=> 3.1.

a) $f(n) = 9n$ and $g(n) = 5n^3$

$\Rightarrow f \in o(g)$ because $\lim\limits_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\Rightarrow \frac{9n}{5n^3} = 0$

$\Rightarrow g \in \omega(f)$ because $\lim\limits_{n \to \infty} \frac{g(n)}{f(n)} = \infty$

$\Rightarrow \lim\limits_{n \to \infty} \frac{5n^3}{8n} = \frac{5}{8} n^2$

$\Rightarrow \lim\limits_{n \to \infty} \frac{5}{8} n^2 = \infty$

$$\Rightarrow \lim_{n \to \infty} \frac{9n^{0.8} + 2n^{0.3} + 14\log n}{n^{0.5}}$$

b)

$$\Rightarrow \frac{9n^{0.8}}{n^{0.5}} + \frac{2n^{0.3}}{n^{0.5}} + \frac{14\log n}{n^{0.5}}$$

$$\Rightarrow 9n^{0.3} + \underbrace{2n^{-0.2} + \frac{14\log n}{n^{0.5}}}_{\text{this will approach zero}}$$

$$\Rightarrow \lim_{n \to \infty} 9n^{0.3}$$

$$\Rightarrow \infty$$

$\therefore f \in \omega(g)$

$$\therefore \lim_{n \to \infty} \frac{n^{0.5}}{9n^{0.8} + 2n^{0.3} + 14\log n} \doteq$$

$$\Rightarrow \frac{n^{0.5}}{9n^{0.8}} + \frac{n^{0.5}}{2n^{0.3}} + \frac{n^{0.5}}{14\log(n)}$$

$$\Rightarrow \underbrace{\frac{1}{9} n^{-0.3}}_{0} + \underbrace{\frac{1}{2} n^{0.2}}_{\to \infty} + \underbrace{\frac{1}{14} \frac{n^{0.5}}{\log(n)}}_{0}$$

$$\Rightarrow \frac{1}{9n^{0.3}} + - -$$

$$\Rightarrow g \in \omega(f)$$

c) $f(n) = n^2 / \log n$ and $g(n) = n\log n$

$$\Rightarrow \lim_{n \to \infty} \frac{\frac{n^2}{\log n}}{n\log n} \Rightarrow \frac{n^2}{\log n} \times \frac{1}{n\log n} \Rightarrow \frac{\cancel{n}^{2}}{\cancel{\log}} \frac{n}{\log n} \times \frac{1}{\log n}$$

$$\Rightarrow \frac{10^n}{n} \times \frac{1}{n} \Rightarrow \frac{10^n}{n^2}$$

$$\Rightarrow \infty$$

$\therefore f \in \omega(g)$

$$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \frac{n^2}{10^n} \quad g \in o(f)$$

$$\Rightarrow 0$$

a))

$\Rightarrow$ $f(n) = \log(3n)^3$ and $g(n) = 9\log n$

$$\Rightarrow \lim_{n\to\infty} \frac{\log(3n)^3}{9\log n} \Rightarrow \frac{3\log 3n}{9\log n} = \frac{1}{3}\frac{\log 3n}{\log n}$$

$$\Rightarrow 10^{\left(\frac{\log 3n}{3\log n}\right)}$$

$$\Rightarrow \frac{3n}{10^3 n} = \frac{3n}{1000n} = \frac{3}{1000}$$

or

$$\Rightarrow \frac{3\log 3n}{9\log n}$$

$$\Rightarrow \frac{\log 3 + \log n}{3\log n} \Rightarrow \frac{\log 3}{3\log n} + \frac{\log n}{3\log n}$$

$$\Rightarrow \frac{\log 3}{3\log n} + \frac{1}{3}$$

$$\Rightarrow \frac{1}{3}\underbrace{\frac{\log 3}{\log n}}_{0} + \frac{1}{3}$$

$$\Rightarrow \frac{1}{3}$$

$\therefore$ $f\in$ since $\frac{1}{3}$ is

$$0 < \frac{1}{3} < \infty$$

$f \in \Theta(g)$

$$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} \Rightarrow \frac{9\log n}{\log(3n)^3} = \frac{9\log n}{3\log 3n} \Rightarrow \frac{3\log n}{\log 3n} + \frac{3\log n}{\log n}$$

$$\Rightarrow 3\underbrace{\frac{\log n}{\log 3}}_{\infty} + 3$$

$$\Rightarrow \infty$$

$f \in \omega(g)$

(2)

=> for i to n-1

min_idx = i

~~for j to n+~~

~~for j=i+1 to n~~

~~idx~~

for j = i+1 to n

if A[j] < A[min_idx]

min_idx = j

A[min_idx], A[i] = A[i], A[min_idx]

b. loop invariant:

Base case $j = i$ ~~After~~ Before the loop runs.

The inner loop begins at $j+1$ so $j = i$ is before the loop.

Inductive hypothesis

$\Rightarrow$ Assume that prior to iteration $j+1$ that min_idx is the index of the minimum element of $(a_i, a_{i+1}, \ldots a_j)$

inductive case: $(j \geq i)$

prior to iteration ~~j+1~~ $j+1$, the index of the minimum element of $(a_i, a_{i+1}, \ldots a_j)$

case 1: ~~a_{j+1}~~ $a_{j+1} \geq a_{min\_idx}$

we make no change as min_idx is still the index of the minimum element of $(a_i, a_{i+1}, \ldots a_{j+1})$

case 2: $a_{j+1} < a_{min\_idx}$

since $a_{j+1} < a_{min\_idx}$, we set the min_idx to $j+1$ and min_idx is still the index of the minimum element of (a_i, a_{i+1}, a_{j+1})

$\Rightarrow$ Since before and after m is the index of the minimum element, the Algorithm is correct.