

# interval analysis

abstract domain:

- $\perp$

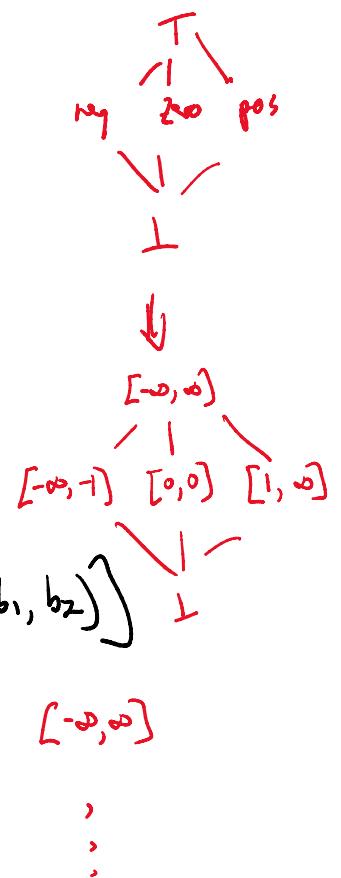
- $[a, b]$  s.t.  $a \in \mathbb{Z} \cup \{-\infty\}$   
 $b \in \mathbb{Z} \cup \{\infty\}$

$$a \leq b$$

$$(T = [-\infty, \infty])$$

$$[a_1, b_1] \cup [a_2, b_2] = [\min(a_1, a_2), \max(b_1, b_2)]$$

$$[a_1, b_1] + [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$$



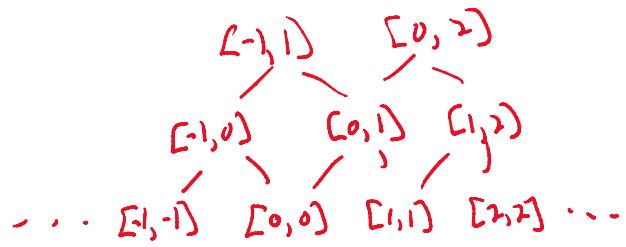
## WIDENING

- widening operator  $\nabla$   
 $L \text{ join } H$

- there are many possible widening operators for an abstract domain

- one example widening operator  
 for the interval domain

$$\cdot L \nabla X = X \nabla \perp = X$$



- $\perp \triangleright X = X \triangleright \perp = X$
- $[a_1, b_1] \triangleright [a_2, b_2] = [a_3, b_3]$  s.t.

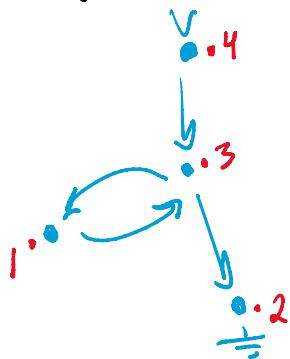
$$L_{a_3} = a_1 \text{ if } a_1 \leq a_2, \text{ else } -\infty$$

$$L_{b_3} = b_1 \text{ if } b_1 \geq b_2, \text{ else } \infty$$

- to keep precision, only apply  $\triangleright$  at loop headers, otherwise use  $\sqcup$

to detect loop headers in CFG:

post-order labeling starting from "entry"



if child node has already been visited but <sup>not</sup> yet labeled, it is a loop header

first-order DFA : abstracts values

second-order DFA : abstracts control-flow (<sup>execution</sup> paths)

faint analysis, asks "how did this string  
... ?"

L faint analysis asks "how did this string  
get to this system call?"

we abstract traces just we abstract values

### reaching definitions

problem stmt: for each "use" of a variable  
what "def"s of that variable may "reach"  
this program point?

L "use": require variable's value

L "def": definition, ie, assignment

L "reach": there is some path from def to use  
that does not definitely redefine  
that variable

DFA: ~~abstract domain~~ + abstract semantics + abstract execution

elements of abstract domain: sets of program points  
representing defs

$$\perp = \{\}$$

T = set of all program points

U = Set U

abstract domain =  $\wp(\text{PP})$  where PP is the set  
of program points  
ordered by set inclusion

abstract store

map variables to sets of program points

↳ for each type, create a fake variable that  
stands for all heap objects of that type

for type  $T$  let reachable-types( $T$ ) be  
the set of types "reachable" via ptr  
dereference and/or struct field accesses from  $T$ ,  
excluding struct ≠ function types

example : struct foo {  
 f1: &bar  
 f2: &int  
}

of lab: &foo

struct bar {  
 f3: &(int) → int  
 f4: &&int  
}

↳ reachable-types(&foo) =  
 $\{ \&bar, \&int, \&(\text{int}) \rightarrow \text{int}, \&\&int, \text{int} \}$

$\{ \&\text{bar}, \&\text{int}, \&(\text{int}) \rightarrow \text{int}, \&\&\text{int}, \text{int} \}$

let PTRS be all pointer-typed globals,  
parameters, & locals of the function being analyzed.

- for  $T \in \text{reachable-types(PTRS)}$ , create a  
fake variable representing heap objects  
of that type
- put all fake variables in 'addr\_taken'