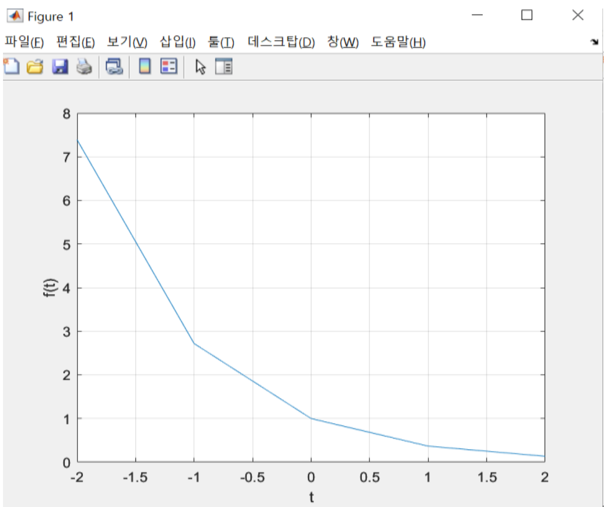
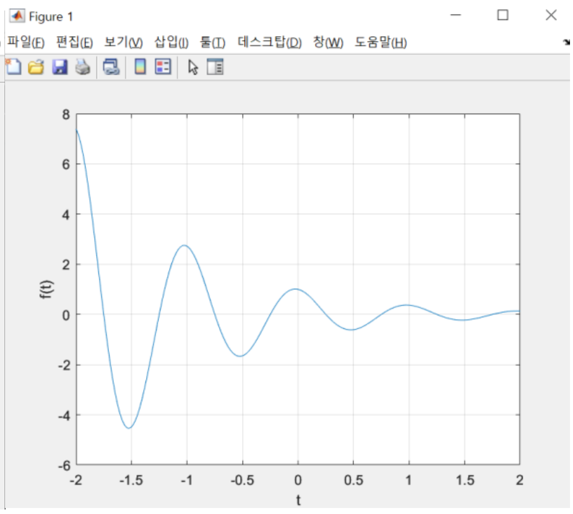
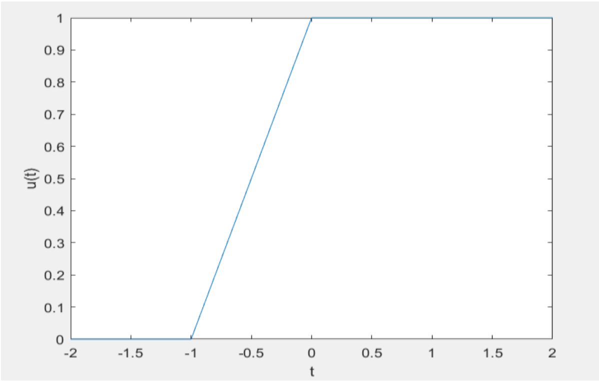
1. Matlab Session

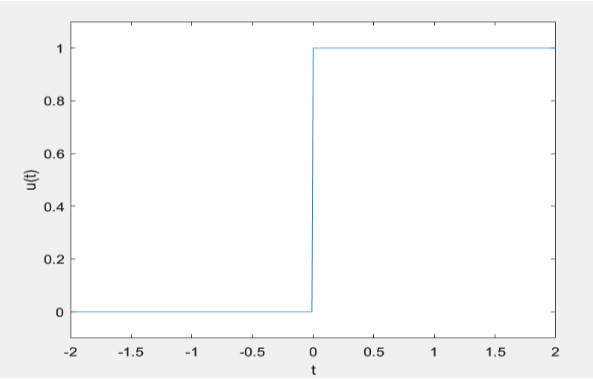
1-1.

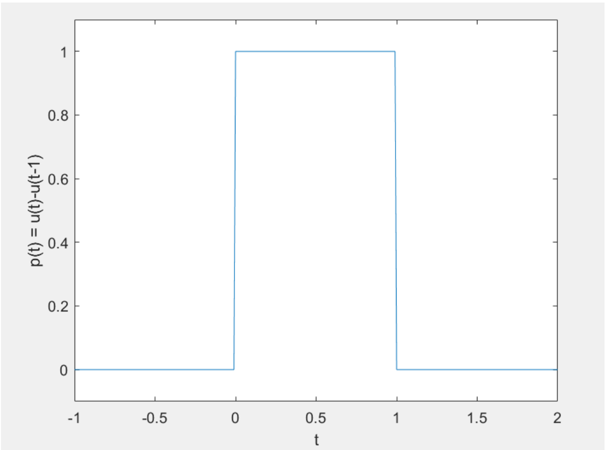




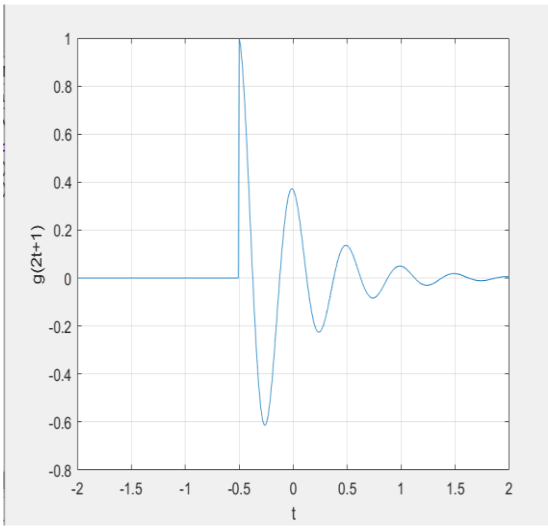
1-2.

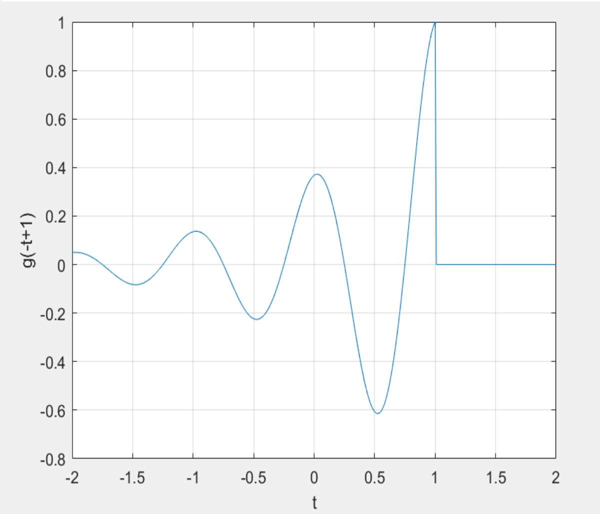


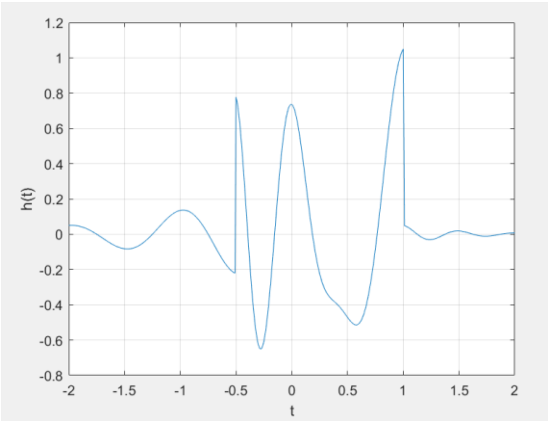




1-3.







1-4.

```
x = inline('exp(-t).*((t>=0)&(t<1))','t');
t = (0:0.01:1);
E_x =sum(x(t).*x(t)*0.01)
x_squared = inline('exp(-2*t).*((t>=0)&(t>1))','t');
E_x = quad(x_squared,0,1)
g_squared = inline('exp(-2*t).*(cos(2*pi*t).*^2).*(t>=0)','t');
t = (0:0.001:100);
E_g = sum(g_squared(t)*0.001)
E_g = quad(g_squared, 0,100)
```

g_squared	E_g

1x1 double

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.2567											
2												
3												
4												
5												
6												
7												

g_squared	E_g

1x1 double

	1	2	3	4	5	6	7	8	9	10
1	0.2562									
2										
3										
4										
5										
6										

2. 연습문제

– 1.1–1

1.1-1 Find the energies of the signals illustrated in Fig. P1.1-1. Comment on the effect on energy of sign change, time shifting, or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

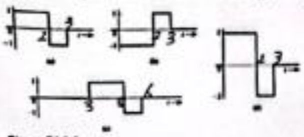


Figure P1.1-1

Energy ⇒ 비주기 함수.

(a) $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt=\int_0^2 1^2dt+\int_2^3 1^2dt=3.$

(b) $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt=\int_0^2 1^2dt+\int_2^3 1^2dt=3.$

(c) $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt=\int_3^5 1^2dt+\int_5^6 1^2dt=3.$

(d) $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt=\int_0^2 4dt+\int_2^3 4dt=12.$

∴ energy는 sign change나 time shifting 되어도 바뀌지않음.
doubling하면 제곱배인 4배가 되고,
K를 곱하면 K^2배가 된다.

– 1.1–5

1.1.5 Determine the power and the rms value for each of the following signals:

a. $5 + 10\cos(100t + \pi/3)$

b. $10\cos(100t + \pi/3) + 16\sin(150t + \pi/5)$

c. $(10 + 2\sin 3t)\cos 10t$

d. $10\cos 5t \cos 10t$

e. $10\sin 5t \cos 10t$

f. $e^{j\alpha t}\cos\omega_0 t$

곱 → 합·차

① $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

② $\cos\alpha\sin\beta = \frac{1}{2}(\sin(\alpha+\beta) - \sin(\alpha-\beta))$

③ $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

④ $\sin\alpha\sin\beta = -\frac{1}{2}(\cos(\alpha+\beta) - \cos(\alpha-\beta))$

a. $P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left|5 + 10\cos\left(100t + \frac{\pi}{3}\right)\right|^2 dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(25 + \underbrace{100\cos\left(100t + \frac{\pi}{3}\right)}_{0\ (\because\ \text{한주기 적분})} + 100\underbrace{\cos^2\left(100t + \frac{\pi}{3}\right)}_{=\frac{1+\cos\left(200t+\frac{2\pi}{3}\right)}{2}}\right) dt$

0 (∵ 한주기 적분)

$\therefore \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (25 + 50)\, dt = 75. \Rightarrow rms = \sqrt{75} = 5\sqrt{3}.$

b. $P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left|10\cos\left(100t + \frac{\pi}{3}\right) + 16\sin\left(150t + \frac{\pi}{5}\right)\right|^2 dt.$

$= \lim \frac{1}{T}\int \left(100\underbrace{\cos^2\left(100t + \frac{\pi}{3}\right)}_{=\frac{1+\cos\left(200t+\frac{2\pi}{3}\right)}{2}} + 320\underbrace{\sin\left(150t + \frac{\pi}{5}\right)\cos\left(100t + \frac{\pi}{3}\right)}_{=\frac{1}{2}\sin\left(250t + \frac{\pi}{3} + \frac{\pi}{5}\right) - \frac{1}{2}\sin\left(-50t + \frac{\pi}{3} - \frac{\pi}{5}\right)} + 256\underbrace{\sin^2\left(150t + \frac{\pi}{5}\right)}_{=\frac{1-\cos\left(300t+\frac{2\pi}{5}\right)}{2}}\right) dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 50 + 128\, dt = 178 \Rightarrow rms = \sqrt{178}.$

$$c.\ \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|(10+2\sin 3t)\cos 10t\right|^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}(10\cos 10t+2\sin 3t\cos 10t)^2 dt.$$

$$=\lim\frac{1}{T}\int\Big(100\underbrace{\cos^2 10t}_{=\frac{1+\cos 20t\,\to 0}{2}}+40\sin 3t\underbrace{\cos^2 10t}_{=\frac{1+\cos 20t\,\to 0}{2}}+4\underbrace{\underbrace{\sin^2 3t}_{=\frac{1-\cos 6t}{2}}\cdot\underbrace{\cos^2 10t}_{=\frac{1+\cos 20t}{2}}}_{=1-\cos 6t\,\to 0\,+\cos 20t\,\to 0\,-\cos 6t\cos 20t\,\to 0}\Big)dt$$

$$\therefore 51 \Rightarrow rms=\sqrt{51}$$

$d.$

$$\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|10\cos 5t\cos 10t\right|^2 dt=\lim\frac{1}{T}\int\left(10\cdot\frac{1}{2}\cdot 2\cos 5t\cos 10t\right)^2 dt$$

$$=\lim\frac{1}{T}\int\frac{100}{4}\cdot(\cos 15t+\cos 5t)^2 dt.$$

$$=\lim\frac{1}{T}\int\frac{100}{4}\cdot\Big(\cos^2 15t\,\to\tfrac{1}{2}+2\cos 5t\cos 15t\,\to 0+\cos^2 5t\,\to\tfrac{1}{2}\Big)dt.$$

$$\therefore 25 \Rightarrow rms=5.$$

$$e.\ \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|10\sin 5t\cos 10t\right|^2 dt=\lim\frac{1}{T}\int\frac{100}{4}(\sin 15t-\sin 5t)^2 dt$$

$$=\lim\frac{1}{T}\int\frac{100}{4}\Big(\sin^2 15t\,\to\tfrac{1}{2}-2\sin 15t\sin 5t\,\to 0+\sin^2 5t\,\to\tfrac{1}{2}\Big)dt.$$

$$=25 \Rightarrow rms=5$$

$$f.\ \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|e^{j\alpha t}\cos\omega_0 t\right|^2 dt=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\underbrace{|e^{j\alpha t}|^2}_{=|\cos\alpha t+j\sin\alpha t|^2=\sqrt{\cos^2\alpha t+\sin^2\alpha t}=1}\cdot\cos^2(\omega_0 t)\,dt.$$

$$=\frac{1}{2}.\ \Rightarrow rms=\frac{1}{\sqrt{2}}.$$

1.2-3 In Fig. P1.2-3, express signals $x_1(t)$, $x_2(t)$ $x_3(t)$, $x_4(t)$, and $x_5(t)$ in terms of signal $x(t)$ and its time-shifted, time-scaled, or time-reversed versions.

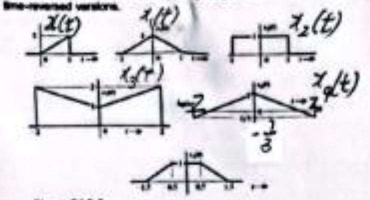
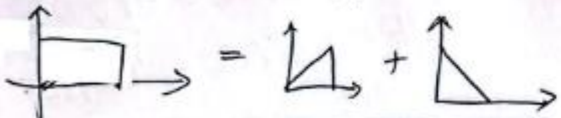
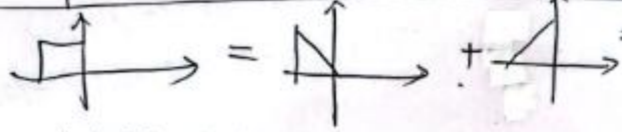


Figure P1.2-3

$x_1(t) = x(t+1) + x(-t+1)$.



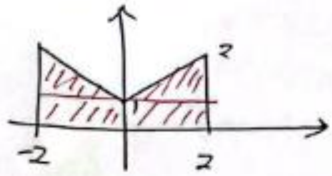
$= x(t) + x(-t+1)$



$= x(-t) + x(t+1)$

$\therefore x_2(t) = x(-t) + x(t+1) + x(t) + x(-t+1)$

or → t축을 2배로 늘리기.

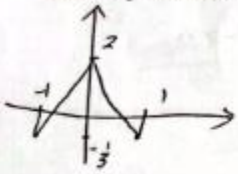


$= x(\frac{t}{2}) + x(-\frac{t}{2}) + x_2(\frac{t}{2})$

$= 2x(\frac{t}{2}) + 2x(-\frac{t}{2}) + x(\frac{t}{2}+1) + x(-\frac{t}{2}+1)$

$= x_3(t)$

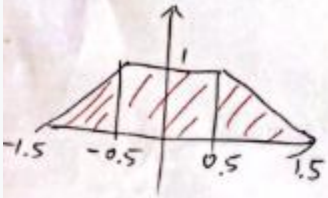
이 함수를 생각해보자.



$= \frac{4}{3}x_1(t) - \frac{1}{3}x_2(t) = x(t+1) + x(-t+1) - \frac{1}{3}x(-t) - \frac{1}{3}x(t)$

↓ t축으로 2배늘림

$\therefore x(\frac{t}{2}+1) + x(-\frac{t}{2}+1) - \frac{1}{3}x(-\frac{t}{2}) - \frac{1}{3}x(\frac{t}{2})$

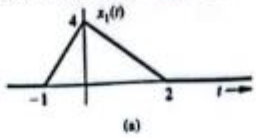


$= x_2(2t) + x(t+1.5) + x(-t+1.5)$

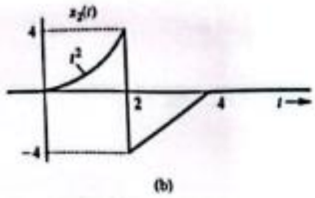
$= x(-2t) + x(2t+1) + x(2t) + x(-2t+1) + x(t+1.5) + x(-t+1.5)$

- 1.4-2

1.4.2 Express each of the signals in Fig. P1.4-2 by a single expression valid for all t



(a)



(b)

Figure P1.4-2

(a) 왼쪽/오른쪽으로 나눠서 생각한다.

왼쪽 $= 4(t+1)\left(u(t+1) - u(t)\right)$

오른쪽 $= (-2t+4)\left(u(t) - u(t-2)\right)$

$\therefore x_1(t) = 4(t+1)\left(u(t+1)-u(t)\right) + (-2t+4)\left(u(t)-u(t-2)\right)$

$= 4(t+1)u(t+1) - 6t\,u(t) + (2t-4)u(t-2)$

(b) 왼쪽 $= t^2\left(u(t) - u(t-2)\right)$

오른쪽 $= 2(t-4)\left(u(t-2) - u(t-4)\right)$

$\therefore x_2(t) = t^2u(t) - (t^2-2t+8)u(t-2) - (2t-8)u(t-4)$

1.5.7 Consider the signal $y(t) = \frac{1}{5}x(-2t-3)$ shown in Figure P1.5-7.

a. Does $y(t)$ have an odd portion, $y_o(t)$? If so, determine and carefully sketch $y_o(t)$. Otherwise, explain why no odd portion exists.

b. Determine and carefully sketch the original signal $x(t)$.

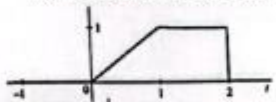
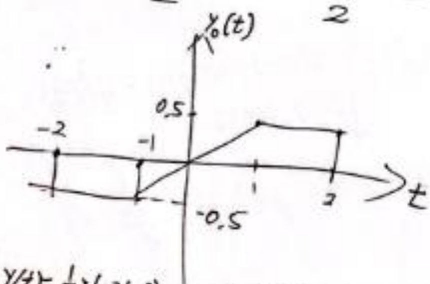


Figure P1.5-7: $y(t) = \frac{1}{5}x(-2t-3)$.

(a) $y_o(t) = \dfrac{y(t)-y(-t)}{2} \rightarrow y(-t) = \frac{1}{5}x(2t-3)$

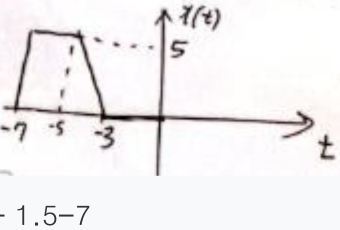
$\therefore y_o(t) = \dfrac{y(t)-y(-t)}{2} = \dfrac{\frac{1}{5}x(-2t-3) - \frac{1}{5}x(2t-3)}{2} = \dfrac{x(-2t-3) - x(2t-3)}{10}$



(b) $y(t) = \frac{1}{5}x(-2t-3) \Longrightarrow 5y(t) = x(-2t-3) \xrightarrow{t=t-1.5} 5y(t-1.5) = x(-2t) \xrightarrow{t=-\frac{1}{2}t} 5y(-\frac{1}{2}t-1.5) = x(t)$

$\therefore x(t) = 5y(-\frac{1}{2}t-1.5)$

y축대칭하고, y(t)를 5배하고, t축으로 −3만큼 평행이동하고, t를 2배로 늘린 꼴



$\ldots - (2t-8)u(t-4)$.

Figure P1.4-2

1.5.7 Consider the signal $y(t) = (\frac{1}{5})x(-2t-3)$ shown in Figure P1.5-7.

a. Does $y(t)$ have an odd portion, $y_O(t)$? If so, determine and carefully sketch $y_O(t)$. Otherwise, explain why no odd portion exists.

b. Determine and carefully sketch the original signal $x(t)$.

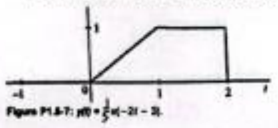
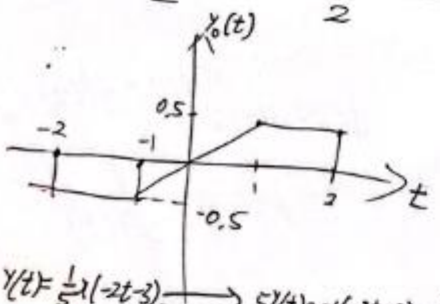


Figure P1.5-7: $y(t) = \frac{1}{5}x(-2t-3)$.

(a) $y_o(t) = \dfrac{y(t)-y(-t)}{2} \rightarrow y(-t) = \frac{1}{5}x(2t-3)$

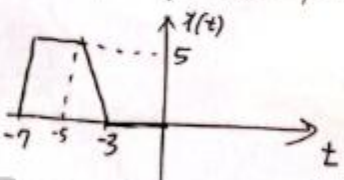
$\therefore y_o(t) = \dfrac{y(t)-y(-t)}{2} = \dfrac{\frac{1}{5}x(-2t-3) - \frac{1}{5}x(2t-3)}{2} = \dfrac{x(-2t-3)-x(2t-3)}{10}$



(b) $y(t) = \frac{1}{5}x(-2t-3) \Longrightarrow 5y(t) = x(-2t-3) \xrightarrow{t=t-1.5} 5y(t-1.5) = x(-2t) \xrightarrow{t=-\frac{1}{2}t} 5y(-\frac{1}{2}t-1.5) = x(t)$

$\therefore x(t) = 5y(-\frac{1}{2}t-1.5)$.

y축 대칭하고, y(t)를 5배 하고, t축으로 -3만큼 평행이동하고, t을 2배로 늘린 꼴



1.7.1 For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, determine which of the systems are linear and which are nonlinear.

a. $\frac{dy}{dt} + 2y(t) = x^2(t)$

b. $\frac{dy}{dt} + 3ty(t) = t^2x(t)$

c. $3y(t) + 2 = x(t)$

d. $\frac{dy}{dt} + y^2(t) = x(t)$

e. $\left(\frac{dy}{dt}\right)^2 + 2y(t) = x(t)$

f. $\frac{dy}{dt} + (\sin t)y(t) = \frac{dx}{dt} + 2x(t)$

g. $\frac{dy}{dt} + 2y(t) = x(t)\frac{dx}{dt}$

h. $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

a. $x_1, y_1, K_1 \Rightarrow \frac{dy_1}{dt} + 2y_1(t) = x_1^2(t) \longrightarrow K_1\frac{dy_1}{dt} + 2K_1y_1(t) = K_1x_1^2(t)$

$x_2, y_2, K_2 \Rightarrow \frac{dy_2}{dt} + 2y_2(t) = x_2^2(t) \longrightarrow K_2\frac{dy_2}{dt} + 2K_2y_2(t) = K_2x_2^2(t)$

$\oplus = \frac{d(K_1y_1 + K_2y_2)}{dt} + 2(K_1y_1(t) + K_2y_2(t)) = K_1x_1^2(t) + K_2x_2^2(t)$

$y(t) = K_1y_1 + K_2y_2$ 가 나오고, $x(t) = \sqrt{K_1x_1^2(t) + K_2x_2^2(t)}$ 의 꼴이 나와야하는데,

이게 linear려면 $x(t) = K_1x_1 + K_2x_2$ 가 나와야 하므로 nonlinear이다.

b. $x_1, y_1, K_1 \Rightarrow \frac{dy_1}{dt} + 3ty_1 = t^2x_1 \longrightarrow K_1\frac{dy_1}{dt} + K_1 3t y_1 = K_1t^2x_1$

$x_2, y_2, K_2 \Rightarrow \frac{dy_2}{dt} + 3ty_2 = t^2x_2 \longrightarrow K_2\frac{dy_2}{dt} + K_2 3t y_2 = K_2t^2x_2$

$\oplus = \frac{d}{dt}(K_1y_1 + K_2y_2) + 3t(K_1y_1 + K_2y_2) = t^2(K_1x_1 + K_2x_2)$

$\therefore$ linear

c. $x_1, y_1, K_1 \Rightarrow 3y_1 + 2 = x_1 \longrightarrow K_1 3y_1 + K_1 \cdot 2 = K_1x_1$

$x_2, y_2, K_2 \Rightarrow 3y_2 + 2 = x_2 \longrightarrow K_2 \cdot 3y_2 + K_2 \cdot 2 = K_2x_2$

$\oplus = 3(K_1y_1 + K_2y_2) + \underbrace{2(K_1 + K_2)}_{\text{쓸데기없는것포함}} = (K_1x_1 + K_2x_2)$

$\therefore$ nonlinear

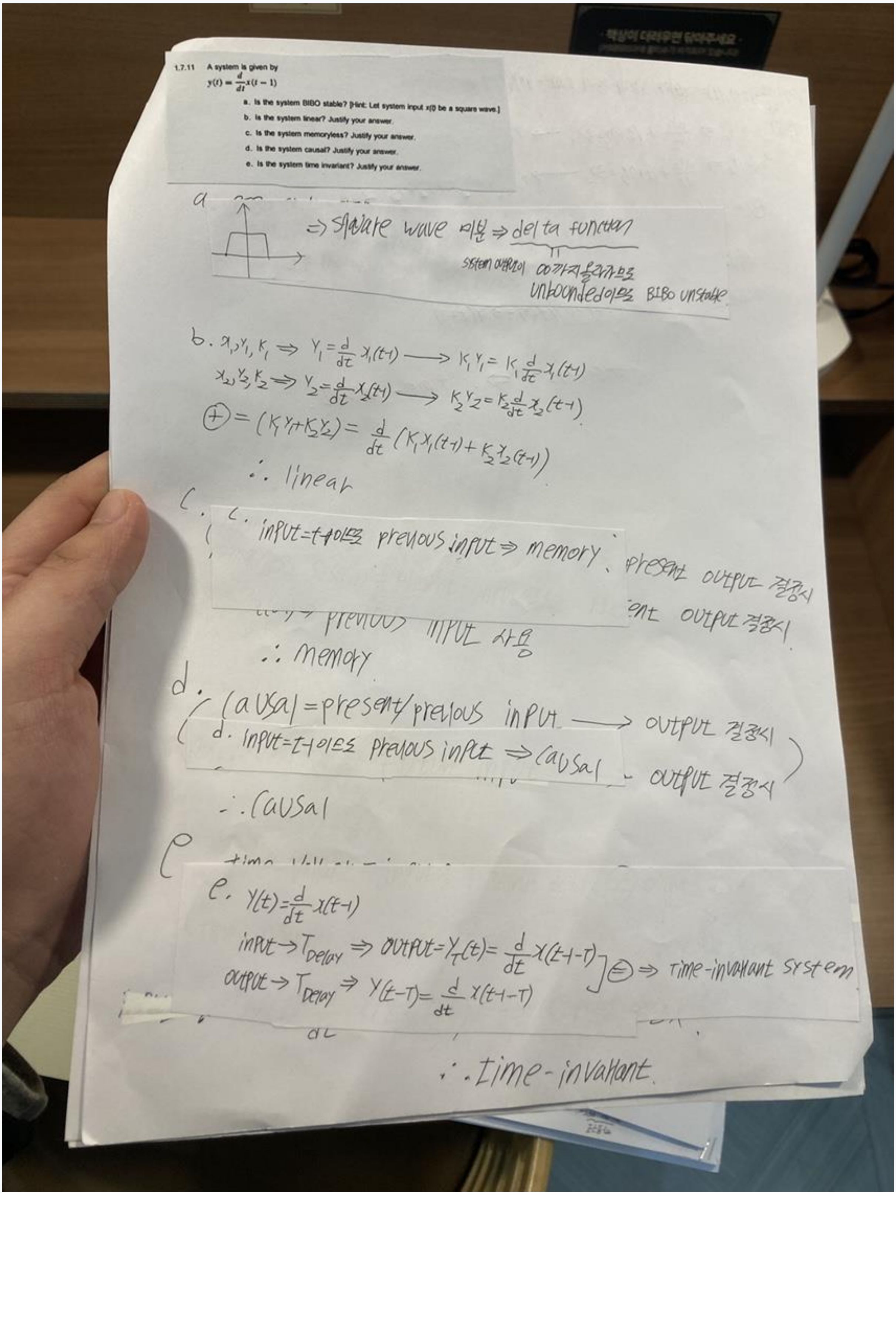
d. $x_1, y_1, K_1 \Rightarrow \frac{dy_1}{dt} + y_1^2 = x_1 \longrightarrow K_1\frac{dy_1}{dt} + K_1y_1^2 = K_1x_1$

$x_2, y_2, K_2 \Rightarrow \frac{dy_2}{dt} + y_2^2 = x_2 \longrightarrow K_2\frac{dy_2}{dt} + K_2y_2^2 = K_2x_2$

$\oplus = \frac{d}{dt}(K_1y_1 + K_2y_2) + \underbrace{(K_1y_1^2 + K_2y_2^2)}_{\neq\, (K_1y_1 + K_2y_2)^2} = (K_1x_1 + K_2x_2)$

$\therefore$ nonlinear.

− 1.7−11



1.7.11 A system is given by

$$y(t) = \frac{d}{dt}x(t-1)$$

a. Is the system BIBO stable? [Hint: Let system input $x(t)$ be a square wave.]

b. Is the system linear? Justify your answer.

c. Is the system memoryless? Justify your answer.

d. Is the system causal? Justify your answer.

e. Is the system time invariant? Justify your answer.

a.

⇒ square wave 미분 ⇒ $\underbrace{\text{delta function}}_{\text{II}}$

system output이 ∞까지 올라가므로

unbounded이므로 BIBO unstable

b. $x_1, y_1, k_1 \Rightarrow y_1 = \frac{d}{dt}x_1(t-1) \longrightarrow k_1 y_1 = k_1 \frac{d}{dt} x_1(t-1)$

$x_2, y_2, k_2 \Rightarrow y_2 = \frac{d}{dt}x_2(t-1) \longrightarrow k_2 y_2 = k_2 \frac{d}{dt} x_2(t-1)$

$+ = (k_1 y_1 + k_2 y_2) = \frac{d}{dt}(k_1 x_1(t-1) + k_2 x_2(t-1))$

∴ linear

c. input = t-1이므로 previous input ⇒ memory

present output 결정시

ent output 결정시

previous input 사용

∴ memory

d. causal = present/previous input ⟶ output 결정시

d. input = t-1이므로 previous input ⇒ causal

output 결정시

∴ causal

e. $y(t) = \frac{d}{dt}x(t-1)$

input → T_{Delay} ⇒ output $= y_T(t) = \frac{d}{dt}x(t-1-T)$

output → T_{Delay} ⇒ $y(t-T) = \frac{d}{dt}x(t-1-T)$

= ⇒ time-invariant system

∴ time-invariant