Week 5.1

Tuesday, February 6, 2024 4:34 PM

- abstract domain should be complete noetherian lattice

every subset of elements has a $\sqcup$ & $\sqcap$

finite ascending chains

partial order s.t. every two elements has $\sqcup$ & $\sqcap$

- abstract semantics should be monotone

$x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$

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strings: alphabet Σ

strings $\in \Sigma^*$

abstract domain

- elements of lattice?
- what is $\top, \bot$?
- what is $\sqcup$ & $\sqcap$?
- is the lattice noetherian?
- what is α & γ?

abstraction function

concretization function

abstract semantics

- define $\hat{f}$
- prove monotone

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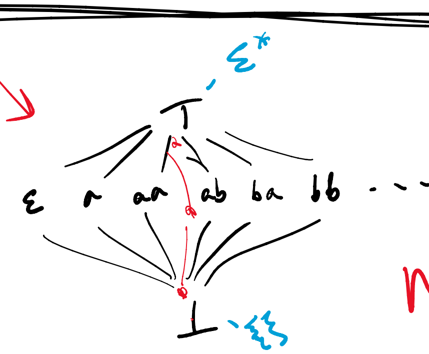
constants analysis



noetherian

abstract domain

$$S = \Sigma^* \cup \{\bot, \top\}$$

$$\begin{aligned}
\sqcup = \quad & \bot \sqcup s = s \sqcup \bot = s \\
& \top \sqcup s = s \sqcup \top = \top \\
& s_1 \sqcup s_1 = s_1 \\
& s_1 \sqcup s_2 = \top, \text{ s.t. } s_1 \neq s_2
\end{aligned}$$

$$\alpha : \mathcal{P}(\Sigma^*) \to S$$

$$\alpha(x) = \begin{cases} \bot & \text{if } x = \{\} \\ s & \text{if } x = \{s\} \\ \top & \text{otherwise} \end{cases}$$

$s_1 \sqcup s_2 = \top$, s.t. $s_1 \neq s_2$

$\sqcap$: $\bot \sqcap s = s \sqcap \bot = \bot$

$\top \sqcap s = s \sqcap \top = s$

$s_1 \sqcap s_1 = s_1$

$s_1 \sqcap s_2 = \bot$, s.t. $s_1 \neq s_2$

$\sqsubseteq$: $s_1 \sqsubseteq s_2$ iff $s_1 \sqcup s_2 = s_2$

$\alpha(x) = \begin{cases} s & \text{if } x = \{s\} \\ \top & \text{otherwise} \end{cases}$

$\gamma : S \to \mathcal{P}(\Sigma^*)$

$\gamma(x) = \begin{cases} \{\} & \text{if } x = \bot \\ \{s\} & \text{if } x = s \\ \Sigma^* & \text{if } x = \top \end{cases}$

$\boxed{\text{prefix analysis}}$

example : {abcd, abef}

$[ab] \Rightarrow \{ab.*\}$

$s \in \underline{S} = \Sigma^* \cup [\Sigma^*] \cup \{\bot\}$

$\top = [\varepsilon] = \Sigma^*$

empty string

$\sqcup$: $\bot \sqcup s = s \sqcup \bot = s$

$s_1 \sqcup s_1 = s_1$

$s_1 \sqcup s_2$ s.t. $s_1 \neq s_2$ = $[lcp(s_1, s_2)]$

foo = "foo"

[foo] = "foo" . Σ^*

$[\varepsilon]$ = "" . Σ^*

"abcd" $\sqcup$ "abef" = [ab]

$\sqcap$: · $s_1 \sqcap s_2 = s_1$ if $s_1 \sqcup s_2 = s_2$

· $s_1 \sqcap s_2 = \bot$ otherwise

"abcd" $\sqcap$ "abef"

$[a]$ = "a", "aa", "ab", "aaa", "aab", ...

$[aa]$ = "aa", "aaa", "aab", ...

✗ $[a] \leq [aa]$

✓ $[aa] \leq [a]$

$[\varepsilon]$

|

$[f]$

|

$[fo]$

|

" ...

YES noetherian

$[fo]$
$|$
$[foo]$
$|$
"foo"
$|$
$\bot$

$$\alpha(x) = \begin{cases} \bot & \text{if } x = \{\} \\ s & \text{if } x = \{s\} \\ [lcp(x)] & \text{otherwise} \end{cases}$$

$$\gamma(x) = \begin{cases} \{\} & x = \bot \\ \{s\} & x = s \\ \{s \mid lcp(s, [pre]) = pre\} & x = [pre] \end{cases}$$

regular expression analysis

↳ $\emptyset$ empty language

ε empty string

$a \in \Sigma$ character

$$\boxed{\begin{array}{c} r_1 \mid r_2 \\ r_1 \cdot r_2 \\ r^* \end{array}}$$

abstract domain

normalized

S = all regular expressions

$\ldots = \emptyset$

multiple reg. exp. all same language

$\cdot\ a^*$

$\mid aa^*$

S = all regular expressions

$\hookrightarrow \bot = \emptyset$

$\hookrightarrow \top = \Sigma^*$

- a^*
- $\varepsilon \mid aa^*$
- $\varepsilon \mid a \mid aaa^*$
- ⋮

$\sqcup$: $S_1 \sqcup S_2 = S_1 \mid S_2$

$\sqcap$: $S_1 \sqcap S_2 = S_1 \,\&\, S_2$

$\sqsubseteq$: $S_1 \sqsubseteq S_2$ iff $S_1 \sqcup S_2 = S_2$

$\hookrightarrow$ language containment

$r_1 \sqsubseteq r_2$ if $L(r_1) \subseteq L(r_2)$

not noethrian

given any finite regular language, we can add ∞ more strings to get "higher" elements of the domain

- "a"
- "a", "aa"
- "a", "aa", "aaa"
- ⋮

all subsets have a $\sqcup$ & $\sqcap$

~~not complete lattice~~

$\sqcup\{01, 0011, 000111, \ldots\} = 0^n1^n$

$\hookrightarrow$ not regular!

NEED WIDENING