K-means

April 4, 2019

[K-means clustering]

1. Apply K-means clustering to MNIST training dataset with different K = 5, 10, 15, 20 and present the following results for each K.
2. Visualize K centroid images for each category.
3. Plot the training energy per optimization iteration.
4. Plot the training accuracy per optimization iteration.
5. Plot the testing accuracy per optimization iteration.

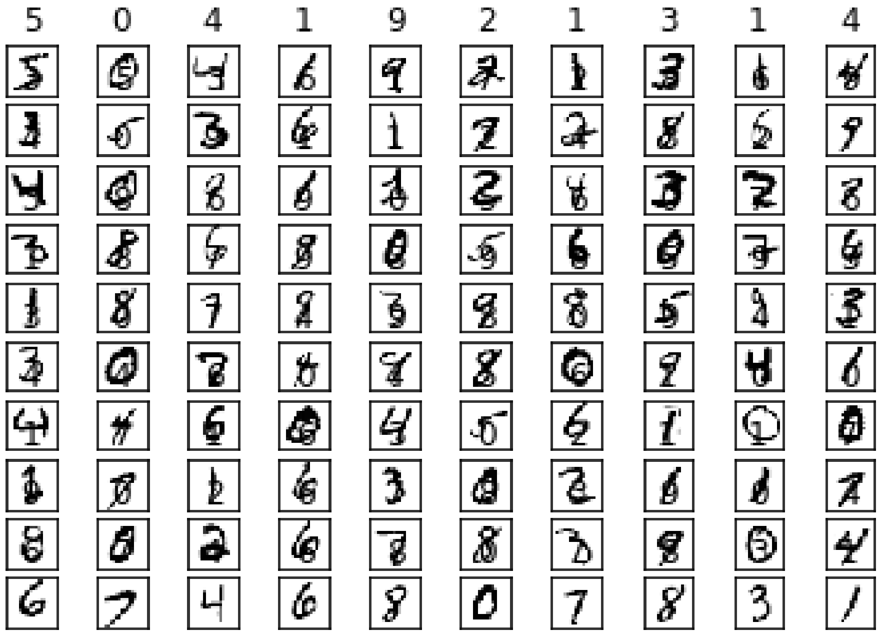
[energy]
$\sum_{k=1}^{K} \|x_i - c_{k_i}\|^2$ where k_i denotes the category of x_i, and c_{k_i} denotes the centroid of category x_i.
[accuracy]
$\frac{\sum_{k=1}^{K} m_k}{N}$ where N denotes the total number of data, and m_k denotes the number of data with majority for category k.

Previous what we did.

```
In [1]: import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd
```

```
In [2]: file_data                = "mnist_train.csv"
        handle_file        = open(file_data, "r")
        data                        = handle_file.readlines()
        handle_file.close()
        size_row        = 28    # height of the image
        size_col          = 28    # width of the image
        num_image        = len(data)
        count               = 0  # count for the number of images

        def normalize(data):

            data_normalized = (data - min(data)) / (max(data) - min(data))

            return(data_normalized)

        def distance(x, y):
```

```
    d = (x - y) ** 2
    s = np.sum(d)
    # r = np.sqrt(s)
    return(s)

list_image  = np.empty((size_row * size_col, num_image), dtype=float)
list_label  = np.empty(num_image, dtype=int)
for line in data:

    line_data   = line.split(',')
    label       = line_data[0]
    im_vector   = np.asfarray(line_data[1:])
    im_vector   = normalize(im_vector)

    list_label[count]       = label
    list_image[:, count]    = im_vector

    count += 1

f1 = plt.figure(1)

for i in range(100):

    label       = list_label[i]
    im_vector   = list_image[:, i]
    im_matrix   = im_vector.reshape((size_row, size_col))

    plt.subplot(10, 10, i+1)
    plt.title(label)
    plt.imshow(im_matrix, cmap='Greys', interpolation='None')

    frame   = plt.gca()
    frame.axes.get_xaxis().set_visible(False)
    frame.axes.get_yaxis().set_visible(False)

plt.show()
```



```
In [3]: avg_list_image =[[0]*784]*10

        cnt1=[0,0,0,0,0,0,0,0,0,0]


        for i in range(60000):
            if(list_label[i]== 0):                         #label = 0
                    avg_list_image[0] += list_image[:,i]
                    cnt1[0]+=1
            elif (list_label[i]== 1):                      #label = 1
                    avg_list_image[1] += list_image[:, i]
                    cnt1[1]+=1
            elif (list_label[i]== 2):                      #label = 2
                    avg_list_image[2] += list_image[:, i]
                    cnt1[2]+=1
            elif (list_label[i]== 3):                      #label = 3
                    avg_list_image[3] += list_image[:, i]
                    cnt1[3]+=1
            elif (list_label[i]== 4):                      #label = 4
                    avg_list_image[4] += list_image[:, i]
                    cnt1[4]+=1
            elif (list_label[i]== 5):                      #label = 5
                    avg_list_image[5] += list_image[:, i]
                    cnt1[5]+=1
            elif (list_label[i]== 6):                      #label = 6
                    avg_list_image[6] += list_image[:, i]
```

```
                cnt1[6]+=1
        elif (list_label[i]== 7):                      #label = 7
                avg_list_image[7] += list_image[:, i]
                cnt1[7]+=1
        elif (list_label[i]== 8):                      #label = 8
                avg_list_image[8] += list_image[:, i]
                cnt1[8]+=1
        else:
                avg_list_image[9] += list_image[:, i]  #label = 9
                cnt1[9]+=1

    for i in range(10):
        avg_list_image[i]= avg_list_image[i]/cnt1[i]
```

To here, It was the last lab until this part. Let's start K-means.

I defined some functions for computing K-means.

```
In [66]: def average_image(avg_label,store_func,b,cnt,z,avg_label_pre,label_cnt,list_accuracy,

             num=0
             p=0
             c=0
             m=0

             while(1):
                 for j in range(data_len):
                     avg_label[j] = kmeans_label(b[j],store_func,z)

                 num+=1

                 if c<len(list_energy):
                     list_energy[p]=energy_func(store_func,b,avg_label)
                     c+=1
                     p+=1
                     list_accuracy[m]=training_accuracy(list_label, avg_label,acc_list,data_le
                     m+=1


                 if(np.array_equal(avg_label,avg_label_pre)):
                     break

                 store_func=[0]*len(store_func)
                 cnt=[0]*len(cnt)
                 avg_label_pre=np.copy(avg_label)

                 for k in range(10000):
                     store_func[avg_label[k]]+= b[k]
```

```
            cnt[avg_label[k]]+=1

        for l in range(len(cnt)):
            if(cnt[l]!=0):
                store_func[l] =store_func[l]/cnt[l]



    return  print("It iterated ",num,"times based on K-means algorithms ")



def kmeans_label(x,y,z):
    for i in range(len(y)):
        z[i]=distance(x,y[i])
    return np.argmin(z)

def energy_func(store_func,b,avg_label):
    sum1=0

    for i in range(10000):
        d=(b[i]-store_func[avg_label[i]])**2
        sum1+=np.sum(d)

    return(sum1)

def arg_max(acc_list,A_Z):

    if len(A_Z)==None:
        return 0

    up_cnt=0
    acc_list=[0]*10

    for j in range(len(A_Z)):
        for k in range(10):
            if A_Z[j]==k:
                acc_list[k]+=1

    a=np.argmax(acc_list)

    for l in range(len(A_Z)):
        if A_Z[l]==a:
            up_cnt+=1

    return up_cnt

def training_accuracy(list_label, avg_label,acc_list,data_len):
```

```python
A=[]
B=[]
C=[]
D=[]
E=[]
F=[]
G=[]
H=[]
I=[]
J=[]
K=[]
L=[]
M=[]
N=[]
O=[]
P=[]
Q=[]
R=[]
S=[]
T=[]


for i in range(data_len):
    if avg_label[i]==0:
        A=np.append(A,list_label[i])
    elif avg_label[i]==1:
        B=np.append(B,list_label[i])
    elif avg_label[i]==2:
        C=np.append(C,list_label[i])
    elif avg_label[i]==3:
        D=np.append(D,list_label[i])
    elif avg_label[i]==4:
        E=np.append(E,list_label[i])
    elif avg_label[i]==5:
        F=np.append(F,list_label[i])
    elif avg_label[i]==6:
        G=np.append(G,list_label[i])
    elif avg_label[i]==7:
        H=np.append(H,list_label[i])
    elif avg_label[i]==8:
        I=np.append(I,list_label[i])
    elif avg_label[i]==9:
        J=np.append(J,list_label[i])
    elif avg_label[i]==10:
        K=np.append(K,list_label[i])
    elif avg_label[i]==11:
        L=np.append(L,list_label[i])
    elif avg_label[i]==12:
        M=np.append(M,list_label[i])
    elif avg_label[i]==13:
        N=np.append(N,list_label[i])
    elif avg_label[i]==14:
        O=np.append(O,list_label[i])
    elif avg_label[i]==15:
        P=np.append(P,list_label[i])
    elif avg_label[i]==16:
        Q=np.append(Q,list_label[i])
    elif avg_label[i]==17:
        R=np.append(R,list_label[i])
    elif avg_label[i]==18:
        S=np.append(S,list_label[i])
    else:
        T=np.append(T,list_label[i])

sum=0
sum+=arg_max(acc_list,A)
sum+=arg_max(acc_list,B)
sum+=arg_max(acc_list,C)
sum+=arg_max(acc_list,D)
sum+=arg_max(acc_list,E)
sum+=arg_max(acc_list,F)
sum+=arg_max(acc_list,G)
sum+=arg_max(acc_list,H)
sum+=arg_max(acc_list,I)
sum+=arg_max(acc_list,J)
sum+=arg_max(acc_list,K)
sum+=arg_max(acc_list,L)
sum+=arg_max(acc_list,M)
sum+=arg_max(acc_list,N)
sum+=arg_max(acc_list,O)
sum+=arg_max(acc_list,P)
sum+=arg_max(acc_list,Q)
sum+=arg_max(acc_list,R)
sum+=arg_max(acc_list,S)
sum+=arg_max(acc_list,T)


return (sum/data_len)*100
```

1 Apply K-means clustering to MNIST training dataset with different K = 5, 10, 15, 20 and present the following results for each K.

When k=5, It shows how many iterations have done.

```
In [5]: #k=5
        b=np.array(list_image.T)
        store_func=np.array([avg_list_image[0],avg_list_image[2],avg_list_image[4],avg_list_ima

        z5=np.array([0]*5)
        cnt2=[0]*5
        avg_label=np.array([0]*60000)
        avg_label_pre=np.array([0]*60000)
        label_cnt=0
        list_energy5=[0]*50
        list_accuracy5=[0]*50
        acc_list=np.array([0]*10)
        data_len=60000

        average_image(avg_label,store_func,b,cnt2,z5,avg_label_pre,label_cnt,list_accuracy5,lis
```

```
It iterated  53 times based on K-means algorithms
```

When k=10, It shows how many iterations have done.

```
In [6]: #k=10
        store_func2=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_list_im
        cnt3=[0]*10
        z10=np.array([0]*10)
        avg_label10=np.array([0]*60000)
        avg_label_pre10=np.array([0]*60000)
        label_cnt10=0
        list_energy10=[0]*50
        list_accuracy10=[0]*50

        average_image(avg_label10,store_func2,b,cnt3,z10,avg_label_pre10,label_cnt10,list_accur
```

```
It iterated  69 times based on K-means algorithms
```

When k=15, It shows how many iterations have done.

```
In [7]: #k=15
        store_func3=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_list_im
        cnt4=[0]*15
        z15=np.array([0]*15)
        avg_label15=np.array([0]*60000)
        avg_label_pre15=np.array([0]*60000)
        label_cnt15=0
        list_energy15=[0]*100
        list_accuracy15=[0]*100

        average_image(avg_label15,store_func3,b,cnt4,z15,avg_label_pre15,label_cnt15,list_accur
It iterated  116 times based on K-means algorithms
```

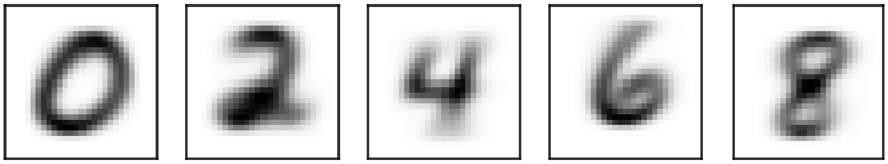
When k=20, It shows how many iterations have done.

```
In [8]: #k=20,
        store_func4=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_list_i
        cnt5=[0]*20
        z20=np.array([0]*20)
        avg_label20=np.array([0]*60000)
        avg_label_pre20=np.array([0]*60000)
        label_cnt20=0
        list_energy20=[0]*100
        list_accuracy20=[0]*100

        average_image(avg_label20,store_func4,b,cnt5,z20,avg_label_pre20,label_cnt20, list_accu
```

```
It iterated  132 times based on K-means algorithms
```

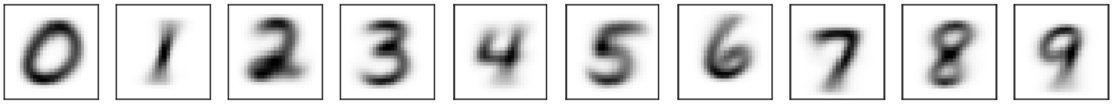
2 Visualize K centroid images for each category.

```
In [9]: for i in range(5):
            im_matrix_avg   = store_func[i].reshape((size_row, size_col))

            plt.figure(1, figsize=(12,3.2))
        #    plt.text(22,25.5,"%c" %'A', fontsize=12)
            plt.subplot(1, 10, i+1)
            plt.imshow(im_matrix_avg, cmap='Greys', interpolation='None')

            frame   = plt.gca()
            frame.axes.get_xaxis().set_visible(False)
            frame.axes.get_yaxis().set_visible(False)

        plt.show()
```

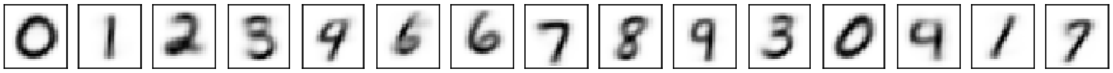


It shows like this. Because I didn't use the random Centroid Number at first. I chose Centroid of 0,2,4,6,8 from the last assignment!! Please notice that.

```
In [10]: for i in range(10):
             im_matrix_avg   = store_func2[i].reshape((size_row, size_col))

             plt.figure(1, figsize=(12,3.2))
         #    plt.text(22,25.5,"%d" %(i-1), fontsize=12)
             plt.subplot(1, 10, i+1)
             plt.imshow(im_matrix_avg, cmap='Greys', interpolation='None')

             frame    = plt.gca()
             frame.axes.get_xaxis().set_visible(False)
             frame.axes.get_yaxis().set_visible(False)

         plt.show()
```



```
In [34]: for i in range(15):
             im_matrix_avg   = store_func3[i].reshape((size_row, size_col))

             plt.figure(1, figsize=(12,3.2))
         #    plt.text(22,25.5,"%d" %(i-1), fontsize=12)
             plt.subplot(1, 15, i+1)
             plt.imshow(im_matrix_avg, cmap='Greys', interpolation='None')

             frame    = plt.gca()
             frame.axes.get_xaxis().set_visible(False)
             frame.axes.get_yaxis().set_visible(False)

         plt.show()
```



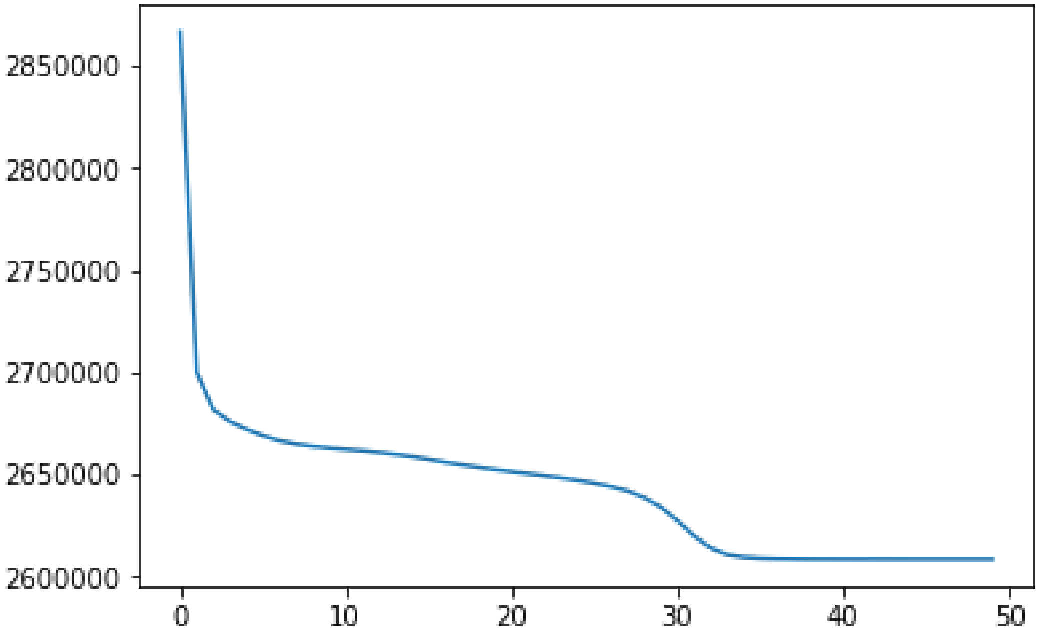
```
In [38]: for i in range(20):
             im_matrix_avg   = store_func4[i].reshape((size_row, size_col))

             plt.figure(1, figsize=(12,3.2))
         #    plt.text(22,25.5,"%d" %(i-1), fontsize=12)
             plt.subplot(1, 20, i+1)
             plt.imshow(im_matrix_avg, cmap='Greys', interpolation='None')
```

```
    frame    = plt.gca()
    frame.axes.get_xaxis().set_visible(False)
    frame.axes.get_yaxis().set_visible(False)

plt.show()
```

3 Plot the training energy per optimization iteration.

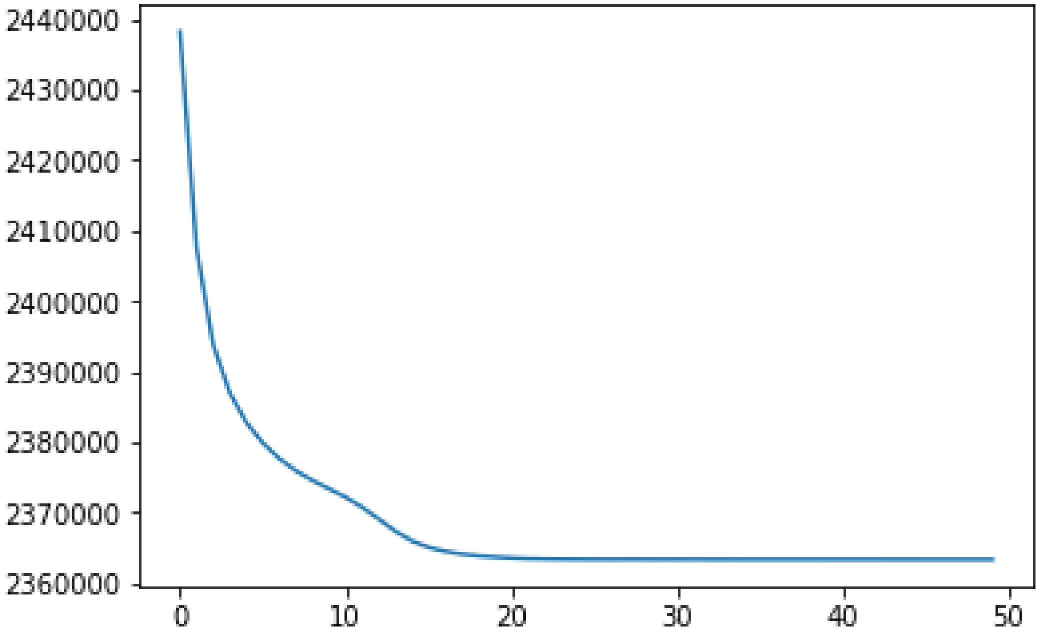
```
In [40]: x=list(range(50))
         y1=np.copy(list_energy5)

         plt.plot(x,y1)
```

```
Out[40]: [<matplotlib.lines.Line2D at 0x2a0d04a3358>]
```

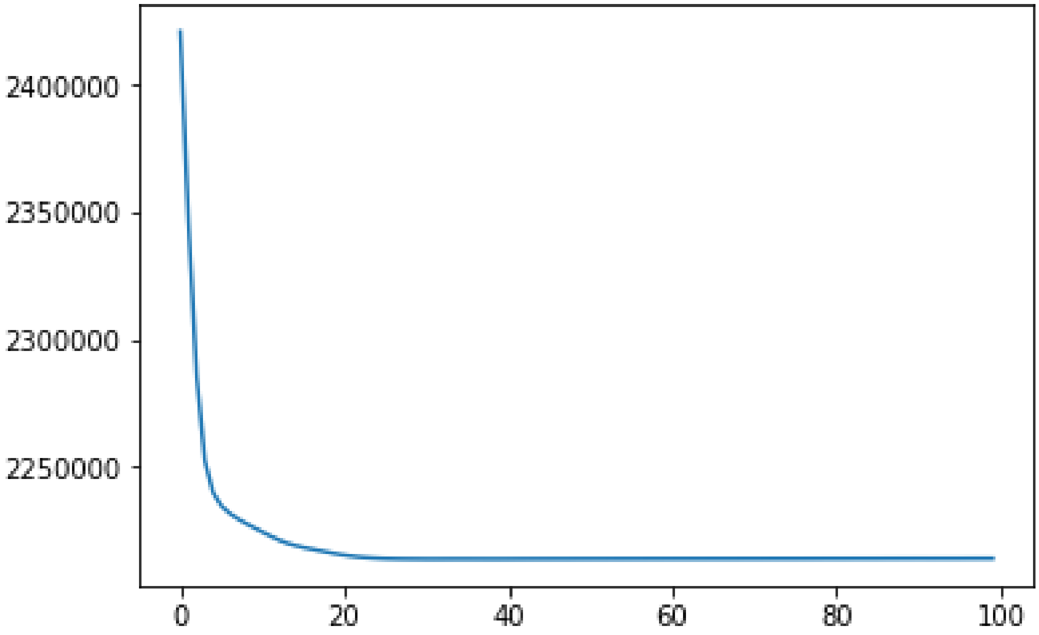


```
In [12]: x2=list(range(50))
         y2=np.copy(list_energy10)


         plt.plot(x2,y2)
```

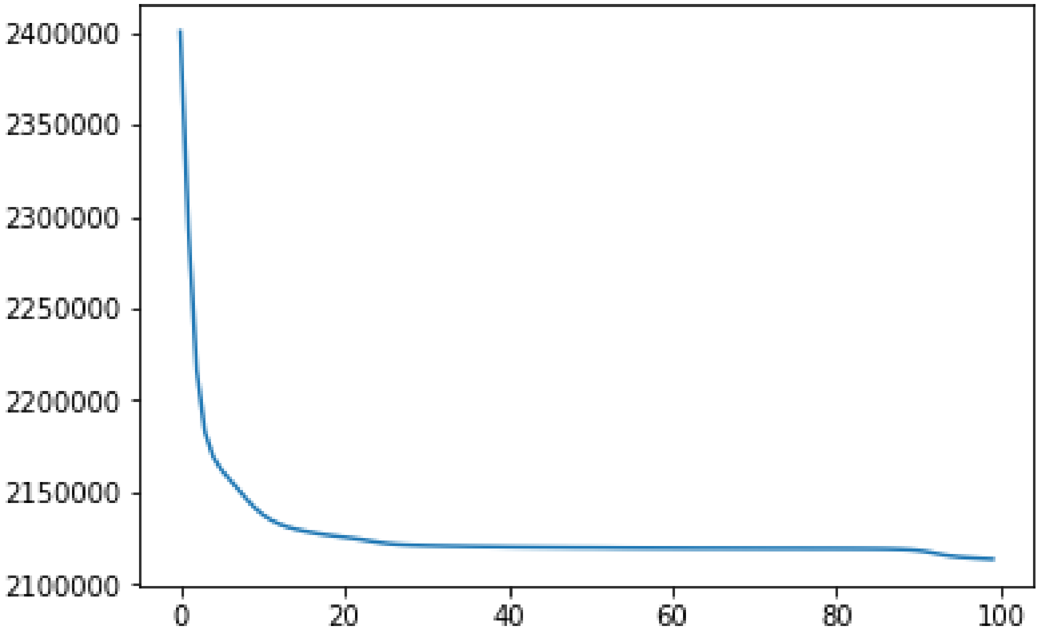
```
Out[12]: [<matplotlib.lines.Line2D at 0x2a0cfce9be0>]
```



```
In [14]: y3=np.copy(list_energy15)


         plt.plot(y3)
Out[14]: [<matplotlib.lines.Line2D at 0x2a0cff94d30>]
```



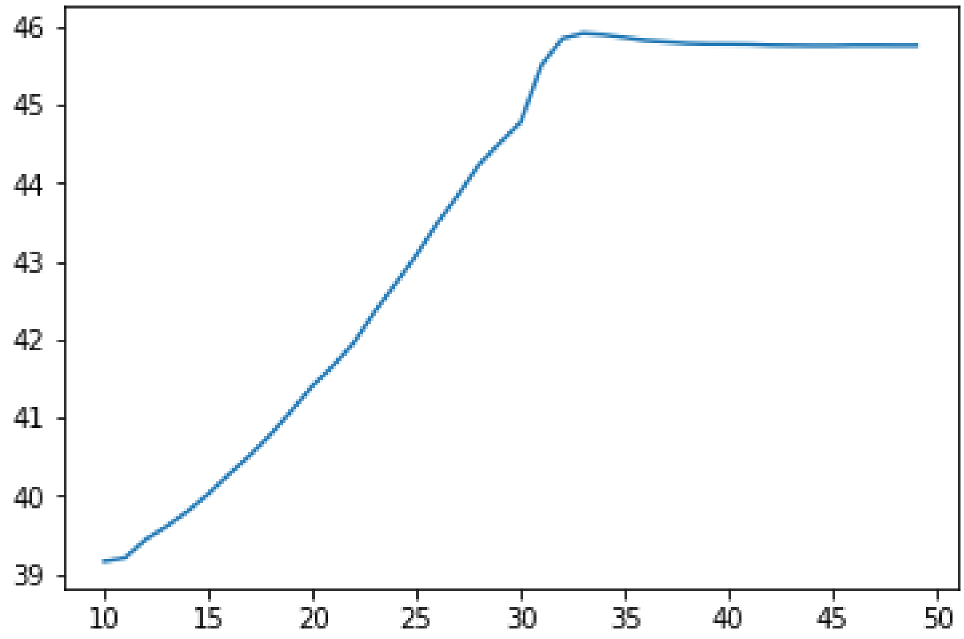
```
In [15]: y4=np.copy(list_energy20)


         plt.plot(y4)

Out[15]: [<matplotlib.lines.Line2D at 0x2a0cfff02b0>]
```



4 Plot the training accuracy per optimization iteration.

```
In [21]: print("When k=5, The Accuracy is ",training_accuracy(list_label, avg_label,acc_list,6

When k=5, The Accuracy is  45.755 %
```

```
In [41]: x5_accuracy=list(range(10,50))
         y5_accuracy=np.copy(list_accuracy5[10:])
         plt.plot(x5_accuracy,y5_accuracy)

Out[41]: [<matplotlib.lines.Line2D at 0x2a0d04fd1d0>]
```

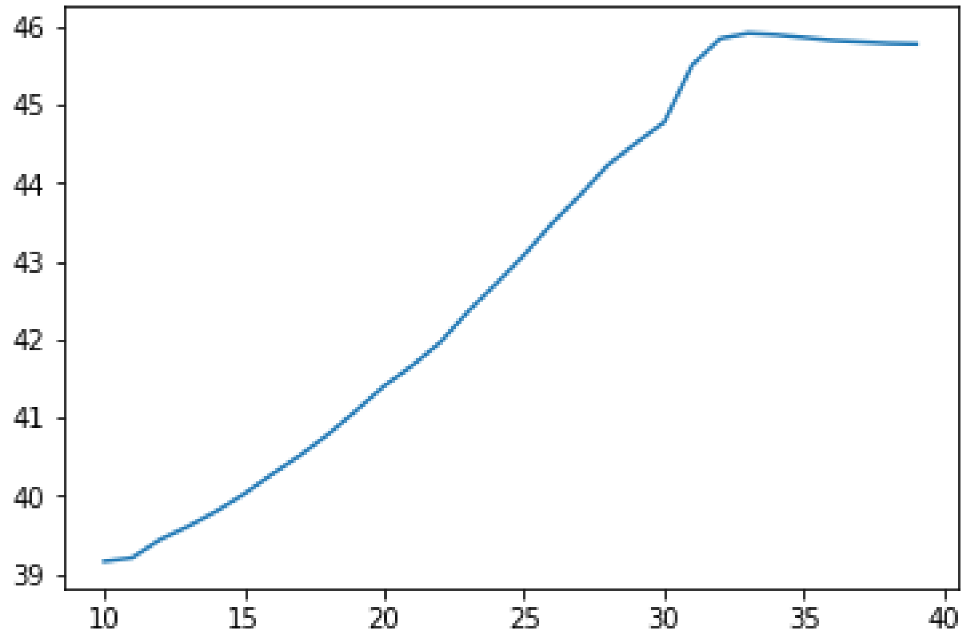


```
In [25]: print("When k=10, The Accuracy is ",training_accuracy(list_label, avg_label10,acc_lis
```

```
When k=10, The Accuracy is  58.901666666666664 %
```

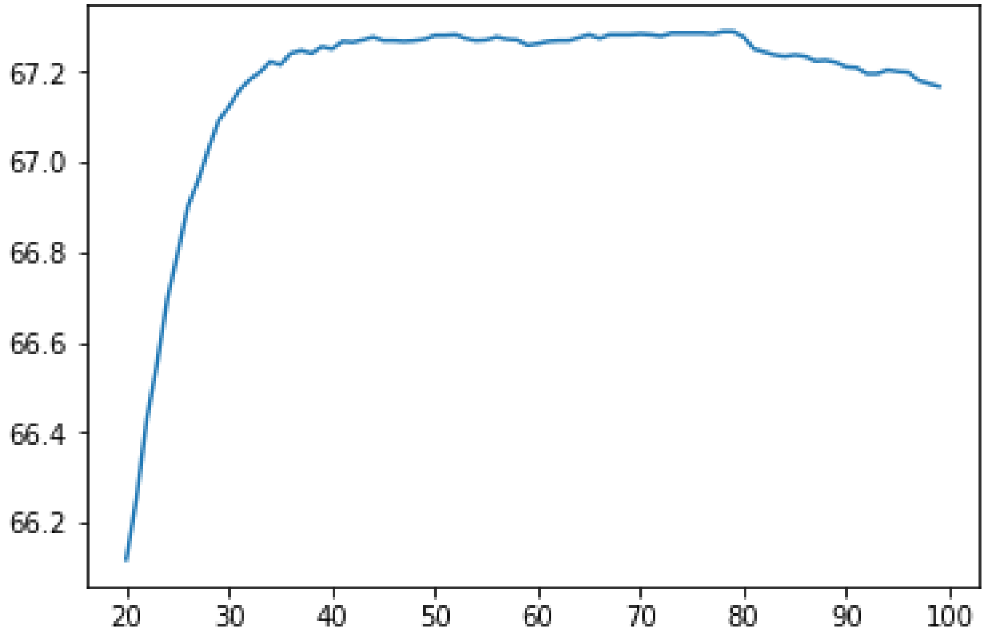
```
In [45]: x10_accuracy=list(range(10,40))
         y10_accuracy=np.copy(list_accuracy5[10:40])
         plt.plot(x10_accuracy,y10_accuracy)
```

```
Out[45]: [<matplotlib.lines.Line2D at 0x2a0d15fee10>]
```



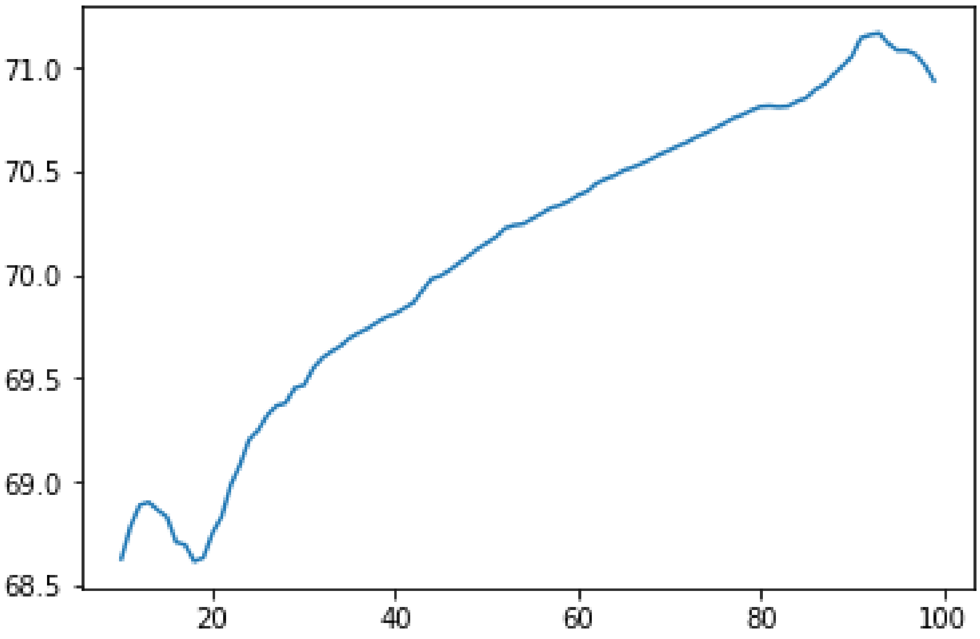
```
In [27]: print("When k=15, The Accuracy is ",training_accuracy(list_label, avg_label15,acc_lis

When k=15, The Accuracy is  67.14833333333333 %
```

```
In [43]: x15_accuracy=list(range(20,100))
         y15_accuracy=np.copy(list_accuracy15[20:])
         plt.plot(x15_accuracy,y15_accuracy)

Out[43]: [<matplotlib.lines.Line2D at 0x2a0d1533940>]
```



```
In [32]: print("When k=20, The Accuracy is ",training_accuracy(list_label, avg_label20,acc_lis

When k=20, The Accuracy is  70.19833333333332 %
```

```
In [47]: x20_accuracy=list(range(10,100))
         y20_accuracy=np.copy(list_accuracy20[10:])
         plt.plot(x20_accuracy,y20_accuracy)

Out[47]: [<matplotlib.lines.Line2D at 0x2a0d1662a90>]
```



5 Plot the testing accuracy per optimization iteration.

In [61]:
```
file_data1                = "mnist_test.csv"
handle_file1        = open(file_data1, "r")
data1                        = handle_file1.readlines()
handle_file1.close()
size_row1        = 28    # height of the image
size_col1          = 28    # width of the image
num_image1        = len(data1)
count1               = 0  # count for the number of images


list_image1  = np.empty((size_row1 * size_col1, num_image1), dtype=float)
list_label1  = np.empty(num_image1, dtype=int)

for line in data1:

    line_data1   = line.split(',')
    label1       = line_data1[0]
    im_vector1   = np.asfarray(line_data1[1:])
    im_vector1   = normalize(im_vector1)

    list_label1[count1]       = label1
    list_image1[:, count1]    = im_vector1
```

```
                count1 += 1

            f2 = plt.figure(1)
```

```
<Figure size 432x288 with 0 Axes>
```

```
In [62]: c=np.array(list_image1.T)
         acc_list1=[0]*10
```

```
In [67]: #k=5
         store_func_1=np.array([avg_list_image[0],avg_list_image[2],avg_list_image[4],avg_list_

         z5_1=np.array([0]*5)
         cnt2_1=[0]*5
         avg_label_1=np.array([0]*10000)
         avg_label_pre_1=np.array([0]*10000)
         label_cnt_1=0
         list_energy5_1=[0]*50
         list_accuracy5_1=[0]*50
         data_len1=10000

         average_image(avg_label_1,store_func_1,c,cnt2_1,z5_1,avg_label_pre_1,label_cnt_1,list_
```

```
It iterated  44 times based on K-means algorithms
```

```
In [70]: #k=10
         store_func2_1=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_lis
         cnt3_1=[0]*10
         z10_1=np.array([0]*10)
         avg_label10_1=np.array([0]*60000)
         avg_label_pre10_1=np.array([0]*60000)
         label_cnt10_1=0
         list_energy10_1=[0]*50
         list_accuracy10_1=[0]*50

         average_image(avg_label10_1,store_func2_1,c,cnt3_1,z10_1,avg_label_pre10_1,label_cnt1
```

```
It iterated  40 times based on K-means algorithms
```

```
In [80]: #k=15
         store_func3_1=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_lis
         cnt4_1=[0]*15
         z15_1=np.array([0]*15)
         avg_label15_1=np.array([0]*60000)
         avg_label_pre15_1=np.array([0]*60000)
         label_cnt15_1=0
         list_energy15_1=[0]*100
         list_accuracy15_1=[0]*100

         average_image(avg_label15_1,store_func3_1,c,cnt4_1,z15_1,avg_label_pre15_1,label_cnt1
```

```
It iterated  42 times based on K-means algorithms
```

```
In [88]: #k=20 ,
         store_func4=np.array([avg_list_image[0],avg_list_image[1],avg_list_image[2],avg_list_
         cnt5_1=[0]*20
         z20_1=np.array([0]*20)
         avg_label20_1=np.array([0]*60000)
         avg_label_pre20_1=np.array([0]*60000)
         label_cnt20_1=0
         list_energy20_1=[0]*100
         list_accuracy20_1=[0]*100

         average_image(avg_label20_1,store_func4_1,c,cnt5_1,z20_1,avg_label_pre20_1,label_cnt2
```

```
[1, 2, 3, 4, 5, 6]
```

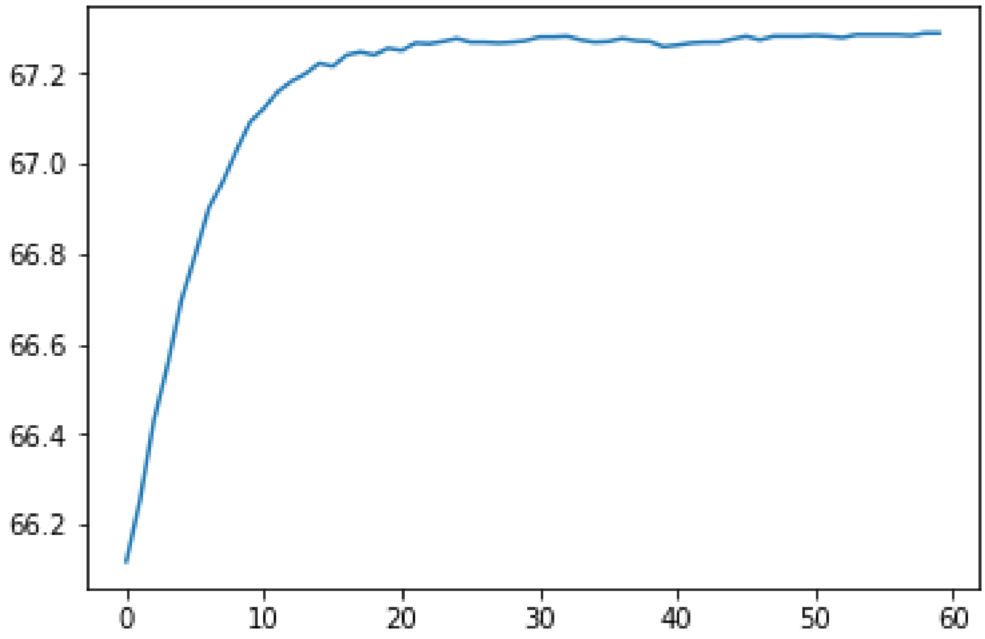
K=5

```
In [68]: print("When k=5, The Accuracy is ",training_accuracy(list_label1, avg_label_1,acc_lis
```

```
When k=5, The Accuracy is  45.67 %
```

```
In [84]: y5_1=np.copy(list_accuracy15[20:80])
         plt.plot(y5_1)
```

```
Out[84]: [<matplotlib.lines.Line2D at 0x2a0d01c2668>]
```



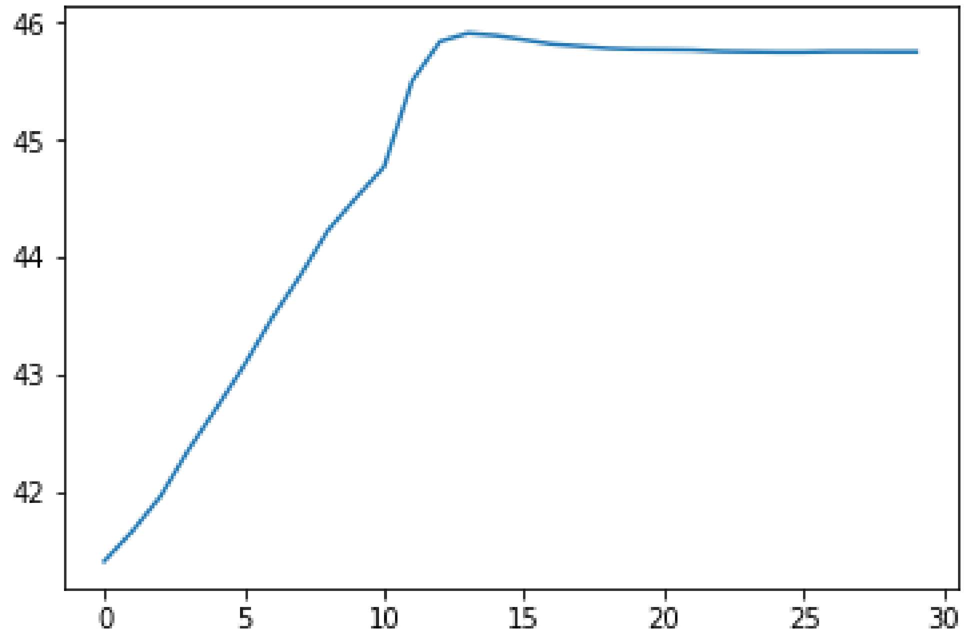
K10

```
In [71]: print("When k=10, The Accuracy is ",training_accuracy(list_label1, avg_label10_1,acc_
```

```
When k=10, The Accuracy is  60.95 %
```

```
In [90]: y10_accuracy=np.copy(list_accuracy5[20:65])
         plt.plot(y10_accuracy)
```

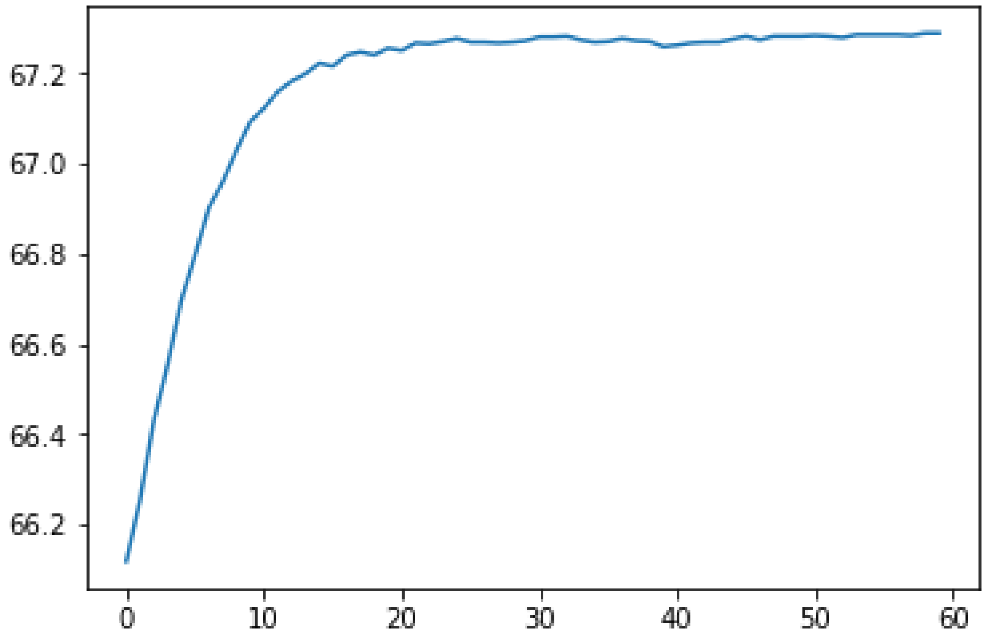
```
Out[90]: [<matplotlib.lines.Line2D at 0x2a0c1143748>]
```



```
In [94]: print("When k=15, The Accuracy is ",training_accuracy(list_label1, avg_label15_1,acc_l

When k=15, The Accuracy is  66.24 %
```

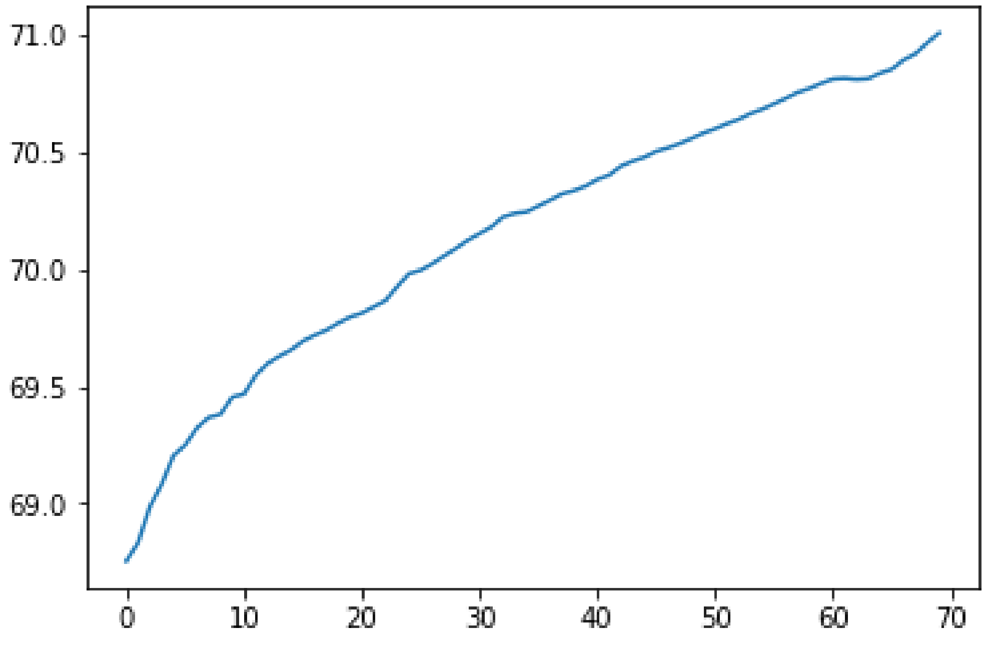
K=15

```
In [91]: y15_accuracy=np.copy(list_accuracy15[20:80])
         plt.plot(y15_accuracy)

Out[91]: [<matplotlib.lines.Line2D at 0x2a0c12278d0>]
```



K=20

```
In [93]: y20_accuracy=np.copy(list_accuracy20[20:90])
         plt.plot(y20_accuracy)

Out[93]: [<matplotlib.lines.Line2D at 0x2a0ce9fd588>]
```


71.0
70.5
70.0
69.5
69.0
0
10
20
30
40
50
60
70