Beam Bending Lab

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Experimental

The experiment was performed using a cantilevered aluminum beam (see all dimensions in *Appendix 1*). The deflection was measured using a micrometer and the strain was measured using a Wheatstone bridge configured strain gauge. The beam was preloaded with a hook that was used to hold the weights. Successive increases of 0.1lbs weights were added onto the beam at once until 0.7lbs was reached. Then the weights were incrementally removed until only the hook remained. Deflection and strain gauge measurements were taken at each weight change.

Results

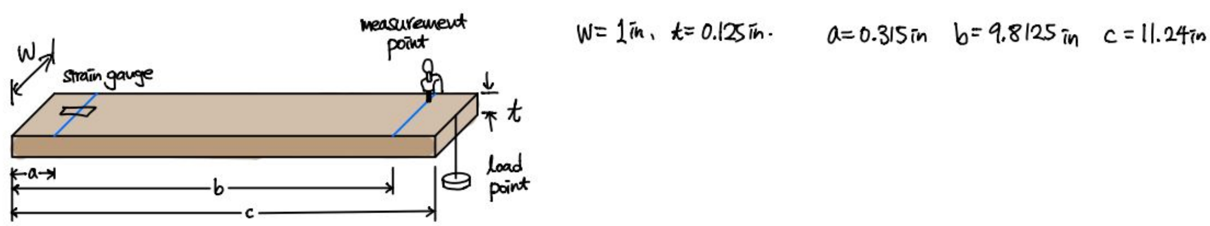


Figure 1: Experimental setup. (credit: YoungWoong Cho)

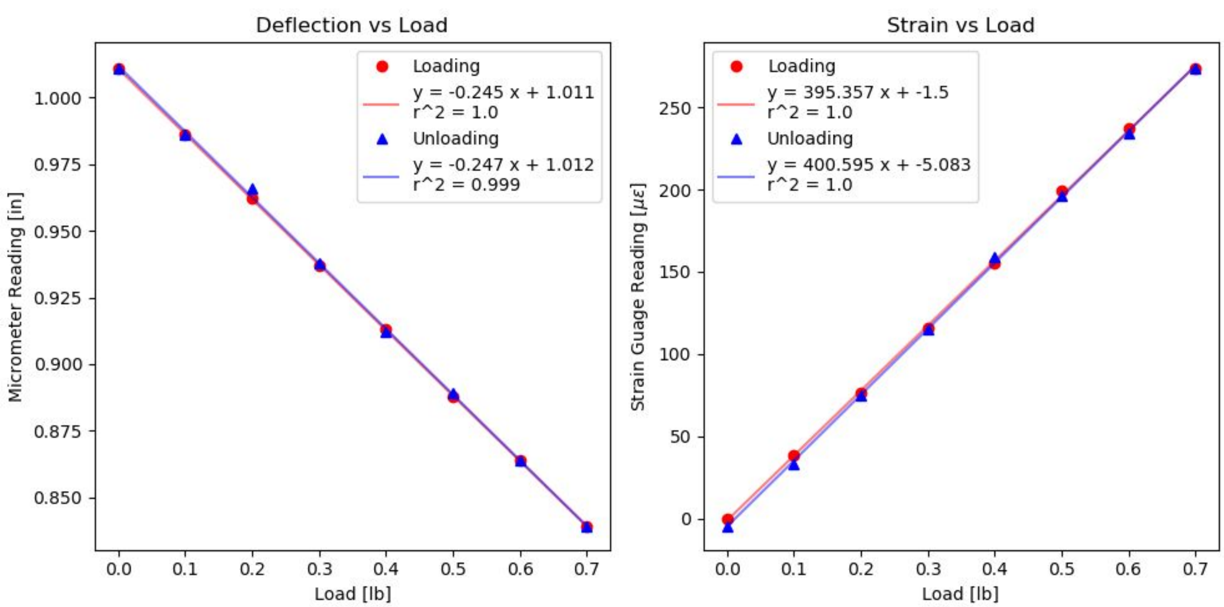


Figure 2: Plots of the deflection vs load and the strain vs load. Each plot shows the linear function and the r^2 value. Note 1: The plots show r^2 = 1.0 might be slightly lower, but were

rounded because of how close to 1.0 they came out. Note 2: Because of how close the values were, the loading and unloading points are difficult to distinguish. (credit: Jared Jacobowitz)

Analysis: E(strain) and E(y)

E(strain) Derivation:

$$d = \frac{1}{2}t \quad [in]$$

$$I = \frac{1}{12}wt^3 \quad [in^4]$$

$$\sigma = \frac{Md}{I} = \frac{P(c-a)d}{I} = \epsilon E \rightarrow \epsilon(P) = \frac{(c-a)d}{IE}P \rightarrow \frac{\mathrm{d}\epsilon}{\mathrm{d}P} = \frac{(c-a)d}{IE}$$

$$\therefore E = \frac{(c-a)d}{I(\mathrm{d}\epsilon/\mathrm{d}P)} \quad \left[\frac{in \cdot in}{in^4 \cdot lb^{-1}} = lb_f/in^2 = psi\right]$$

Using the average of the two slopes for $\mathrm{d}\epsilon/\mathrm{d}F = 39.8 \times 10^{-5}\, lb^{-1}$ and $I = 1.63 \times 10^{-4}\, in^4$ then $E(\mathrm{d}\epsilon/\mathrm{d}F) = 10.5Mpsi$. (see *Appendix 3* for hand calculations)

$E(\mathrm{d}\epsilon/\mathrm{d}F) = 10.5Mpsi$. (see Appendix 3 for hand calculations).

E(y) Derivation:

Starting with $y = -Pb^2(3c-b)/(6EI)$ from Appendix D from the textbook (see *Appendix 3* for hand derivation of this), the equation can be differentiated and manipulated as such:

$$\mathrm{d}y = \frac{-\mathrm{d}Pb^2}{6EI}(3c-b)$$

$$\therefore E = \frac{-b^2}{6I(\mathrm{d}y/\mathrm{d}P)}(3c-b) \quad \left[\frac{in^2 \cdot in}{in^4 \cdot in \cdot lb^{-1}} = lb_f/in^2 = psi\right]$$

Plugging in the average slope $\mathrm{d}y/\mathrm{d}P = -0.246\, in/lb$, then $E(\mathrm{d}y/\mathrm{d}P) = 9.58Mpsi$.

Discussion

The experiment had a couple of sources of error, though none of them were very large. The first error source was in the measurement of the test setup, but this was done mainly using a micrometer, which has a very small uncertainty. The second source is in the weights used. To our knowledge, the weights were not precision weights. The third source was the strain gauge. There was slightly drifting on the readings, though this is not seen in the data, so it would not have contributed significantly error, but the gauge was only accurate to the one's place, so there is some uncertainty to the value. Despite all these errors being small, they accumulate.

Hyoungseob Kim performed error propagations for each method of computing the modulus of elasticity. Using the strain vs load slope the error was ±0.35Mpsi and using the deflection vs load slope the error was ±0.47Mpsi. Using the ranges of the uncertainty, the calculated values were compared to the known values of 7075 and 6061 aluminum with Young's modulus of 10.4Mpsi and 10.1Mpsi, respectively. These values were from the back of the textbook in Appendix B. The calculated value was compared to these two aluminum alloys because they are very common, but there is always some variance in the actual modulus for a given sample. For the strain vs load, when compared to 7075 aluminum, the percent difference ranged from -4.8% to +2.1% and for 6061 it ranged from -7.9% to -0.9%. For the deflection vs load, when compared to 7075 aluminum, the percent difference ranged from +3.3% to +12.4% and for 6061 it ranged from +0.5% to +5.1%. These differences are quite modest and reasonable, including the actual value within or very close to the calculated value.

Comparing the two calculated values, the percent difference ranges of the strain vs load were smaller than the deflection vs load. This is surprising since the strain gauge had a much higher uncertainty than the micrometer. The reason for the lower precision of the micrometer might have been due to difficulty in feeling when the micrometer's anvil was in contact with the beam. This concern was expressed by Prof. Wootton who performed this experiment for us.

With the largest load, the beam experienced 3ksi of stress. Comparing this to the known stress values in Appendix B of the textbook, the beam was not at all close to yielding. This is consistent with the data that shows the beam returning to the exact same position upon removing the load.

Appendix 1: Data Tables

Experimental Setup Measurements

Experimental Parameters	Value	Units	Uncertainty
Material	Aluminum	N/A	N/A
Thickness	0.125	in	0.002
Width	1.00	in	0.01
Distance Clamp to Load point	11.24	in	0.01
Distance Camp to measurement point	9.8125	in	0.01
Distance Clamp to Strain Gauge center	0.315	in	0.01

Table 1: Measurements for beam dimensions and instrument locations (with uncertainty).

Logged Data

UNITS	lbs	lbs	in	με
# of Weights	Weight Added	Cumulative Load	Micrometer Reading	Strain Gauge Reading
Hook	0	0	1.011	0

1	0.1	0.1	0.986	38
2	0.1	0.2	0.962	76
3	0.1	0.3	0.937	116
4	0.1	0.4	0.913	155
5	0.1	0.5	0.888	199
6	0.1	0.6	0.864	237
7	0.1	0.7	0.839	274
7	0	0.7	0.839	274
6	-0.1	0.6	0.864	234
5	-0.1	0.5	0.889	196
4	-0.1	0.4	0.912	159
3	-0.1	0.3	0.938	115
2	-0.1	0.2	0.966	75
1	-0.1	0.1	0.986	33
Hook	-0.1	0	1.011	-5

Table 2: Table of the micrometer and strain gauge measurements for each cumulative load.

Appendix 2: Roles of Each Author

YoungWoong Cho - Drew test setup, logged data for *Appendix 1*, and provided hand calculations of HW8 and HW9.

Jared Jacobowitz - Setup document, created plots, wrote the *Experimental, Analysis,* and *Discussion* sections, and inserted data for *Appendix 1*.

Hyoungseob Kim - Logged data for *Appendix 1* and provided error propagation calculations for HWs 8 and 9.

Appendix 3: HW8 and HW9 Hand Calculations

Derivation of the modulus of elasticity from the experimental strain data (credit: YoungWoong Cho).

From the lab dataset, the best fit slope of strain vs load is 39.5 and 40.1. Considering that the strain is measured in micro scales and the load is increasing by 0.1, the average slope is

$$\frac{d\varepsilon}{dF_{ave}} = \frac{1}{2}\left[\frac{(39.5\times10^{-6})}{0.1} + \frac{(40.1\times10^{-6})}{0.1}\right] = 39.8\times10^{-5}\ /lbs$$

From $E = \frac{\sigma}{\varepsilon}$ and $\sigma = \frac{My}{I}$, $\quad E\varepsilon = \frac{My}{I} = \frac{V(c-a)\cdot\frac{1}{2}t}{\frac{1}{12}wt^3} = \frac{6P(c-a)}{wt^2} \quad \Rightarrow \varepsilon(P) = \frac{6(c-a)}{wt^2 E}P$

$$\frac{d\varepsilon}{dP} = \frac{6(c-a)}{wt^2 E} \Rightarrow E = \frac{6(c-a)}{wt^2\left(\frac{d\varepsilon}{dP}\right)} = \frac{6(11.24in - 0.315in)}{(1in)(0.125in)^2(39.8\times10^{-5}/lbs)} = 10.54\times10^6\ psi$$

$\Rightarrow E_{AR} = 10.54\times10^6\ psi$

Error Propagation of the modulus of elasticity from the experimental strain data (credit: Hyoungseob Kim).

$$\delta(\text{strain vs load})_L = \sqrt{\left(\frac{\partial y}{\partial x}\delta x\right)^2 + (0)^2} = 0$$

$\delta P = 0\,\text{lb}$ $\delta w = 0.01\,\text{in}$ (width) $\delta d_m = 0.01\,\text{in}$ (distance to measurement point)

$\delta t = 0.02\,\text{in}$ (thickness) $\delta d_L = 0.01\,\text{in}$ (distance to load point) $\delta d_s = 0.01\,\text{in}$ (distance to strain gauge)

$$\delta(\text{strain vs load})_V = \sqrt{\left(\frac{\partial y}{\partial x}\delta x\right)^2 + (0)^2} = 0$$

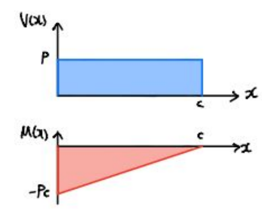
there is no error propagation in slope since there is no error on weight

$$\delta E = \sqrt{\left(\frac{\partial E}{\partial d_L}\delta d_L\right)^2 + \left(\frac{\partial E}{\partial d_s}\delta d_s\right)^2 + \left(\frac{\partial E}{\partial w}\delta w\right)^2 + \left(\frac{\partial E}{\partial t}\delta t\right)^2 + \cancel{\left(\frac{\partial E}{\partial\left(\frac{\partial \varepsilon}{\partial P}\right)}\delta\frac{\partial \varepsilon}{\partial P}\right)^2}^{\,0}}$$

$$= \left[\left[\frac{6\cdot 0.01}{(1)(0.125)^2(32.8\times10^{-5})}\right]^2 + \left[\frac{-6\cdot 0.01}{(1)(0.125)^2(32.8\times10^{-5})}\right]^2 + \left[-\frac{6(11.24-0.315)(0.01)}{(1)^2(0.125)^2(32.8\times10^{-5})}\right]^2 + \left[-\frac{12\cdot(11.24-0.315)(0.02)}{(1)(0.125)^3(32.8\times10^{-5})}\right]^2\right]^{1/2}$$

$$\delta E = 3.54\times10^5\ \text{Psi} = \boxed{\pm 0.354\ \text{MPsi}}$$

Derivation of the modulus of elasticity from the experimental deformation data (credit: YoungWoong Cho).



Since the load is being exerted at the opposite end of the beam, $\begin{cases} V(x) = P \\ M(x) = Px - Pc \end{cases}$

Now derive an equation that relates load to deflection.

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} = \frac{P}{EI}x - \frac{Pc}{EI} \quad\Rightarrow\quad \frac{dy}{dx} = \int \frac{M(x)}{EI}dx = \frac{P}{2EI}x^2 - \frac{Pc}{EI}x + c_1$$

Using the boundary condition $\left.\frac{dy}{dx}\right|_{x=0} = 0$, $\quad \left.\frac{dy}{dx}\right|_{x=0} = c_1 = 0 \quad\Rightarrow\quad \frac{dy}{dx} = \frac{P}{2EI}x^2 - \frac{Pc}{EI}x$

$$y(x) = \int \frac{dy}{dx}dx = \int\left(\frac{P}{2EI}x^2 - \frac{Pc}{EI}x\right)dx = \frac{P}{6EI}x^3 - \frac{Pc}{2EI}x^2 + c_0$$

Using the boundary condition $y(0) = 0$, $\quad y(0) = c_0 = 0 \quad\Rightarrow\quad y(P,x) = \frac{P}{6EI}x^3 - \frac{Pc}{2EI}x^2$

Since the deflection is measured at the point b from the clamp, $y(P) = P\left(\frac{b^3}{6EI} - \frac{b^2c}{2EI}\right)$

Therefore the slope of the deflection-force curve is $\frac{dy}{dP} = \frac{b^3}{6EI} - \frac{b^2c}{2EI} = \frac{1}{E}\cdot\frac{b^2}{6I}(b-3c)$

$\Rightarrow E = \left(\frac{dy}{dP}\right)^{-1}\frac{b^2}{6I}(b-3c)$ with $b = 9.8125\,\text{in}$, $c = 11.24\,\text{in}$, $I = \frac{1}{12}wt^3 = \frac{1}{12}(1\,\text{in})(0.125\,\text{in})^3 = 1.63\times10^{-4}\,\text{in}^4$

From the lab dataset, the best fit slope of deflection vs. load is -0.0247 and -0.0245. Considering that the load is increasing by 0.1, the average slope is

$$\left(\frac{dy}{dP}\right)_{ave} = \frac{1}{2}\left[\frac{(-0.0247)}{0.1} + \frac{(-0.0245)}{0.1}\right] = -0.246 \text{ in/lbs}$$

$$\Rightarrow E = \left(\frac{dy}{dP}\right)^{-1} \frac{b^2}{6I}(b-3c) = \left(\frac{1}{-0.246 \text{ in/lbs}}\right) \frac{(9.8125 \text{ in})^2}{6(1.63\times10^{-4} \text{ in}^4)}(9.8125 \text{ in} - 3\cdot 11.24 \text{ in})$$

$$= 9.568\times10^6 \text{ psi} \qquad \Rightarrow E_{AE} = 9.568\times10^6 \text{ psi}$$

Error propagation of the modulus of elasticity from the experimental deformation data(credit: Hyoungseob Kim).

$\delta P = 0$ lb $\quad \delta W = 0.01$ in (width) $\quad \delta d_m = 0.01$ in (distance to measurement point)

$\delta t = 0.02$ in (thickness) $\quad \delta d_L = 0.01$ in (distance to load point) $\quad \delta d_s = 0.01$ in (distance to strain gauge)

$\delta \frac{\partial y}{\partial P} = 0$ since there is no error in weights

$$\delta E = \sqrt{\left(\frac{\partial E}{\partial d_m}\delta d_m\right)^2 + \left(\frac{\partial E}{\partial w}\delta w\right)^2 + \left(\frac{\partial E}{\partial t}\delta t\right)^2 + \left(\frac{\partial E}{\partial d_s}\delta d_s\right)^2}$$

$$= \left[\left[\frac{0.01}{0.246}\frac{9.8125}{6(1.63\times10^{-4})}\left[2(9.8125-3\cdot11.24) - 9.8125(9.8125-3\cdot11.24)\right]\right]^2 + \left[\frac{0.01}{0.246}\left(\frac{9.8125^2}{6(\frac{1}{12}(0.125)^3)}\right)(9.8125-3\cdot11.24)\right]^2\right.$$

$$\left. + \left[\frac{0.002}{0.246}\left(\frac{9.8125^2(-3)}{6(\frac{1}{12}(1)(0.125)^4)}\right)(9.8125-3\cdot11.24)\right]^2 + \left[\frac{1}{0.246}\left(\frac{9.8125^2}{6(1.63\times10^{-4})}(-3)(0.01)\right)^2\right]\right]^{1/2}$$

$$\delta E = 4.7\times10^5 \text{ Psi} = \boxed{\pm 0.47 \text{ MPsi}}$$