Linear Algebra_Quiz 3 (Sep 21(Mon) / 2020)

1.P.93

5. Let V be a subspace of $\mathbf{R}^n$. Let W be the set of elements of $\mathbf{R}^n$ which are perpendicular to every element of V. Show that W is a subspace of $\mathbf{R}^n$. This subspace W is often denoted by $V^\perp$, and is called V **perp**, or also the **orthogonal complement of** V.

p.109

4. Let (a, b) and (c, d) be two vectors in $\mathbf{R}^2$.
   (i) If $ad - bc \neq 0$, show that they are linearly independent.
   (ii) If they are linearly independent, show that $ad - bc \neq 0$.
   (iii) If $ad - bc \neq 0$ show that they form a basis of $\mathbf{R}^2$.

p.110

In each of the following cases, exhibit a basis for the given space, and prove that it is a basis.

16. The space of symmetric $n \times n$ matrices.

p.121

1. Find the rank of the following matrices.

(i) $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 4 & 2 & 3 \end{pmatrix}$

2. Let A be a triangular matrix

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$$

Assume that none of the diagonal elements is equal to 0. What is the rank of A?

(New Problems)

(증명하시오)

1.

Theorem 3.4.2. *Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a set of at least two vectors ($n \geq 2$) in a vector space V. Then S is linearly dependent if and only if one of the vectors in S can be written as a linear combination of the rest.*

2. (증명하시오) Any matirx A, row rank A = columns rank A

3.

Determine bases for $\mathcal{N}(A)$ and $\mathcal{N}(A^T A)$ when

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, determine the ranks and nullities of the matrices A and $A^{\mathrm{T}} A$.