ASSIGNMENT
01

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QUESTION NO (01)

A, B, C are position vectors

$4\hat{i} + 4j + k, \quad -4\hat{i} + 3j - 4k, \quad 4i - j - 2k$

(a)

find equation of plan ABC and answere in form of $ax + by + cz = d$

now A $(4, -4, 1)$
B $(-4, 3, -4)$
C $(4, -1, -2)$

$\vec{AB} = \vec{OB} - \vec{OA}$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$$\boxed{l = -8i + 7j - 5k}$$

$\vec{AC} = \vec{OC} - \vec{OA}$

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$$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$$

$$= 3j - 3k$$

$$n = \vec{AB} \times \vec{AC}$$

$$= \begin{bmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$$

$$= i(-4+15) - (24-0)j + (-24)k$$

$$= -6i - 24j - 24k$$

$$\boxed{= i + 4j + 4k}$$

$$d = a \cdot n = (4i - 4j + k)(i + 4j + 4k)$$

$$= 4 - 16 + 4 = -8$$

equation of plane

$$r \cdot n = d$$

$$r(i + 4j + 4k) = -8$$

$$(xi + 4j + zk) \cdot (i + 4j + 4k) = -8$$

$x + 4y + 4z = -8$

$$\boxed{x + 4y + 4z + 8 = 0}$$

(b)

$$\frac{d}{|n|} = \frac{+8}{\sqrt{1^2 + 4^2 + 4^2}}$$

$$= \frac{8}{3.3} = 1.39$$

(c)

Line OD : $r = a + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$$\Rightarrow 2\lambda + 12\lambda - 12\lambda = -8$$

$$2\lambda = -8$$

$$\boxed{\lambda = -4}$$

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QUESTION NO (03)

(a)

$L_1 \Rightarrow ti + j \qquad -2i - j$

$L_2 \Rightarrow j + tk \qquad -2j + k$

shortest dist b/w L_2 and L_1 is $\sqrt{21}$

$r_1 \Rightarrow OA + \lambda AB$

$r_2 \Rightarrow OA + \lambda AB$

$r_1 = ti + j \quad + \lambda 2i - j$

$$L_1 = r_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -10 \\ 0 \end{bmatrix}$$

$$L_2 \Rightarrow \vec{r_2} = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{(b_1 \times b_2)}$$

$$= \frac{(-i+2j+4k)\cdot(ti+tk)}{\sqrt{21}}$$

$$\sqrt{21} = t+4 \;/\sqrt{21}$$

$$21 = 5t$$

$$\boxed{t = 21/5}$$

(b)

$$r_1 = \frac{21}{5}i + j + \lambda(2i-j)$$

$$r_2 = j - \frac{21}{5}\hat{k} + u(-2j+k)$$

$$\pi_1 = r = OA + \lambda AB + uAC$$

$$\vec{r} = -\frac{21}{5}i + j + \lambda(-2i-j) + u(-2j+k)$$

(c)

$$d_2 = 5x - 6y + 5z = 0$$

$$d_2 = \frac{x-0}{0} \qquad d_2 = \frac{y-1}{2}$$

$$d_2 = \frac{2-4.2}{1}$$

from L_2 direction vector is

$= \{0, -2, 1\}$

from π_2 normal vector is

$(5, -6, 7)$

$\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{|a||b|}$

$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ 21/5 \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$= -6 + 29.4 = 23.4$

$|a| = \sqrt{1^2 + \left(\frac{21}{5}\right)^2} = 23.4$

$|b| = \sqrt{5^2 + 6^2 + 7^2}$

$\boxed{b = 0.49}$

$\theta = \cos^{-1}\left(\dfrac{23.4}{4.3 \times 10.49}\right)$

$\boxed{\theta = 59.34°}$

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(d)

$\bar{n}_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}$ $\bar{n}_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$\bar{n}_1 \cdot \bar{n}_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$= -21 - 6$

$= -27$

$|a| = \sqrt{(-21/5)^2 + 1^2} = 4.3$

$|b| = \sqrt{5^2 + (6^2) + 7^2} = 10.49$

$\theta = \cos^{-1}\left(\frac{-27}{4.3 \times 10.49}\right)$ $\boxed{\theta = 126.78}$

$\theta = 180 - 126.78$

$\boxed{= 53.23^\circ}$ acute angle.

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QUESTION NO (05)

(a)

Mid point

$$\left(\frac{-2-6}{2}\right)\left(\frac{-1-3}{2}\right) = \frac{-8}{2}, \frac{-4}{2}$$

$C(-4, -2)$

eq of circle $= (x+h)^2 + (y-k)^2 = r^2$ — 1

$(x+4)^2 + (y+2)^2 = r^2$

$(-2+4)^2 + (1+2)^2 = r^2$

$4+1 = r^2$ $\boxed{r^2 = 5}$

$(x+4)^2 + (y+2)^2 = 5$

(b)

eq of circle

$(x-h)^2 + (y-k)^2 = r^2$

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$x = 0$, $y = b$ now at $(4, 0)$

$(4)^2 + (-b)^2 = r^2$

$16 + b^2 = r^2$ — (1)

at $(0, 2)$

$0^2 + (2-b)^2 = r^2$

$(2-b)^2 = r^2$ — (2)

now (1) & (11)

$16 + b^2 = (2-b)^2$

$16 + \cancel{b^2} - \cancel{b^2} - 4 + 4b = 0$

$\boxed{b = -3}$

now put $b = -3$ in (1)

$r^2 = 4^2 + (-3)^2$

$\boxed{r = \pm 5}$

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Part d

$x^2 = 24y$

now as we know

$x^2 = 4ay$

$4a = 24$

$a = 8$

so focus is $f(a, 0) = F(0, 0)$

and eq of direction is

$\boxed{x = -a}$ $\boxed{x = -8}$

(e)

elipse $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

compare with eq of elipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^4}$

$a = 5 \quad b = 4$

$c = \sqrt{a^2 - b^2} = \sqrt{25-16}$

$\boxed{= \pm 3}$

$F_1 = (3, 0) \quad F_2 = (-3, 0)$

length of major axis $= 2a$

$= 2(5) = 10$

(f)

major axis $= 10$

minor axis $= 8$

$2a = 10$

$a = 5$

$2b = 8$

$b = 4$

equation of ellipse along x-axis

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

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QUESTION 02

$A = (7i + 4j - k)$ $\quad B(11k + 3j) + c(i + 6j + 3k)$

$D = 2i + 7j + \lambda k$

$AB = 4i - 1j + k$

$CD = 0i - j + (3 - \lambda)k$

$L_1 =$ line $AB = OA + \lambda B$

$L_2 =$ line $OC + CD$

$$L_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 2 \\ b \\ -3 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ -1 \\ 3\lambda \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{b_1 \times b_2}$$

$$D = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$\lambda = 49\lambda^2 + 1 - 24\lambda$$

(b)

AB × AD

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & -1 & 1 \\ 5 & 3 & 2 \end{vmatrix}$$

$= 15 - 13\hat{j} + 7k$

$= -5(2) - 13(7) + (7)(1)t$

$= -10 - 9 + 7t$

for plane

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & -1 & 1 \\ 4 & 3 & 3 \end{vmatrix}$$

$= -8\hat{i} - 15\hat{j} + 17k$

$$= -8(x-11) - 15(y-3) + 17(x-0)$$

$$\Rightarrow 8x + 15y - 17x - 133$$

(c)

$$\theta = \cos^{-1}\left(\frac{3.18}{\sqrt{25+169+49} \cdot \sqrt{64+225+289}}\right)$$

$$= \cos^{-1}\left(\frac{318}{15.58 \cdot 84.04}\right)$$

$$= \frac{318}{374.5}$$

$$\boxed{\theta = 31.89^\circ}$$