MODULE – 1.

→ **Basic Graphic System :–**

It has 5 main elements —

* Input Devices
* Processor
* Memory
* Frame Buffer (image formed here)
* Output Devices.

→ Pixels are the smallest adressable graphical unit represented on the computer screen

→ **Raterisation**: The process of representing continuous picture or graphics object as a collection of discrete pixels is called rasterisation.

→ **Applications :–**

* Graphs & charts
* Data Visualisation
* Computer-Aided Design.
* Virtual Reality.
* Education & Training
* Computer Art
* Movies
* Games
* Graphical User Interface

→ **Video - Display Devices :–**

1. Cathode - Ray Tubes —

* invented by Karl Ferdinand Braun.
* beam of electrons directed ~~forth~~ from cathode (–) to phospher-coated anode (+).
* directed by magnetic focusing and deflection coils in vaccum filled tube
* Horizontal and vertical deflection direct the electron beam to any point on the screen.

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* Characteristics of CRT:
   (a) Intensity is proportional to the no. of electrons repelled in beam per second.
   (b) Brightness is the max no. of points that can be displayed without overlapping
   (c) Focusing forces the electron beam to converge to a point on the monitor screen.
   (d) Deflection directs the electron beam horizontally or vertically to any point.
   (e) Aspect Ratio is the ratio of horizontal pixels to vertical pixels for an ~~eq~~ equal length line.

2. Raster-Scan Display —
   * The electron beam scans the screen from top to bottom one row at a time. Each row is called a scan line.
   * The electron beam is turned on and off to produce a collection of dots painted one row at a time. These will form an image.
   * A raster is a matrix of pixels covering the ~~sc~~ screen area and is composed of raster lines.
   * The image is formed in a frame buffer containing the total screen area and where each memory location corresponds to a pixel.

3. Random-Scan Display —
   * Random scan systems are also called vector, stroke-writing, or calligraphic displays.
   * The electric beam directly draws the picture in any specified order.

* Picture is stored in a display list, refresh display file, vector file, or display program as a set of drawing commands.
* Advantages :
  - ⓐ High Resolution
  - ⓑ Easy Animation
  - ⓒ Requires Little Memory.
* Disadvantages :
  - ⓐ Requires intelligent electron beam.
  - ⓑ limited screen density, limited to simple, line-based images.
  - ⓒ limited colour capability

3. Color CRT ~~Moint~~ Monitor —

* it is designed as RGB monitors also called full-color system or true color system.
* Two methods :
  - ⓐ Random scan → uses beam penetration.
    - 2 layers (Red, Green) phosphors.
    - low speed electron beam to get red.
    - high speed electron beam to get green
    - intemediate speed electrom beam to get both yellow & orange
    - color is controlled by electron beam voltage; only produces a restricted set of colors.
  - ⓑ Raster scan →
    - uses a shadow mask with 3 electron guns : R, G, B.
    - color is produced by adjusting the

~~intesit~~ intensity level of each electron beam
— produces a wide range of colors.

4. Flat Panel Display —
* these are the video devices that are thinner, lighter and require less power.
* Emissive panel : electrical energy into light.
eg — plasma panels, thin-film electroluminescent display device, light-emitting diodes.
* Non-emissive : light into graphics using optical effects ; eg — LCD.

→ OpenGL State :—
* state machine.
* functions are of two types —
(a) Primitive generating
— can cause output if primitive is visible.
— how vertices are processed and appearance of primitive are controlled by the state.
(b) State changing
— Transformation functions.
— Attribute functions.

→ Classes for OpenGL Functions :—
* Primitives — draws points, lines, segments, polygons, text, curves, surfaces.
* Attributes — specify display characteristics of objects: color, fill, line width, font
* Viewing — determines aspects of view: position and angle of camera, view port size.

* Transformations — change appearance or characteristics of objects : Rotate, scale, translate
* Input — Handle keyboard, mouse, etc.
* Control — communicate with window system.
* Query — Get display information

→ **OpenGL interface :—**

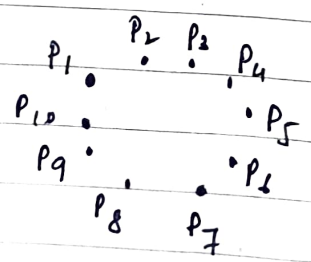
* Graphics Utility Interface (GLU).
    - creates common objects like spheres.
* GL Utility Toolkit (GLUT).
    - provides generic interface to window system.
* GLX for Unix/Linux & wgl for windows
    - provide low-level glue to window system.

→ **Using Libraries :—**

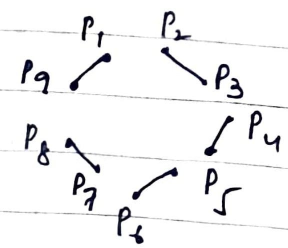
* Header files :
    ⓐ #include <GL/glut.h>
    ⓑ #include <GL/glu.h>
    ⓒ #include <GL/glew.h>

→ **Primitives :—**

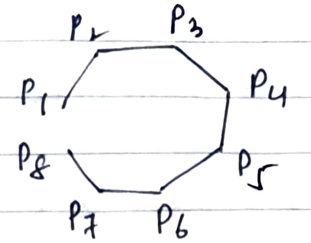
* GL_POINTS



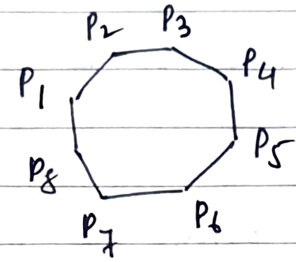
* GL_LINES



* GL_LINE_STRIP



* GL_LINE_LOOP



* OpenGL Primitives —

ⓐ GL_POINTS
ⓑ GL_LINES
ⓒ GL_LINE_LOOP
ⓓ GL_LINE_STRIP
ⓔ GL_TRIANGLES
ⓕ GL_POLYGON.
ⓖ GL_TRIANGLE_FAN
ⓗ GL_TRIANGLE_STRIP.
ⓘ GL_QUAD_STRIP

* Special Polygon —

ⓐ GL_TRIANGLE_STRIPE
ⓑ GL_QUAD_STRIPE
ⓒ GL_TRIANGLE_FAN.

Digital Differential Analyzer Algorithm

Step 1: Read start and end coordinates.

Step 2: Calculate Δx, Δy and m.

Step 3: Calculate the no. of points between start and end coordinates.

Step 4: Find the next point based on the 3 cases :—

CASE I — $m < 1$

then, $x_{p+1} = 1 + x_p$.

$y_{p+1} = m + y_p$.

CASE II — $m = 0$

then, $x_{p+1} = 1 + x_p$.

$y_{p+1} = 1 + y_p$.

CASE III — $m > 1$..

then, $x_{p+1} = \frac{1}{m} + x_p$.

$y_{p+1} = 1 + y_p$

Step 5: Repeat step 4 until end point is reached.

Bresenham's Line Drawing Algorithm.

Step 1: Input two line end points, storing the left end point in (x_0, y_0).

Step 2: Plot the point (x_0, y_0)

Step 3: Calculate the constant $\Delta x, \Delta y, 2\Delta y$ and $(2\Delta y - 2\Delta x)$ and get the first value for the decision parameter as:

$$P_0 = 2\Delta y - \Delta x.$$

Step 4: At each x_k along the line, starting at $k=0$, perform the following test. If $P_k < 0$, the next point to plot is $(x_k + 1, y_k)$ and:

$$\boxed{P_{k+1} = P_k + 2\Delta y}$$ ($P_k \rightarrow$ decision parameter)

otherwise, the next point to plot is $(x_k + 1, y_k + 1)$ and:

$$\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x}$$

Step 5: Repeat step 4 $(\Delta x - 1)$ times.

Mid-Point Circle Algorithm: (8 way symmetry)

Step 1: Read the inputs of the circle, that is, radius(r) and centre of the circle (x, y).

Step 2: Initially the value of $x=0$ and $y=r$ $\therefore [(x,y)=(0,r)]$

Step 3: Plot the pixel $x+x_c$ and $y+y_c$.

Step 4: Find out the decision parameter $P_K = 1-r$.

Step 5: Start the loop depending on two conditions—

CASE I — if $P_K > 0$

$$P_{K+1} = P_K + 2(x_{K+1}) + 1 - 2y_{K+1}$$

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y_K - 1.$$

CASE II — if $P_K < 0$

$$P_{K+1} = P_K + 2x_{K+1} + 1$$

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y$$

Step 6: Repeat step 5 until $x=y$.

MODULE – 2

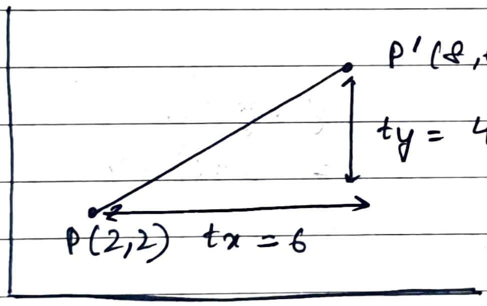
→ 2D Transformations :–

* Transformations are the operations applied to geometrical description of an object to change its position, orientation, or size are called geometric transformations.

* Transformations are needed to manipulate the initially created object and to display the modified object without having to redraw it

* Translation —

(a) It moves all points in an object along the same straight-line path to new positions.



(b) $P'_x = P_x + t_x.$
$P'_y = P_y + t_y.$

(c) Matrix Form :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}.$$

Q. Given a circle C with $r=10$ and center coordinates (1,4). Apply the translation with distance 5 towards X axis and 1 towards y-axis. Obtain new coordinates of C without changing the radius

→ old coordinates of C = (1,4).
translation vector = (5,1).

$\therefore$ $x' = x + t_x$ $\quad$ | $\quad$ $y' = y + t_y$

$= 1 + 5 = 6$ $\quad$ | $\quad$ $= 4 + 1 = 5$.

$\therefore$ New coordinates $= (x', y') = (6, 5)$.

Q. Given a square with coordinate points A(0,3), B(3,3), C(3,0), D(0,0). Apply the translation with distance 1 towards x axis and 1 towards y axis. Obtain the new coordinates of the square.

→ Old coordinates = A(0,3), B(3,3), C(3,0), D(0,0).

Translation vector = (1,1)

A' $= x + t_x$. $\qquad$ $y + t_y$

$= 0 + 1 = 1$ $\qquad$ $= 3 + 1 = 4$.

$\therefore$ A'(1,4)

B' $\quad x + t_x$. $\qquad$ $y + t_y$

$= 3 + 1 = 4$ $\qquad$ $= 3 + 1 = 4$.

$\therefore$ B' = (4,4)

C' $\quad x + t_x$ $\qquad$ $y + t_y$

$= 3 + 1 = 4$ $\qquad$ $= 0 + 1 = 1$.

$\therefore$ C'(4,1).

D' $\quad x + t_x$ $\qquad$ $y + t_y$

$= 0 + 1 = 1$ $\qquad$ $= 0 + 1 = 1$

$\therefore$ D'(1,1).

* Rotation :-

(a) It is a process of rotating an object ~~from one~~ with respect to an angle in 2D plane

(b) repositions all points in an object along a circular path in the plane centered at the pivot point.

(c) $p'_x = p_x \cos\theta - p_y \sin\theta$
$p'_y = p_x \sin\theta + p_y \cos\theta$.

(d) Matrix Form :

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q. Given a line segment with starting point as (0,0) and ending point as (4,4). Apply 30° rotation anticlockwise direction on the ~~long~~ line segment and find out the new coordinates of the line.

→ old $(x,y) = (4,4)$
Rotation angle $(\theta) = 30°$.

$p'_x = x\cos\theta - y\sin\theta$
$= 4 \cdot \cos 30° - 4 \cdot \sin 30°$.
$= 4 \cdot \sqrt{3}/2 - 4 \cdot 1/2 = 2\sqrt{3} - 2$.
$= 2(\sqrt{3} - 1) = 1.46$.

$p'_y = \cancel{y\cos\theta} \cdot x\sin\theta + y\cos\theta$.
$= 4 \cdot \sin 30° + 4 \cdot \cos 30°$.
$= 4 \times \frac{1}{2} + 4 \times \frac{\sqrt{3}}{2} = 2 + 2\sqrt{3}$
$= 2(1+\sqrt{3}) = 5.46$.

∴ coordinates after rotation at 30° = (1.46, 5.46).

Alternatively,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4\cos 30^\circ - 4\sin 30^\circ \\ 4\sin 30^\circ + 4\cos 30^\circ \end{bmatrix} = \begin{bmatrix} 4\times \sqrt{3}/2 - 4\times 1/2 \\ 4\times 1/2 + 4\times \sqrt{3}/2 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Hence, new coordinates are $(1.46, 5.46)$.

* Scaling —

ⓐ It changes the size of the objects and involves two scale factors, S_x and S_y for the x- and y-coordinates respectively.

ⓑ Scales are about the origin

ⓒ $P'_x = S_x \cdot P_x.$ | scaling factor > 1 : size ↑.

$P'_y = S_y \cdot P_y.$ | scaling factor < 1 : size ↓.

ⓓ Matrix form :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Q. Given a square object with coordinates points A(0,3), (3,3), C(3,0), D(0,0). Apply the scaling parameter 2 towards x-axis and 3 towards y-axis and obtain the new coordinates of the objects

→ old coordinates : A(0,3), B(3,3), C(3,0), D(0,0).

$S_x = 2.$; $S_y = 3$

For coordinate A(0,3) —

$x' = S_x . x = 2 \times 0 = 0$

$y' = S_y . y = 3 \times 3 = 9.$

∴ new coordinates : A(0,9).

For coodinates B(3,3) —

$x' = S_x . x = 3 \times 2 = 6.$

$y' = S_y . y = 3 \times 3 = 9.$

∴ new coordinates : B(6,9).

For coodinate C(3,0) —

$x' = S_x . x = 3 \times 2 = 6.$

$y' = S_y . y = 0 \times 3 = 0.$

∴ new coordinates : C(6,0).

For coordinates D(0,0) —

$x' = S_x . x = 2 \times 0 = 0$

$y' = S_y . y = 3 \times 0 = 0$

∴ new coordinates : D(0,0).

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Q1. Translate a ~~state~~ triangle with vertices at original coordinates (10,20), (10,10), (20,10) by $t_x = 5$, $t_y = 10$.

→ old coordinates = (10,20), (10,10), (20,10).
Translation vector = (5, 10).

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} + \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \end{bmatrix} + \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 25 \\ 20 \end{bmatrix}.$$

Hence, new coordinates : (15,30), (15,20), (25,20)

Q2. Scale a triangle w.r.t origin, with vertices at original coordinates (10,20), (10,10), (20,10) by $S_x = 2$ and $S_y = 1.5$.

→ old coordinates = (10,20), (10,10), (20,10).
$S_x = 2$ and $S_y = 1.5$.

$x_1' = S_x \cdot x = 10 \times 2 = 20$
$y_1' = S_y \cdot y = 20 \times 1.5 = 30$
$\therefore (x_1', y_1') = (20, 30)$.
$x_2' = S_x \cdot x = 10 \times 2 = 20$
$y_2' = S_y \cdot y = 10 \times 1.5 = 15$.
$\therefore (x_2', y_2') = (20, 15)$.

$x_3' = S_x \cdot x = \cancel{6}\, 20 \times 2 = 40.$

$y_3' = S_y \cdot y = 10 \times 1.5 = 15.$

$\therefore (x_3', y_3') = (40, 15).$

Hence, new coordinates are : $(20,30), (20,15), (40,15)$.

Q3. Rotate a triangle about the origin with vertices at original coordinates $(10,20), (10,10), (20,10)$ by $30°$.

→ old coordinates $= (10,20), (10,10), (20,10)$

Rotation angle $(\theta) = 30°$.

$\therefore P_x' = P_x \cos\theta - P_y \sin\theta.$

$= 10 \cdot \cos 30° - 20 \sin 30°.$

$= \cancel{10}^{5} \cdot \sqrt{3}/\cancel{2} - \cancel{20}^{10} \cdot 1/\cancel{2}$

$= 5\sqrt{3} - 10 = 5(\sqrt{3} - 2). = -1.34.$

$P_y' = P_x \sin\theta + P_y \cos\theta.$

$= 10 \cdot \cos 30° + 20 \cdot \sin 30°$

$= \cancel{10}^{5} \cdot \sqrt{3}/\cancel{2} + \cancel{20}^{10} \cdot \frac{1}{\cancel{2}} = 5\sqrt{3} + 10$

$= 5(\sqrt{3} + 2) = 18.66.$

$\therefore$ new coordinates : $(-1.34, 18.66)$.

$\therefore P_x' = 10 \cos 30° - 10 \sin 30°$

$= \cancel{10}^{5} \times \sqrt{3}/\cancel{2} - {}^{5}\cancel{10} \times \frac{1}{\cancel{2}}$

$= 5\sqrt{3} - 5 = 5(\sqrt{3} - 1) = 3.66.$

$\therefore P_y' = 10 \sin 30° + 10 \cos 30°.$

$= \cancel{10}^{5} \times \frac{1}{\cancel{2}} + \cancel{10}^{5} \times \sqrt{3}/\cancel{2}$

$= 5 + 5\sqrt{3} = 5(1 + \sqrt{3}) = 13.66.$

$\therefore$ new coordinates : $(3.66, 13.66)$.

$$P_x' = 20 \cdot \cos 30^\circ - 10 \sin 30^\circ.$$
$$= \cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}} - \cancel{10}^{5} \times \frac{1}{\cancel{2}}$$
$$= 10\sqrt{3} - 5 = 5(2\sqrt{3}-1) = 12.32.$$

$$P_y' = 20 \sin 30^\circ + 20 \cos 30^\circ.$$
$$= \cancel{20}^{10} \times \frac{1}{\cancel{2}} + \cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}}.$$
$$= 10 + 10\sqrt{3} = 10(1+\sqrt{3}) = 27.32.$$

$\therefore$ new coordinates : $(12.32, 27.32)$

* Reflection —

(a) It is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180° about the reflection axis

(b) $\left.\begin{array}{l} x' = x. \\ y' = -y. \end{array}\right\}$ Reflection on x-axis | $\left.\begin{array}{l} x' = -x \\ y' = y \end{array}\right\}$ Reflection on y-axis

(c) Matrix form :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$ } Reflection on x-axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ } Reflection on y-axis

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Q. Given a triangle with coordinate points A(3,4), B(6,4), C(5,6). Apply the reflection on x-axis and obtain the new coordinates of the object.

→ old coordinates : A(3,4), B(6,4), C(5,6).
Reflection has to be taken on x-axis.
for coordinate A(3,4) ——
$x' = x = 3$
$y' = -y = -4$
$\therefore (x', y') = (3, -4)$.

For coordinate B(6,4) ——
$x' = x = 6$
$y' = y = -4$
$\therefore (x', y') = (6, -4)$.

For coordinate C(5,6) ——
$x' = x = 5$
$y' = -y = 6$
$\therefore (x', y') = (5, -6)$.

$\therefore$ New coordinates are : A(3,−4), B(6,−4) and C(5,−6).

* Shearing ——

(a) It is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused ~~of~~ to slide over each other.

(b) Two types of shearing :
- Shearing in x-axis

$x' = x + sh_x \cdot y$.
$y' = y$

Matrix form :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

- Shearing in y-axis

$x' = x.$

$y' = y + sh_y \cdot x.$

Matrix form :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Q. Given a triangle with points (1,1), (0,0) and (1,0). Apply shear parameter 2 on x-axis and 2 on y-axis and find out the new coordinates of the object

→ old coordinates : A(1,1), B(0,0), C(1,0).

shearing in x-axis :—

For coordinate A(1,1) —

$x' = x + sh_x \cdot y.$

$= 1 + 2 \cdot 1 = 3.$

$y' = y = 1.$ ∴ A'(3,1).

For coordinates B(0,0) —

$x' = x + sh_x \cdot y.$

$= 0 + 2.0 = 0.$

$y' = y = 0$ ∴ B'(0,0).

For coordinate C(1,0) —

$x = x + sh_x \cdot y$

$= 1 + 2.0 = 1$ ∴ C'(1,0).

$y' = y = 0$

New coordinates after shearing on x-axis:
A(3,1), B(0,0), C(1,0).

Shearing on y-axis:—

~~x' = x =~~ For coordinate A(1,1) —

$x' = x = 1$

$y' = y + sh_y \cdot x$

$= 1 + 2 \cdot 1 = 3.$

$\therefore$ ~~x~~ $A'(1,3)$.

For coordinate B(0,0) —

$x' = x = 0.$

$y' = y + sh_y \cdot x.$

$= 0 + 2.0 = 0$. $\quad \therefore B'(0,0)$.

For coordinate C(1,0) —

$x' = x = 1.$

$y' = y + sh_y \cdot x.$

$= 0 + 2.1 = 2.$ $\quad \therefore C'(1,2)$.

New coordinate after shearing on y-axis:
A(1,3), B(0,0), C(1,2).

→ 2D Viewing :—

MC : Modeling coordinates
↓ Apply model transformations
WC : World coordinates
↓ Determine visible parts.
VC : Viewing coordinates
↓ To standard coordinates
NC : Normalised coordinates
↓ clip and determine pixels
DC : Device coordinates

~~* OpenGL 2D viewing — glMatrixMode(GL_PROJECTION);~~

* OpenGL 2D viewing —

step 1 : glMatrixMode (GL_PROJECTION);

step 2 : specify the 2D clipping window
glOrtho2D (xwmin, xwmax, ywmin, ywmax);

step 3 : specify the viewport
glViewport(xwmin, ywmin, vpWidth, vpHeight);

In short —

```
glMatrixMode (GL_PROJECTION);
glOrtho2D(xwmin, xwmax, ywmin, ywmax);
glViewport (xvmin, xvmax, vpwidth, vpHeight);
```

→ Coordinate systems :—

* Screen coordinate — the coordinate system used to address the screen (device coordinates)

* World coordinate — A user-defined application specific coordinate system having its own units of measure, axis, origin, etc.

* Window — the rectangular region of the world that is visible

* Viewport — the rectangular region of the screen space that is used to display the window.

MODULE — 3.

→ **3D Transformations :—**

* Translation — A point can be translated in 3D by adding translation coordinate (t_x, t_y, t_z) to the original coordinate (x, y, z) to get new coordinate (x', y', z').

(a) Matrix Form :

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

(b)
$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z.$$

Q: Given a 3D object with coordinate points A(0,3,1), B(3,3,2), C(3,0,0), D(0,0,0). Apply the translation with distance 1 towards x-axis, 1 towards y-axis, 2 towards z-axis and obtain new coordinates of the object

→ Old coordinates : A(0,3,1), B(3,3,2), C(3,0,0), D(0,0,0).

- $(t_x, t_y, t_z) = (1, 1, 2)$.

For coordinate A(0,3,1) —

$x' = x + t_x$ $= 0 + 1 = 1.$

$y' = y + t_y$ $= 3 + 1 = 4$

$z' = z + t_z$ $= 1 + 2 = 3.$

∴ A'(1,4,3).

For coordinate B(3,3,2) —

$x' = x + t_x$ $= 3 + 1 = 4$

$y' = y + t_y$ $= 3 + 1 = 4$

$z' = z + t_z$ $= 2 + 2 = 4.$

∴ B'(4,4,4)

For coordinate $C(3,0,0)$ —

$x' = x + t_x$
$= 3+1 = 4$

$y' = y + t_y$
$= 0+1 = 1$

$z' = z + t_z$
$= 0+2 = 2$

$\therefore C'(4,1,2)$.

For coordinate $D(0,0,0)$ —

$x' = x + t_x$
$= 0+1 = 1$

$y' = y + t_x$
$= 0+1 = 1$

$z' = z + t_z$
$= 0+2 = 2$.

$\therefore D'(1,1,2)$.

Hence, new coordinates are : $A(1,4,3)$, $B(4,4,4)$, $C(4,1,2)$, $D(1,1,2)$.

* Rotation —

(a) It is a process of rotation of an object w.r.t to an angle in a 3D plane.

(b) For x-axis rotation —

$x' = x$

$y' = y\cos\theta - z\sin\theta$.

$z' = y\sin\theta + z\cos\theta$.

Matrix ~~Mode~~ Form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & \\ & & & \\ & & & \end{bmatrix}$$

Matrix form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(c) For y-axis rotation —

$x' = z\sin\theta + x\cos\theta$

$y' = y$

$z' = y\cos\theta - x\sin\theta$.

Matrix form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(d) For z-axis rotation —

$x' = x\cos\theta - y\sin\theta$.

$y' = x\sin\theta - y\cos\theta$.

$z' = z$

Matrix form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Q: Given a homogeneous point (1,2,3). Apply rotation 90° towards x, y and z axes and find out the new coordinates points.

→ old coordinates : (1,2,3)

$\theta = 90°$.

For x-axis rotation —

$x' = x = 1$.

$y' = y\cos\theta - z\sin\theta$.

$= 2.\cos 90° - 3.\sin\theta$.

$= 2.0 - 3.1 = -3$.

$z' = y\sin\theta + z\cos\theta$

$= 2\sin 90^\circ + 3\cos 90^\circ.$

$= 2.1 + 3.0 = 2.$

new coordinates after x-rotation = $(1, -3, 2)$.

For y-axis rotation —

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

$$= \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1+0+0+0 \\ 0+2+0+0 \\ 0+0+3+0 \\ 0+0+0+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

* Scaling —

$x' = x \cdot S_x$.

$y' = y \cdot S_y$

$z' = z \cdot S_z$.

Matrix form:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

Q:- Given a 3D object with coordinate points A(0,3,3), B(3,3,6), C(3,0,1), D(0,0,0). Apply the scaling parameters 2 towards x-axis, 3 towards y-axis and 3 towards z-axis and obtain the new coordinate of the object.

→ old coordinates : A(0,3,3), B(3,3,6), C(3,0,1), D(0,0,0).

$S_x = 2$; $S_y = 3$; $S_z = 3$.

For coordinates A(0,3,3) —

$x' = x \cdot S_x = 0 \times 2 = 0$.

$y' = y \cdot S_y = 3 \times 3 = 9$.

$z' = z \cdot S_z = 3 \times 3 = 9$.

new coordinates : A'(0, 9, 9).

For coordinates B(3,3,6) —

$x' = x \cdot S_x = 2 \times 3 = 6$.

$y' = y \cdot S_y = 3 \times 3 = 9$.

$z' = z \cdot S_z = 6 \times 3 = 18$.

new coordinates : B'(6, 9, 18).

For coordinates $C(3,0,1)$ —

$x' = x \cdot S_x = 3 \times 2 = 6$

$y' = y \cdot S_y = 0 \times 3 = 0$

$z' = z \cdot S_z = 1 \times 3 = 3$

new coordinates : $C'(6,0,3)$.

For coordinates $D(0,0,0)$ —

$x' = x \cdot S_x = 0 \times 2 = 0$.

$y' = y \cdot S_y = 0 \times 3 = 0$.

$z' = z \cdot S_z = 0 \times 3 = 0$.

new coordinates : $D'(0,0,0)$.

Hence new coordinates after scaling are :
$A(0,9,9)$, $B(6,9,18)$, $C(6,0,3)$, $D(0,0,0)$.

* Reflection —

(a) Reflection Relative to XY Plane :

$x' = x$

$y' = y$

$z' = -z$.

Matrix Form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(b) Reflection Relative to YZ Plane :

$x' = -x$

$y' = y$

$z' = z$.

Matrix Form :

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

(C) Reflection Relative to XZ Plane:

$x' = x$

$y' = -y$

$z' = z.$

Matrix Form!

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

Q: Given 3D triangle with coordinate points A(3,4,1), B(6,4,2), C(5,6,3). Apply the reflection on the XY plane and ~~font~~ find out the new coordinates of the object.

→ old coordinates : A(3,4,1), B(6,4,2), C(5,6,3).

Reflection Relative to XY Plane :

For coordinate A(3,4,1) —

$x' = x = 3$

$y' = y = 4$

$z' = -z = -1$

∴ A'(3,4,−1).

For coordinate B(6,4,2) —

$x' = x = 6$

$y' = y = 4$

$z' = -z = -2$

∴ B'(6,4,−2)

For coordinate $C(5,6,3)$ —

$x' = x = 5$

$y' = y = 6$

$z' = -z = -3.$

$\therefore C'(5,6,3).$

Hence, new coordinates after reflection are:

$A(3,4,-1)$, $B(6,4,-2)$ and $C(5,6,-3)$.

* Shearing —

ⓐ Shearing in x-axis:

$x' = x$

$y' = y + sh_y \cdot x$

$z' = z + sh_z \cdot x$

Matrix form:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_y & 1 & 0 & 0 \\ sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ⓑ Shearing in y-axis:

$x' = x + sh_x \cdot y.$

$y' = y$

$z' = z' + sh_z \cdot y$

Matrix form:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$