Linear Algebra_Quiz 6 (Oct 19(Mon) / 2020)

1)P.190

7. Let V be a finite dimensional vector space with a positive definite scalar product. Let W be a subspace. Show that

$$V = W + W^{\perp} \qquad \text{and} \qquad W \cap W^{\perp} = \{0\}.$$

In the terminology of the preceding chapter, this means that V is the direct sum of W and its orthogonal complement. [Use Theorem 2.3.]

2)P.190

10. Let A be a symmetric $n \times n$ matrix. Let $X, Y \in \mathbf{R}^n$ be eigenvectors for A, that is suppose that there exist numbers a, b such that $AX = aX$ and $AY = bY$. Assume that $a \neq b$. Prove that X, Y are perpendicular.

3) P.207

1. Compute the following determinants.

(e) $\begin{vmatrix} -1 & 5 & 3 \\ 4 & 0 & 0 \\ 2 & 7 & 8 \end{vmatrix}$

4) P.208

2. Compute the following determinants.

(b) $\begin{vmatrix} -1 & 1 & 2 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 4 & 1 & 2 \\ 3 & 1 & 5 & 7 \end{vmatrix}$

5)P.208

*(b) If $x_1, \ldots, x_n$ are numbers, then show by induction that

$$\begin{vmatrix} 1 & x_1 & \cdots & x_1^{n-1} \\ 1 & x_2 & \cdots & x_2^{n-1} \\ 1 & x_n & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{i<j} (x_j - x_i),$$

the symbol on the right meaning that it is the product of all terms $x_j - x_i$ with $i < j$ and i, j integers from 1 to n. This determinant is called the **Vandermonde** determinant V_n. To do the induction easily, multiply each column by x_1 and subtract it from the next column on the right, starting from the right-hand side. You will find that

$$V_n = (x_n - x_1) \cdots (x_2 - x_1) V_{n-1}.$$

6)p.209

6. Find the determinants of the following matrices.

(i) Let A be a triangular $n \times n$ matrix, say a matrix such that all components below the diagonal are equal to 0.

$$A = \begin{pmatrix} a_{11} & & & \\ 0 & a_{22} & & * \\ 0 & 0 & & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}.$$

What is $D(A)$?

7) p.209

12. Let c be a number and let A be an $n \times n$ matrix. Show that

$$D(cA) = c^n D(A).$$

8)p.214

Compute the ranks of the following matrices.

8. $$\begin{pmatrix} 3 & 1 & 1 & -1 \\ -2 & 4 & 3 & 2 \\ -1 & 9 & 7 & 3 \\ 7 & 4 & 2 & 1 \end{pmatrix}$$

9) P.217

1. Solve the following systems of linear equations.

(c) $$\begin{aligned} 4x + y + z + w &= 1 \\ x - y + 2z - 3w &= 0 \\ 2x + y + 3z + 5w &= 0 \\ x + y - z - w &= 2 \end{aligned}$$

10) P.217

2. Write down explicitly the inverse of a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

assuming that $ad - bc \neq 0$.