Part III
Superconducting Qubit architecture and Hardware

Chapter 13
Lagrangian Mechanics and Hamiltonian Mechanics

13.1 Introduction

Most of us have learned the basics of Newtonian mechanics. Newtonian mechanics is also called *vectorial mechanics* because it studies the motion of bodies under the influence of vector euantities such as $force$. However, Newtonian mechanics is not convenient in solving certain problems. There are other frameworks in theoretical physics called analytical mechanics such as **Lagrangian mechanics** and **Hamiltonian mechanics**. They are equivalent to Newtonian mechanics and they use scalar quantities such as the kinetic energy and potential energy of a system to derive the equations of motion of the system. In many problems, they appear to be more elegant and succincy than Newtonian mechanics. More importantly, the concpets in analytical mechanics can be *generalized* to hyperspace/phase space, in which we do not live. Moreover, Hamiltonian mechanics allow us to transition from classical mechanics to quantum mechanics more "smoothly." In this chapter, we will learn the *skills* of using Lagrangian and Hamiltonian mechanics. Readers are expected to learn the rules only. Readers may refer to [1] if they are interested in having a deeper appreciation of analytical mechanics.

13.1.1 Learning Outcomes

Be able to write down the Lagrangian and Hamiltonian of a given physical system; be able to derive the equation of motion of a system based on its Lagrangian and Hamiltonian.

13.1.2 Teaching Videos

- Search for Ch13 in this playlist
  - `https://tinyurl.com/3yhze3jn`

- Other Videos
  - `https://youtu.be/Ydj2hintCkc`
  - `https://youtu.be/u2SgXmf2SvQ`
  -`https://youtu.be/IdSF_064ZSo`

13.2 Lagrangian Mechanics

13.2.1 Generalized Coordinates and Belocities

For the purposes in the following chapters, we only consider point particles, conservative forces, and non-relativistic mechanics. Let us consider a system comprised of N particle. We know that if the coordinates of each particle and the velocity of each particle are known at a given time, the system has a well-defined state. This is because the acceleration of a particle depends on the force exerted on it. And the force is the spatial derivative of its potential, which is a function of its coordinates. Therefore, if we know their positions and velocities, we know their accelerations and, thus, can deduce their past and future states.

For N particles, inour real space, there are $3N$ independent coordinates due to the three orthogonal directions. Therefore, they are the collection of $\vec{q} = \{q_1, q_2, \cdots, q_{3N}\}$, where we write it as a $3N$-dimensional vector. Similarly, it has $3N$ independent velocities, $\vec{\dot{q}} = \{\dot{q_1}, \dot{q_2}, \cdots, \dot{q_{3N}}\}$, where

$$\dot{q_i} = \frac{dq_i}{dt}. \tag{13.1}$$

Besides using real spatial coordinates and velocities to uniquely determine the state of a system, one may also use other $3N$ quantities to determine its coordinates as long as they also give the system $3N$ degrees of freedom [2]. Such quantities are called the **generalized coordinates**. The time derivatives (Eq.(13.1)) of the generalized coordinates are called the **generalized velocities**. For the formalism we will discuss later, it is easier to think and understand using spatial coordinates and velocities but we need to keep in mind and accept the fact that they are applicable to generalized coordinates and velocities, too.

13.2.2 Lagrangian and Lagrange's Equations

We will

References

1. Goldstein, H., Poole, C. & Safko, J. *Classical Mechanics* (Pearson, 2001).
2. Landau, L. & Lifshitz, E. *Mechanics: Volume 1 (Course of Theoretical Physics S)* (Butterworth-Heinemann, 1976).