CS540 Introduction to Artificial Intelligence
Lecture 22

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Guess Average Game Derivation

Motivation

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Traveler's Dilemma Example

Motivation

- Two identical antiques are lost. The airline only knows that its value is at most v dollars, so the airline asks their owners (travelers) to report its value (integers larger than or equal to some integer $x > 1$). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive x dollars in reward from the other traveler.
- The best response of to v is $\max\left(v-1, x\right)$, and the only mutual best response is both report x.
- This result is inconsistent with experimental observations.

Traveler's Dilemma Example Derivation

Motivation

Normal Form Games

Definition

- In a simultaneous move game, a state represents one action from each player.
- The costs or rewards, sometimes called payoffs, are written in a payoff table.
- The players are usually called the ROW player and the COLUMN player.
- If the game is zero-sum, the convention is: ROW player is MAX and COLUMN player is MIN.

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}(s_{MIN}) = \arg\max_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \arg\min_{s \in S_{MIN}} c(s_{MAX}, s)$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c\left(s_i, s\right) > c\left(s_{i'}, s\right) \;\; \forall\, s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c\left(s, s_i\right) < c\left(s, s_{i'}\right) \;\; \forall\, s \in S_{MAX}$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Weakly Dominated and Dominant Strategy

Definition

- An action s_i weakly dominates another $s_{i'}$ if it leads to a better state or a state with the same payoff no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c\left(s_i, s\right) \geqslant c\left(s_{i'}, s\right) \;\; \forall\, s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c\left(s, s_i\right) \leqslant c\left(s, s_{i'}\right) \;\; \forall\, s \in S_{MAX}$$

- The action $s_{i'}$ is called weakly dominated.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

–	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.

Prisoner's Dilemma Derivation

Discussion

Public Good Game

Discussion

- You received one free point for this question and you have two choices.
- A: Donate the point.
- B: Keep the point.
- Your final grade is the points you keep plus twice the average donation.

Properties of Nash Equilibrium

Discussion

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Rationalizable actions (the set of Nash equilibria is a subset of this) can be found be iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

Normal Form of Sequential Games

Discussion

- Sequential games can have normal form too, but the solution concept is different.
- Nash equilibria of the normal form may not be a solution of the original sequential form game.

Fixed Point Algorithm

Description

- For small games, it is possible to find all the best responses. The states that are best responses for all players are the solutions of the game.
- For large games, start with a random action, find the best response for each player and update until the state is not changing.

Fixed Point Diagram

Definition

Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Rock Paper Scissors Example

Discussion

- There are no pure strategy Nash equilibria.
- Playing each action (rock, paper, scissors) with equal probability is a mixed strategy Nash.

Rock Paper Scissors Example Derivation

Discussion

Battle of the Sexes Example

Discussion

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

–	Bach	Stravinsky
Bach	A (x, y)	B $(0, 0)$
Stravinsky	C $(0, 0)$	D (y, x)

Battle of the Sexes Example Diagram

Discussion

Battle of the Sexes Example Derivation

Discussion

Volunteer's Dilemma

Discussion

- On March 13, 1964, Kitty Genovese was stabbed outside the apartment building. There are 38 witnesses, and no one reported. Suppose the benefit of reported crime is 1 and the cost of reporting is $c < 1$.
- Suppose every witness uses the same mixed strategy of not reporting with probability p and reporting with probability $1 - p$. Then the mixed strategy Nash equilibrium is characterized by the following expression.

$$p^{37} \cdot 0 + \left(1 - p^{37}\right) \cdot 1 = 1 - c \Rightarrow p = c^{\frac{1}{37}}$$

Volunteer's Dilemma Derivation

Discussion

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Fixed Point Nash Equilibrium

Algorithm

- Input: the payoff table $c\left(s_i, s_j\right)$ for $s_i \in S_{MAX}, s_j \in S_{MIN}$.
- Output: the Nash equilibria.
- Start with random state $s = \left(s_{MAX}, s_{MIN}\right)$.
- Update the state by computing the best response of one of the players.

$$\begin{aligned} \text{either } s' &= \left(br_{MAX}\left(s_{MIN}\right), br_{MIN}\left(br_{MAX}\left(s_{MIN}\right)\right)\right) \\ \text{or } s' &= \left(br_{MAX}\left(br_{MIN}\left(s_{MAX}\right)\right), br_{MIN}\left(s_{MAX}\right)\right) \end{aligned}$$

- Stop when $s' = s$.