Linear Algebra(26(MON), OCT, 2020)

1)P.238

7. Let V be a finite dimensional vector space. Let A, B be linear maps of V into itself. Assume that $AB = BA$. Show that if v is an eigenvector of A, with eigenvalue λ, then Bv is an eigenvector of A, with eigenvalue λ also if $Bv \neq O$.

2)p.249

2. Let A be a triangular matrix,

$$A = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

What is the characteristic polynomial of A, and what are its eigenvalues?

3)P.250

7. Find the eigenvalues and a basis for the eigenspaces of the following matrices.

(a) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

4) P.250

14. Let $A: V \to V$ be a linear map of V into itself, and let $\{v_1, \ldots, v_n\}$ be a basis of V consisting of eigenvectors having distinct eigenvalues $c_1, \ldots, c_n$. Show that any eigenvector v of A in V is a scalar multiple of some v_i.

5)p.259

3. Let V be a finite dimensional vector space with a positive definite scalar product. Let $A: V \to V$ be a symmetric linear map. We say that A is **positive definite** if $\langle Av, v \rangle > 0$ for all $v \in V$ and $v \neq 0$. Prove:
   (a) If A is positive definite, then all eigenvalues are > 0.
   (b) If A is positive definite, then there exists a symmetric linear map B such that $B^2 = A$ and $BA = AB$. What are the eigenvalues of B? [*Hint:* Use a basis of V consisting of eigenvectors.]