11)

Suppose an atom of iron at rest emits an X-ray photon of energy 6.4 keV. Calculate the "recoil" momentum and kinetic energy of the atom. (*Hint:* Do you expect to need classical or relativistic kinetic energy for the atom? Is the kinetic energy likely to be much smaller than the atom's rest energy?)

12)

What is the minimum X-ray wavelength produced in bremsstrahlung by electrons that have been accelerated through 2.50×10^4 V?

13)

An atom absorbs a photon of wavelength 375 nm and immediately emits another photon of wavelength 580 nm. What is the net energy absorbed by the atom in this process?

14)

A photon of wavelength 7.52 pm scatters from a free electron at rest. After the interaction, the electron is observed to be moving in the direction of the original photon. Find the momentum of the electron.

15)

A hydrogen atom is moving at a speed of 125.0 m/s. It absorbs a photon of wavelength 97 nm that is moving in the opposite direction. By how much does the speed of the atom change as a result of absorbing the photon?

16)

Before a positron and an electron annihilate, they form a sort of "atom" in which each orbits about their common center of mass with identical speeds. As a result of this motion, the photons emitted in the annihilation show a small Doppler shift. In one experiment, the Doppler shift in energy of the photons was observed to be 2.41 keV. (*a*) What would be the speed of the electron or positron before the annihilation to produce this Doppler shift? (*b*) The positrons form these atom-like structures with the nearly "free" electrons in a solid. Assuming the positron and electron must have about the same speed to form this structure, find the kinetic energy of the electron. This technique, called "Doppler broadening," is an important method for learning about the energies of electrons in materials.

17)

A photon of energy E interacts with an electron at rest and undergoes pair production, producing a positive electron (positron) and an electron (in addition to the original electron):

$$\text{photon} + e^- \rightarrow e^+ + e^- + e^-$$

The two electrons and the positron move off with identical momenta in the direction of the initial photon. Find the kinetic energy of the three final particles and find the energy E of the photon. (*Hint:* Conserve momentum and total relativistic energy.)

18)

11. A certain crystal is cut so that the rows of atoms on its surface are separated by a distance of 0.352 nm. A beam of electrons is accelerated through a potential difference of 175 V and is incident normally on the surface. If all possible diffraction orders could be observed, at what angles (relative to the incident beam) would the diffracted beams be found?

19)

13. Sound waves travel through air at a speed of 330 m/s. A whistle blast at a frequency of about 1.0 kHz lasts for 2.0 s. (*a*) Over what distance in space does the "wave train" representing the sound extend? (*b*) What is the wavelength of the sound? (*c*) Estimate the precision with which an observer could measure the wavelength. (*d*) Estimate the precision with which an observer could measure the frequency.

20)

16. Estimate the signal processing time that would be necessary if you want to design a device to measure frequencies to a precision of no worse than 10,000 Hz.

21)

The Σ^* particle has a rest energy of 1385 MeV and a lifetime of 2.0×10^{-23} s. What would be a typical range of outcomes of measurements of the Σ^* rest energy?

22)

20. A pi meson (pion) and a proton can briefly join together to form a Δ particle. A measurement of the energy of the πp system (Figure 4.28) shows a peak at 1236 MeV, corresponding to the rest energy of the Δ particle, with an experimental spread of 120 MeV. What is the lifetime of the Δ?

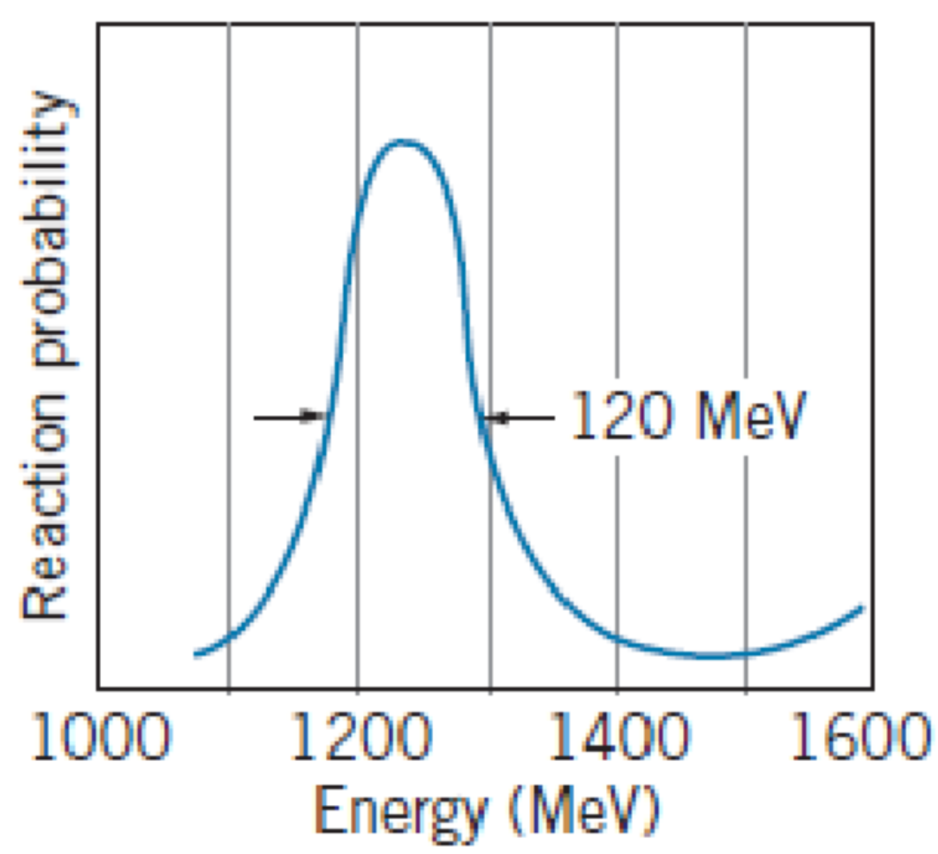


FIGURE 4.28 Problem 20.

23)

A beam of thermal neutrons (see Problem 2) emerges from a nuclear reactor and is incident on a crystal as shown in Figure 4.29. The beam is Bragg scattered, as in Figure 3.5, from a crystal whose scattering planes are separated by 0.247 nm. From the continuous energy spectrum of the beam we wish to select neutrons of energy 0.0105 eV. Find the Bragg-scattering angle that results in a scattered beam of this energy. Will other energies also be present in the scattered beam at that angle?

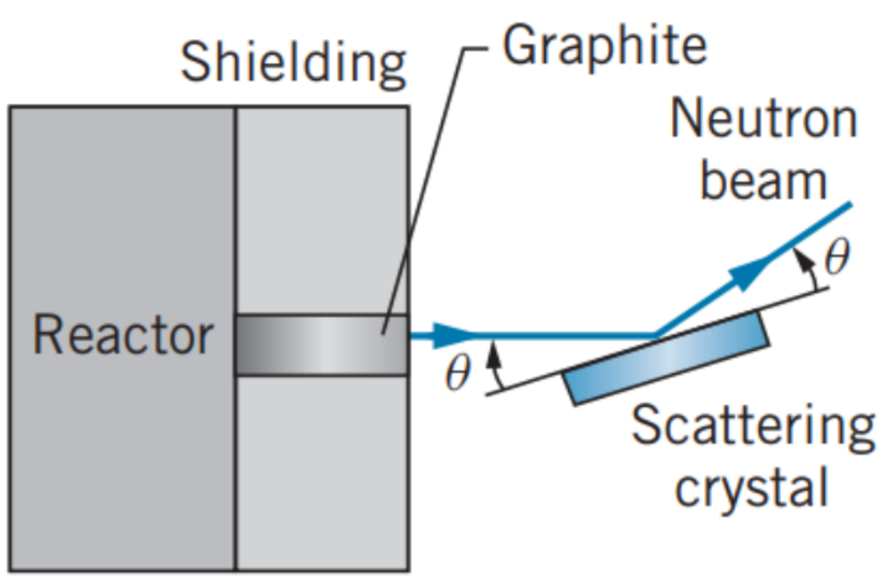


FIGURE 4.29

24)

36. In a metal, the conduction electrons are not attached to any one atom, but are relatively free to move throughout the entire metal. Consider a cube of copper measuring 1.0 cm on each edge. (*a*) What is the uncertainty in any one component of the momentum of an electron confined to the metal? (*b*) Estimate the average kinetic energy of an electron in the metal. (Assume $\Delta p = [(\Delta p_x)^2 + (\Delta p_y)^2 + (\Delta p_z)^2]^{1/2}$.) (*c*) Assuming the heat capacity of copper to be 24.5 J/mole · K, would the contribution of this motion to the internal energy of the copper be important at room temperature? What do you conclude from this? (See also

25)

X-ray photons of wavelength 0.02480 nm are incident on a target and the Compton-scattered photons are observed at 90.0°. (a) What is the wavelength of the scattered photons? (b) What is the momentum of the incident photons? Of the scattered photons? (c) What is the kinetic energy of the scattered electrons? (d) What is the momentum (magnitude and direction) of the scattered electrons?

26)

In Compton scattering, calculate the maximum kinetic energy given to the scattered electron for a given photon energy.

27)

The neutrons produced in a reactor are known as *thermal neutrons,* because their kinetic energies have been reduced (by collisions) until $K \cong \frac{3}{2}kT$ where T is room temperature. (a) What is the kinetic energy of such neutrons? (b) What is their de Broglie wavelength? Because this wavelength is of the same order as the lattice spacings of the atoms of a solid, neutron diffraction (like X-ray and electron diffraction) is a useful means of studying solid lattices.

28)

(a) Show that the group velocity and phase velocity are related by:

$$v_{\text{group}} = v_{\text{phase}} - \lambda \frac{dv_{\text{phase}}}{d\lambda}$$

(b) When white light travels through glass, the phase velocity of each wavelength depends on the wavelength. (This is the origin of dispersion and the breaking up of white light into its component colors—different wavelengths travel at different speeds and have different indices of refraction.) How does v_{phase} depend on λ? Is $dv_{\text{phase}}/d\lambda$ positive or negative? Therefore, is $v_{\text{group}} > v_{\text{phase}}$ or $< v_{\text{phase}}$?

29)

Show that dE/dp=v remains valid when E represents the relativistic kinetic energy of the particle.

30)

A particle is trapped in an infinite one-dimensional well of width L. If the particle is in its ground state, evaluate the probability to find the particle (a) between $x = 0$ and $x = L/3$; (b) between $x = L/3$ and $x = 2L/3$; (c) between $x = 2L/3$ and $x = L$.

31)

Using the normalization condition, show that the constant A has the value $(m\omega_0/\hbar\pi)^{1/4}$ for the one-dimensional simple harmonic oscillator in its ground state.

32)

A two-dimensional harmonic oscillator has energy $E = \hbar\omega_0(n_x + n_y + 1)$, where n_x and n_y are integers beginning with zero. (a) Justify this result based on the energy of the one-dimensional oscillator. (b) Sketch an energy-level diagram similar to Figure 5.7, showing the values of E and the quantum numbers n_x and n_y. (c) Show that each level is degenerate, with degeneracy equal to $n_x + n_y + 1$.

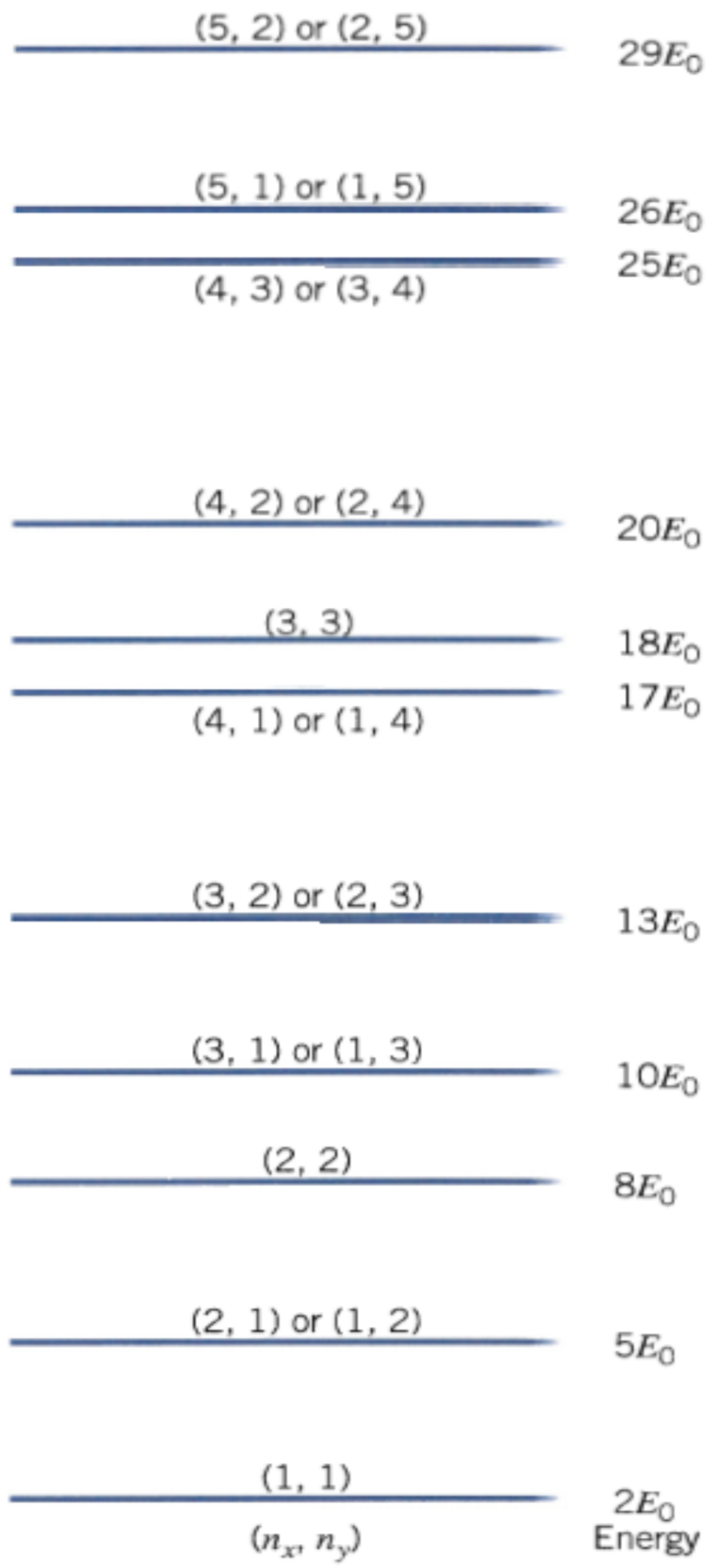


FIGURE 5.7 The lower permitted energy levels of the particle confined to the two-dimensional box.

33)

(a) Write down the wave functions for the three regions of the potential energy barrier (Figure 5.15) for $E < U_0$. You will need six coefficients in all. Use complex exponential notation. (b) Use the boundary conditions at $x = 0$ and at $x = a$ to find four relationships among the six coefficients. (Do not try to solve these relationships.) (c) Suppose particles are incident on the barrier from the left. Which coefficient should be set to zero? Why?

FIGURE 5.15 A barrier of height U_0 and width a.