Problem 4-2

a)

$\Rightarrow T(n) = 36T(n/6) + 2n$

$\Rightarrow T(n) =$ $\quad n^{\log_b a} = n^{\log_6 36} = n^2$

$a = 36$

$b = 6.$

$f(n) = 2n$

$\Rightarrow$ case 1: $f(n) = O(n^{2-\varepsilon})$ for $\varepsilon = 1$.

$T(n) = \Theta(n^2)$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

$n^{\log_b a} = n^{1.46}$

$f(n) = O(n^{1.46-\varepsilon})$ for $\varepsilon = 0.26$

$T(n) = \Theta(n^{1.46})$

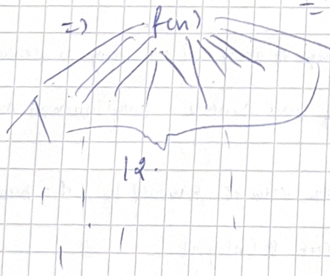
c) $T(n) = 12(n/2) + n^2 \log n$

$a = 12$

$b = 2$

$f(n) = n^2 \lg n$

$\Rightarrow$



$\rightarrow n^2 \lg n$

$\rightarrow \frac{12}{2} n^2 \log n = 6 n^2 \lg n$

$\rightarrow \frac{12 \times 12}{4} = 12 \times 3 = 36 n^2 \log n$

$\Rightarrow n^2 \log$

$\Rightarrow$ ~~$n^2 \log$~~ $n^2 \lg n \, (1 + 6 + 6^2 + 6^3 + \dots)$

$\Rightarrow$ in this case

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 12})$ ~~$= \Theta(n^{3.5})$~~

d. $\Rightarrow T(n) = 3T(n/5) + T(n/2) + 2^n$.

lower Bou

$\Rightarrow$ ~~T(n)~~

$\Rightarrow 3T(n/5) \simeq T(n/2)$

$\Rightarrow T(n) = 2T(n/2) + 2^n$

$\Rightarrow T(n) = 4T(n/4) + 2^n$

$\Rightarrow T(n) = 8T(n/8) + 2^n$

$\Rightarrow$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + 2^n$

$\Rightarrow$

$\Rightarrow$ Base Case. $\frac{n}{2^k} = 1$

$n = 2^k$

$k = \log_2 n = \log_2 2^k$

$\log_2 n = k$

$\Rightarrow$

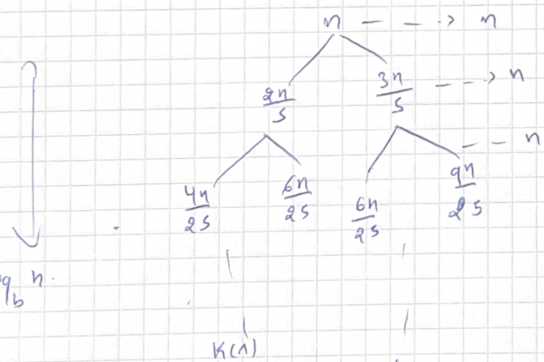
$T(n) = 2^{\log_2 n} T(1) + 2^n$

$\Rightarrow 2^{\log_2 n}$

$f(n) = 2^n$

$\Rightarrow$ Case 3: $T(n) = \theta(2^n)$

$\Rightarrow$ e. $T(n) = T(2n/5) + T(3n/5) + \theta(n)$



$n = \log_b n$.

Higher Bound:

$\Rightarrow \frac{3^k}{5^k} n = 1$

$n = \left(\frac{5}{3}\right)^k$

$\log_{5/3} n = \log_{5/3} \left(\frac{5}{3}\right)^k$

$\Rightarrow k = \log_{5/3} n$

$\Rightarrow$ lower Bound $= n \log_{5/2} n$

$\Rightarrow$ lower Bound: (Left most path)

$\Rightarrow$

$\frac{2^k}{5^k} n = 1$

$n = \left(\frac{5}{2}\right)^k$

$\log_{5/2} n = \log_{5/2} k$

$k = \log_{5/2} n$

higher Bound $= n \log_{5/3} n$

$\Rightarrow$ only differ by a constant factor

$\boxed{T(n) = \theta(n \log n)}$