

Uninformed Search
oooo

BFS
oooooooooooo

DFS
oooooooooooooooooooo

CS540 Introduction to Artificial Intelligence

Lecture 15

Young Wu

Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

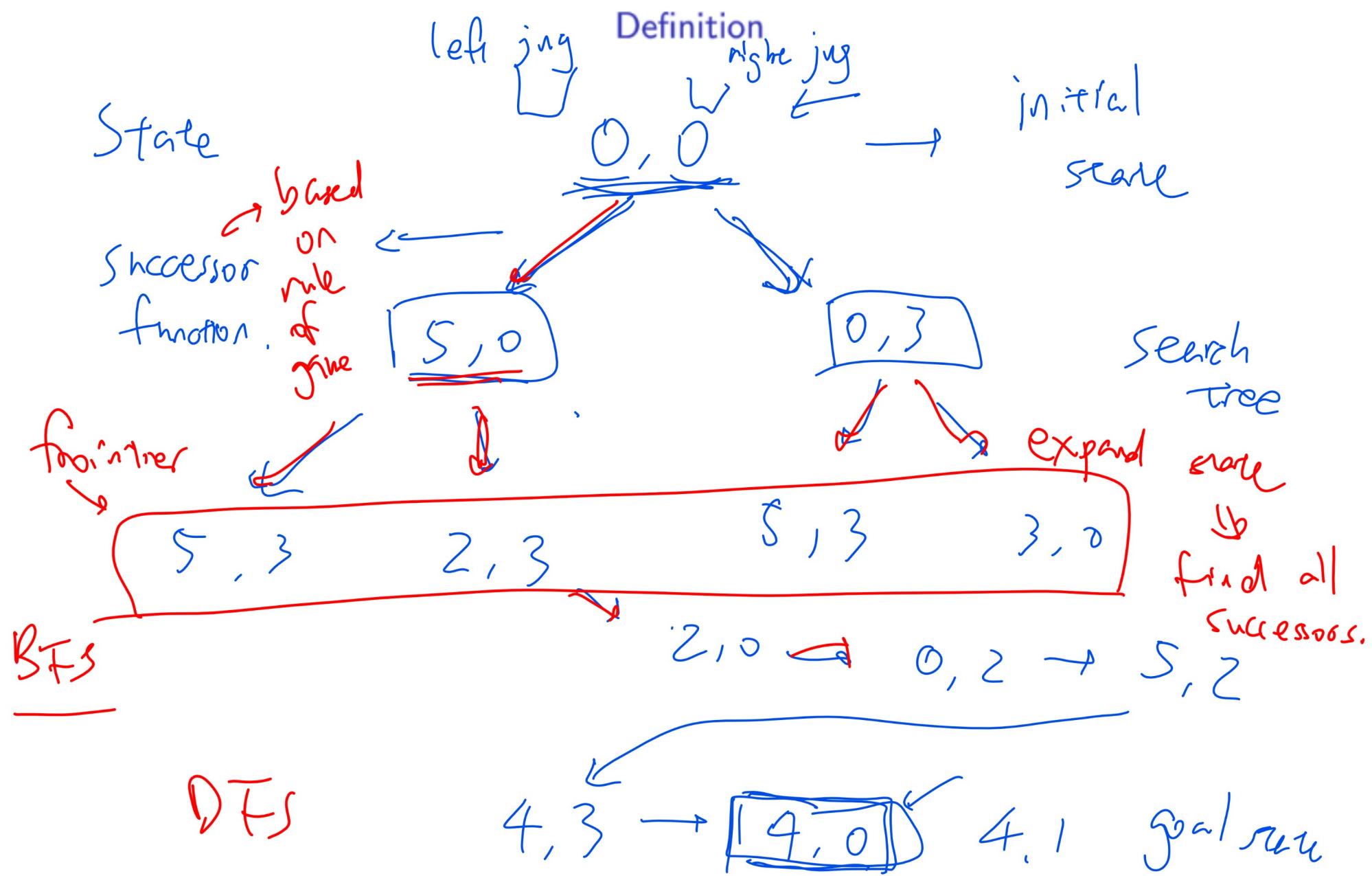
July 8, 2020

Uninformed Search
•○○○

BFS
○○○○○○○○○○

DFS
○○○○○○○○○○○○○○○○○○○○○○○

Water Jugs Example



Complexity

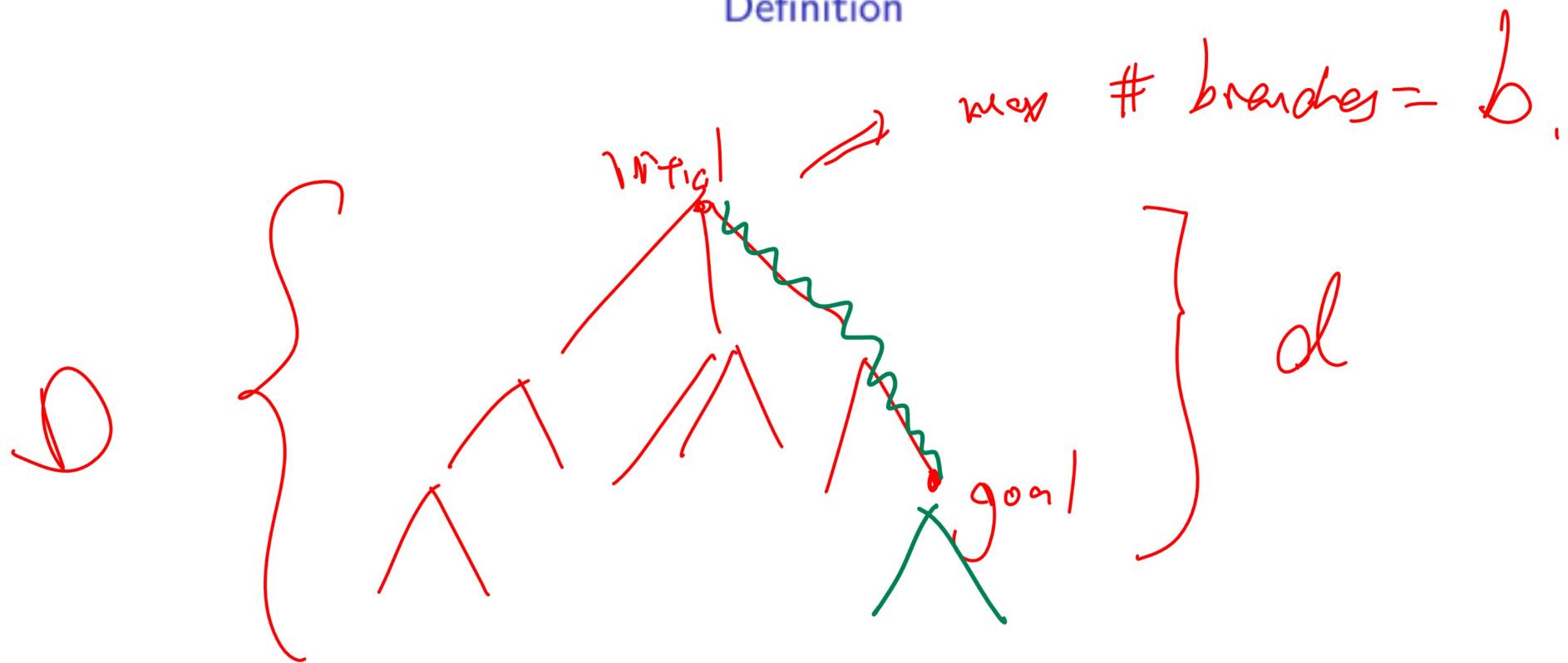
Definition

- The time complexity of a search strategy is the worst case maximum number of vertices expanded.
- The space complexity of a search strategy is the worst case maximum number of states stored in the frontier at a single time.
- Notation: the goals are d edges away from the initial state. This means assuming a constant cost of 1, the optimal solution has cost d . The maximum depth of the graph is D .
- Notation: the branching factor is b , the maximum number of actions associated with a state.

$$b = \max_{s \in V} |s'(s)|$$

Search Tree Diagram

Definition



Uninformed Search
oooo

BFS
●oooooooooo

DFS
oooooooooooooooooooo

Breadth First Search

Description

- Use Queue (FIFO) for the frontier.
- Remove from the front, add to the back.

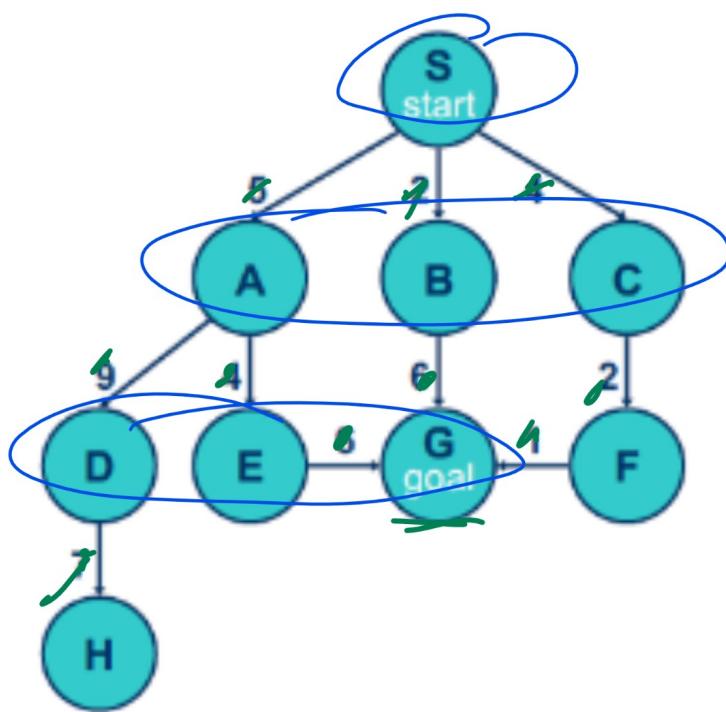
Uninformed Search
oooo

BFS
o●oooooooooooo

DFS
oooooooooooooooooooo

BFS Simple Example

Quiz



Queues: ~~S, A, B, C, D, E,~~

deQ from the front
enQ successors.

~~S, F~~
H, G,

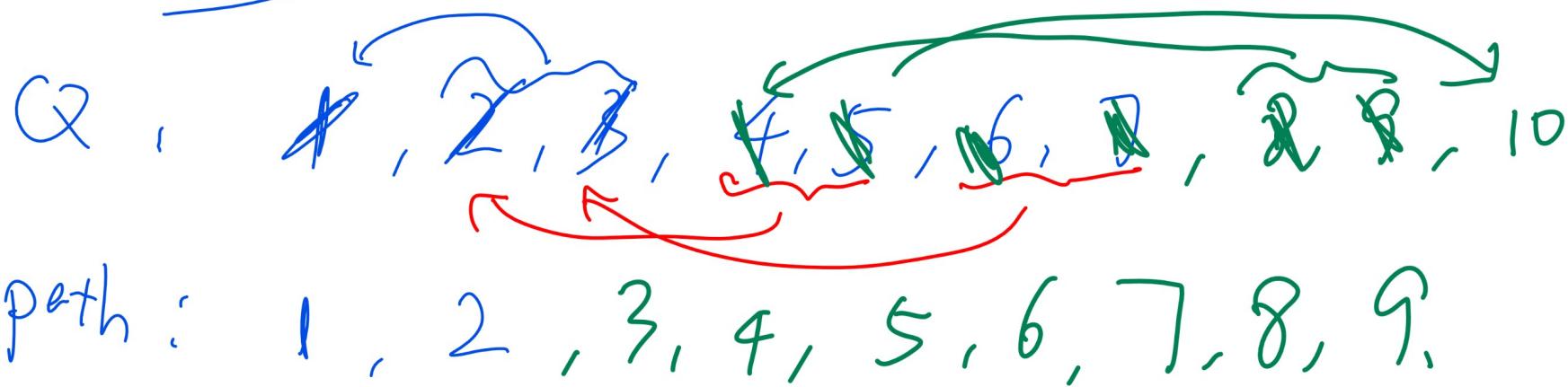
[Stop when deQ G]

Expansion path: S, A, B, C, D, E, G

BFS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a BFS expansion sequence?



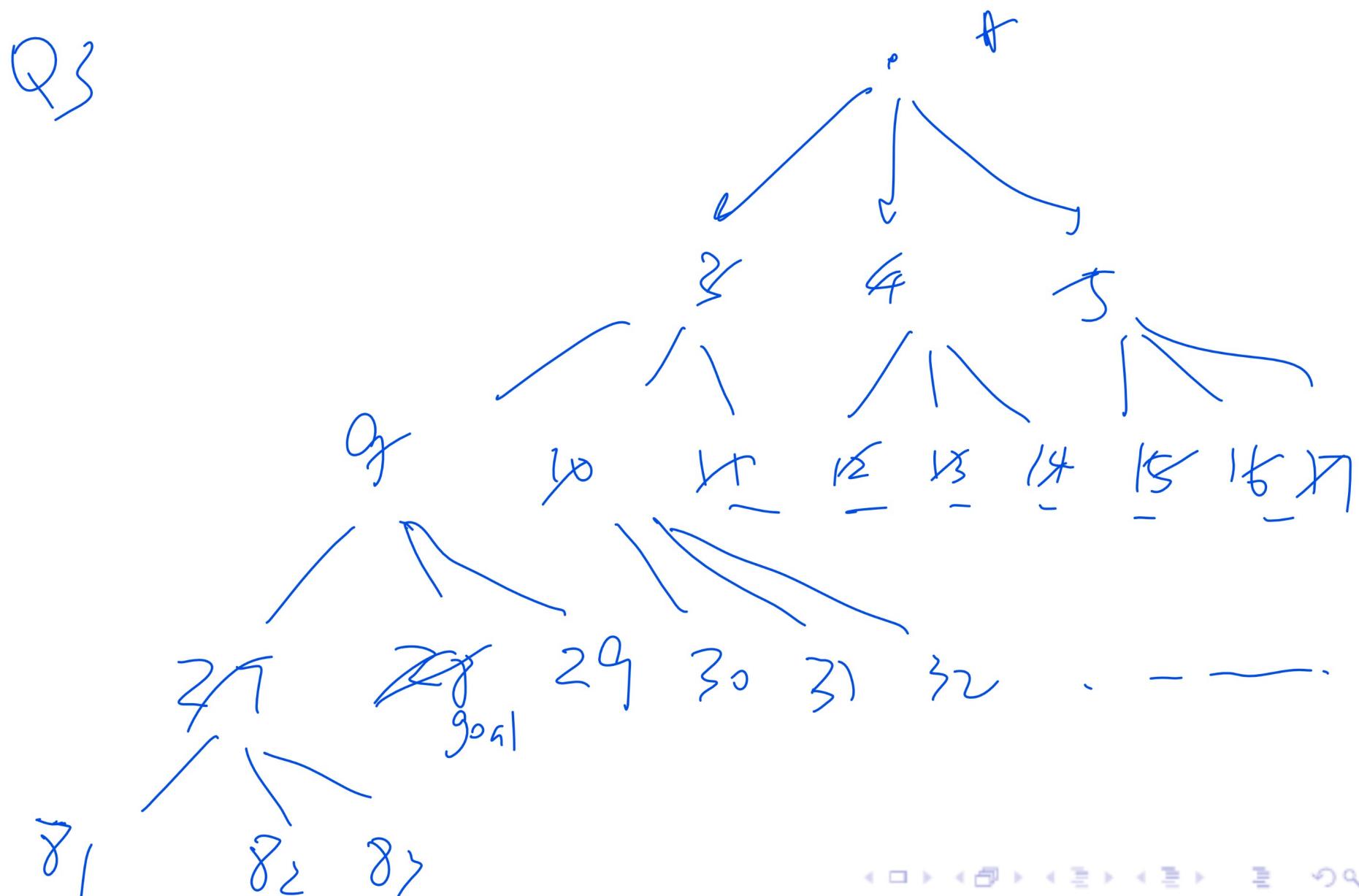
Uninformed Search
oooo

BFS
oooo●ooooooo

DFS
oooooooooooooooooooo

BFS Example 1 Diagram

Quiz



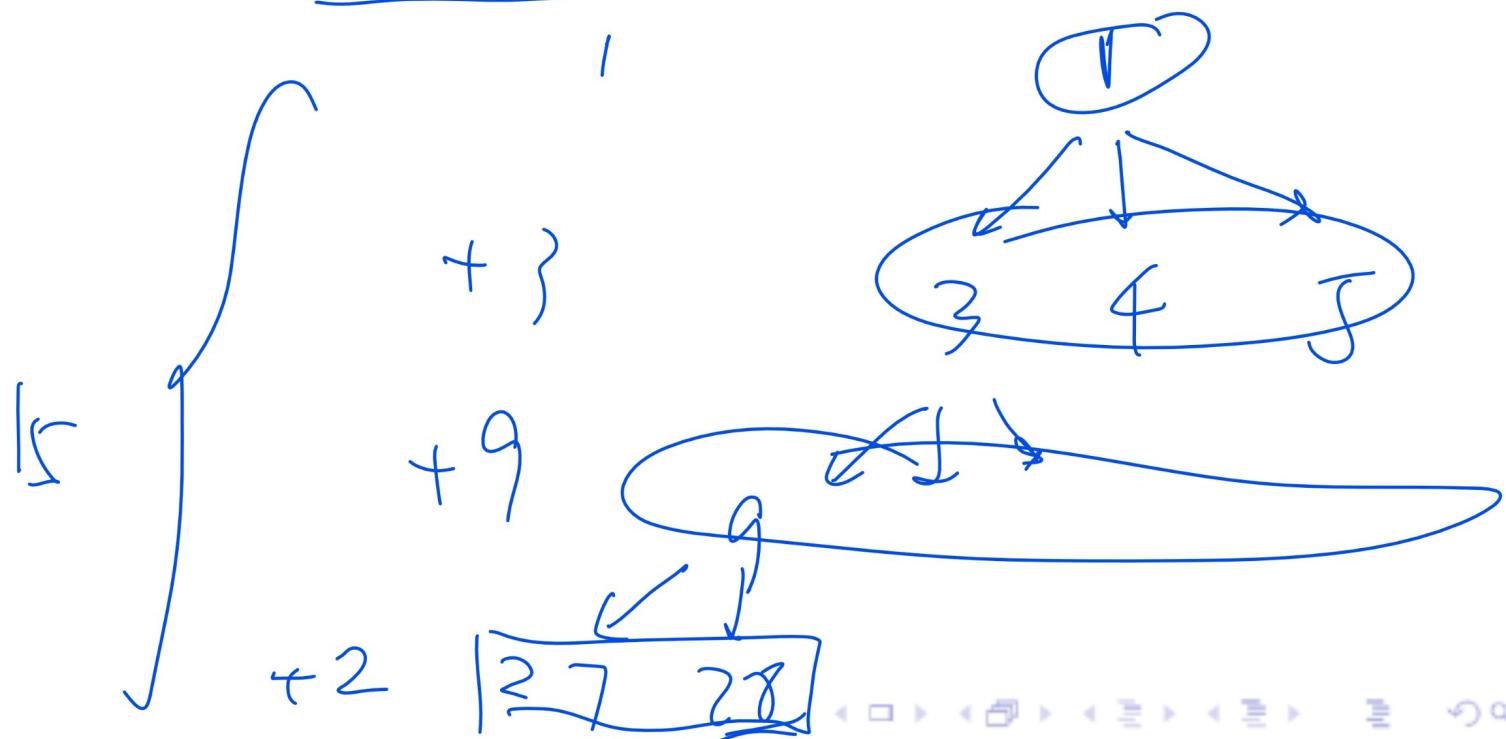
BFS Example 2

Quiz

Q3

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a BFS search? Include both the initial and goal states.

- A: 4
- B: 5
- C: 14
- D: 15
- E: 28



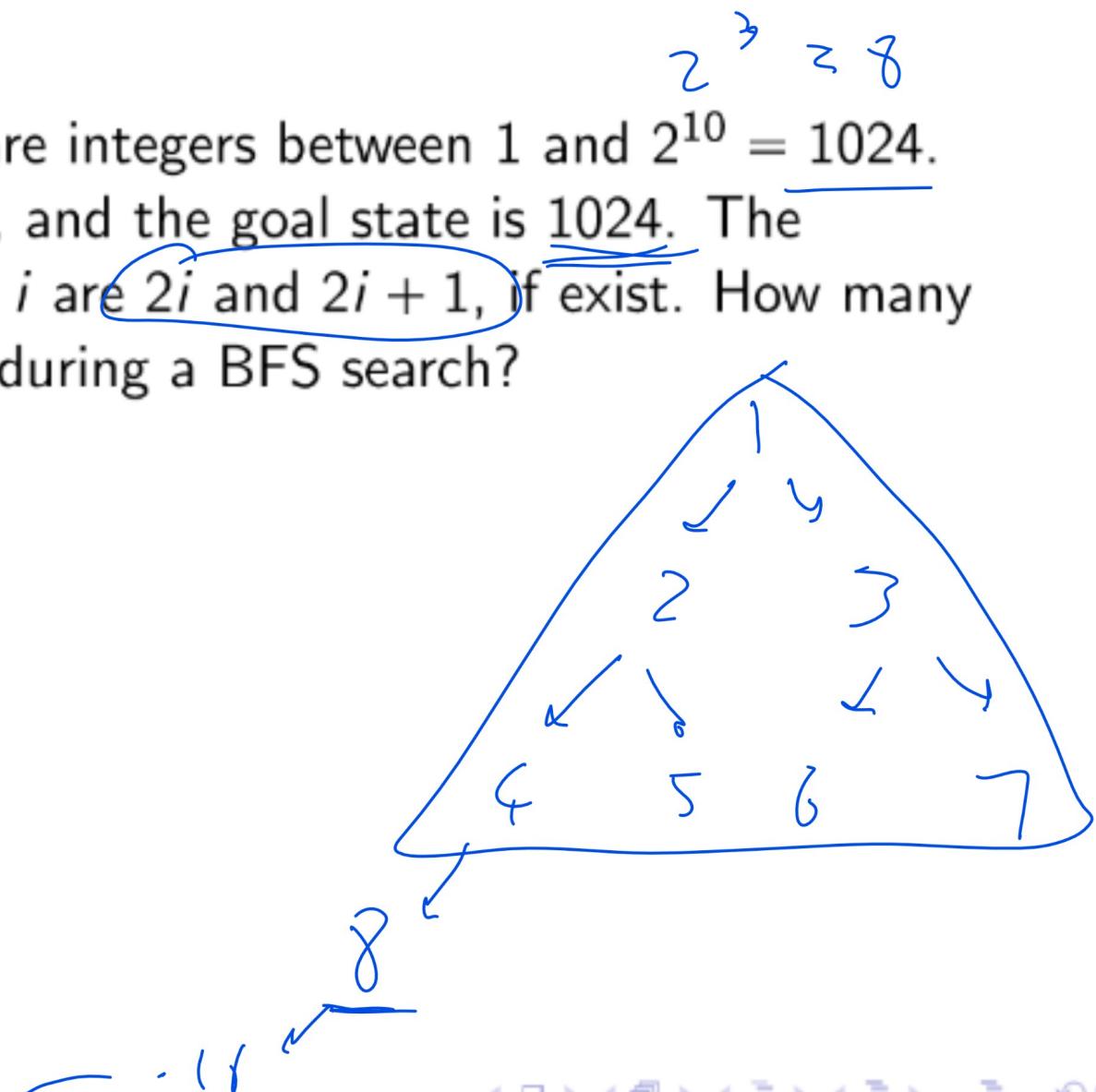
BFS Example 3

Quiz

Q4

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024



Uninformed Search
oooo

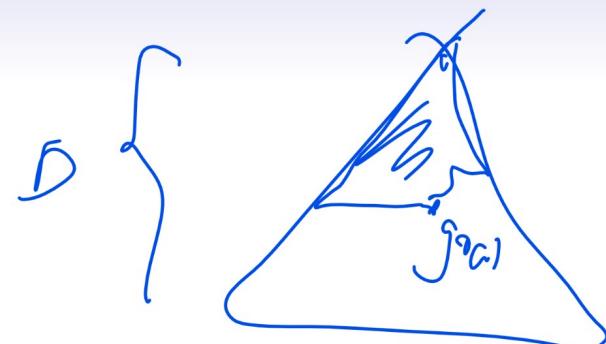
BFS
ooooooo•ooo

DFS
oooooooooooooooooooo

BFS Example 4

Q5 (last)

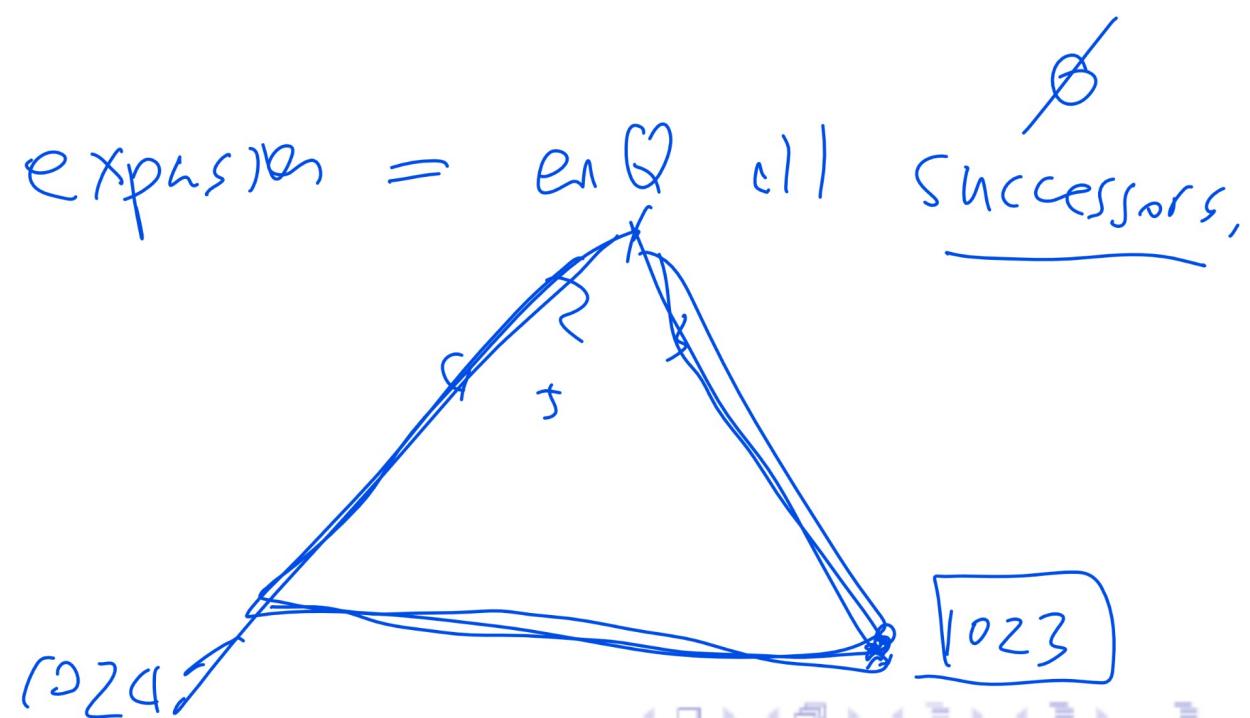
Quiz



- Suppose the states are integers between 1 and $2^{10} - 1 = \underline{1023}$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12

- D: 1023
- E: 1024



Breadth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- EnQueue initial states.

$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_0$$

$$Q = Q + s'(s)$$

Uninformed Search
oooo

BFS
oooooooo●○

DFS
oooooooooooooooooooo

Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

Breadth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = b + b^2 + \dots + b^d$$

- Space complexity: the worst case is storing all vertices at depth d in the frontier.

$$S = b^d$$

Uninformed Search
oooo

BFS
oooooooooooo

DFS
●oooooooooooooooooooo

Depth First Search

Description

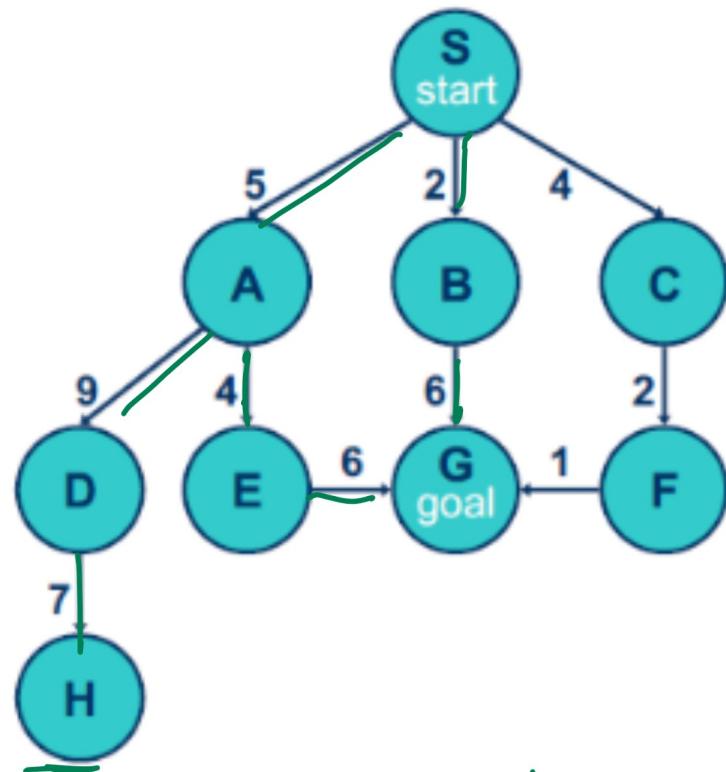
- Use Stack (LIFO) for the frontier.
- Remove from the front, add to the front.

Uninformed Search
oooo

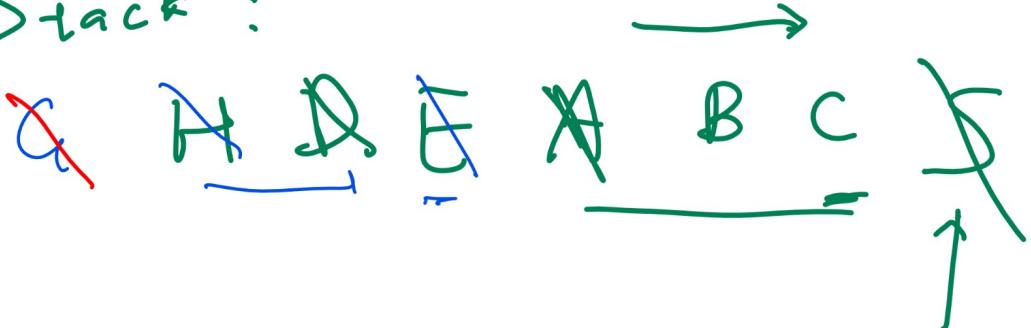
BFS
oooooooooooo

DFS
o●oooooooooooooooooooo

DFS Simple Example



Quiz
Stack :



note
BFS

A B C

Stop when pop G

for PS
mg

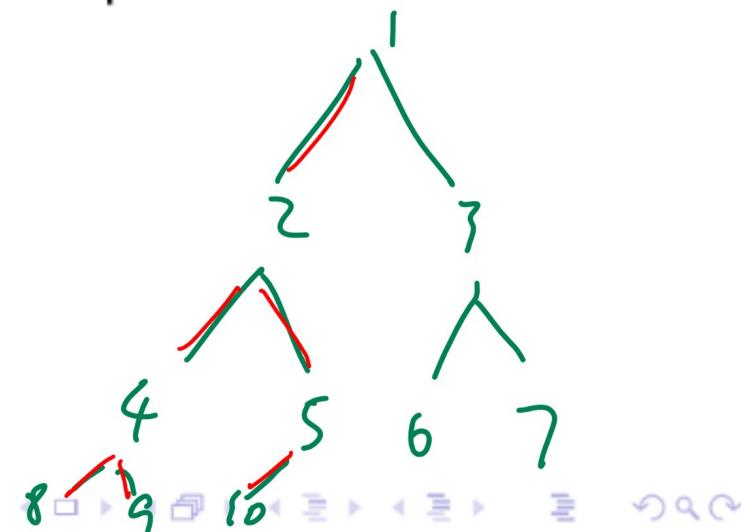
expansion path: S, A, D, H, E, G

DFS Example 1

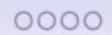
Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a DFS expansion sequence?

1, 2, 4, 8, 9, 5, 10



Uninformed Search



BFS



DFS



DFS Example 1 Diagram

Quiz

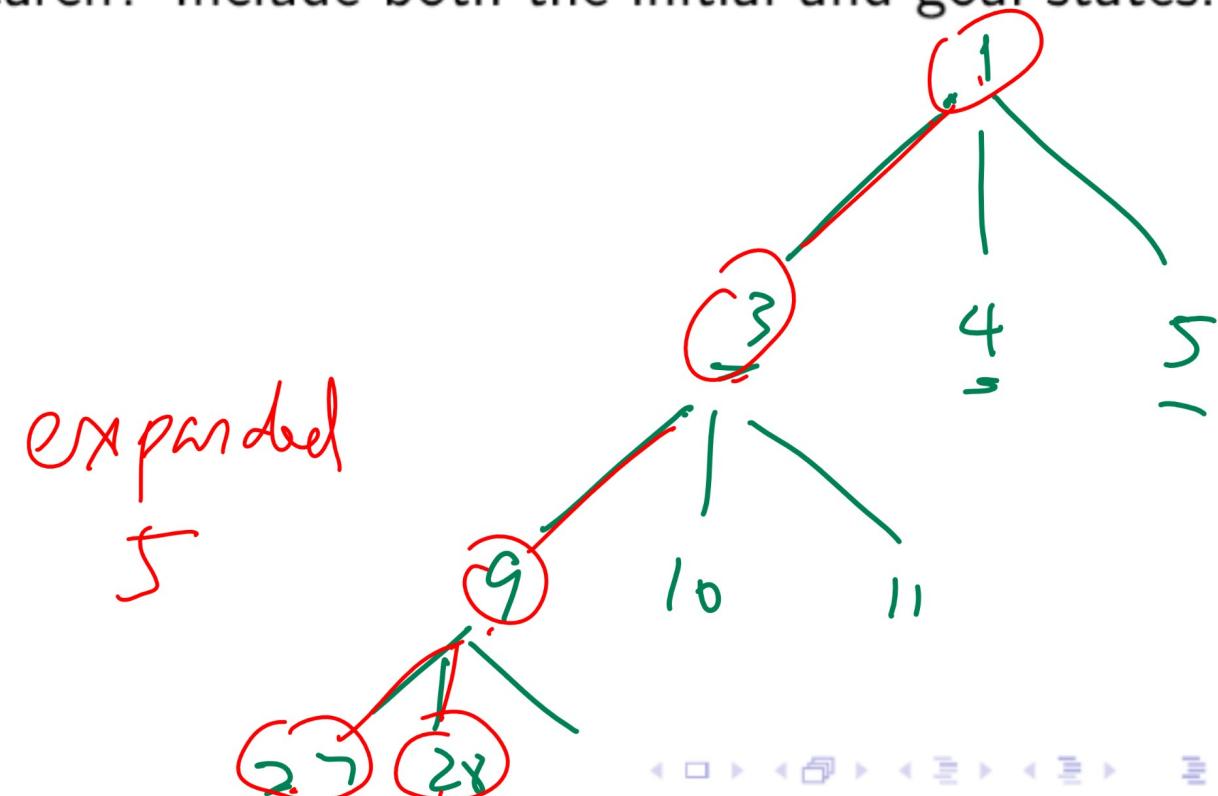
DFS Example 2

Quiz

Q 1

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a DFS search? Include both the initial and goal states.

- A: 4
- B: 5
- C: 14
- D: 15
- E: 28



DFS Example 3

Quiz

Q2

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

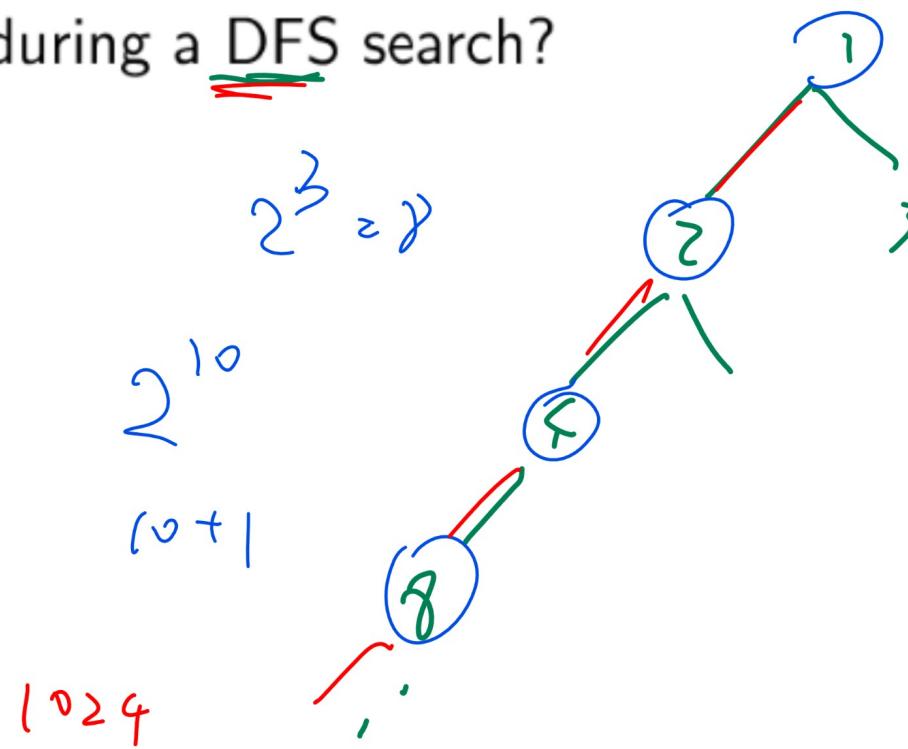
A: 10

B: 11

C: 12

D: 1023

E: 1024



DFS Example 4

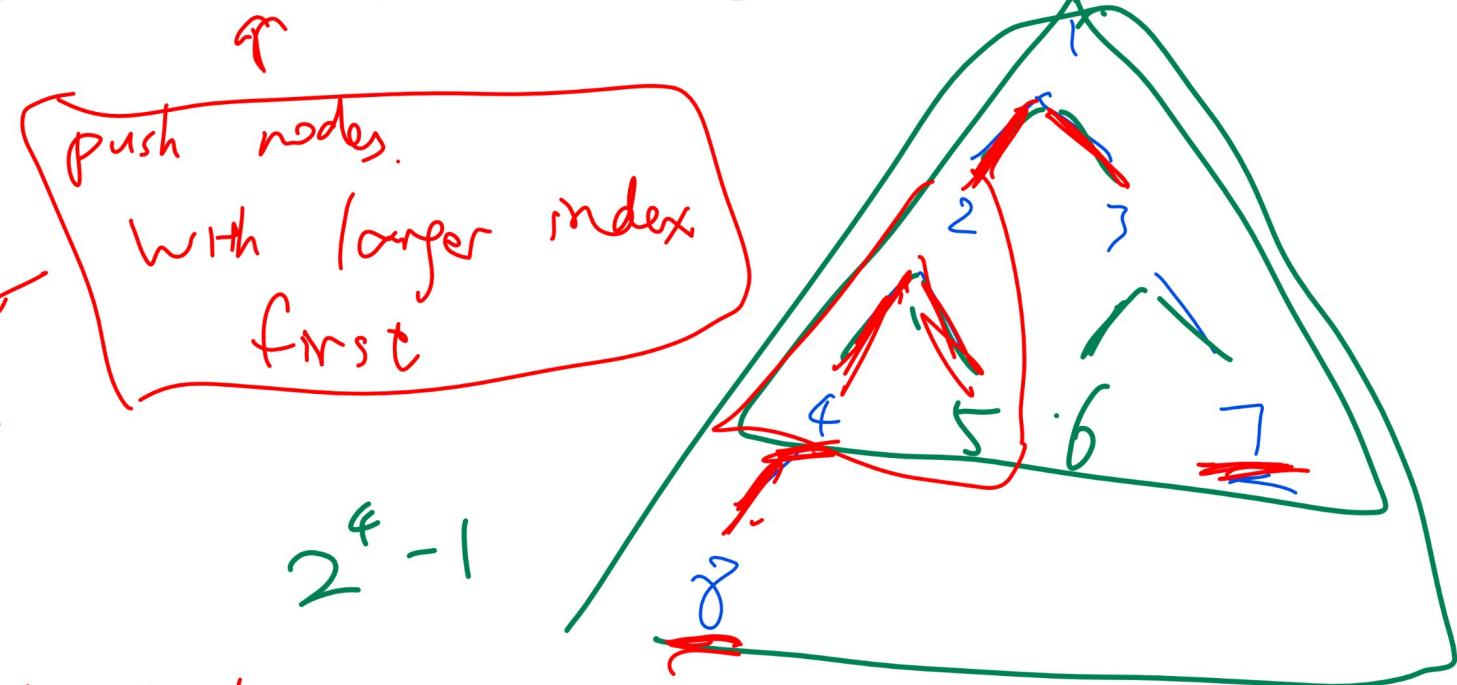
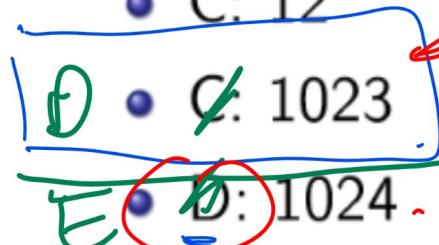
Quiz

Q3

put on final

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

- A: 10
- B: 11
- C: 12



Depth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Push initial states.

$$S = I$$

- While S is not empty and goal is not popped, pop S and push its successors.

$$s = S_0$$

$$S = s'(s) + S$$

Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

Depth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

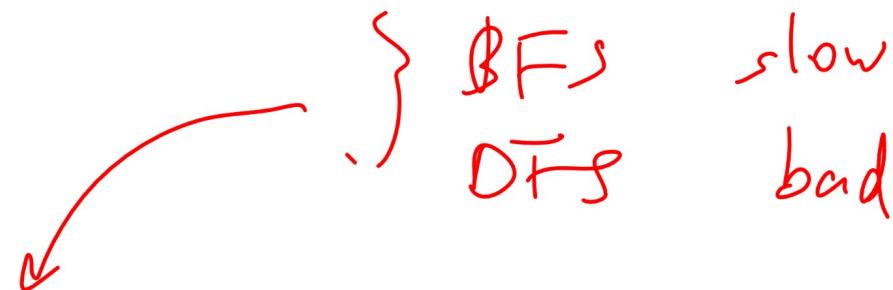
$$T = b^{D-d+1} \dots + b^{D-1} + b^D$$

- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

$$S = (b - 1) D + 1$$

Iterative Deepening Search

Description



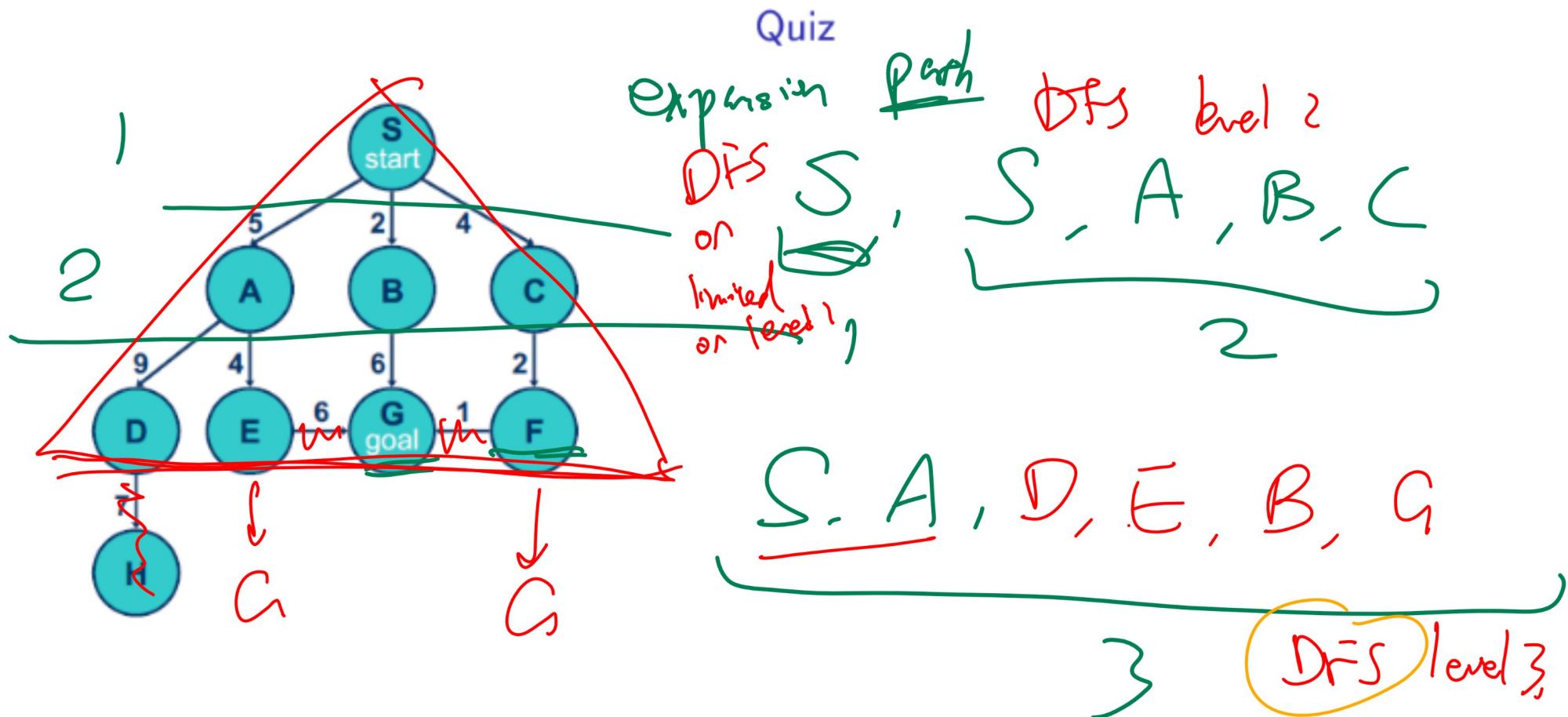
- DFS but stop if path length $> \underline{1}$]]
- repeat DFS but stop if path length $> \underline{2}$]]
- ...
- repeat DFS but stop if path length $> \underline{\underline{d}}$

Uninformed Search
oooo

BFS
oooooooooooo

DFS
oooooooooooo•oooooooo

IDS Simple Example



IDS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?

Same.

Uninformed Search

○○○○

BFS

○○○○○○○○○○

DFS

○○○○○○○○○○○●○○○○○

IDS Example 1 Diagram

Quiz

IDS Example 2

Quiz

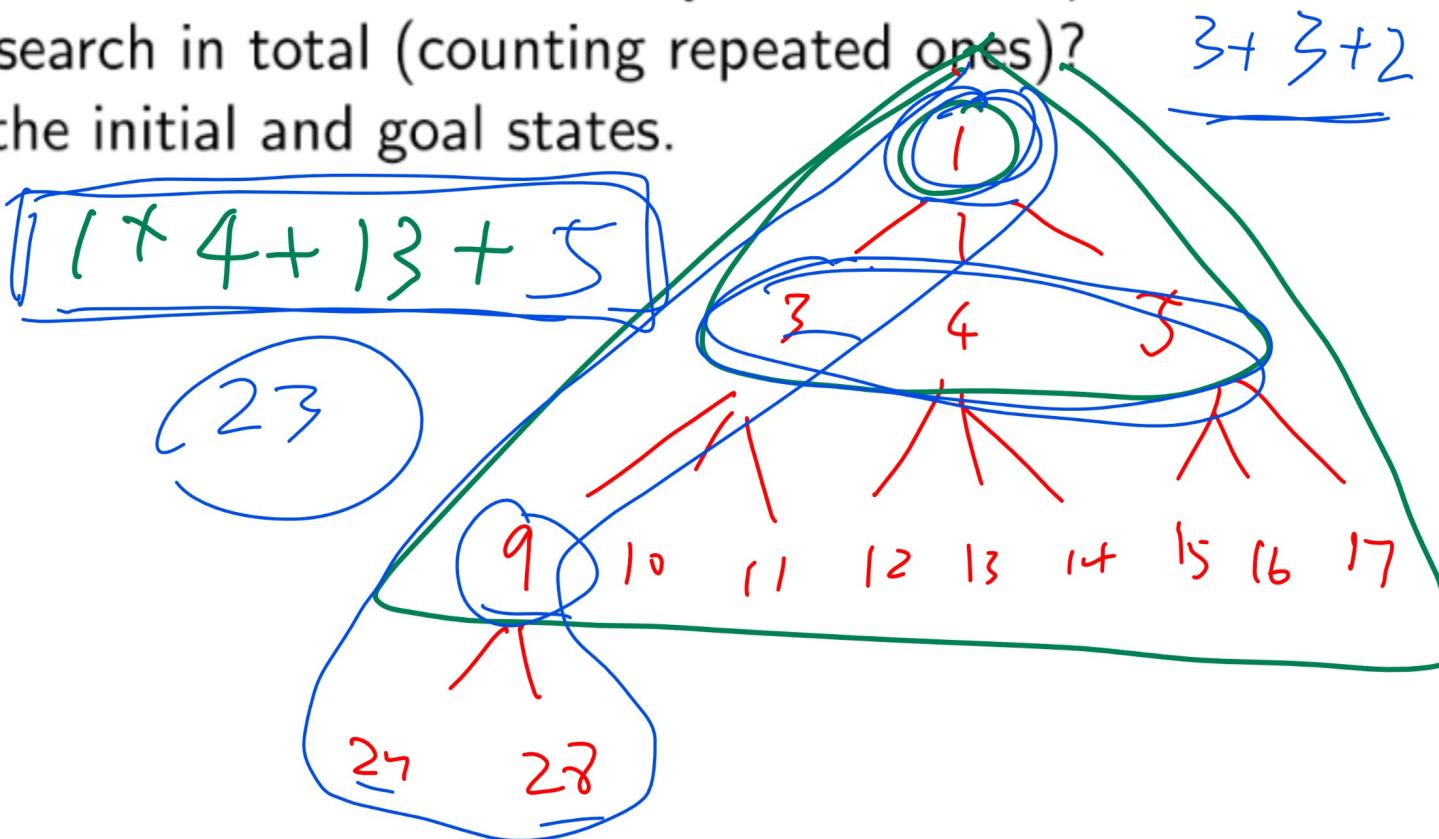
- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many unique states are expanded during a IDS search? Include both the initial and goal states.
- A: 4
- B: 5
- C: 14
- D: 15
- E: 28

~~BFS~~ Example 3
~~IDS~~ Quiz

Q4

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a IDS search in total (counting repeated ones)?
Include both the initial and goal states.

- A: $15 + 8$
- B: $15 + 9$
- C: $15 + 13$
- D: $15 + 17$
- E: $15 + 18$



Iterative Deepening Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Perform DFS on the digraph restricted to vertices with depth ≤ 1 from the initial state.
- Perform DFS on the digraph restricted to vertices with depth ≤ 2 from the initial state.
- Repeat until the goal is dequeued.

Uninformed Search
○○○○

BFS
○○○○○○○○○○

DFS
○○○○○○○○○○○○○○●○○

Iterative Deepening Search Performance

Discussion

- IDS is complete.
- IDS is optimal with $c = 1$.

Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = db + (d - 1) b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b - 1) d$$

Uninformed Search
○○○○

BFS
○○○○○○○○○○

DFS
○○○○○○○○○○○○○○○○●

Configuration Space

Discussion