

# CS540 Introduction to Artificial Intelligence

## Lecture 1

Young Wu

Based on lecture slides by Jerry Zhu and Yingyu Liang

May 23, 2019

# Grading

Admin

- Quizzes: best 10 of 11 weeks, 2 points each.
- Math homework: use them to replace quiz grades.
- Programming homework: best 10 of 11 weeks, 4 points each.
- Exams: one midterm and one final, 20 points each.

# Quizzes

Admin

- Download Socrative, the room number is CS540S1 or CS540S2.
  - Default login for Socrative is your wisc email ID.
  - If someone else tries to hack your account, please email or post on Piazza.
  - 1 point for Participation questions.
  - 1 point for Graded questions.
  - Points rounded up to one of {0, 0.5, 1, 1.5, 2}.
  - Quiz questions can show up any time during the lecture.

# Test

## Quiz (Graded)

- A: Don't choose this
- B: Don't choose this
- C: Choose this
- D: Don't choose this
- E: Don't choose this

# Quizzes on Canvas

Admin

- If there is any problem with your device or the app, you can submit your answers on paper or Canvas before the end of the lecture.

# Guess Average Game

## Quiz (Participation)

- Write down an integer between 0 and 100 that is the closest to two thirds ( $2/3$ ) of the average of everyone's (including yours) integers.
- A: 0 – 20
- B: 21 – 40
- C: 41 – 60
- D: 61 – 80
- E: 81 – 100

# Guess Average Game, Again

## Quiz (Participation)

- Write down an integer between 0 and 100 that is the closest to two thirds ( $2/3$ ) of the average of everyone's (including yours) integers.
- A: 0 – 10
- B: 11 – 20
- C: 21 – 30
- D: 31 – 60
- E: 61 – 100

# Math Homework

Admin

- Due in 1 week Sunday (Monday morning is okay).
- Grade yourself: one of {1, 1.5, 2}
- 1 means you attempted something but you know it's completely incorrect.
- 1.5 means you attempted something but you know it's not completely correct.
- 2 means you think everything is correct and you give me permission to share it with other students as a sample solution.
- Put 2.5 if you already got 2 for the Quiz and just want me to share your (hopefully) correct solutions with other students.

# Programming Homework

Admin

- Due in 1 week Sunday (if you don't want spoilers).
- Can submit any time before Sunday in 3 weeks (we will post our solutions in Java, Python, or Matlab after the 1 week due date).
- You can fix your code and output and resubmit after the due dates to replace the previous grade.
- 2 points for output (auto-graded).
- 2 points for code (only check for correctness and plagiarism).
- You can submit output without code to get 2 if you use (steal) code from other people.
- If you are caught submitting someone else's code or output, you cannot resubmit.

# Quiz (Participation)

## Favorite Programming Language

- What is your favorite programming language (choose one)?
- A: Java
- B: Python
- C: Matlab
- D: Other
- E: None

# Midterm and Final

Admin

- Two alternative dates, attend either one. The second one is harder.
- 40 Multiple Choice questions: around half will be math and statistics related questions, the other half will be algorithm related questions.

# (Not recommended) Ways to Get B+

Admin

- Not attending any lecture and not doing any math homework.
- Not learning any math and statistic for exams.
- Not attending one of the exams.
- Not doing any programming: use the code from other people every week.

# Only Way to Get A Admin

- Do everything.

# Textbook

Admin

- SS is available for free online.
- If you are planning to take 760, 761, 861 in the future, it is highly recommended that you read the first few chapters of this book.
- Otherwise, you can skip all the error bound, VC dimension related materials.

# Admin

Admin

- Math and Stat Review posted under W1.
- Complete slides (with diagrams and quiz questions etc) will be posted Thursday or Friday.
- Homework will be posted on Friday (due in 9 days, not 2 days).
- Exact due dates are on Canvas: programming homework can be submitted two weeks late (except for the last two homework (one week late)).

# Questions

Admin

- Questions?

# Supervised Learning

## Motivation

- Supervised learning:

Data	Features (Input)	Output	-
Sample	$\{(x_{i1}, \dots, x_{im})\}_{i=1}^n$	$\{y_i\}_{i=1}^n$	find "best" $\hat{f}$
-	observable	known	-
New	$(x'_1, \dots, x'_m)$	$y'$	guess $\hat{y} = \hat{f}(x')$
-	observable	unknown	-

# Training and Test Sets

## Motivation

- Supervised learning:

Data	Features (Input)	Output	-
Training	$\{(x_{i1}, \dots, x_{im})\}_{i=1}^{n'}$	$\{y_i\}_{i=1}^{n'}$	find "good" $\hat{f}$
-	observable	known	-
Validation	$\{(x_{i1}, \dots, x_{im})\}_{i=n'}^n$	$\{y_i\}_{i=n'}^n$	find "best" $\hat{f}$
-	observable	known	-
Test	$(x'_1, \dots, x'_m)$	$y'$	guess $\hat{y} = \hat{f}(x')$
-	observable	unknown	-

# Loss Function

## Motivation

- An objective function is needed to select the "best"  $\hat{f}$ . An example is the squared distance between the predicted and the actual  $y$  value:

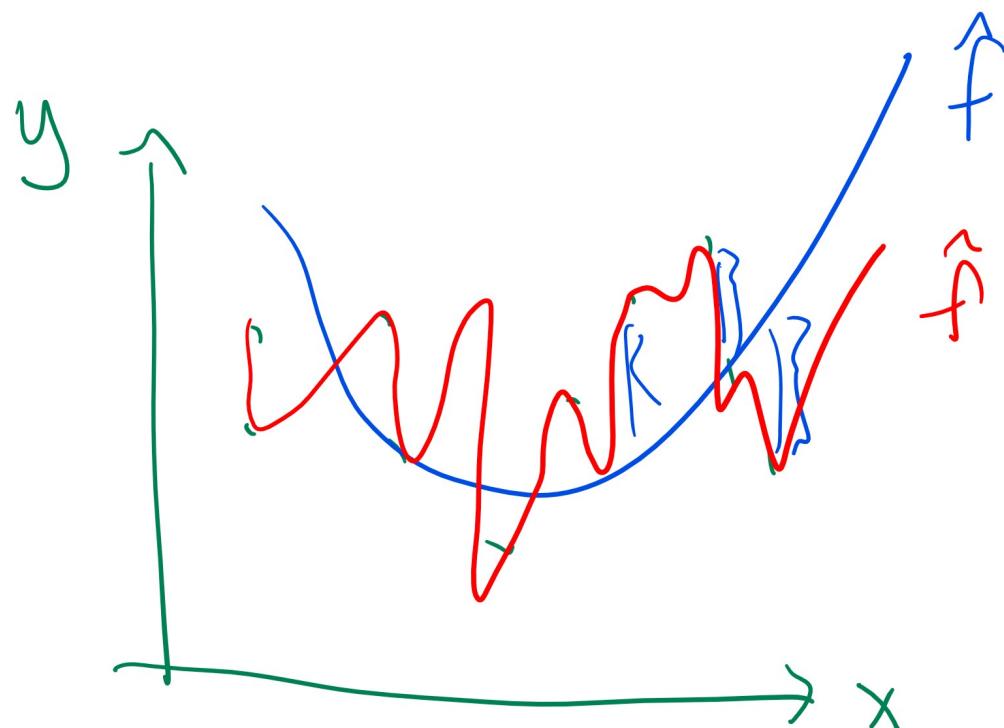
$$\hat{f} = \arg \min_f \frac{1}{2} \sum_{i=1}^n (f(x_i) - y_i)^2$$

*actual*  
*prediction*

- The objective function is called the cost function (or the loss function), and the objective is to minimize the cost.
- A training data point  $x_i$  is also called an instance.

# Function Space Diagram

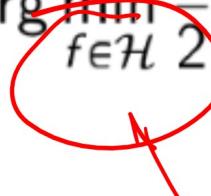
Motivation



# Hypothesis Space

## Motivation

- There are too many functions to choose from.
- There should be a smaller set of functions to choose  $\hat{f}$  from.

$$\hat{f} = \arg \min_{f \in \mathcal{H}} \frac{1}{2} \sum_{i=1}^n (f(x_i) - y_i)^2$$


- The set  $\mathcal{H}$  is called the hypothesis space.

# Linear Regression

## Motivation

- For example,  $\mathcal{H}$  can be the set of linear functions. Then the problem can be rewritten in terms of coefficients (parameters).

$$(\hat{w}_1, \dots, \hat{w}_m, \hat{b}) = \arg \min_{w_1, \dots, w_m, b} \frac{1}{2} \sum_{i=1}^n (a_i - y_i)^2$$

where  $a_i = w_1 x_{i1} + w_2 x_{i2} + \dots + w_m x_{im} + b$

- $\{w_1, \dots, w_m\}$  are called weights.  $b$  is called bias.
- The problem is called (least squares) linear regression.

# Activation Function

## Motivation

- Suppose  $\mathcal{H}$  is the set of functions that are compositions between another function  $g$  and linear functions.

$$(\hat{w}_0, \hat{w}_1, \dots, \hat{w}_m, \hat{b}) = \arg \min_{w_1, \dots, w_m, b} \frac{1}{2} \sum_{i=1}^n (a_i - y_i)^2$$

where  $a_i = g(w_1x_{i1} + w_2x_{i2} + \dots + w_mx_{im} + b)$

$g(x_{i1}, x_{i2}, \dots)$

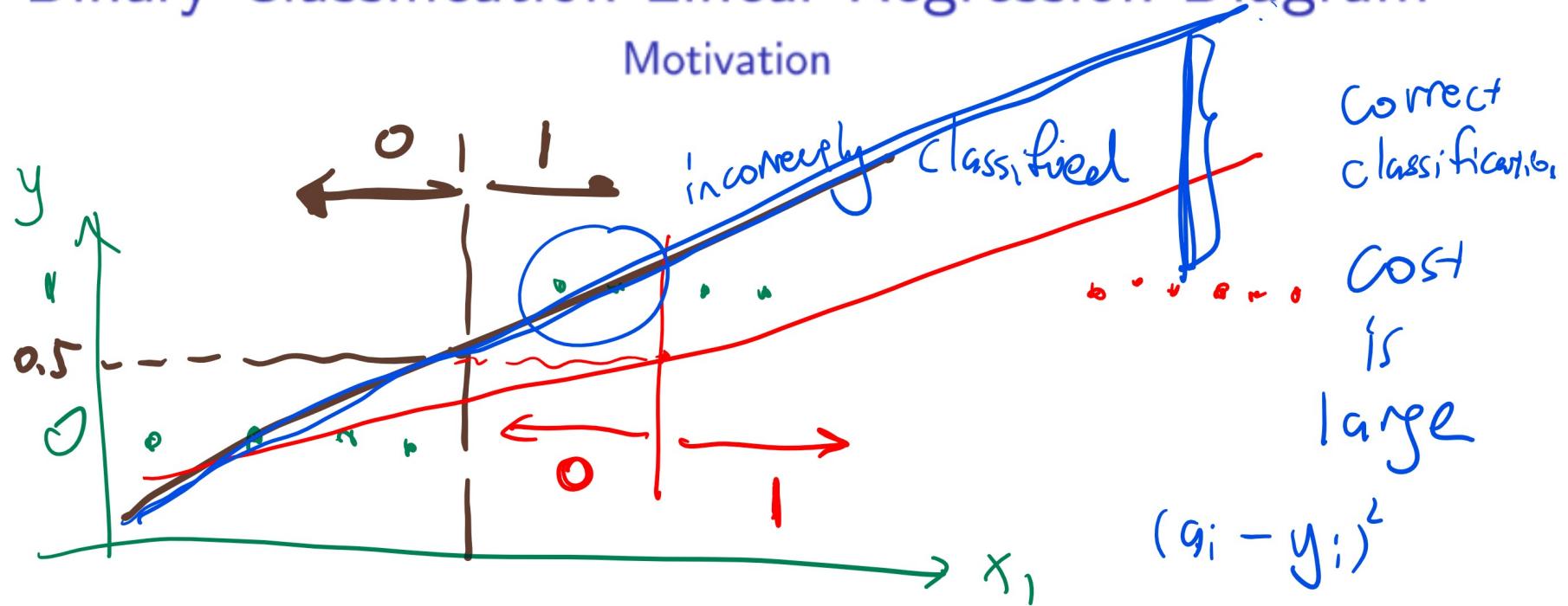
- $g$  is called the activation function.

# Binary Classification

## Motivation

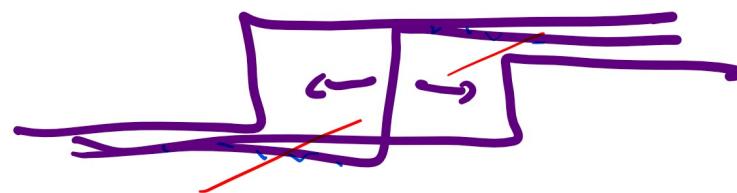
- If the problem is binary classification,  $y$  is either 0 or 1, and linear regression is not a great choice.
- This is because if the prediction is either too large or too small, the prediction is correct, but the cost is large.

# Binary Classification Linear Regression Diagram



# Linear Threshold Unit

## Motivation



- One simple choice is to use the step function as the activation function:

$$g(\boxed{\cdot}) = \mathbb{I}_{\{\boxed{\cdot} \geq 0\}} = \begin{cases} 1 & \text{if } \boxed{\cdot} \geq 0 \\ 0 & \text{if } \boxed{\cdot} < 0 \end{cases}$$

- This activation function is called linear threshold unit (LTU).

$$\mathbb{I}_{\{x \geq 0\}} = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Same



## Objective Function for LTU

Quiz (Graded)

up to constant

- Which ones (multiple) of the following functions are equivalent to the squared error for binary classification?

$$\begin{cases} 1 & \text{if } a_i \approx y_i \\ 0 & \text{if } a_i \neq y_i \end{cases}$$

$$C = \sum_{i=1}^n (a_i - y_i)^2, y_i \in \{0, 1\}$$

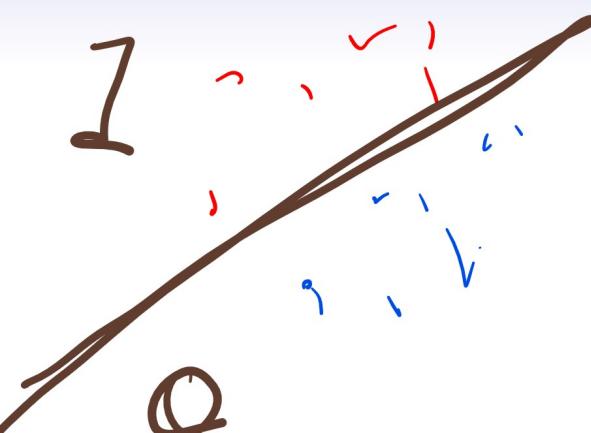
- A:  $\sum \mathbb{1}_{\{a_i = y_i\}}$
- B:  $\sum \mathbb{1}_{\{a_i \neq y_i\}}$
- C:  $\sum |a_i - y_i|$
- D:  $\sum \max\{0, 1 - a_i y_i\}$
- E:  $\sum \max\{0, 1 - (2 \cdot a_i - 1)(2 \cdot y_i - 1)\}$

✓ ✓

✓

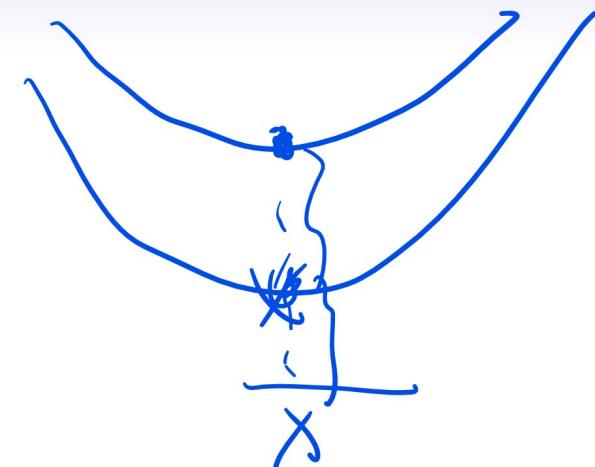
$y_i$	$a_i$	$C$
0	0	0
1	0	1
0	1	1
1	1	0

$y_i$	$a_i$	$C$
0	0	0
1	0	1
0	1	1
1	1	0



## Perceptron Algorithm

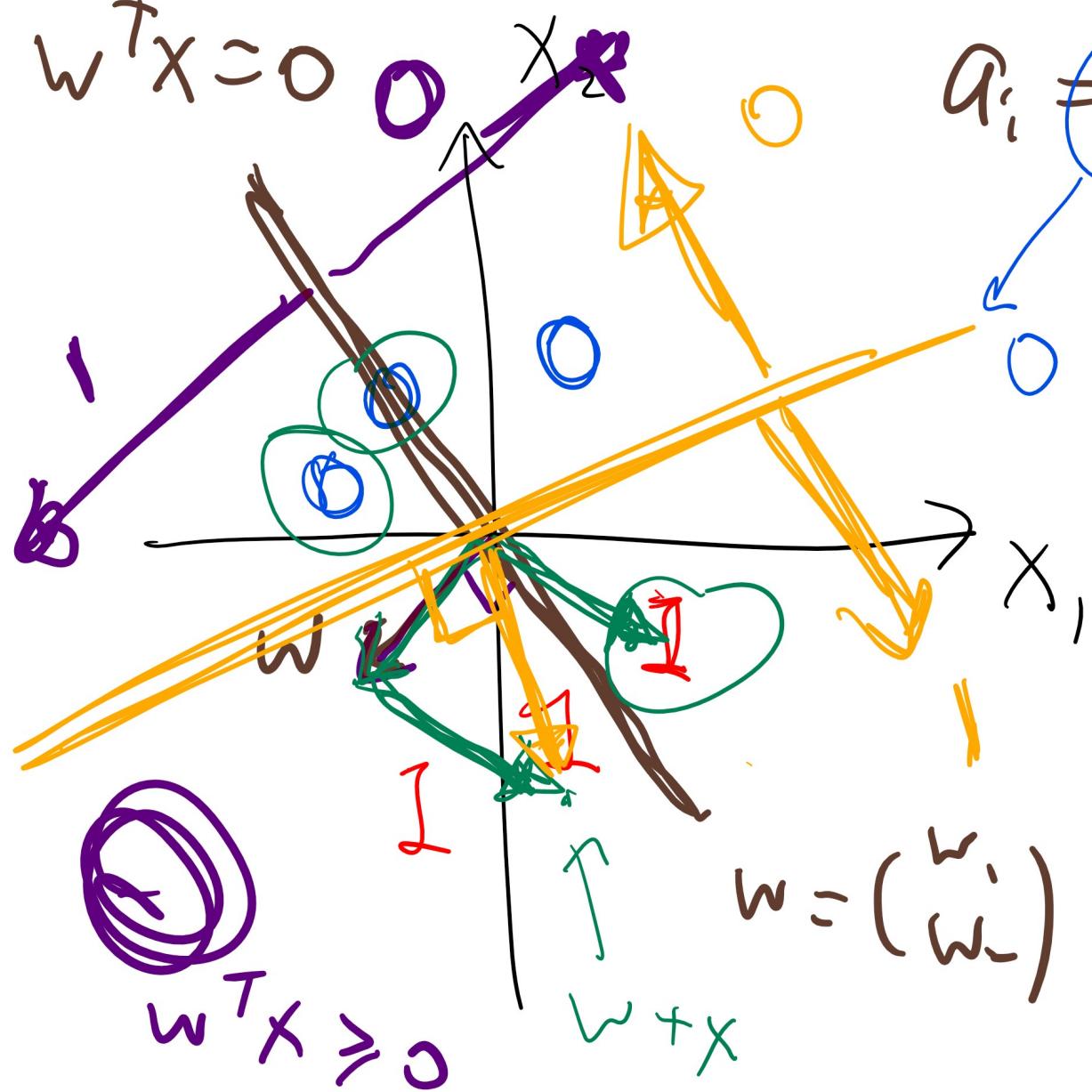
### Description



- Initialize random weights.
- Evaluate the activation function at one instance  $x_i$  to get  $\hat{y}_i$ .
- If the prediction  $\hat{y}_i$  is 0 and actual  $y_i$  is 1, increase the weights by  $x_i$ .
- If the prediction  $\hat{y}_i$  is 1 and actual  $y_i$  is 0, decrease the weights by  $x_i$ .
- Repeat for all data points and until convergent.

# Perceptron Algorithm Diagram, 0 Example

Description



$$a_i = \begin{cases} 1 & \{w_1 x_1 + w_2 x_2 \geq 0\} \\ -1 & \{w_1 x_1 + w_2 x_2 < 0\} \end{cases}$$

no bias

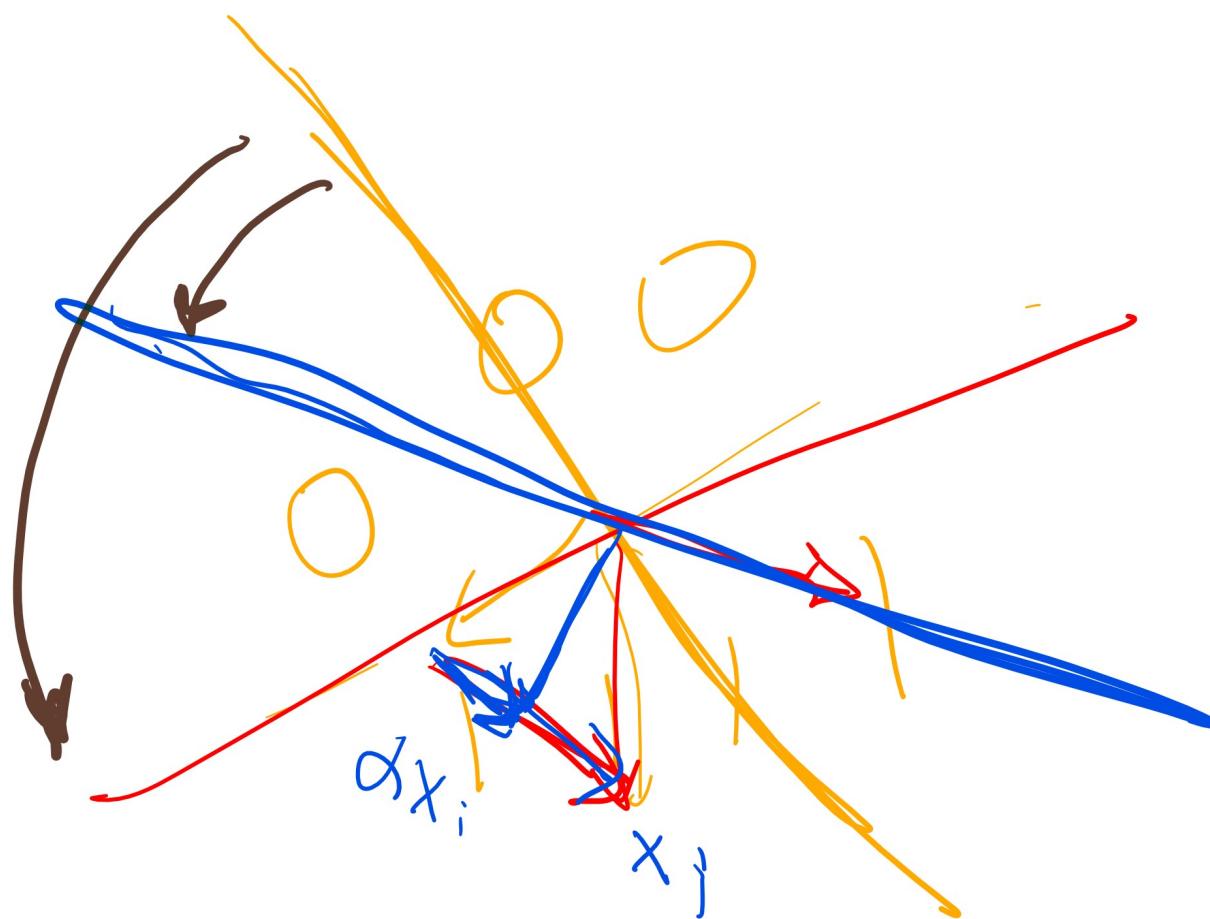
$$\begin{array}{c} > 0 \\ \geq 0 \\ < 0 \\ \leq 0 \end{array} \quad \begin{array}{c} > 0 \\ > 0 \\ < 0 \\ < 0 \end{array}$$

$$\begin{array}{c} \leq 0 \\ \leq 0 \end{array} \quad \begin{array}{c} w_1 x_1 + w_2 x_2 \\ = - \end{array}$$

$$w^T x = (w_1, w_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

# Perceptron Algorithm Diagram, 1 Example

Description



# Perceptron Algorithm, Part 1

## Algorithm

- Inputs: instances:  $\{x_i\}_{i=1}^n$  and  $\{y_i\}_{i=1}^n$
- Outputs: weights and biases:  $w_1, \dots, w_m$ , and  $b$
- Initialize the weights,

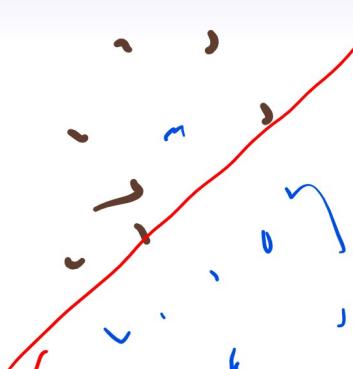
$$w_1, \dots, w_m, b \sim \text{Unif } [0, 1]$$

- Evaluate the activation function at a single data point  $x_i$ ,

$$a_i = \mathbb{1}_{\{w^T x_i + b \geq 0\}}$$

## Perceptron Algorithm, Part 2

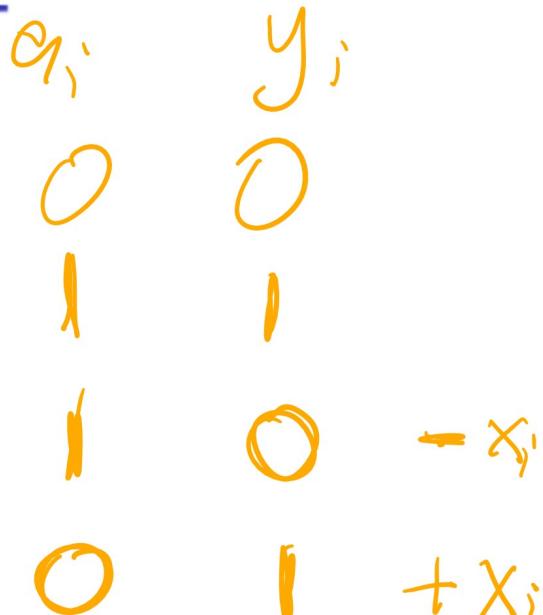
### Algorithm



- Update weights using the following rule,

$$w = w - \alpha(a_i - y_i)x_i$$

$$b = b - \alpha(a_i - y_i)$$



- Repeat the process for every  $x_i, i = 1, 2, \dots, n$
- Repeat until  $a_i = y_i$  for every  $i = 1, 2, \dots, n$

Learning rate

If  $(x, y)$  is not  
linearly separable  $\Rightarrow$

$0 < \alpha \leq 1$   
not converge. (Stop)

# Learning Rate

## Discussion

- The learning rate  $\alpha$  controls how fast the weights are updated.
- They can be constant for each update or they can change (usually decrease) for each update.
- For perceptron learning, it is typically set to 1.

Q7

## Perceptron Algorithm

Quiz (Graded)

0, 2

$$q_i = \begin{cases} 1 & w_1 x_1 + w_2 x_2 \\ & + w_3 x_3 + b \geq 0 \end{cases}$$

- 2017 May Final Exam Q3
- Let the learning rate be  $\alpha = 0.2$ . Currently  $w = [0.2 \ 0.7 \ 0.9]^T$ ,  $b = -0.7$ , and  $x_i = [0 \ 0 \ 1]^T$  and  $y_i = 0$ . What is the updated weights  $\begin{bmatrix} w \\ b \end{bmatrix}$ ?

- A:  $[0 \ 0.5 \ 0.9 \ -0.7]^T$
- B:  $[0.2 \ 0.7 \ 1.1 \ -0.5]^T$
- C:  $[0.2 \ 0.7 \ 0.7 \ -0.9]^T$
- D:  $[0.4 \ 0.9 \ 0.9 \ -0.7]^T$
- E: none of the above

$$\begin{cases} w = w - \alpha(q_i - y_i)x_i \\ b = b - \alpha(q_i - y_i) \end{cases}$$

$$-0.7 - 0.2(1 - 0) \\ -0.9$$