

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Functions

TEAM INFDEV

Hogeschool Rotterdam
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Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Introduction

Lecture topics

- So far we have shown how data representation can be abstracted away
- Building useful containers only once makes it possible to reuse their definition
- Many data structures (tuples, lists, maps, sets, etc.) become thus a new layer of abstraction

Lecture topics

- Manipulating these data structures happens in user code
- Often, user code needs to perform operations that are similar to each other
- Similar operations should not require rewriting everything every time

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Problem discussion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Introduction

- Consider many operation on lists
 - finding or removing a specific element in a container
 - computing the length of a list
 - removing all elements that satisfy a condition
 - ...

Lenght of a list

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 cnt = 0
2 x = l
3 while not(x.IsEmpty):
4     cnt = cnt + 1
5     x = x.Tail
6 print("List l contains " + str(cnt) + " elements.")
```

Introduction

Lenght of a list

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 cnt = 0
2 x = l
3 while not(x.IsEmpty):
4     cnt = cnt + 1
5     x = x.Tail
6 print("List l contains " + str(cnt) + " elements.")
```

Introduction

- What does `l` contain?
- What do we do with the values of the list?
- Do they even matter?

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Introduction

- Suppose that we now have another list, k
- We wish to know its length
- How do we do it?

Lenght of a list

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 cnt = 0
2 x = k
3 while not(x.IsEmpty):
4     cnt = cnt + 1
5     x = x.Tail
6 print("List_k contains " + str(cnt) + " elements.")
```

Introduction

```
1 cnt = 0
2 x = k
3 while not(x.IsEmpty):
4     cnt = cnt + 1
5     x = x.Tail
6 print("List_k contains " + str(cnt) + " elements.")
```

Introduction

- Looks suspiciously like the previous code block
- Why?

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

General idea

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Adding our own layers

- The goal of this lecture is to add a new layer of abstraction to our programs
- We wish to reuse **implementations**, not only data structures
- This layer of abstraction is called **functions**

Adding our own layers

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 +-----+
2 | ...      |
3 +-----+
4 | Functions      |
5 +-----+
6 | data structures      |
7 +-----+
8 | if, for, while, variables |
9 +-----+
10 | (Python) runtime      |
11 +-----+
12 ...
```

Description

- A function is a collection of instructions and variables
- Some instructions and variables are fixed inside its **body**
- Other instructions and variables come from outside the function, and thus are not fixed; these are called **parameters** of the function
- We try to strike the right balance between flexibility and work done
- The function returns a final result that can be recovered by the code that uses the function

Blueprint of a function (NOT ACTUAL PYTHON CODE!)

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 length of a list l:  
2     cnt = 0  
3     x = l  
4     while not(x.IsEmpty):  
5         cnt = cnt + 1  
6         x = x.Tail  
7     return cnt as the final result
```

Description

Blueprint of a function (NOT ACTUAL PYTHON CODE!)

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 length of a list l:  
2   cnt = 0  
3   x = l  
4   while not(x.IsEmpty):  
5       cnt = cnt + 1  
6       x = x.Tail  
7   return cnt as the final result
```

Description

- `length` is the **function name**
- `l` is the only **parameter**
- Lines 2 through 6 are **fixed**
- `cnt` is the **final result**

Using the function

- Code that needs the length of a function can now simply invoke function `length`
- The resulting code will simply be `l_len = length(l)`
- `l_len` will be assigned with the value returned by the function

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Technical details

Introduction

- A function can be defined in Python quite easily
- The syntax is:
 - `def <<name>>(<<parameters>>):a`
 - `body`
 - `return <<result>>`
- Inside a function we can put whatever instructions we need
 - `if`
 - `for`
 - `...`

^aParameters might be none, thus we can write simply `()`

^bMultiple parameters are separated by a comma, thus
`(<<p1>>, <<p2>>, ..., <<pn>>)`

Using the function

- After we declare a function, we can use it
- The syntax is quite simple
 - `<<name>>(<<parameters>>)` to just call the function and ignore the result
 - `<<v>> = <<name>>(<<parameters>>)` to call the function and assign the result to the `<<v>>` variable
- After calling the function, we enter the local environment of the function
- Variables, the PC, etc. are separate from those of the calling site

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC
9

H


```
1 def length(l):  
2     cnt = 0  
3     x = l  
4     while not(x.IsEmpty):  
5         cnt = cnt + 1  
6         x = x.Tail  
7     return cnt  
8  
9 print(length(Node(10, Empty)))
```

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC
9

H


```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```

S

PC	length	PC	I
9	nil	2	ref(1)

H

0	1
[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	I
9	nil	2	ref(1)

H

0	1
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 10; T \mapsto \text{ref}(0)]$

```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```


Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S	PC	length	PC	I
	9	nil	2	ref(1)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```

S	PC	length	PC	I	cnt
	9	nil	3	ref(1)	0

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S	PC	push	PC	I	cnt
	9	length	3	ref(1)	0

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S	PC	push	PC	I	cnt
	9	length	3	ref(1)	0

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```

S	PC	length	PC	I	cnt	x
	9	nil	4	ref(1)	0	ref(1)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

After a few steps...

S	PC	length	PC	l	cnt	x
	9	nil	7	ref(1)	1	ref(0)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2   cnt = 0
3   x = l
4   while not(x.IsEmpty):
5     cnt = cnt + 1
6     x = x.Tail
7   return cnt
8
9 print(length(Node(10, Empty)))

```

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

After a few steps...

S	PC	length	PC	l	cnt	x
	9	nil	7	ref(1)	1	ref(0)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2     cnt = 0
3     x = l
4     while not(x.IsEmpty):
5         cnt = cnt + 1
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7     return cnt
8
9 print(length(Node(10, Empty)))

```

Do we still need all the local variables of the function?

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

After a few steps...

S	PC	length	PC	l	cnt	x
	9	nil	7	ref(1)	1	ref(0)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2     cnt = 0
3     x = l
4     while not(x.IsEmpty):
5         cnt = cnt + 1
6         x = x.Tail
7     return cnt
8
9 print(length(Node(10, Empty)))

```

Do we still need all the local variables of the function?
Where do we put the result?

Runtime example

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

After a few steps...

S	PC	length	PC	l	cnt	x
	9	nil	7	ref(1)	1	ref(0)

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

```

1 def length(l):
2     cnt = 0
3     x = l
4     while not(x.IsEmpty):
5         cnt = cnt + 1
6         x = x.Tail
7     return cnt
8
9 print(length(Node(10, Empty)))

```

**Do we still need all the local variables of the function?
Where do we put the result?**

S	PC	length
	9	1

H	0	1
	[E \mapsto true]	[E \mapsto false; V \mapsto 10; T \mapsto ref(0)]

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Syntax and semantics

- We will now describe how Python functions work precisely
- This is a **fundamental** bit of knowledge that determines if you really do learn how to program or not
- This **absolutely requires** a lot of focus to get
- Please panic a bit on the inside

Subtleties that make functions “fun” to use

- About variables
 - Variables and parameters inside a function have precise **scope** (visibility)
 - Primitive values given as parameters can be **changed only locally** to the function
 - References given as parameters can be **permanently changed** from within the function
 - Global variables defined outside the function may be **read but not changed** from within the function^a
- About behaviour
 - A function may **call itself**, in a process known as **recursion**
 - A function may **get as parameters and return other functions**, in a process known as **higher order functions**

^aUnless you use some tricks we strongly discourage

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Local and global variables (basics of scope)

- The parameters of a function are added to the list of accessible variables
- They are only visible from inside the function
- Global variables are also visible from inside the function

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Local and global variables (basics of scope)

- Every call to a function generates a new value of the stack memory S
- This contains (private copy of) all local variables
- The heap memory H remains the same
- The original stack memory (the **global variables**) remains accessible, just read-only

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Local and global variables (basics of scope)

- Every call to a function also reserves some special locations in the stack
- The local PC of the function
- The local variables of the function
- The returned value when the function is done

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 print(f(30))
8 x = 2
9 print(f(10))
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 print(f(30))
8 x = 2
9 print(f(10))
```

Local and global variables (basics of scope)

- x is a global variable, visible outside and inside the function
- z is a local variable, visible only inside the function
- **What does this program print?**

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 print(f(30))
8 x = 2
9 print(f(10))
```

Local and global variables (basics of scope)

- x is a global variable, visible outside and inside the function
- z is a local variable, visible only inside the function
- **What does this program print?**
- 10, 30, 20

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC
1

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```


Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC
1

H


```

1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))

```

S

PC	x
6	1

H

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x
6	1

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x
6	1

H


```

1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))

```

S

PC	x	f	PC	z
6	1	nil	4	10

H

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f	PC	z
6	1	nil	4	10

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```

Locals and globals

Functions

TEAM INFDEV

Introduction

Problem discussion

General idea

Technical details

Assignments

Conclusion

S

PC	x	f	PC	z
6	1	nil	4	10

H


```

1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))

```

S

PC	x	f
7	1	10

H

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f
7	1	10

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f
7	1	10

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```

S

PC	x
8	2

H

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f	PC	z
8	2	nil	4	10

H


```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
```


Locals and globals

Functions

TEAM INFDEV

Introduction

Problem discussion

General idea

Technical details

Assignments

Conclusion

S

PC	x	f	PC	z
8	2	nil	4	10

H


```

1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))

```

S

PC	x	f
8	2	20

H

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
9 print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
9 print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program do?**

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 x = 1
2
3 def f(z):
4     return x * z
5
6 print(f(10))
7 x = 2
8 print(f(10))
9 print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program do?**
- Crash with `NameError: name 'z' is not defined`

```
1 def f(z):  
2     z = z + 1  
3     return z * 2  
4  
5 print(f(10))  
6 print(f(30))
```

Local and global variables (basics of scope)

- `z` is a local variable, visible only inside the function

```
1 def f(z):  
2     z = z + 1  
3     return z * 2  
4  
5 print(f(10))  
6 print(f(30))
```

Local and global variables (basics of scope)

- `z` is a local variable, visible only inside the function
- **What does this program print?**

Locals and globals

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 def f(z):  
2     z = z + 1  
3     return z * 2  
4  
5 print(f(10))  
6 print(f(30))
```

Local and global variables (basics of scope)

- z is a local variable, visible only inside the function
- **What does this program print?**
- 22, 62

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Shadowing

- The parameters of a function have priority over globals
- They supersede global variables of the same name


```
1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))
```

Shadowing

- `x` is a global variable, potentially visible inside the function
- `x` is also a local variable of the function, which has priority over the global `x`

```
1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))
```

Shadowing

- `x` is a global variable, potentially visible inside the function
- `x` is also a local variable of the function, which has priority over the global `x`
- **What does this program print?**

```
1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))
```

Shadowing

- x is a global variable, potentially visible inside the function
- x is also a local variable of the function, which has priority over the global x
- **What does this program print?**
- 20, 40

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x
6	1

H


```
1 x = 1
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3 def f(x):
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5
6 print(f(10))
7 print(f(20))
```

Shadowing

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x
6	1

H


```

1 x = 1
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3 def f(x):
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5
6 print(f(10))
7 print(f(20))

```

S

PC	x	f	PC	x
6	1	nil	4	10

H

S

PC	x	f	PC	x
6	1	nil	4	10

H


```
1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))
```

S

PC	x	f	PC	x
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1 x = 1
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3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))

```

S

PC	x	f
7	1	20

H

S

PC	x	f
7	1	20

H


```
1 x = 1
2
3 def f(x):
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```


Shadowing

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f
7	1	20

H


```

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4     return x * 2
5
6 print(f(10))
7 print(f(20))

```

S

PC	x	f	PC	x
7	1	nil	4	20

H

S

PC	x	f	PC	x
7	1	nil	4	20

H


```
1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))
```

Shadowing

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	x	f	PC	x
7	1	nil	4	20

H


```

1 x = 1
2
3 def f(x):
4     return x * 2
5
6 print(f(10))
7 print(f(20))

```

S

PC	x	f
8	1	40

H

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Recursion

- (Recursive) functions are all functions that call themselves in their bodies
- This is based on the principle of induction and in general a very powerful technique
- This leads to a compacter and often more easily correct representation
 - Code is not easier to read, especially to the untrained eye

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Recursion

- Remember that calling a function creates a new instance of stack memory
- Recursive functions do this a lot
- Each recursive call has its own environment

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 def length(l):  
2     if l.IsEmpty:  
3         return 0  
4     else:  
5         return length(l.Tail) + 1
```

Recursion

- How many 1's shall we have?

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

```
1 def length(l):  
2     if l.IsEmpty:  
3         return 0  
4     else:  
5         return length(l.Tail) + 1
```

Recursion

- How many 1's shall we have?
- As many as the nodes of the initial value

Functions

TEAM INFDEV

Introduction

Problem discussion

General idea

Technical details

Assignments

Conclusion

S

PC
7

H


```
1 def length(l):  
2     if l.IsEmpty:  
3         return 0  
4     else:  
5         return length(l.Tail) + 1  
6  
7 print(length(Node(1,Node(2,Empty))))
```


Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC
7

H


```

1 def length(l):
2     if l.IsEmpty:
3         return 0
4     else:
5         return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC	length	PC	l
7	nil	2	ref(2)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	l
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$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC	length	PC	I
7	nil	2	ref(2)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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6
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```

S

PC	length	PC	I
7	nil	5	ref(2)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	I
7	nil	5	ref(2)

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```

S	PC	length	PC	I
	7	nil	5	ref(2)

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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2   if l.isEmpty:
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6
7 print(length(Node(1, Node(2, Empty))))

```

S	PC	length	PC	I	length	PC	I
	7	nil	5	ref(2)	nil	2	ref(1)

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	l	length	PC	l
7	nil	5	ref(2)	nil	2	ref(1)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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```

S

PC	length	PC	I	length	PC	I
7	nil	5	ref(2)	nil	2	ref(1)

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0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

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```

S

PC	length	PC	I	length	PC	I
7	nil	5	ref(2)	nil	5	ref(1)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

S

PC	length	PC	I	length	PC	I	length	PC	I
7	nil	5	ref(2)	nil	5	ref(1)	nil	2	ref(0)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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7 print(length(Node(1, Node(2, Empty))))

```


Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S	PC	length	PC	I	length	PC	I	length	PC	I
	7	nil	5	ref(2)	nil	5	ref(1)	nil	2	ref(0)

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

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1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
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6
7 print(length(Node(1, Node(2, Empty))))

```

S	PC	length	PC	I	length	PC	I	length	PC	I
	7	nil	5	ref(2)	nil	5	ref(1)	nil	3	ref(0)

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	I	length	PC	I	length	PC	I
7	nil	5	ref(2)	nil	5	ref(1)	nil	3	ref(0)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	I	length	PC	I	length	PC	I
7	nil	5	ref(2)	nil	5	ref(1)	nil	3	ref(0)

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.isEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC	length	PC	I	length	PC	I	length
7	nil	5	ref(2)	nil	5	ref(1)	0

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

S

PC	length	PC	l	length	PC	l	length
7	nil	5	ref(2)	nil	5	ref(1)	0

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC	length	PC	I	length	PC	I	length
7	nil	5	ref(2)	nil	5	ref(1)	0

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.isEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC	length	PC	I	length
7	nil	5	ref(2)	0+1

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length	PC	l	length
7	nil	5	ref(2)	1

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S	PC	length	PC	l	length
	7	nil	5	ref(2)	1

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S	PC	length
	7	2

H	0	1	2
	$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length
7	2

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```


Recursion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

S

PC	length
7	2

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

```

1 def length(l):
2   if l.IsEmpty:
3     return 0
4   else:
5     return length(l.Tail) + 1
6
7 print(length(Node(1, Node(2, Empty))))

```

S

PC
8

H

0	1	2
$[E \mapsto \text{true}]$	$[E \mapsto \text{false}; V \mapsto 2; T \text{ mapsto } \text{ref}(0)]$	$[E \mapsto \text{false}; V \mapsto 1; T \text{ mapsto } \text{ref}(1)]$

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Assignments

Build and test, on paper...

- A function `add` that increments all elements of a list by a fixed value:
 - `add(10, Node(1,Node(2,Node(3,Empty)))) -> Node(11,Node(12,Node(13,Empty)))`
- A function `filterEven` that removes all odd elements from a list:
 - `filterEven(Node(1,Node(2,Node(3,Empty)))) -> Node(2,Empty)`
- A function `sum` that adds all elements of a list:
 - `sum(Node(1,Node(2,Node(3,Empty)))) -> 6`

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Conclusion

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

Lecture topics

- Often, user code needs to perform operations that are similar to each other
- Through the mechanism of function definition, we can recycle code
- Functions can encode algorithms in many way
 - Simple code abstractions to avoid repetition
 - Recursive problems

Functions

TEAM
INFDEV

Introduction

Problem
discussion

General idea

Technical
details

Assignments

Conclusion

The best of luck, and thanks for the
attention!