Linear Algebra_Quiz 2 (Sep 14(Mon) / 2020)

Exercies II, p.47

10.

(b) Define a matrix A to be **skew-symmetric** if ${}^tA = -A$. Show that for any square matrix A, the matrix $A - {}^tA$ is skew-symmetric.

P.60

12. Let X be a column vector having all its components equal to 0 except the j-th component which is equal to 1. Let A be an arbitrary matrix, whose size is such that we can form the product AX. What is AX?

P.61

22. Let A, B be two square matrices of the same size. We say that A is **similar** to B if there exists an invertible matrix T such that $B = TAT^{-1}$. Suppose this is the case. Prove:
    (a) B is similar to A.
    (b) A is invertible if and only if B is invertible.
    (c) ${}^tA$ is similar to ${}^tB$.
    (d) Suppose $A^n = O$ and B is an invertible matrix of the same size as A. Show that $(BAB^{-1})^n = O$.

23. Let A be a square matrix which is of the form

$$\begin{pmatrix} a_{11} & * & \cdots\cdots & * \\ 0 & a_{22} & * \quad \cdots & * \\ \vdots & & \ddots & \vdots \\ & & & * \\ 0 & \cdots\cdots & 0 & a_{nn} \end{pmatrix}.$$

The notation means that all elements below the diagonal are equal to 0, and the elements above the diagonal are arbitrary. One may express this property by saying that

$$a_{ij} = 0 \quad \text{if} \quad i > j.$$

Such a matrix is called **upper triangular**. If A, B are upper triangular matrices (of the same size) what can you say about the diagonal elements of AB?

P.76 In each of for following cases find a row equivalent matrix in row echelon form.

2. (a) $\begin{pmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix}$.

P.85 1. Using elementary row operations, find inverses for the following matrices.

(e) $\begin{pmatrix} -1 & 5 & 3 \\ 4 & 0 & 0 \\ 2 & 7 & 8 \end{pmatrix}$

(Note : For another way of finding inverses, see the chapter on determinants.)

P.87

1. (a) Let $A = (a_{ij})$, $B = (b_{jk})$ and let $AB = C$ with $C = (c_{ik})$. Let C^k be the k-th column of C. Express C^k as a linear combination of the columns of A. Describe precisely which are the coefficients, coming from the matrix B.