Combating the Growing Demand of the Working Age: A Population Aging Model

Presented by

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Chen Zihan, Feng Emily, Zhao Zhenyi

Part I

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Problem Description

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Background/

Population aging is one of the most pressing challenges in modern society. The shrinking workforce and growing retired population intensify this issue.

Question: How do these shifts impact the sustainability of the labor market?

W Problem Description

Objective/

Our research introduces a mathematical model to simulate shifts among three population groups: those below working age, those of working age, and those above working age.

Purpose: By analyzing the changes in population structure, we aim to assess the severity of aging and the gap between younger generations and retirees.

Part II

Mathematical Model

Mathematical Model

In this paper, we define the age categories as follows:

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$B(t)$: Individuals under working age (aged 0 to 17 years)

$W(t)$: Individuals of working age (aged 18 to 66 years)

$R(t)$: Individuals over working age (retired) (aged 67 years and above)

Mathematical Model

$$\frac{dF}{dt} = Rate\ In - Rate\ Out$$

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where F represents the net flow rate.

$$\frac{dB(t)}{dt} = rB(t) - \alpha B(t) \tag{1}$$

$$\frac{dW(t)}{dt} = \gamma W(t) - \beta W(t) + \alpha B(t) \tag{2}$$

$$\frac{dR(t)}{dt} = -\delta R(t) + \beta W(t) \tag{3}$$

Mathematical Model

$$\frac{dB(t)}{dt} = rB(t) - \alpha B(t) \qquad (1)$$

$$\frac{dW(t)}{dt} = \gamma W(t) - \beta W(t) + \alpha B(t) \qquad (2)$$

$$\frac{dR(t)}{dt} = -\delta R(t) + \beta W(t) \qquad (3)$$

—

where:

r : Growth rate of the population below working age

α : Proportion of individuals transitioning to working age per year

β : Proportion of individuals retiring per year

γ : Immigration factor of individuals per year

δ : Death rate factor of the retired population per year

Mathematical Model

$$\frac{dB(t)}{dt} = rB(t) - \alpha B(t) \qquad (1)$$

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Determining r , α:

r : Growth rate of the population below working age

$$r = \frac{n_0}{\sum_{i=0}^{17} n_i} = \frac{1.09}{21.76} \approx 0.05.$$

α : Proportion of individuals transitioning to working age per year

$$\alpha = \frac{n_{17}}{\sum_{i=0}^{17} n_i} = \frac{1.31}{21.76} \approx 0.06.$$

Mathematical Model

$$\frac{dW(t)}{dt} = \gamma W(t) - \beta W(t) + \alpha B(t) \qquad (2)$$

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Determining γ , β:

β : Proportion of individuals retiring per year

$$\beta = \frac{n_{66}}{\sum_{n=18}^{66} n} = \frac{1.22}{63.16} \approx 0.0193.$$

γ : Immigration factor of individuals per year

$$\gamma = \frac{N_{immigrant}}{\sum_{n=18}^{66} n_i \times N_{total}} = \frac{2.89}{63.16\% \times 334.9} \approx 0.0137$$

Mathematical Model

$$\frac{dR(t)}{dt} = -\delta R(t) + \beta W(t) \qquad (3)$$

—

Determining δ:

δ : Death rate factor of the retired population per year

$$\delta = \frac{N_{death}}{\sum_{n=67}^{101} n_i \times N_{total}} = \frac{2.310745}{0.1648 \times 334.9} \approx 0.041867754$$

Part III

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Numerical Solution

Numerical Solution

$$\frac{dB(t)}{dt} = rB(t) - \alpha B(t) \qquad (1)$$

$$B(t) = 72.832e^{-0.01t}$$

$$\frac{dW(t)}{dt} = \gamma W(t) - \beta W(t) + \alpha B(t) \qquad (2)$$

$$W(t) = \frac{4.36992}{-0.0044}e^{-0.01t} + 1204.69e^{-0.0056t}$$

$$\frac{dR(t)}{dt} = -\delta R(t) + \beta W(t) \qquad (3)$$

$$R(t) = e^{-0.041867754t}\left(641.08e^{0.0362678t} - 601.488e^{0.0318678t} + 15.59952\right)$$

Part IV

Conclusion

Solution Plots

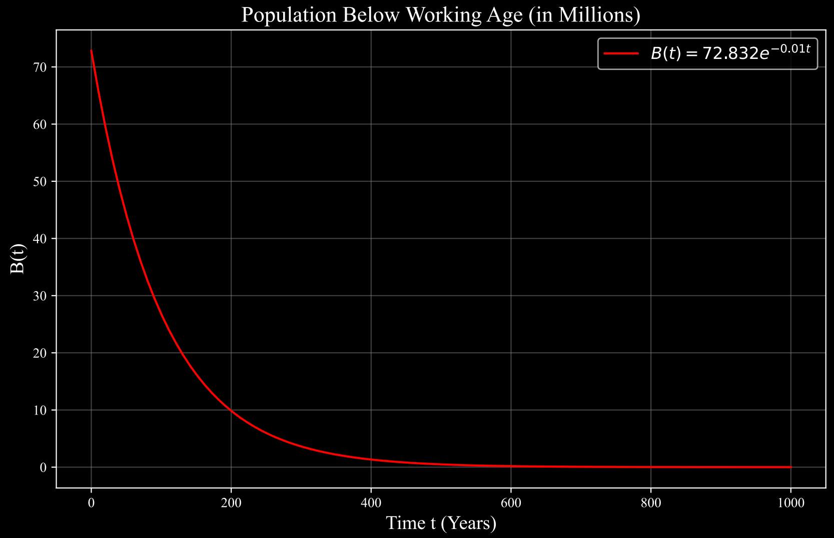


Figure 1. Plot of $B(t)$

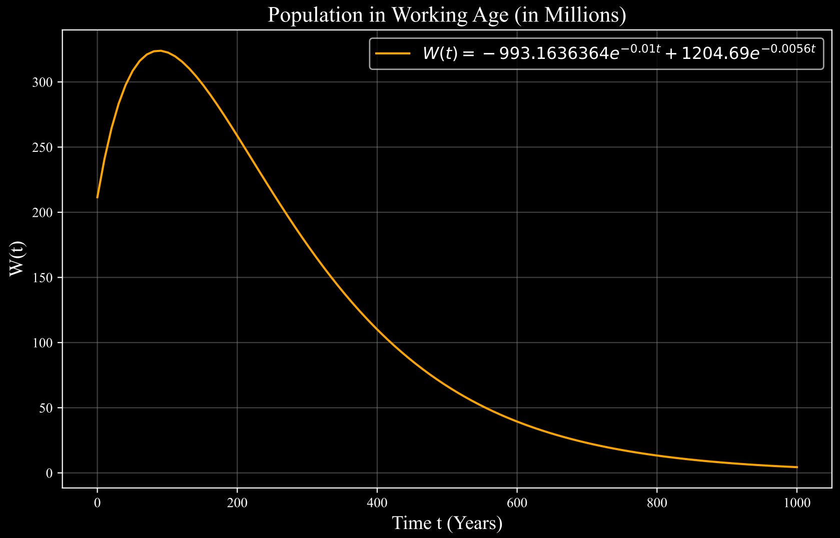


Figure 2. Plot of $W(t)$

Solution Plots

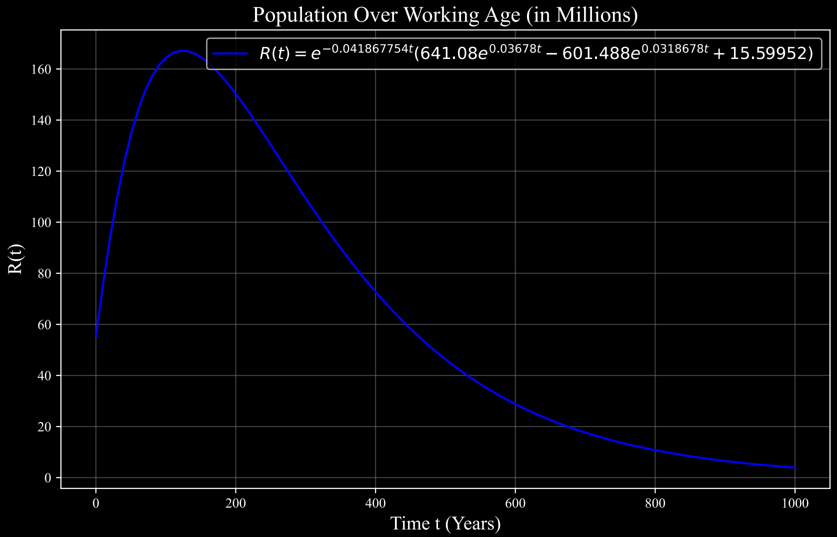


Figure 3. Plot of $R(t)$

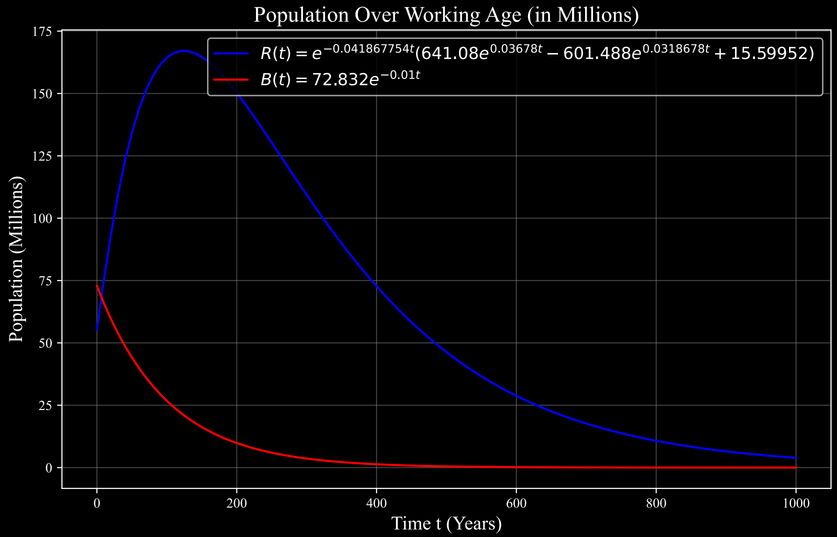
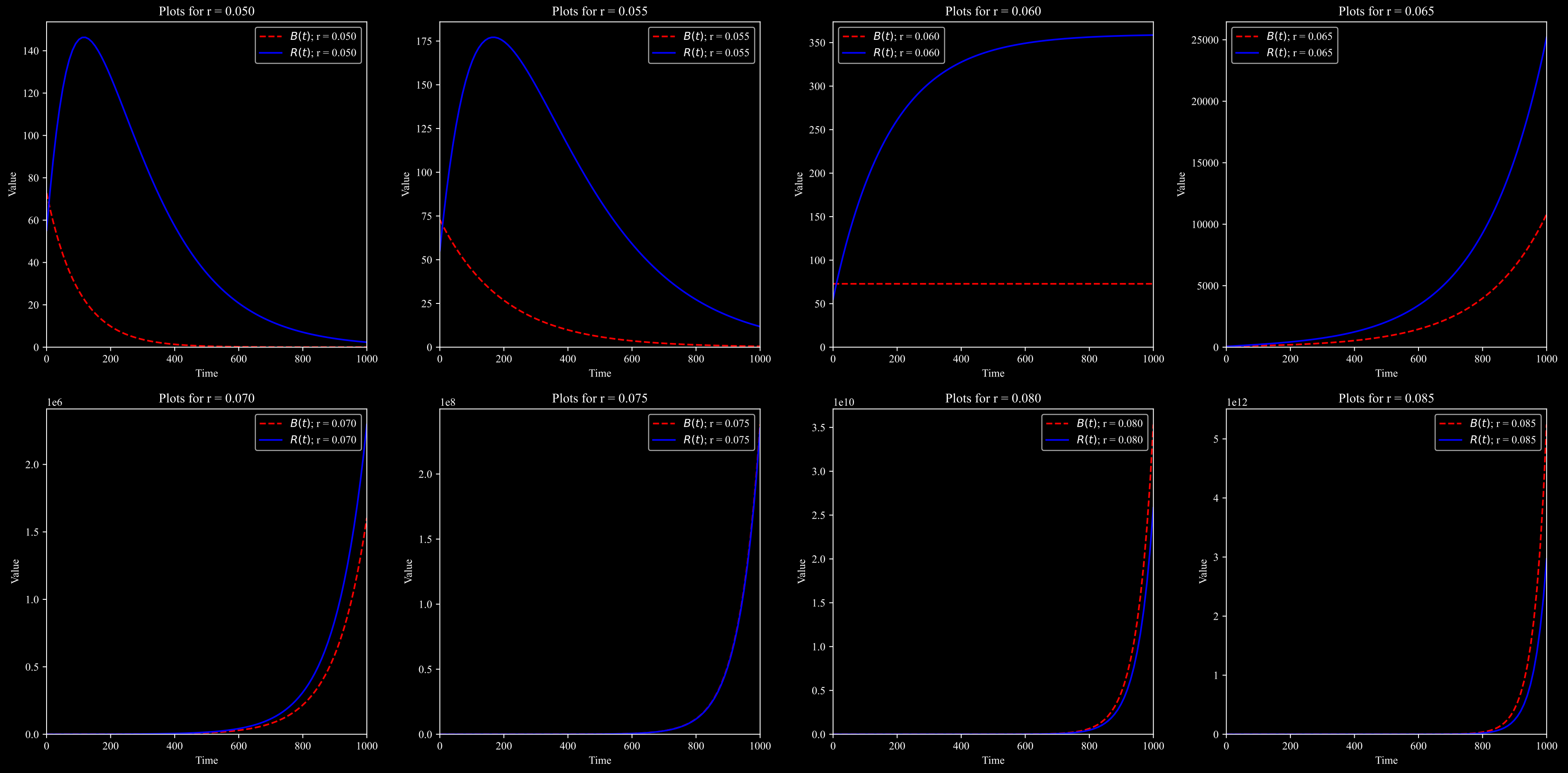


Figure 4. Plot of $R(t)$ v.s $B(t)$

Solution Plots

Figure 5. Population Plots with Various Growth Rates



Part VI

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Future Work

Many Thanks

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