## Linear Algebra\_Quiz 2 (Sep 14(Mon) / 2020 )

Exercies II, p.47

10.

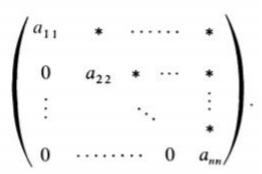
(b) Define a matrix A to be **skew-symmetric** if  ${}^{t}A = -A$ . Show that for any square matrix A, the matrix  $A - {}^{t}A$  is skew-symmetric.

P.60

12. Let X be a column vector having all its components equal to 0 except the j-th component which is equal to 1. Let A be an arbitrary matrix, whose size is such that we can form the product AX. What is AX?

P.61

- 22. Let A, B be two square matrices of the same size. We say that A is **similar** to B if there exists an invertible matrix T such that  $B = TAT^{-1}$ . Suppose this is the case. Prove:
  - (a) B is similar to A.
  - (b) A is invertible if and only if B is invertible.
  - (c) 'A is similar to 'B.
  - (d) Suppose  $A^n = O$  and B is an invertible matrix of the same size as A. Show that  $(BAB^{-1})^n = O$ .
- 23. Let A be a square matrix which is of the form



The notation means that all elements below the diagonal are equal to 0, and the elements above the diagonal are arbitrary. One may express this property by saying that

$$a_{ij} = 0$$
 if  $i > j$ .

Such a matrix is called **upper triangular**. If A, B are upper triangular matrices (of the same size) what can you say about the diagonal elements of AB?

P.76 In each of for following cases find a row equivalent matrix in row echelon form.

2. (a) 
$$\begin{pmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{pmatrix}$$
 (b)  $\begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix}$ .

P.85 1. Using elementary row operations, find inverses for the following matrices.

$$\begin{pmatrix} -1 & 5 & 3 \\ 4 & 0 & 0 \\ 2 & 7 & 8 \end{pmatrix}$$

(Note: For another way of finding inverses, see the chapter on determinants.)

P.87

1. (a) Let  $A = (a_{ij})$ ,  $B = (b_{jk})$  and let AB = C with  $C = (c_{ik})$ . Let  $C^k$  be the k-th column of C. Express  $C^k$  as a linear combination of the columns of A. Describe precisely which are the coefficients, coming from the matrix B.