1 Many-to-Many Matching with Rational Inattention

When player i of type t meets player j of type s, the match utility ε_i^j is drawn from $F_t^s(\varepsilon)$, not observable to player i, but could be learned costly through signal x.

The cost of a signal x is proportional to $\mathcal{I}(\varepsilon, x) = \text{ Entropy } (\varepsilon) - \mathbb{E}_{x[\text{ Entropy } (\varepsilon|x)]}$, let the cost parameter be $\dfrac{1}{\lambda.}$

1.1 One-to-one matching case $(K = 1)$:

With complete information:

Player i can observe the match utility ε_i^j, then acceptance rule is a threshold rule:

$$\begin{cases} A & \text{if } \varepsilon_i^j > x_t^s = V_{t,\emptyset} \\ R & \text{otherwise} \end{cases}$$

where $V_{t,\emptyset}$ is the continuation value of a type t.

With rational inattention:

Player i chooses a signal with distribution $f(x|\varepsilon) : \mathbb{R} \to \Delta(X)$ and a strategy $\sigma : X \to \{A, R\}$ that maximizes:

$$\max_{f_i,\sigma_i} E\left[u_i(\sigma_i, \sigma_{-i}, \varepsilon_i, \varepsilon_{-i}) - \frac{1}{\lambda} I(\varepsilon, x)\right]$$

where $u_i(A, A, \varepsilon_i, \varepsilon_{-i}) = \varepsilon_{-i}$

and $u_i(A, R, \varepsilon_i, \varepsilon_{-i}) = u_i(R, A, \varepsilon_i, \varepsilon_{-i}) = u_i(R, R, \varepsilon_i, \varepsilon_{-i}) = V_{t,\emptyset}$

This problem can be simplified to choosing a choice rule $\pi_i(\varepsilon)$ that maximizes:

$$\max_{\pi_i} \mathbb{E}\left[u_i(\sigma_i, \sigma_{-i}, \varepsilon_i, \varepsilon_{-i}) - \frac{1}{\lambda} I(\varepsilon_{-i}, x)\right]$$

where $\pi_i(\varepsilon) = \mathbb{P}\{\sigma_i(x) = A|\varepsilon_{-i} = \varepsilon\}$

Substituting in the utility functions and the opponent's choice rule:

$$\max_{\pi_i} \mathbb{E}_{\varepsilon\left[\pi_i(\varepsilon_{-i})\pi_{-i(\varepsilon_i)}\varepsilon_{-i} + \left(1 - \pi_i(\varepsilon_{-i})\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset} - c_I I(\varepsilon_{-i}, \sigma_i)\right]}$$

The solution is given by:

$$\pi_i\left(\varepsilon_{-i}\right)=\frac{q_i e^{u_i(A,\varepsilon_{-i})}}{q_i e^{u_i(A,\varepsilon_{-i})}+\left(1-q_i\right)e^{u_i(R,\varepsilon_{-i})}}$$

where q_i maximizes: $\mathbb{E}_{\pi_0}\left[\log\left(q_i e^{u_i(A,\varepsilon_{-i})}+\left(1-q_i\right)e^{u_i(R,\varepsilon_{-i})}\right)\right]$

Substituting in the utility functions:

$$\pi_i\left(\varepsilon_{-i}\right)=\frac{q_i e^{\mathbb{E}_{\varepsilon_i}\left[\pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}\right]}}{q_i e^{\mathbb{E}_{\varepsilon_i}\left[\pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}\right]}+\left(1-q_i\right)e^{V_{t,\emptyset}}}$$

where q_i maximizes: $\mathbb{E}_{\varepsilon_{-i}}\left[\log\left(q_i e^{\pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}}+\left(1-q_i\right)e^{V_{t,\emptyset}}\right)\right]$

The derivative has the same sign as: $e^{\pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}}-e^{V_{t,\emptyset}}$

Therefore, $q_i=$

$$\begin{cases}0 & \text{if } \pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}>V_{t,\emptyset}\\ 1 & \text{if } \pi_{-i(\varepsilon_i)}\varepsilon_{-i}+\left(1-\pi_{-i(\varepsilon_i)}\right)V_{t,\emptyset}<V_{t,\emptyset}\end{cases}$$

or, $q_i=$

$$\begin{cases}0 & \text{if } \varepsilon_{-i}>V_{t,\emptyset}\\ 1 & \text{if } \varepsilon_{-i}<V_{t,\emptyset}\end{cases}$$

also, $\pi_i\left(\varepsilon_{-i}\right)=$

$$\begin{cases}0 & \text{if } \varepsilon_{-i}>V_{t,\emptyset}\\ 1 & \text{if } \varepsilon_{-i}<V_{t,\emptyset}\end{cases}$$

$$\pi_t^s=\mathbb{E}_{\varepsilon[\pi_i(\varepsilon_{-i})]}=F_t^s\left(V_{t,\emptyset}\right)$$

2 Many-to-Many Matching with Learning

2.1 Complete Information case (match utility is type):

Each player has types $t \in \mathcal{T}$, meets with each other in continuous time τ

Denote the value of type s at time $\tau V_{s,\tau}$ and the value of a match between s and s' is $U_s^{s'}$

Parameters c_s is the cost of waiting and $\rho_s^{s'}$ the rate at which a type s meets with a type s'

2.2 Values

Let $\lambda_s^{s'}$ be the rate of arrival of offers (meeting and acceptance)

$$
\begin{aligned}
V_{s,\tau} &= \mathbb{E}_{s'}\left[\lambda_s^{s'} d\tau \max\left\{V_{s,\tau+d\tau}, U_s^{s'}\right\} + \left(1-\lambda_s^{s'} d\tau\right)\left(V_{s,\tau+d\tau}\right)\right] - c_s d\tau \\
c_s d\tau &= \mathbb{E}_{s'}\left[\lambda_s^{s'} d\tau \max\left\{U_s^{s'} - V_{s,\tau+d\tau}, 0\right\} + V_{s,\tau+d\tau} - V_{s,\tau}\right] \\
c_s &= \mathbb{E}_{s'}\left[\lambda_s^{s'} \max\left\{U_s^{s'} - V_{s,\tau+d\tau}, 0\right\}\right] + \frac{V_{s,\tau+d\tau} - V_{s,\tau}}{d\tau} \\
c_s &= \mathbb{E}_{s'}\left[\lambda_s^{s'} \max\left\{U_s^{s'} - V_{s,\tau}, 0\right\}\right] + \dot{V}_{s,\tau}
\end{aligned}
$$

2.3 Continuous Type

With continuous types s and in equilibrium:

$$
c_s = \int_{\underline{s}}^{\bar{s}} \lambda_s^{s'} \max\left\{U_s^{s'} - V_s, 0\right\} dF\left(s'\right)
$$

Suppose each player with type s uses the threshold strategy $x^\star(s)$, when meeting with a type s':

$$
\begin{cases}
A & \text{if } s' \geq x^\star(s) \\
R & \text{otherwise}
\end{cases}
$$

Then the values should be equal at threshold: $V_s = U_s^{x^\star(s)}$

and $\lambda_s^{s'} = \left(1 - F\left(x^\star\left(s'\right)\right)\right)\rho_s^{s'}$

$$
\begin{aligned}
c_s &= \int_{\underline{s}}^{\bar{s}} \rho_s^{s'}\left(1 - F\left(x^\star\left(s'\right)\right) \max\left\{U_s^{s'} - U_s^{x^\star(s)}, 0\right\} dF\left(s'\right)\right. \\
&= \left(1 - F\left(x^\star\left(s'\right)\right)\right) \int_{x^\star(s)}^{\bar{s}} \rho_s^{s'}\left(U_s^{s'} - U_s^{x^\star(s)}\right) dF\left(s'\right) \\
&= \left(1 - F\left(x^\star\left(s'\right)\right)\right) \int_{x^\star(s)}^{\bar{s}} \left(1 - F\left(s'\right)\right) \frac{\partial}{\partial s'}\left(\rho_s^{s'}\left(U_s^{s'} - U_s^{x^\star(s)}\right)\right) ds'
\end{aligned}
$$

This equation charaterizes the set of thresholds $x^\star(s)\ \forall\, s \in \mathcal{T}$ on both sides of the market

2.4 Discrete Types

With discrete types s:

The same threshold strategy is used but $x^\star(s)$ is determined by:

$$U_s^{x^\star(s)-1} \leq V_s \leq U_s^{x^\star(s)}$$

$$c_s = \sum_{s'=x^\star(s)}^{\bar{s}} \rho_s^{s'} \left(1 - F\left(x^\star(s')\right)\right) \left(U_s^{s'} - U_s^{x^\star(s)}\right) p_s$$

where $p_s = dF(s) = \mathbb{P}\{S = s\}$

2.5 Two Types

With two types $s \in \{h, l\}$:

Either $x^\star(h) = h$ or $x^\star(h) = l$

Case 1 : $x^\star(h) = h$:

$$c_h = \rho_h^h p_h \left(U_h^h - V_h\right) p_h$$

$$\Rightarrow V_h = U_h^h - \frac{c_h}{\rho_h^h p_h^2}$$

$$\text{then } U_h^l \leq V_h \leq U_h^h \text{ implies}$$

$$\Rightarrow U_h^l \leq U_h^h - \frac{c_h}{\rho_h^h p_h^2} \leq U_h^h$$

$$\Rightarrow 0 \leq \frac{c_h}{\rho_h^h p_h^2} \leq U_h^h - U_h^l$$

The above condition gives the equilibrium with only h-h and l-l matchings

Case 2 : $x^{\star}(h) = l$:

$$c_h = \rho_h^h \left(U_h^h - V_h\right) p_h + \rho_h^l \left(U_h^l - V_h\right) p_l$$
$$\Rightarrow V_h = \frac{\rho_h^h U_h^h p_h + \rho_h^l U_h^l p_l - c_h}{\rho_h^h p_h + \rho_h^l p_l}$$
$$\Rightarrow V_h = U_h^l + \frac{\rho_h^h \left(U_h^h - U_h^l\right) p_h - c_h}{\rho_h^h p_h + \rho_h^l p_l}$$

then $V_h \leq U_h^l$ implies

$$\Rightarrow U_h^l + \frac{\rho_h^h \left(U_h^h - U_h^l\right) p_h - c_h}{\rho_h^h p_h + \rho_h^l p_l} \leq U_h^l$$
$$\Rightarrow \rho_h^h \left(U_h^h - U_h^l\right) p_h - c_h \leq 0$$
$$\Rightarrow U_h^h - U_h^l \leq \frac{c_h}{\rho_h^h p_h}$$

The above condition gives the equilibrium with both $h-h, h$-l and l-l matchings

2.6 Complete Information case (match utility is random)

Each player meets at rate $\lambda_{s'}$. During a meeting with type s', match utility is drawn from a type specific distribution $\varepsilon_{s'} \sim F_{s'}(\varepsilon)$.

2.7 Value

$$V_{s,\tau} = \sum_{s'} d\tau \lambda_{s'} \pi^s_{s'} \int_0^\infty \max\{V_{s,\tau+d\tau}, \varepsilon_{s'} dF_{s'}(\varepsilon)\} + \left(1 - \sum_{s'} d\tau \lambda_{s'} \pi^s_{s'}\right) V_{s,\tau+d\tau} - c_s d\tau$$

$$c_s = \sum_{s'} \lambda_{s'} \pi^s_{s'} \int_0^\infty \max\{\varepsilon_{s'} - V_s, 0\} dF_{s'}(\varepsilon)$$

$$c_s = \sum_{s'} \lambda_{s'} \pi^s_{s'} \int_{x_s}^\infty (\varepsilon_{s'} - x_s) dF_{s'}(\varepsilon)$$

Special case: $F_{s'} \sim \text{ Exp } (\beta_{s'})$

$$\int_{x_s}^\infty (\varepsilon_{s'} - x_s) dF_{s'}(\varepsilon) = \frac{e^{x_s \beta_{s'}}}{\beta_{s'}}$$

$$\pi^s_{s'} = 1 - F_s(x_{s'}) = e^{-x_{s'} \beta_s}$$

$$\therefore c_s = \sum_{s'} \lambda_{s'} e^{-x_{s'} \beta_s} \frac{e^{x_s \beta_{s'}}}{\beta_{s'}}$$

If there is only one type:

$$c = \lambda e^{-x\beta} \frac{e^{-x\beta}}{\beta}$$

$$\Rightarrow x = \frac{1}{2\beta} \log\left(\frac{\lambda}{c\beta}\right)$$

If there are two types $s \in \{h, l\}$:

$$\begin{cases} c_h = \lambda_h e^{-x_h \beta_h} \dfrac{e^{-x_h \beta_h}}{\beta_h} + \lambda_l e^{-x_l \beta_h} \dfrac{e^{-x_h \beta_l}}{\beta_l} \\ c_l = \lambda_h e^{-x_h \beta_l} \dfrac{e^{-x_l \beta_h}}{\beta_h} + \lambda_l e^{-x_l \beta_l} \dfrac{e^{-x_l \beta_l}}{\beta_l} \end{cases}$$

$$\Rightarrow x_s = \frac{1}{2\beta_s} \log\left(\frac{2\lambda_s}{c_s \beta_s}\right) \text{ for } s \in \{h, l\}$$

2.8 Incomplete Information case (match utility is random, type is observable)

The type of a player is given by his own type s and her belief p over her types, p_s being the belief that he is type s. Summamize the types by $A = (s,p) \in (T, \Delta(T))$. Note that A can also be summarized by his history of acceptance and rejections $A = (s, A(s_1), A(s_2), ..., A(s_n))$ where s_i the type of person she met in her i-th meeting and A(s) $\in \{a, r\}$.

$$\text{Each player with belief p uses the strategy } \begin{cases} a & \text{if } \varepsilon_{s'} > x_p \\ r & \text{otherwise} \end{cases}$$

2.9 Value

$$V_{s,A,\tau} = \sum_{s'} d\tau\lambda_{s'} \sum_s \pi^s_{s'} p_s \int \max\left\{\varepsilon_{s'}, V_{s,A+a(s'),\tau+d\tau} dF_{s'}(\varepsilon)\right.$$

$$+ \sum_{s'} d\tau\lambda_{s'} \sum_s \left(1 - \pi^s_{s'}\right) p_s V_{s,A+r(s'),\tau+d\tau}$$

$$+ \left(1 - \sum_{s'} d\tau\lambda_{s'}\right) V_{s,A,\tau+d\tau} - c_s d\tau$$

in equilibrium,

$$c_s = \sum_{s'} \lambda_{s'} \left((1) + (2) + (3)\right)$$

$$(1) \left(\sum_s \pi^s_{s'} p_s\right) \int_{x_p}^{\infty} \varepsilon_{s'} - x_p dF_{s'}(\varepsilon)$$

$$(2) \left(\sum_s \pi^s_{s'} p_s\right) \left(V_{s,p+a(s')} - V_{s,p}\right)$$

$$(3) \left(1 - \sum_s \pi^s_{s'} p_s\right) \left(V_{s,p+r(s')} - V_{s,p}\right)$$

with belief dynamics,

$$\text{p} + a(s') = \frac{\pi^s_{s'} p_s}{\sum_t \pi^t_{s'} p_t}$$

$$\mathrm{p} + r\left(s'\right) = \frac{\left(1 - \pi_{s'}^{s}\right) p_s}{\sum_t \left(1 - \pi_{s'}^{t}\right) p_t}$$

$$\pi_{s'}^{s} = \int_p \left(1 - F_s\left(x_p\right)\right) \mathbb{P}\left\{p|s'\right\}$$

3 Two Period $Mo\delta$

3.1 $Mo\delta$

In the first period, people meet, observes the type and match utility of the other person s' and $\varepsilon_{s'} \sim F_{s'}$, and decide to accept and receive the match utility or reject the other person and continue to the second.

In the second period, staying single leads to a fixed utility depending on the type of the player u_s.

3.2 Strategies

Everyone starts with the same prior, so they have the same thresholds meeting a type s':

$$\begin{cases} a & \text{if } \varepsilon_{s'} \geq x_{0,s'} \\ r & \text{otherwise} \end{cases}$$

In period two, the reservation utility does not depend on the type of the other person, so:

$$x_{1,a,s'} = x_{1,a,s} \; \forall \; s, s'$$

$$x_{1,r,s'} = x_{1,r,s} \; \forall \; s, s'$$

3.3 Masses

In the first period, let the masses be:

$$m_{0,s} = p_s$$

In the second period:

$$\text{type s who got an acceptance in period 1 } : m_{1,a,s} = \left(\sum_{s'} p_{s'} \cdot F_{s'} \left(x_{0,s'} \right) \right) \left(1 - F_s \left(x_{0,s} \right) \right)$$

$$\text{type s who got a rejection in period 1 } : m_{1,r,s} = F_s \left(x_{0,s} \right)$$

3.4 Beliefs

In the first period everyone has the same belief:

$$p_0 \left(s \right) = p_s$$

In the second period:

$$\text{after an acceptance } : p_{1,a}(s) = \frac{1 - F_s(x_{0,s})}{\sum_t 1 - F_t(x_{0,t})}$$
$$\text{after an rejection } : p_{1,r}(s) = \frac{F_s(x_{0,s})}{\sum_t F_t(x_{0,t})}$$

3.5 Thresholds

In period two, threshold is equal to the value if the person is accepted but chooses to reject:

$$x_{1,a,s} = \sum_t p_{1,a}(t)\, u_t$$
$$x_{1,r,s} = \sum_t p_{1,r}(t)\, u_t$$

In period one, threshold is equal to the value if the person is accepted but chooses to reject:

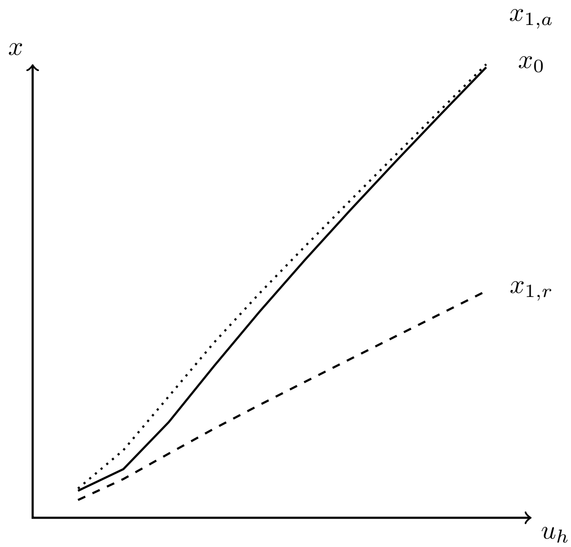
$$\begin{aligned} x_{0,s} = & \sum_t m_{1,a,t} \left(\sum_{t'} \left(p_{1,a}(t')\left(1 - F_{t'}(x_{1,a,t'})\right)\right) \int_{x_{1,a,t}}^{\infty} \varepsilon_t dF_t + \sum_{t'} \left(p_{1,a}(t')\, F_{t'}(x_{1,a,t'})\right) x_{1,a,s} \right) \\ & + \sum_t m_{1,r,t} \left(\sum_{t'} \left(p_{1,a}(t')\left(1 - F_{t'}(x_{1,r,t'})\right)\right) \int_{x_{1,a,t}}^{\infty} \varepsilon_t dF_t + \sum_{t'} \left(p_{1,a}(t')\, F_{t'}(x_{1,r,t'})\right) x_{1,a,s} \right) \end{aligned}$$

Note that $x_{1,a,s}$ is independent of s, so $x_{0,s}$ should be independent of s as well.

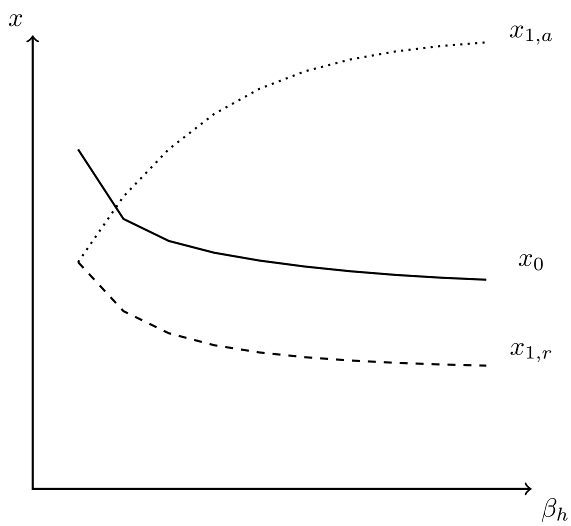
3.6 Numerical Solutions

Consider the case with two types $s \in \{h, l\}$, F_s is exponentially distributed with rate $\beta_{s.}$

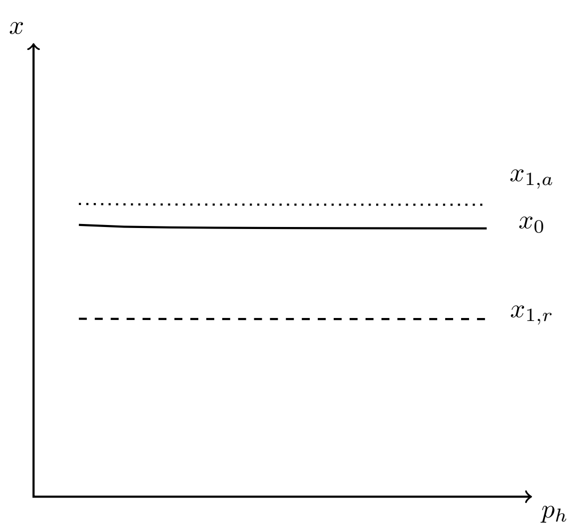
Start with $p_h = p_l = \frac{1}{2}, \beta_l = 1, \beta_h = 2, u_l = 0, u_h = 1$, the following plots changes u_h from 1 to 10, β_h from 1 to 10 and $\frac{1}{p_h}$ from 1 to 10.



x
$x_{1,a}$
x_0
$x_{1,r}$
u_h



x
$x_{1,a}$
x_0
$x_{1,r}$
β_h



x
$x_{1,a}$
x_0
$x_{1,r}$
p_h

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4.1 $Mo\delta$

1. Continum of agents, continuous time, discounting
2. Type $x \in \{H, L\}$ is match utility
3. History of opponent known (later relaxed)

4.2 Results

1. x decrease from rejection
2. x increase from offer by type below reservation
3. Smaller change if unknown history of opponent
4. Larger propotion of L type in market

5 Overlapping Generations $Mo\delta$

5.1 Model 1

When people meet, they observe the age, type and match utility of the other person (t, s, ε) where $t \in \{0, 1\}, s \in \{h, l\}, \varepsilon \sim F_{s.}$

5.2 Strategies

Let $x_{t,z}^{t',s'}$ denote the threshold of a person with age t with history z, when meeting with a person with age t' and type s' . The history $z = \emptyset$ when $t = 0$ and $z \in \{(a, 0), (a, 1), (r, 0), (r, 1)\}$, denoting whether the person is accepted or rejected by an age 0 or 1 person.

At age 0, let the threshold be:

$$\text{For each } (t', s'), x_0^{t',s'}$$

At age 1, reservation utility does not depend on the age and type of the other person:

$$\text{For each } z, x_{1,z}^{t',s'} = x_{1,z}^{\tilde{t},\tilde{s}} \; \forall \; \left(t', s', \tilde{t}, \tilde{s}\right)$$

5.3 Masses

At each time, there are 10 groups of people, for $s \in \{h, l\}$:

$$\begin{aligned}
m_{0,s} &= p_s \\
m_{1,a,0,s} &= \left(\sum_{s'} p_{0,s'} F_{s'}\left(x_0^{0,s'}\right)\right)\left(1 - F_s\left(x_0^{0,s}\right)\right) \\
m_{1,a,1,s} &= \left(\sum_{s'} p_{1,s'} F_{s'}\left(x_0^{1,s'}\right)\right)\left(\sum_{z'} p_{1,z'}\left(1 - F_s\left(x_{1,z'}^{0,s}\right)\right)\right) \\
m_{1,r,0,s} &= F_s\left(x_0^{0,s}\right) \\
m_{1,r,1,s} &= \left(\sum_{z'} p_{1,z'} F_s\left(x_{1,z'}^{0,s}\right)\right)
\end{aligned}$$

5.4 Beliefs

At age 0, everyone has the same belief:

$$p_0\left(s\right) = p_s$$

At age 1, the beliefs from being rejected by age 0 and age 1 person are different:

$$\begin{aligned}
p_{1,a,0}\left(s\right) &= \frac{1 - F_s\left(x_0^{0,s}\right)}{\sum_{\tilde{s}}\left(1 - F_{\tilde{s}}\left(x_0^{0,\tilde{s}}\right)\right)} \\
p_{1,a,1}\left(s\right) &= \frac{\sum_{z'} p_{1,z'}\left(1 - F_s\left(x_{1,z'}^{0,s}\right)\right)}{\sum_{\tilde{s}}\left(\sum_{z'} p_{1,z'}\left(1 - F_{\tilde{s}}\left(x_{1,z'}^{0,\tilde{s}}\right)\right)\right)} \\
p_{1,r,0}\left(s\right) &= \frac{F_s\left(x_0^{0,s}\right)}{\sum_{\tilde{s}} F_{\tilde{s}}\left(x_0^{0,\tilde{s}}\right)} \\
p_{1,r,1}\left(s\right) &= \frac{\sum_{z'} p_{1,z'} F_s\left(x_{1,z'}^{0,s}\right)}{\sum_{\tilde{s}} p_{1,z'}\left(\sum_{z'} F_{\tilde{s}}\left(x_{1,z'}^{0,\tilde{s}}\right)\right)}
\end{aligned}$$

5.5 Thresholds

At age 1,

$$x_{1,z}^{t',s'} = \sum_s p_{1,z}\left(s\right) u_s$$

At age 0,

$$\begin{aligned}
x_0^{t',s'} &= \sum_{\tilde{s},z'} m_{1,\tilde{z}',s} \sum_{s,z} p_{1,z}\left(s\right)\left(1 - F_s\left(x_{1,z}^{1,s}\right)\right)\int_{x_{1,z}^{1\tilde{,}s}}^{\infty} \varepsilon_{\tilde{s}} dF_{\tilde{s}} + \sum_{s,z} p_{1,z}\left(s\right) F_s\left(x_{1,z}^{1,s}\right) x_{1,z}^{1,s} \\
&+ \sum_{\tilde{s}} m_{0\tilde{,}s} \sum_{s,z} p_{1,z}\left(s\right)\left(1 - F_s\left(x_{1,z}^{1,s}\right)\right)\int_{x_{1,z}^{1\tilde{,}s}}^{\infty} \varepsilon_{\tilde{s}} dF_{\tilde{s}} + \sum_{s,z} p_{1,z}\left(s\right) F_s\left(x_{1,z}^{1,s}\right) x_{1,z}^{1,s}
\end{aligned}$$

5.6 Model 2

When people meet, they observe the age and match utility of the other person (t, ε) where $t \in \{0,1\}, \varepsilon \sim F_s$, but not the type of the other person.

5.7 Strategies

Let $x_{t,z}^{t'}$ denote the threshold of a person with age t with history z, when meeting with a person with age t' . The history $z = \emptyset$ when $t = 0$ and $z \in \{(a,0),(a,1),(r,0),(r,1)\}$, denoting whether the person is accepted or rejected by an age 0 or 1 person.

At age 0, let the threshold be:

$$\text{For each } t', x_0^{t'}$$

At age 1, reservation utility does not depend on the age and type of the other person:

$$\text{For each } z, x_{1,z}^{t'} = x_{1,z}^{\tilde{t}} \ \forall \ \left(t', \tilde{t}\right)$$

5.8 Masses

At each time, there are 10 groups of people, for $s \in \{h, l\}$:

$$\begin{aligned}
m_{0,s} &= p_s \\
m_{1,a,0,s} &= \left(\sum_{s'} p_{0,s'} F_{s'}\left(x_0^0\right)\right)\left(1 - F_s\left(x_0^0\right)\right) \\
m_{1,a,1,s} &= \left(\sum_{s'} p_{1,s'} F_{s'}\left(x_0^1\right)\right)\left(\sum_{z'} p_{1,z'}\left(1 - F_s\left(x_{1,z'}^0\right)\right)\right) \\
m_{1,r,0,s} &= F_s\left(x_0^0\right) \\
m_{1,r,1,s} &= \left(\sum_{z'} p_{1,z'} F_s\left(x_{1,z'}^0\right)\right)
\end{aligned}$$

5.9 Beliefs

At age 0, everyone has the same belief:

$$p_0(s) = p_s$$

At age 1, the beliefs from being rejected by age 0 and age 1 person are different:

$$p_{1,a,0}(s) = \frac{1 - F_s\left(x_0^0\right)}{\sum_{\tilde{s}} \left(1 - F_{\tilde{s}}\left(x_0^0\right)\right)}$$

$$p_{1,a,1}(s) = \frac{\sum_{z'} p_{1,z'}\left(1 - F_s\left(x_{1,z'}^0\right)\right)}{\sum_{\tilde{s}} \left(\sum_{z'} p_{1,z'}\left(1 - F_{\tilde{s}}\left(x_{1,z'}^0\right)\right)\right)}$$

$$p_{1,r,0}(s) = \frac{F_s\left(x_0^0\right)}{\sum_{\tilde{s}} F_{\tilde{s}}\left(x_0^0\right)}$$

$$p_{1,r,1}(s) = \frac{\sum_{z'} p_{1,z'} F_s\left(x_{1,z'}^0\right)}{\sum_{\tilde{s}} p_{1,z'}\left(\sum_{z'} F_{\tilde{s}}\left(x_{1,z'}^0\right)\right)}$$

5.10 Thresholds

At age 1,

$$x_{1,z}^{t'} = \sum_{s} p_{1,z}(s)\, u_s$$

At age 0,

$$\begin{aligned} x_0^{t'} = & \sum_{\tilde{s},z'} m_{1,\tilde{z}',s} \sum_{s,z} p_{1,z}(s)\left(1 - F_s\left(x_{1,z}^1\right)\right) \int_{x_{1,z}^1}^{\infty} \varepsilon_{\tilde{s}} dF_{\tilde{s}} + \sum_{s,z} p_{1,z}(s)\, F_s\left(x_{1,z}^1\right) x_{1,z}^1 \\ & + \sum_{\tilde{s}} m_{0,\tilde{,}s} \sum_{s,z} p_{1,z}(s)\left(1 - F_s\left(x_{1,z}^1\right)\right) \int_{x_{1,z}^1}^{\infty} \varepsilon_{\tilde{s}} dF_{\tilde{s}} + \sum_{s,z} p_{1,z}(s)\, F_s\left(x_{1,z}^1\right) x_{1,z}^1 \end{aligned}$$

6 Many-to-Many Matching with Learning (Old Model)

6.1 Incomplete Information case:

Let the type of a player $s \in [0, 1]$ be the belief of being high type,

When type s meets another player with type s', she can perfectly observe the actual type, meaning $s' \in \{0, 1\}$

Let the match utilities be $U_s^{s'} = sU_h^{s'} + (1-s) U_l^{s'}$

Need $c_h = c_l =$ c so that the player cannot identify her own type

6.2 Value

The two type incomplete information case is like the continuous type complete information case:

$$c = s\left(\lambda_s^h \max\left\{U_s^h - V_s, 0\right\}\right) + (1-s)\left(\lambda_s^l \max\left\{U_s^l - V_s, 0\right\}\right)$$

$$\text{where } \lambda_s^{s'} = s + (1-s) F_{s'}(s^\star)$$

and F_t is the equilibrium distribution of beliefs of players with true type t

and $s^\star = \max_s \{x^\star(s) = l\}$ is the threshold type who accepts a meeting with l type

Here, the assumption is every player accepts a meeting with h type, and only players with type $s \in [0, s^\star]$ accepts a meeting with l type

6.3 Incomplete Information case (old model):

Each player has one of two types $t \in \{h, l\}$,

When player i meets player j, i observes the number of past meetings j had (or equivalently the age of j) and a sample match utility $u_{i,j}$ drawn from $F_{t_j} \in \{F_h, F_l\}$ depending on j's type,

Then each player chooses one of two actions $\sigma_i \in \{$ accept , reject $\}$:

If both players choose accept, player i obtains match utility $\mathbb{E}\left[F_{t_j}\right] = u_{t_j} \in \{u_h, u_l\}$;

Otherwise, player i continues searching.

6.4 Question:

1. Two types of behavior in equilibrium?

   (a) Players sets low acceptance threshold because high type players have exited the market early

   Here, large proportion of players with high age has low types

   (b) Players sets high acceptance threshold to wait for acceptance from other high type players

   Here, large proportion of players with high age has high types

2. How do acceptance rule change with initial prior?

   (a) More skewed distribution will lead to lower acceptance threshold

   (b) Uniform prior lead to highest acceptance threshold

6.5 Types and masses:

After $k \in \{0, 1, 2, ...\}$ meetings, player i's type is given by a pair (t_i, A_i), where:

$t_i \in$ T is the true type of player i

$A_i \in \{a, r\}^k$ is a vector of length k, the state (or history) of player i

For example, $A_i = \{r, r, a, r, a\}$ means player i met 5 other players, rejected the 1st, 2nd and 4th, and was rejected by 3rd and 5th player

(Note: the type need to specify the age of the other player in the meetings as well)

Then $\sum_k A_i(r)$ is the number of rejections by player i and $\sum_k A_i(a)$ is the number of rejections for player i by other players

Let $m(t, A)$ denote the mass of players with type t and state A,

Then $m(t, k) = \sum_{A \in \mathcal{A}_k} m(t, A)$ is the mass of players with k meetings,

6.6 Strategies and Acceptance Probabilities:

Consider the threshold strategy for player i with type (t, A) meeting player j with age k,

$$\begin{cases} \text{accept} & \text{if } u_{i,j} \geq x_A^k \\ \text{reject} & \text{otherwise} \end{cases}$$

Define acceptance probabilities, the probability of a player with type (t, A) accepting another player with age k

$$\begin{aligned}
\pi_{t,A}^k &= \mathbb{P}\left\{ \text{ accept } | t_i = t, A_i = A, |A_j| = k \right\} \\
\pi_{t,k'}^k &= \mathbb{P}\left\{ \text{ accept } | t_i = t, |A_i| = k', |A_j| = k \right\} \\
&= \sum_{A \in \mathcal{A}_{k'}} \pi_{t,A}^k \frac{m(t, A)}{m(t, k')}
\end{aligned}$$

Acceptance probabilities should be consistent with the threshold strategy and the beliefs about the types.

6.7 Beliefs and Updating Process:

The posterior belief should be consistent with the equilibrium mass of type A:

$$\begin{aligned}
p_A(t) &= \mathbb{P}\left\{ t_i = t | A_i = A \right\} \\
&= \frac{m(t, A)}{\sum_{s \in T} m(s, A)} \\
p_k(t) &= \mathbb{P}\left\{ t_i = t || A_i| = k \right\} \\
&= \frac{m(t, k)}{\sum_{s \in T} m(s, k)}
\end{aligned}$$

Belief is only updated if player i chooses accept and player j with age k chooses reject:

$$\begin{aligned}
p_{A+a}(t) &= p_A(t) \frac{\sum_{B \in \mathcal{A}_k} F_t\left(x_B^{|A|}\right) \frac{m(B)}{m(k)}}{\sum_{s \in T} p_A(s) \sum_{B \in \mathcal{A}_k} F_s\left(x_B^{|A|}\right) \frac{m(B)}{m(k)}} \\
p_{A+r}(t) &= p_A(t)
\end{aligned}$$

Also, these beliefs should be consistent with the equilibrium masses:

$$p_{A+a}(t) = \frac{m(t, A+a)}{\sum_s m(s, A+a)}$$

$$p_{A+r}(t) = \frac{m(t, A+r)}{\sum_s m(s, A+r)}$$

6.8 Values

Acceptance probabilities should determine the thresholds:

$$\sum_{t \in T} p_A(t)\, \pi_{t,A}^k = \sum_{t \in T} p_k(t) \left(1 - F_t\left(x_A^k\right)\right)$$

At the threshold x_A^k = x, the values from accepting and rejection should be the same

$$\sum_{t \in T} \left(\frac{f_t(x)\, p_k(t)}{\sum_{s \in T} f_s(x)\, p_k(s)} \right) \left(\pi_{t,k}^{|A|} u_t + \left(1 - \pi_{t,k}^{|A|}\right)(V_{A+a} - c) \right) = V_{A+r} - c$$

6.9 Equilibrium Flows

The flow into state A should be equal to the flow out of state A for all types t:

$$m(t, A)\, \pi_{t,A}^k \left(1 - \sum_{s \in T} p_k(s)\, \pi_{s,k}^{|A|}\right)) = m(t, A+a) \text{ not correct right now}$$

$$m(t, A)\left(1 - \pi_{t,A}^k\right) = m(t, A+r)$$

(Note: need probability of meeting k)