

# (Linked)Lists

TEAM INFDEV

Hogeschool Rotterdam  
Rotterdam, Netherlands

# Introduction

## Lecture topics

- We will dive deeper into different data structures
- These are already well known and understood
- Perfect for learning how a data structure is designed
- Last week we saw classes and lists (usually called arrays)
- Today we will introduce linked lists

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## Individual meetings

- I liked it
- Not always able to give advice
- **I should** pause more often
- **You should** ask for pauses or to repeat a slide
- ...

# Problem discussion

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## Examples

- All players
- All the employees of the company
- All the trucks on the road
- All the aliens in the spaceship
- All the alien spaceships in the fleet
- ...

# With variables?

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```
1 truck1 = Truck(...)
2 truck2 = Truck(...)
3 ...
4 truck10 = Truck(...)
```

## Examples

# With variables?

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```
1 truck1 = Truck(...)
2 truck2 = Truck(...)
3 ...
4 truck10 = Truck(...)
```

## Examples

- Does this work?
- What if we have more or less than 10 trucks?



# General idea

## Introduction

- To solve this problem, we want to have all the data in a single variable
- The variable contains thus an unknown number of values
  - Might be empty
  - Might have only one element
  - Might have hundreds of elements
  - ...
  - Our list solves these issues
  - However, there is more

## Description

- Today, we will define an open-ended data structure called Linked list (short: list)
- This will be built as a linear chain of **nodes**
- In the simplest implementation, each node has
  - a **value**
  - a reference to the **next** elements
- We never really know how many elements we have in the list until we follow all the references through
- A special case is the empty list, which has no element and no reference to the next elements

```

1  +---+---+   +---+---+   +---+---+   +---+---+
2  | 3 | ----->| 7 | ----->| 4 | ----->|   |   |
3  +---+---+   +---+---+   +---+---+   +---+---+

```

## Description

- Consider a list with elements 3, 7, and 4
- We need four nodes (the last is empty), all referencing the next

## Description

- A list of values is built as either of:
  - An empty list `Empty`
  - A non-empty list containing the current value `v` and the rest of the list `vtail` `Node(v,tail)`
- **A list with three integers would be?**

## Description

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`Node(1,Node(2,Node(3,Empty)))`
- **A list with two integers would be?**

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`Node(1,Node(2,Node(3,Empty)))`
- **A list with two integers would be?**  
`Node(1,Node(2,Empty))`
- **An empty list would be?**

## Description

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- **A list with three integers would be?**  
`Node(1,Node(2,Node(3,Empty)))`
- **A list with two integers would be?**  
`Node(1,Node(2,Empty))`
- **An empty list would be?** `Empty`
- ...



## Description

- A list of values offers us two pieces of information:
  - The value `Value` of the current element of the list
  - The rest `Tail` of the list
- Given a list `x`
  - **We can read print its first value with?**

## Description

- A list of values offers us two pieces of information:
  - The value `Value` of the current element of the list
  - The rest `Tail` of the list
- Given a list `x`
  - **We can read print its first value with?** `x.Value`
  - **We can print its second value with?**

## Description

- A list of values offers us two pieces of information:
  - The value `Value` of the current element of the list
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- Given a list `x`
  - **We can read print its first value with? `x.Value`**
  - **We can print its second value with? `x.Tail.Value`**
  - **We can print its third value with?**

## Description

- A list of values offers us two pieces of information:
  - The value `Value` of the current element of the list
  - The rest `Tail` of the list
- Given a list `x`
  - **We can read print its first value with?** `x.Value`
  - **We can print its second value with?** `x.Tail.Value`
  - **We can print its third value with?** `x.Tail.Tail.Value`
  - ...

# Technical details

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## Introduction

- How is this done in Python?
- We shall build two data structures that, together, make up arbitrary lists
- We begin with the blueprints

# The blueprint (**THIS IS NOT CODE!**)

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```
1  Abstraction Empty =  
2      nothing  
3  
4  Abstraction Node =  
5      Value, which contains the datum of this element of the list  
6      Tail, which contains the remaining nodes of the list
```

## Introduction

# The blueprint (**THIS IS NOT CODE!**)

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```
1 Abstraction Empty =  
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```

## Introduction

- How do we translate this to Python?



# The blueprint (**THIS IS NOT CODE!**)

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```
1 Abstraction Empty =  
2   nothing  
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4 Abstraction Node =  
5   Value, which contains the datum of this element of the list  
6   Tail, which contains the remaining nodes of the list
```

## Introduction

- How do we translate this to Python?
- Each abstraction becomes a class
- Each field is assigned under `__init__` to `self`

# The actual code

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```
1 class Empty:
2     def __str__(self):
3         return "[]"
4
5 class Node:
6     def __init__(self, value, tail):
7         self.Value = value
8         self.Tail = tail
```

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## Examples of list usage

- We now wish to build a list with our data structures
- We will build a list based on the input of the user
- User specifies how many, and which elements must go in the list

# Examples of list usage

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S

PC
1

H


```
1 l = Empty()
2 count = int(input("How many elements?"))
3 for i in range(0, count):
4     v = int(input("Insert the next element"))
5     l = Node(v, l)
```

# Examples of list usage

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1 l = Empty()
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```

S

PC	l	count	i	v
5	ref(0)	5	0	80085

H

0
Empty()

# Examples of list usage

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PC	l	count	i	v
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# Examples of list usage

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```

S

PC	l	count	i	v
3	ref(1)	5	0	80085

H

0	1
Empty()	[ Value $\mapsto$ 80085; Tail $\mapsto$ ref(0)]

# Examples of list usage

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S

PC	l	count	i	v
5	ref(1)	5	1	8078

H

0	1
...	[ Value $\mapsto$ 80085; Tail $\mapsto$ ref(0)]

```
1 l = Empty
2 count = int(input("How many elements?"))
3 for i in range(0, count):
4     v = int(input("Insert the next element"))
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# Examples of list usage

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PC	l	count	i	v
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0	1			
...	[ Value $\mapsto$ 80085; Tail $\mapsto$ ref(0)]			

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1 l = Empty
2 count = int(input("How many elements?"))
3 for i in range(0, count):
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```

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PC	l	count	i	v
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H

0	1	2		
...	...	[ Value $\mapsto$ 8078; Tail $\mapsto$ ref(1)]		

# Examples of list usage

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- We used a for-loop to build a list
- We'd like to use a loop to iterate a list!
- something along the lines of:

```
1 make a list
2 while there is another node in the list:
3     print the first node
4     continue with the next node
```

## Examples of list usage

- We now wish to use the list we just built
- Specifically, we will print all its elements
- **How many elements does it have?**

## Examples of list usage

- We now wish to use the list we just built
- Specifically, we will print all its elements
- **How many elements does it have?**
- Unknown: it is specified by the user!

# Examples of list usage

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S

PC	I
1	ref(2)

H

0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

```

1 x = l
2 while not(x.IsEmpty):
3     print(x.Value)
4     x = x.Tail
    
```

# Examples of list usage

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```

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PC	I	x
2	ref(2)	ref(2)

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0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

# Examples of list usage

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PC	l	x
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H

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1 x = l
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```

What gets printed?

# Examples of list usage

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PC	l	x
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```

1 x = l
2 while not(x.IsEmpty):
3     print(x.Value)
4     x = x.Tail

```

What gets printed?  $H[x][Value] = H[2][Value] = 3$



# Examples of list usage

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PC	l	x
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0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

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What gets printed?  $H[x][Value] = H[2][Value] = 3$

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```

**Where is x.Tail?**

# Examples of list usage

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PC	I	x
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0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

```

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```

Where is **x.Tail**?  $H[x][Tail] = H[2][Tail] = \text{ref}(1)$

# Examples of list usage

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PC	I	x
3	ref(2)	ref(2)

H

0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

```

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2 while not(x.IsEmpty):
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```

Where is **x.Tail**?  $H[x][Tail] = H[2][Tail] = \text{ref}(1)$

S

PC	I	x
4	ref(2)	ref(1)

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0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

# Examples of list usage

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S

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H

0	1	2
$[E \mapsto T]$	$[E \mapsto F; V \mapsto 2; T \mapsto \text{ref}(0)]$	$[E \mapsto F; V \mapsto 3; T \mapsto \text{ref}(1)]$

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# Examples of list usage

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S

PC	l	x
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H

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$[E \mapsto T]$	$[E \mapsto F; V \mapsto 2; T \mapsto \text{ref}(0)]$	$[E \mapsto F; V \mapsto 3; T \mapsto \text{ref}(1)]$

```

1 x = l
2 while not(x.IsEmpty):
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4     x = x.Tail

```

Where is **x.Tail**?  $H[x][\text{Tail}] = H[1][\text{Tail}] = \text{ref}(0)$

# Examples of list usage

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S

PC	I	x
3	ref(2)	ref(1)

H

0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

```

1 x = 1
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```

Where is **x.Tail**?  $H[x][Tail] = H[1][Tail] = \text{ref}(0)$

S

PC	I	x
4	ref(2)	ref(0)

H

0	1	2
[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

# Examples of list usage

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S

PC	l	x
2	ref(2)	ref(0)

H

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[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

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1 x = l
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```

**What is the value of x.IsEmpty?**



# Examples of list usage

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S

PC	l	x
2	ref(2)	ref(0)

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[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

```

1 x = l
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```

**What is the value of `x.IsEmpty`?**  $H[x][IsEmpty] =$   
 $H[0][IsEmpty] = \text{True}$

# Examples of list usage

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S	PC	l	x
	2	ref(2)	ref(0)

H	0	1	2
	[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

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```

**What is the value of `x.IsEmpty`?**  $H[x][IsEmpty] =$   
 $H[0][IsEmpty] = \text{True}$

S	PC	l	x
	5	ref(2)	ref(0)

H	0	1	2
	[ E $\mapsto$ T ]	[ E $\mapsto$ F; V $\mapsto$ 2; T $\mapsto$ ref(0) ]	[ E $\mapsto$ F; V $\mapsto$ 3; T $\mapsto$ ref(1) ]

# In-class homework

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## Implement the following (on paper)

- Read a list from the user input
- Remove all odd numbers
- A “volunteer” runs the steps on paper with the memory model

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## Implement the following (on paper)

- Read a list from the user input
- Sum all its values
- A “volunteer” runs the steps on paper with the memory model

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## Implement the following (on paper)

- Read a list from the user input
- Reverse it
- A “volunteer” runs the steps on paper with the memory model

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## Implement the following (on paper)

- Read two lists from the user input
- Append the second to the first (concatenate them)
- A “volunteer” runs the steps on paper with the memory model

# Conclusion



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## Lecture topics

- What we solved today was the issue of representing multiple data inside a single variable
- We used a simple data structure, the **list**
- We showed how we can consume (use) the list through looping

# This is it!

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The best of luck, and thanks for the  
attention!