

Chapter 12

Electron Spin Qubit in Semiconductor-1-Qubit and 2-Qubit Gates

12.1 Introduction

In Chap.11, we showed how to implement a qubit using electron spin on a silicon substrate. We also demonstrated how to initialize and measure a qubit. In this chapter, we will study how to perform a universal 1-qubit gate and a 2-qubit entanglement gate to fulfill the last two DiVincenzo's criteria (Sect. 1.3). In Chap.10, we showed that by applying a vertical DC magnetic field and a rotating horizontal magnetic field and then *working in the rotating frame*, we would be able to rotate any state on Bloch sphere about any vector. This allows us to build a universal 1-qubit gate (Section 27.4 in [1]) However, in the literature, many silicon qubits are still implemented with the setup in Chap. 9 which means that the qubit is placed in a vertical DC magnetic field and a perturbing and linearly oscillating horizontal magnetic field. This is what we will use in this chapter. We will first summarize an experimental paper on how it implements 1-qubit gate. Then we will discuss the implementation of a 2-qubit entanglement gate with an example.

12.1.1 Learning Outcomes

Be able to describe how a 1-qubit gate can be implemented for silicon spin qubits; understand how a CNOT-gate can be implemented by using a native entanglement gate of silicon spin qubits and other 1-qubit gates.

12.1.2 Teaching Videos

- Search for Ch12 in this playlist
 - <https://tinyurl.com/3yhze3jn>
- Other Videos
 - <https://youtu.be/OJVw4xICV10>
 - https://youtu.be/_CpQ-Uy0Kgo

12.2 1-Qubit Gate Implementation

We will use an example in [2] (with some variation) to demonstrate how to implement electron spin qubit in silicon and the 1-qubit gate. The setup is illustrated in Fig. 12.1.

Firstly, the Hilbert space is created by applying a DC magnetic field pointing to the right. This direction is named the $-\hat{z}$ direction. Therefore, *spin-up means that the spin is pointing to the left, and spin-down means that the spin is pointing to the right*. This is nothing special because the name of direction is completely a human definition.

The system is kept at 50 mK and $B_0 = 1.4T$. This is a spin qubit. To enhance the decoherence time, we need to avoid the spin interacting with the external environment. The electron spin can interact with the nuclear spin in the silicon easily and lose its coherence. Unfortunately, while 95.53% of natural silicon atoms have a zero nuclear spin (92.23% ^{28}Si isotope and 3.1% ^{30}Si isotope), 4.67% of them have a spin (^{29}Si isotope). This is because ^{29}Si has 14 protons and 15 neutrons and their spins are not nullified due to odd number of nucleons. The electron spin will interact strongly with the nucleus of ^{29}Si . Therefore, a highly purified silicon substrate is used, in which ^{29}Si is reduced to 800 ppm (part-per-million). That means, among 1 million silicon atoms in the substrate, only 800 will be ^{29}Si .

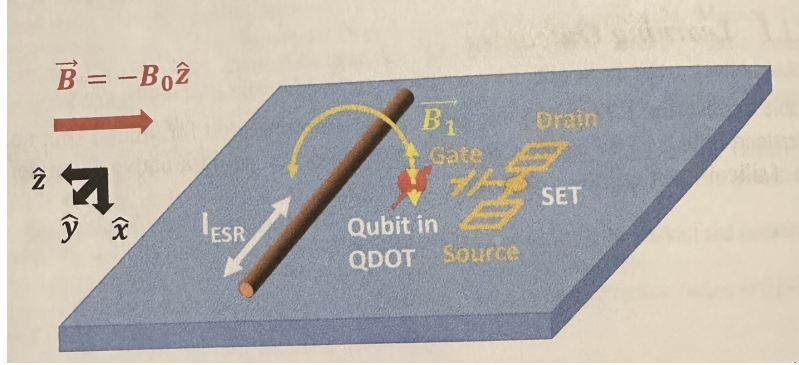


Fig. 12.1 Illustration of how an electron spin qubit is implemented in [2]. The magnetic field due to I_{ESR} is shown and it points perpendicular to the substrate at the qubit

For **readout**, it also uses spin-to-charge conversion as in Sect. 11.6. However, instead of using a quantum point contact (QPC) transistor, it uses a **single electron transistor** (SET) which has its current also very sensitive to the charge occupation in the quantum dot.

To implement a 1-qubit gate, it uses the theory we learned in Sect 9.3. That is by applying a perturbing oscillating magnetic field in the \hat{x} direction. But note that, the \hat{x} direction in Fig.12.1 is perpendicular to the silicon substrate. To achieve the oscillating magnetic field, an AC electric current, I_{ESR} , is passed through a wire next to the qubit. ESR stands for **electron spin resonance** which means that its frequency is the same as the Larmor frequency due to \vec{B} (see Sect.9.4). The current will generate a circular magnetic field about the wire

due to the **Biot-Savart law** (see also Example 24.2). Although it is circular, at the point where it touches the qubit, it is pointing at the tangential direction of the circle and thus it is pointing in the \hat{x} direction and perpendicular to the Si substrate. Since I_{ESR} is an AC current, the magnetic field at the qubit oscillates up and down and has the equation form given in the first term of Eq. (9.5). And as discussed in Sect. 9.6, in the rotating frame, any state will rotate about the \hat{x}' due to Rabi oscillation. Combined with Larmor precession, it can be used to implement any 1-qubit gate.

12.3 2-Qubit Gate for Spin Implementation

A 2-qubit gate is usually much more difficult to implement than a 1-qubit gate. We also need to recall that, to form a universal set of quantum gates, what we need is not an arbitrary 2-qubit gate but an **entanglement gate**, which can be used to create entanglements (see Sect. 4.6). Entanglement is one of the reasons that makes quantum computing superior to classical computing.

A 2-qubit gate requires the interaction between two qubits and the external excitation under an appropriate Hamiltonian. It is not difficult to appreciate that different quantum computer architectures have different "native" entanglement gates because the physics is fairly different in different quantum computing architectures. Here, a **native entanglement gate** is a gate that can be implemented easily using the physics relevant to a given architecture and its controls.

One of the native entanglement gates for an electron spin qubit is a special form of the controlled phase shift gate. We will call it U_{Cphase} [3] to distinguish it from $U_{CPS,\pi}$ in Section 16.4 in [1],

$$U_{Cphase} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi} & 0 & 0 \\ 0 & 0 & e^{i(\pi-\phi)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (12.1)$$

where ϕ is the phase. We will discuss how to implement it soon. We will first show how it can be used to create an entanglement.

12.3.1 Creation of Entanglement

We have learned that we can use $CNOT$ gate, U_{XOR} to make two qubits entangled (Section 15.4 in [1] and Sect.4.6). This is realized with the help of a Hadamard gate, H . This circuit is shown in Fig. 12.2.