ME363

Semester Technical Report

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Jojo Liang, YoungWoong Cho, Eunkyu Q Kim, Henry Son

Professor Luchtenburg

Cooper Union

Introduction

Helicopters are often employed to lift heavy external loads and drop them off in hard to reach areas. In the external loading configuration, the cargo is slung under the helicopter by cargo hooks then dropped off at the desired location. Typical sling loads are bluff bodies which are susceptible to aerodynamic instability. In particular, at high forward airspeeds, sling loads are subjected to yawing oscillations and / or complete rotations, which are often coupled with lateral oscillations (called swaying motion) of increasing or non-decreasing amplitude [1].

In the Spring 2020 session of ME363, we began studying a simplified version of this problem. We examined an autonomous unmanned aerial vehicle carrying a sling load with no aerodynamic coupling [2]. The objectives of the semester was to develop mathematical models that describe the system dynamics and implement Python programs that simulate the motion of the system under different scenarios.

Our work this semester will serve as the foundation for our summer investigation of more complicated drone-load systems. Furthermore, our scripts will allow us to make more realistic models with more complex operating conditions. Possible applications of the models include collaborative drone systems and comparison of how the loaded sling may behave and fail if it were to be modeled as elastic versus inelastic

Modeling the Drone

In our model, we use the physical parameters of the Crazyflie 2.0 nano-quadcopters [2].

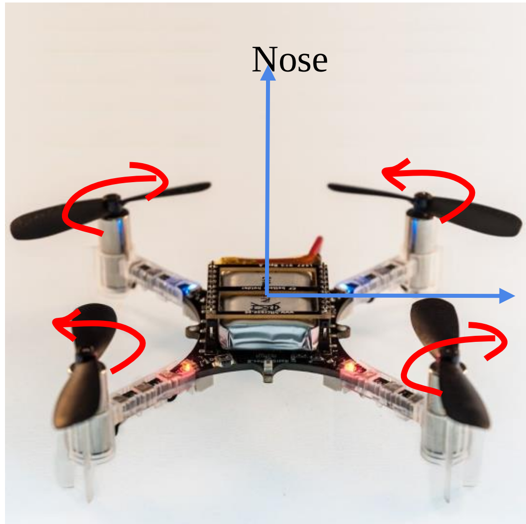
Parameter	**Description**	**Value**
m	Mass of drone	$0.33[kg]$
d	Arm length	$39.73 \times 10^{-3}[m]$
J_c	Mass moment of inertia around each axis of the body frame	$\begin{bmatrix} 1.395 \times 10^{-5} & 0 & 0 \\ 0 & 1.436 \times 10^{-5} & 0 \\ 0 & 0 & 2.173 \times 10^{-5} \end{bmatrix} [kg \times m^2]$
c_T	Rotor thrust coefficient	$3.1582 \times 10^{-10}[N/rpm^2]$
c_M	Rotor moment coefficient	$7.9379 \times 10^{-12}[N/rpm^2]$

With these physical parameters, we are able to calculate the effect of each rotor's rotational speed Ω_i. For instance, thrust and moment provided by each rotor can be found by

$$T_i = c_T \Omega_i^2$$

$$M_i = c_M \Omega_i^2$$

We will also distinguish each of our rotors and their corresponding rotation directions.



Coordinate Frames

The drone-load system is modeled using the moving frame method for multi-body dynamics [3]. First we define a fixed, inertial coordinate frame with its 1-direction $\underline{e}_1^I$ pointed north, 2-direction $\underline{e}_2^I$ pointed east, and 3-direction $\underline{e}_3^I$ pointed down toward the center of the earth. Then we define a non-inertial frame attached to the body of the drone. The body frame's 1-direction $\underline{e}_1^B$ aligns with the drone's nose, 2-direction $\underline{e}_2^B$ points starboard, and 3-direction $\underline{e}_3^B$ points down.

Fig -1: Inertial Frame and Body Frame

The two frames introduced are related by an orthonormal rotation matrix R such that

$$\underline{\underline{e}}^B = \underline{\underline{e}}^I R \quad (1)$$

$$\underline{\underline{e}}^I = \underline{\underline{e}}^B R^{-1}$$

For orthonormal rotation matrices, $R^{-1} = R^T$. Hence,

$$\underline{\underline{e}}^I = \underline{\underline{e}}^B R^T \quad (2)$$

While the inertial frame does not change with respect to time, the body frame translates and rotates as the drone translates and rotates. To characterize the state of the system through time, we must be ready to find how the body frame changes as time elapses $\dot{\underline{\underline{e}}}^B$. By taking the first time derivative of (1) and making a substitution using (2), we find that

$$\dot{\underline{\underline{e}}}^B = \underline{\underline{e}}^B R^T \dot{R}$$

Then we define a useful quantity $\tilde{\omega} = R^T \dot{R}$

$$\dot{\underline{\underline{e}}}^B = \underline{\underline{e}}^B \tilde{\omega}$$

It can also be found that

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_1 & \omega_2 \\ \omega_1 & 0 & -\omega_3 \\ -\omega_2 & \omega_3 & 0 \end{bmatrix}$$

In addition, $\underline{\tilde{\omega}}$ in matrix multiplication can be used in place of the operation $\underline{\omega} \times \_\_$

Equations of Motion

We can now write our equations of motion for the drone center of mass using Euler's Laws for rigid bodies. Euler's Second Law (E2L) relates the rotational momentum $\underline{\dot{H}}$ of a rigid body to applied external moments. In general,

$$\dot{H} = \sum_{i=1}^{n} M_i$$

This law can be applied to measurements made in the drone's body frame. Each thrust T_i provides a moment M_i, and the drone body's mass moment of inertia is represented by the constant matrix J_c

$$J_c \underline{\dot{\omega}} = -\tilde{\omega} J_c \underline{\omega} + \underline{\tau}\text{, where}$$

$$\underline{\tau} = \begin{bmatrix} \frac{d}{\sqrt{2}}[(T_2 + T_3) - (T_1 + T_4)] \\ \frac{d}{\sqrt{2}}[(T_1 + T_2) - (T_3 + T_4)] \\ [(M_2 + M_4) - (M_1 + T_3)] \end{bmatrix}$$

Isolation and integration of $\underline{\dot{\omega}}$ gives the rate of rotation $\underline{\omega}$ of the drone body frame for all time.

Next, we use Euler's First Law to find the translation of the drone center of mass. E1L relates the acceleration of the center of mass $\underline{\ddot{x}}$ of a rigid body to external forces applied to the rigid body.

$$m\underline{\ddot{x}} = \sum_{i=1}^{n} F_i$$

E1L is only applicable to inertial frame measurements. Since measurements taken by on-board sensors are given the body frame, we favor a form of E1L expressed in body frame quantities. In order to convert between the inertial and body frames, we make use of the rotation matrix R.

We rewrite E1L by defining a convenient quantity $\underline{u}^B = R^T \dot{\underline{x}}$ which then yields E1L rewritten in terms of body frame measurements. By our definition of the body frame, each of the rotors provide a thrust T_i in the $-\underline{e}_3^B$ direction. This yields

$$m(\tilde{\omega}\underline{u}^B + \dot{\underline{u}}^B) = R^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + \sum_{i=1}^{4} \begin{bmatrix} 0 \\ 0 \\ -T_i \end{bmatrix}$$

Before we can isolate and integrate $\dot{\underline{u}}^B$ we need to find how R^T evolves with time. We use the Rodrigues' Formula to construct the rotation matrix at discrete points in time using Legendre polynomials such that

$$R = I + \frac{\tilde{\omega}}{||\underline{\omega}||} \sin ||\omega|| t + \frac{\tilde{\omega}^2}{||\underline{\omega}||^2}(1 - \cos ||\underline{\omega}|| t)$$

This method is preferred over simply integrating a differential equation of R with respect to time because numerical computation of integrals yield small errors that accumulate quickly. On the other hand, using Legendre polynomials preserves the orthogonality between the body frame directions and yields more accurate evolutions of the body frame.

Simultaneous Integration

If we wish to solve for translation and rotation simultaneously, we can concatenate E1L and E2L in a matrix equation

$$M\ddot{X} + DM\dot{X} = G,$$

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_c & 0 & 0 \\ 0 & 0 & 0 & 0 & J_c & 0 \\ 0 & 0 & 0 & 0 & 0 & J_c \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

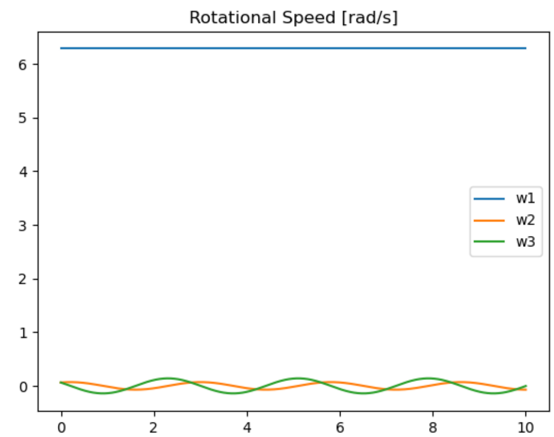
$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega_1 & \omega_2 \\ 0 & 0 & 0 & \omega_1 & 0 & -\omega_3 \\ 0 & 0 & 0 & -\omega_2 & \omega_3 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

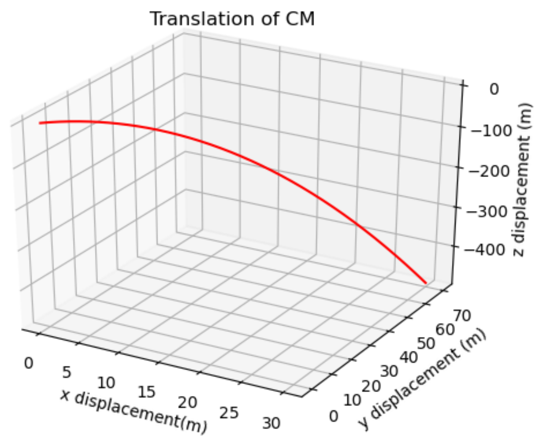
Results

To test our equations of motion, we wrote a few scripts with simple test cases to see if the behavior is as expected.

Center of Mass Translation and Rotation

In the first test case, we gave the a rigid body nonzero initial translational and rotational velocities.

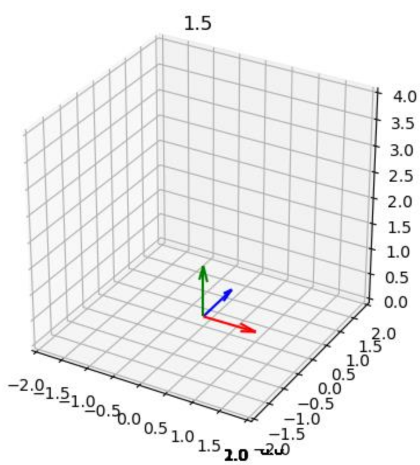


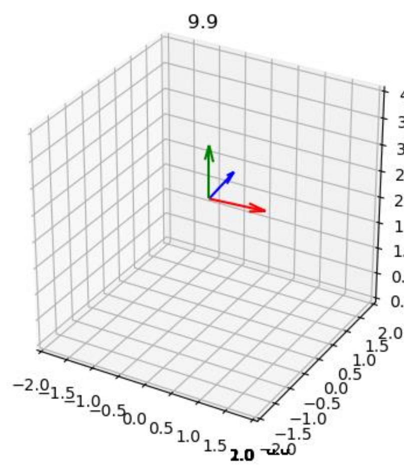


We see, as expected, that the center of mass draws out a parabola as it free falls and rotation about one direction is steady while the other two see oscillation.

Upward Force

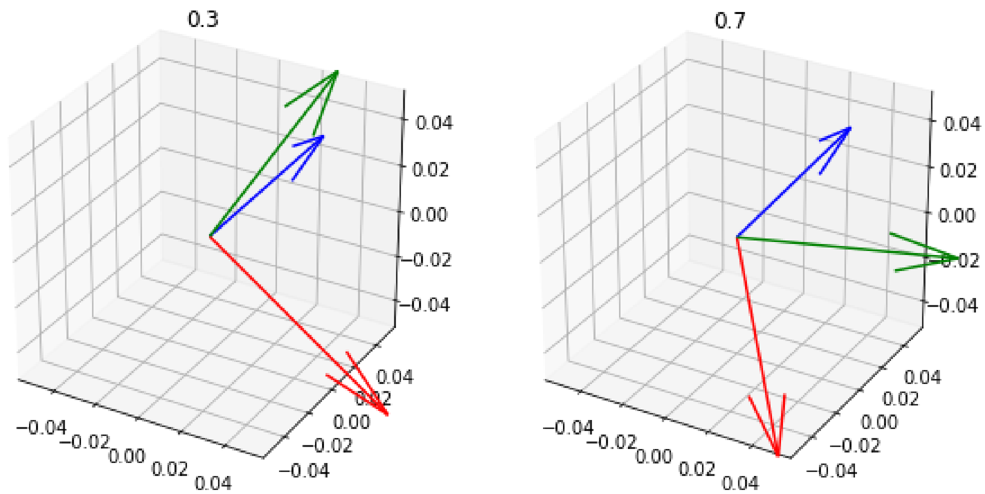
This script implements rotor thrust, but balanced then such that no net moment is exerted on the drone. Hence, we see that the body frame ascends steadily from a purely upward force.





Rotating Body Frame (Rodrigues)

This test case implemented the Rodrigues Formula. The body is subjected to a nonzero initial rotational velocity. We verified the validity of using the Rodrigues Formula in our model by observing that the body frame vectors remain orthogonal to one another through time.



Conclusion

Seeing that the semester's work is largely foundational, there is not much to be drawn from our results except that the model we have so far is valid for the simple cases that we have posed. However, this work will be useful for continuing our work in the Summer 2020 term. Next steps can include developing more complex scenarios that involve collaborating drones, elastic/inelastically slung loads, more sophisticated tasks, rotor failure, ground effect, etc. Furthermore, we may begin to learn and develop controllers for drones in each of those situations.

Cited Works

[1] Cyr, D., Guarino, P., Hitchen, J., Morar, R., Sperry, J., Cowlagi, R.V., Olinger, D.J. and Nyren, D.J., 2016. *Stabilization of Helicopter Sling Loads with Passive and Active Control Surfaces*. In 54th AIAA Aerospace Sciences Meeting (p. 2031).

[2] Pounds, P.E., Bersak, D.R. and Dollar, A.M., 2012. *Stability of small-scale UAV helicopters and quadrotors with added payload mass under PID control.* Autonomous Robots, 33(1-2), pp.129-142.

[3] Murakami, H., 2013, "A Moving Frame Method for Multi-Body Dynamics Using SE(3)". Proceedings of the ASME 2015 International Mechanical Engineering Congress and Exposition, Paper IMECE2015-51192.

[4] Luis, C., & Le Ny, J. (2016). *Design of a trajectory tracking controller for a nanoquadcopter*. Technical report, Mobile Robotics and Autonomous Systems Laboratory, Polytechnique Montreal.