وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog Integrated Systems Design

Lecture 17
Analog Filters Overview

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This lecture is mainly based on: (1) Maxim Integrated App Note 733: A Filter Primer
(2) Berkeley EECS 247 Lecture Notes by H. K.

Why Filters?

- Extraction of desired signal from many (radio, cell phone, ADSL…..)
- Separating signal and noise
- Anti-aliasing or smoothing
- Phase equalization
- Amplifier bandwidth limitations

[H. K., Berkeley EECS 247, 2010]

Historical Perspective

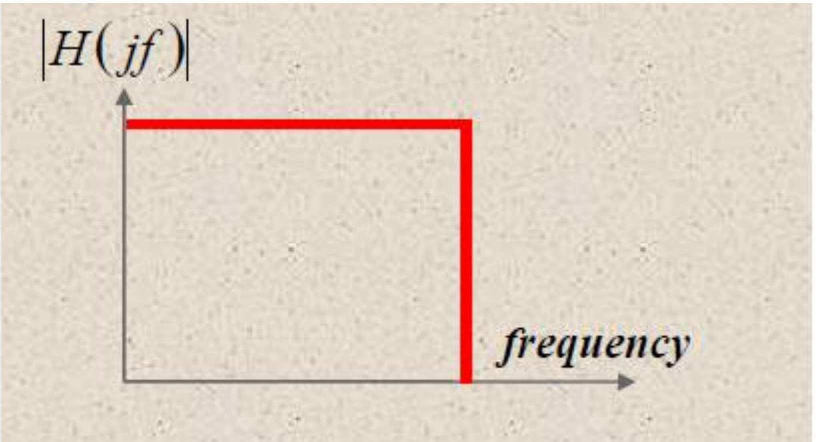
- Driven by the needs of the emerging telephone system in the early part of the twentieth century, the earliest analog filters utilized inductors (coils) and capacitors and were designed using ad hoc methods.
- It was not until the 1940s that a design theory for passive LC filters was developed.
  - This filter synthesis method, however, required extensive computation and had to await the emergence of the digital computer and its widespread use in the 1950s and the 1960s to become adopted, further developed, and utilized.
- The invention of the transistor and the IC resulted in new ways of assembling electronic circuits, which were, however, incompatible with the bulky and heavy inductors used in audio-frequency LC filters.
- The search for inductorless filters thus began and was greatly influenced by the emergence of the IC op amp in the late 1960s and its availability at low cost.

Historical Perspective

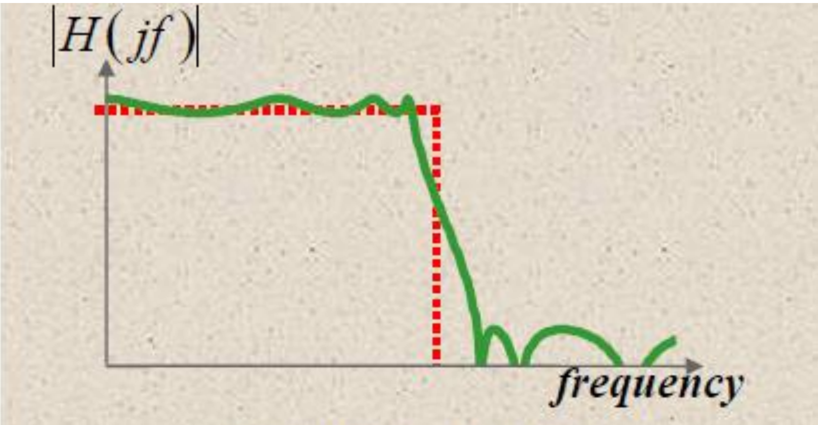
- Extensive research in the 1960s and 1970s resulted in a large repertoire of op amp–RC filters [See Chapter 17 in Sedra/Smith 7th ed.].
- No sooner had op amp–RC filters become a mature and reliable technology than the need arose for analog filters that could be fully integrated.
- Multiple approaches were proposed in the late 1970s, but the switched-capacitor circuit has become the technology of choice for low-frequency applications, mostly because of its compatibility with CMOS.
- For higher-frequency applications approaching the gigahertz range, needed for the burgeoning mobile communications market, two approaches have gained momentum:
  - Transconductance-C (Gm-C) filters
  - Active-LC filters
    - Uses actual physical inductors (in the nH range) fabricated on the IC chip.

Ideal vs Practical Filters (Ex: LPF)

- Ideal filter
  - Flat magnitude response in the passband
  - Brick-wall transition
  - Infinite level of rejection of out-of-band signals
  - What about time domain response?
- Practical filter
  - Ripple in passband magnitude response
  - Limited rejection of out-of-band signals



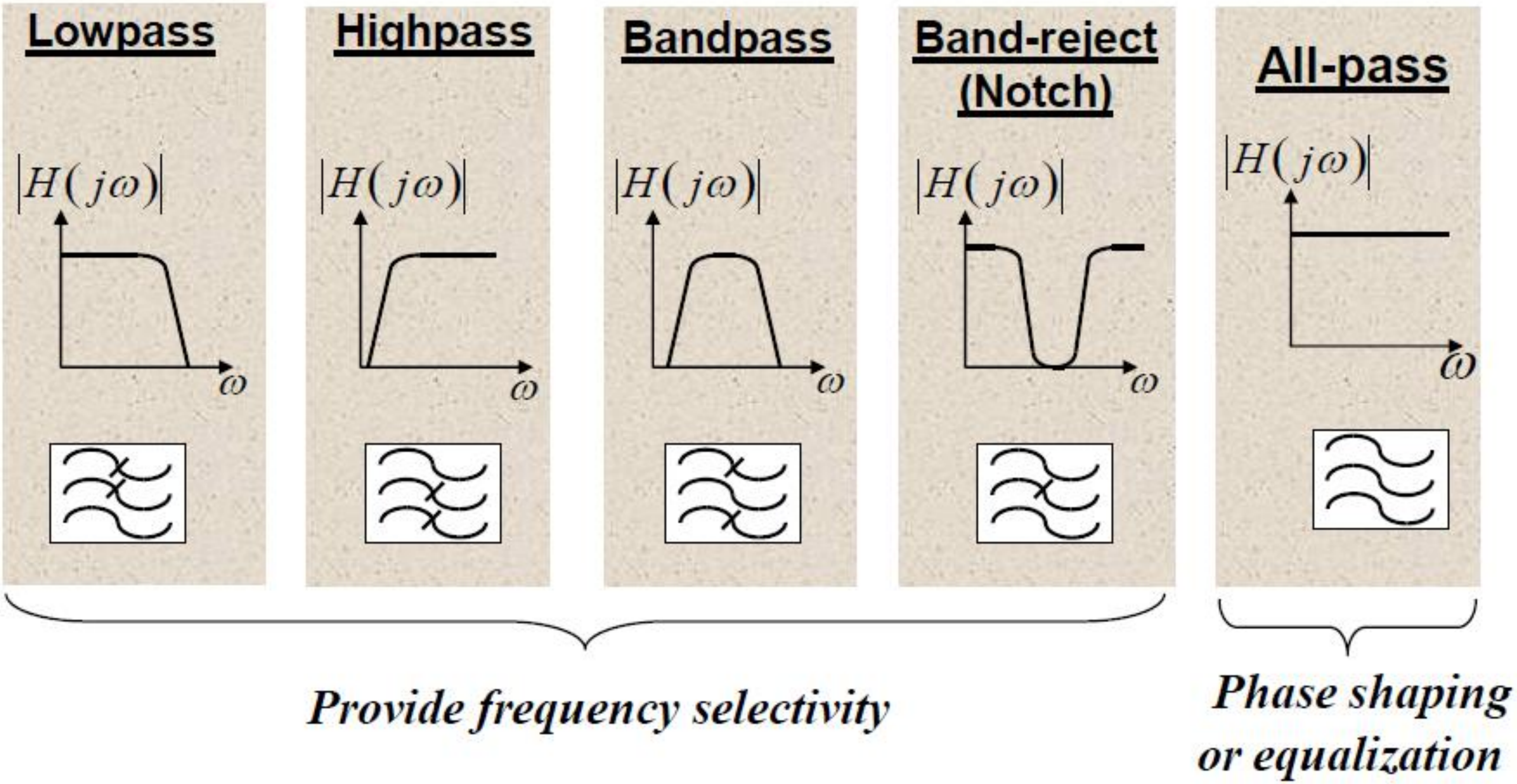
Ideal Lowpass Brick-Wall Filter



More Practical Filter

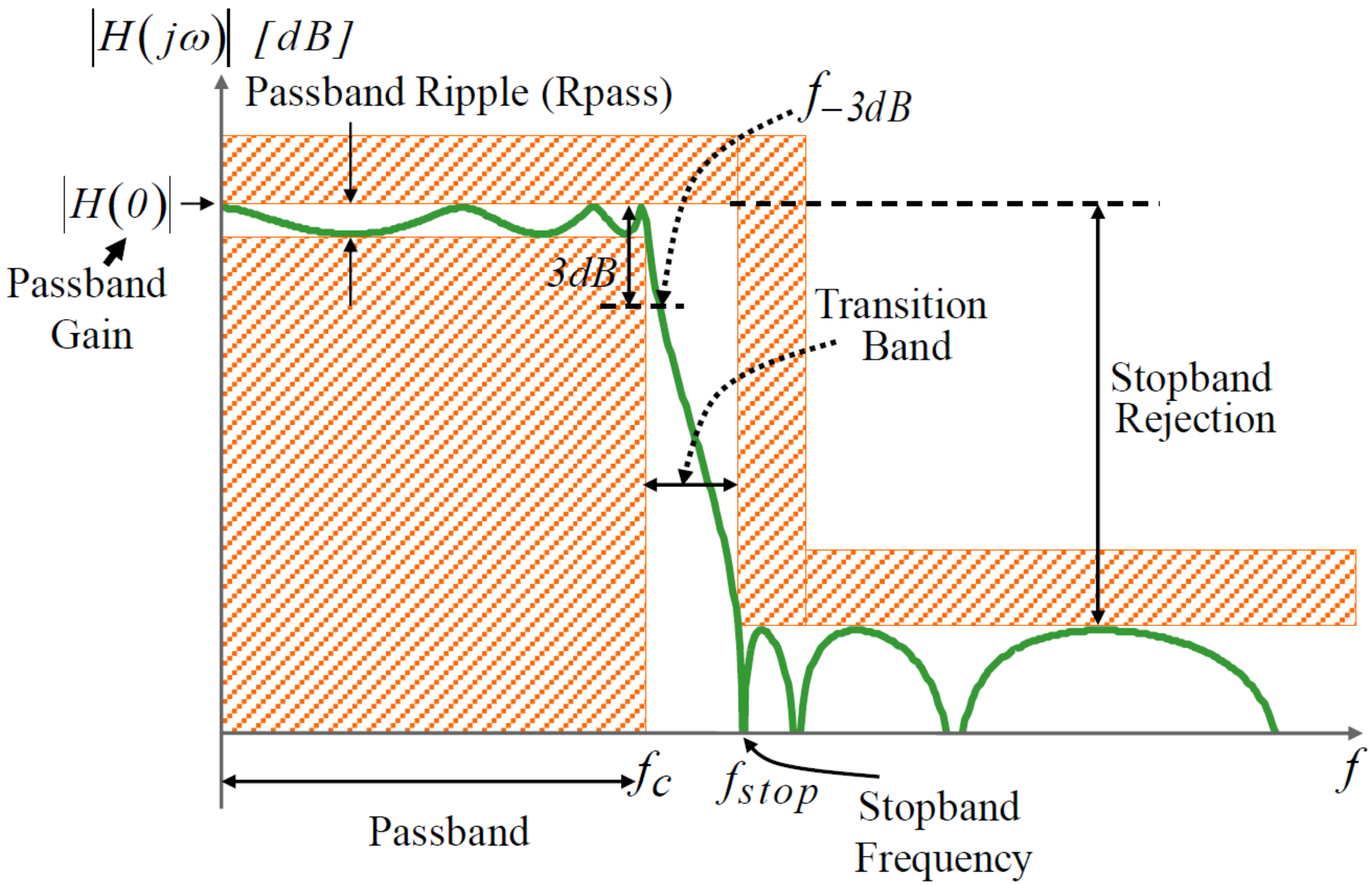
[H. K., Berkeley EECS 247, 2010]

Filter Types



[H. K., Berkeley EECS 247, 2010]

Magnitude Characteristics



 [H. K., Berkeley EECS 247, 2010]

Linear Phase Response

$$\omega t = \theta$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- For all signals to get the same delay we need

$$\Delta t = \frac{\Delta\theta}{\omega} = constant$$

- This can be achieved only if $\Delta\theta$ is a linear function of ω
  - $\Delta\theta$ is the phase shift introduced by the system (the phase response)
  - We need linear phase response to avoid phase distortion

Filter Specifications

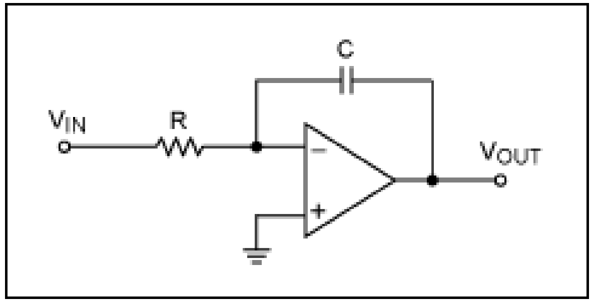
- Magnitude response vs frequency characteristics:
  - Passband ripple
  - Cutoff frequency or -3dB frequency
  - Passband gain
  - Stopband rejection
- Phase characteristics:
  - Group delay (linear phase response to avoid phase distortion)
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Area/pole & Power/pole

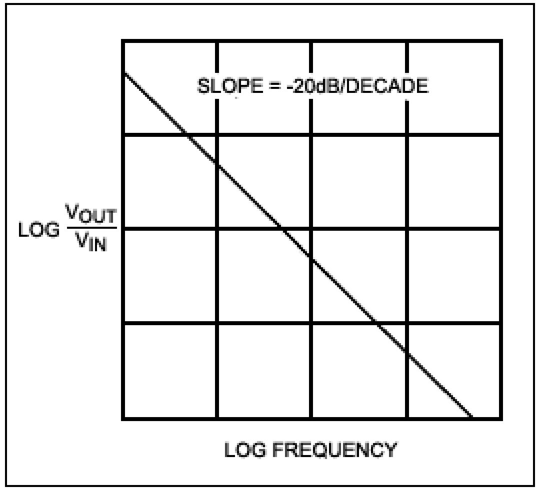
[H. K., Berkeley EECS 247, 2010]

First-Order Active (Lossless) Integrator

- Ideally, the response of an integrator at zero frequency is infinite
  - It has a pole at zero frequency
  - If you apply a DC signal at the input, the output is a linear ramp that grows in amplitude until limited by the power supplies
  - Integrator's gain diminishes with increasing frequency
    - There is a zero at s = ∞

$$\frac{V_{out}}{V_{in}} = -\frac{Z_C}{R} = -\frac{1}{sRC} = -\frac{\omega_o}{s}$$





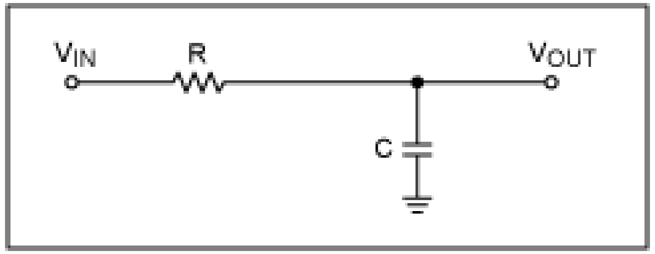
[Maxim Integrated App Note 733]

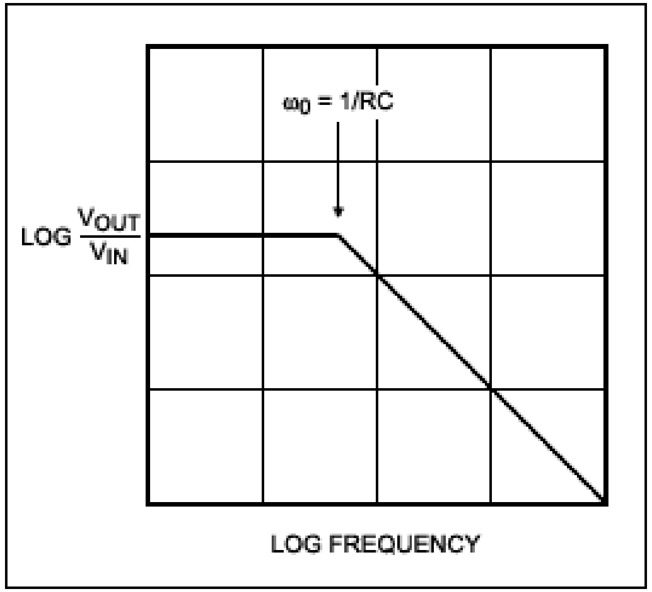
First-Order Passive RC Low-pass Filter

- Similar to first-order active integrator
- But the pole exists at $s = -\omega_o$ instead of DC
- A.k.a. practical (lossy) integrator

$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_C + R} = \frac{1}{1 + sRC} = \frac{\omega_o}{\omega_o + s}$$

- More stable (no infinite output for a real input)





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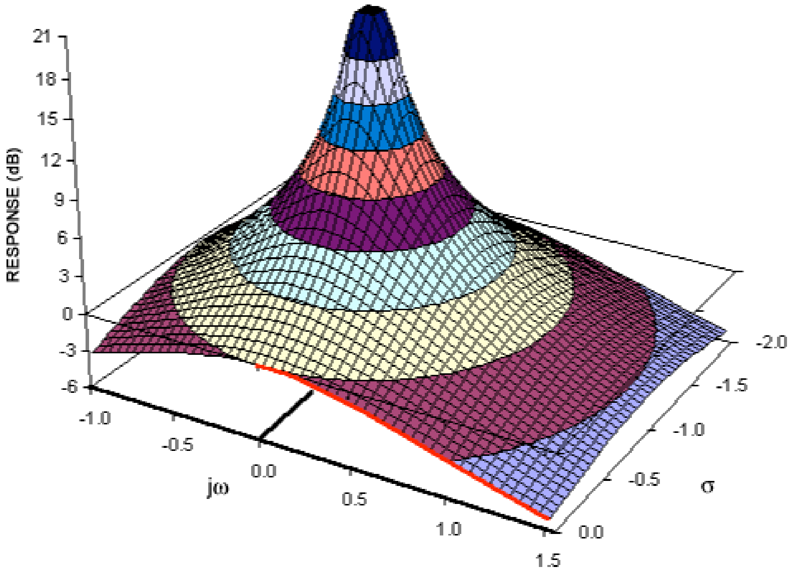
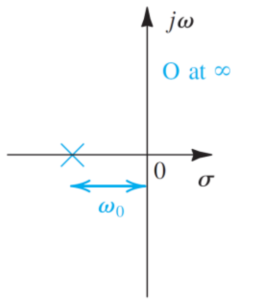
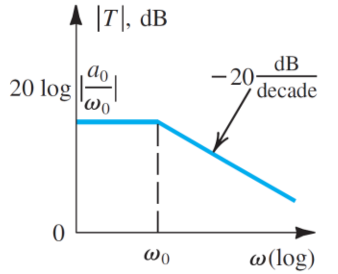
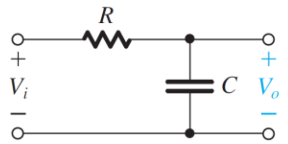
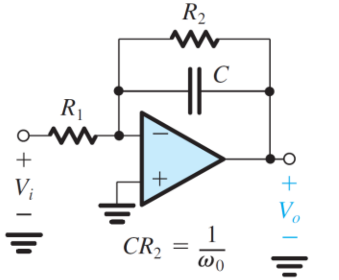
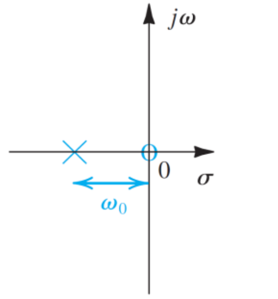
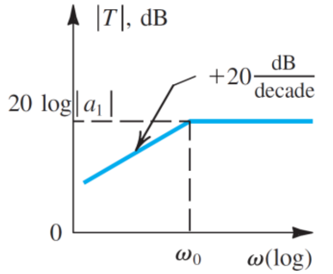
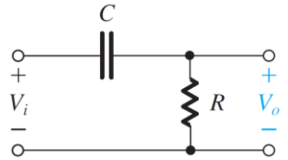
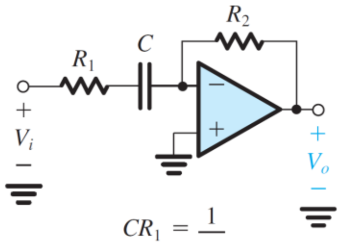
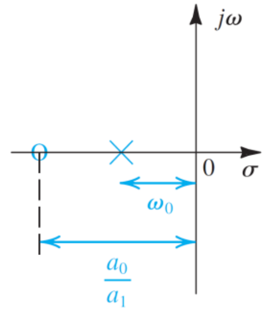
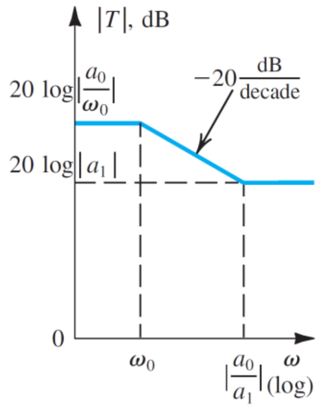
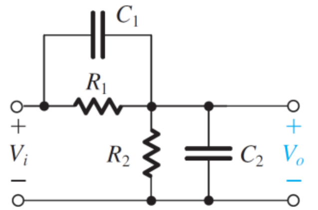
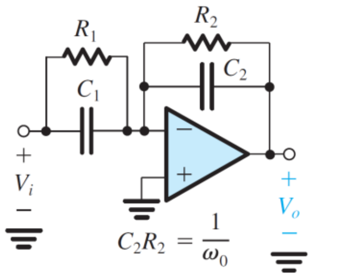


[Maxim Integrated App Note 733]

First-Order Filters

Filter Type and ***T*(*s*)**	***s*-Plane Singularities**	**Bode Plot for |*T*|**	**Passive Realization**	**Op Amp–RC Realization**
(a) Low pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 /\!/ R_2) = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	 $C_2R_2 = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

[Sedra/Smith, 2015]

Second-Order Filter

- A second-order filter has s^2 in the denominator and two poles in the complex plane
- General Form:

$$H(s) = \frac{K(s + z1)(s + z2)}{(s + p1)(s + p2)}$$

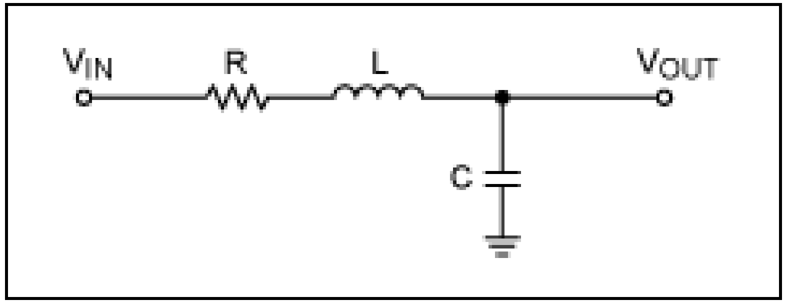
  - Also known as **biquad section**
  - Biquadratic means ratio of two quadratic functions
- Can be built by
  - Using inductance and capacitance in a passive circuit
    - No DC gain, difficult to integrate, coupled filter parameters, poor selectivity
  - An active circuit of resistors, capacitors, and amplifiers
    - The active circuitry replaces the inductor
- Filters constructed with Resistors and Capacitors **only** result in real poles
  - Don’t give interesting transfer functions

Second-Order Passive LC LPF

$$H(s) = \frac{Z_C}{R + Z_L + Z_C} = \frac{1}{LCs^2 + RCs + 1}$$

- Double zero at infinity
- Define: $\omega_o^2 = \frac{1}{LC}$ and $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$
  - ω_0 is the characteristic frequency and Q is the quality factor
  - Lower R means higher Q

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

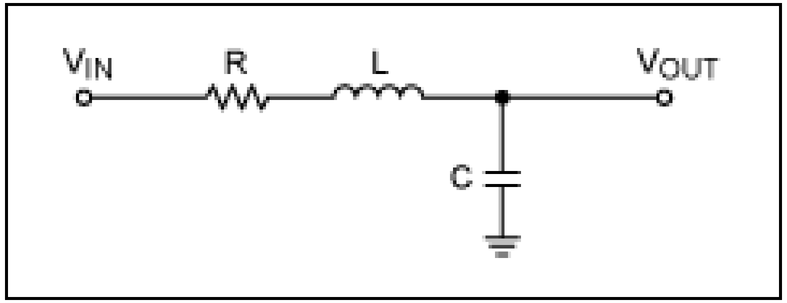


[Maxim Integrated App Note 733]

Second-Order Passive LC LPF

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

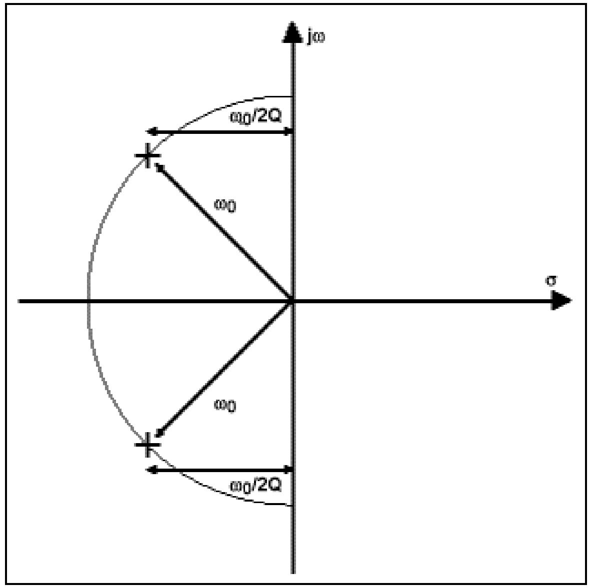
- $\omega_o^2 = \frac{1}{LC}$ and $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$
- The poles occur at $s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2 = 0$
- The term $(b^2 - 4ac)$ equals $\omega_0^2(1/Q^2 - 4)$
  - If $Q < 0.5$: roots are real and negative, simply like two first-order RC filters in cascade
    - This case is not very interesting
  - If $Q > 0.5$: $(b^2 - 4ac)$ is negative and the roots are complex



[Maxim Integrated App Note 733]

Second-Order Passive LC LPF

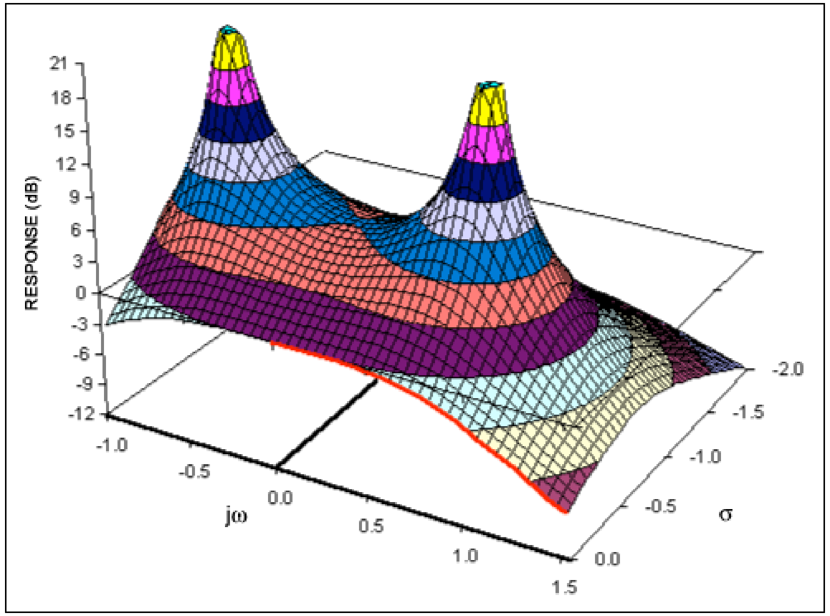
- Q > 0.5: complex-conjugate poles
  - The real part of both roots is $-b/2a = -\omega_0/2Q$
  - The imaginary parts will be equal and opposite in signs
  - Poles lie at a distance of ω_0 from the origin
  - Increasing the Q moves the poles in a semicircle away from each other and toward the $j\omega$ axis
  - When Q = 0.5, the poles meet at $-\omega_0$ on the negative-real axis

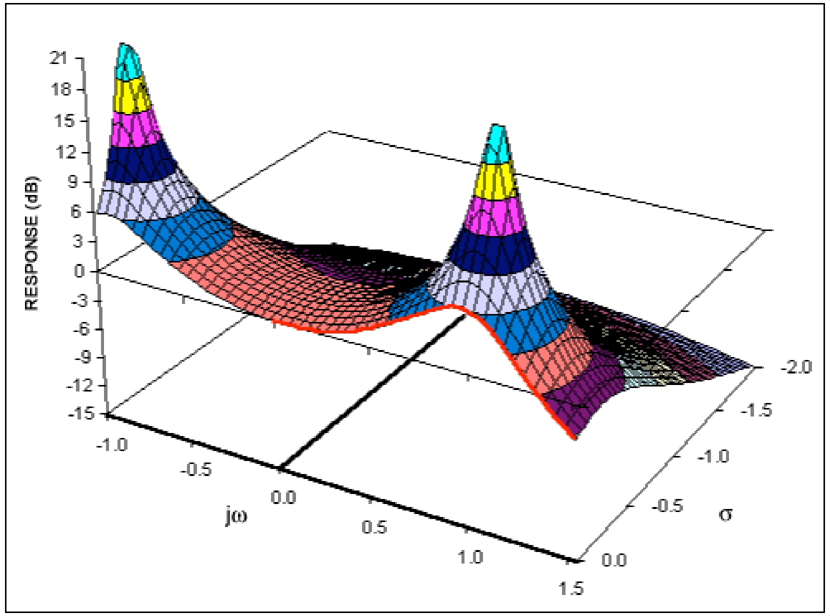


 [Maxim Integrated App Note 733]

Second-Order Passive LC LPF

- Ex: $Q = 0.707$: the response is a low-pass filter (maximally flat response)
- Ex: $Q = 2$: Peaking at the high end of the passband.
  - Transient response: ringing at the filter output
  - Lower values of Q result in less ringing (more damping)
- If Q becomes infinite ($R = 0$), the poles reach the $j\omega$ axis, causing an infinite frequency response (instability and continuous oscillation) at $s = \omega_0$





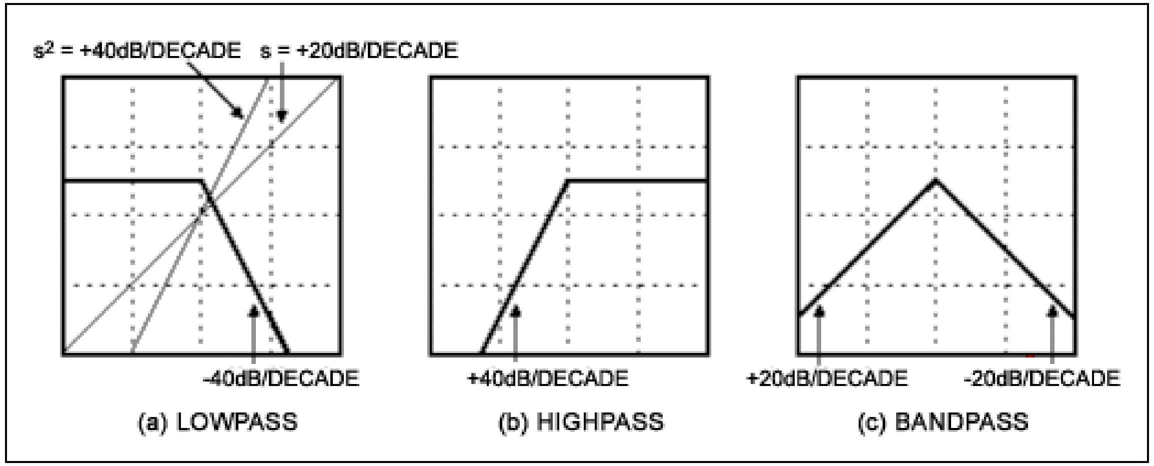
[Maxim Integrated App Note 733]

Second-Order Passive LC HPF

- To change a low-pass filter into a high-pass filter, turn the s plane inside out, making low frequencies high and high frequencies low
- Apply the transformation $s = \omega_0^2/s$, so that $s = \infty$ when $\omega_0^2/s = 0$, and vice versa.
  - At ω_0 the old and new values of s are identical.

$$H(s) = \frac{\omega_o^2}{\frac{\omega_o^4}{s^2} + \frac{\omega_o^3}{Qs} + \omega_o^2} = \frac{s^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

- Same as LPF, except that the numerator is s^2 instead of ω_0^2

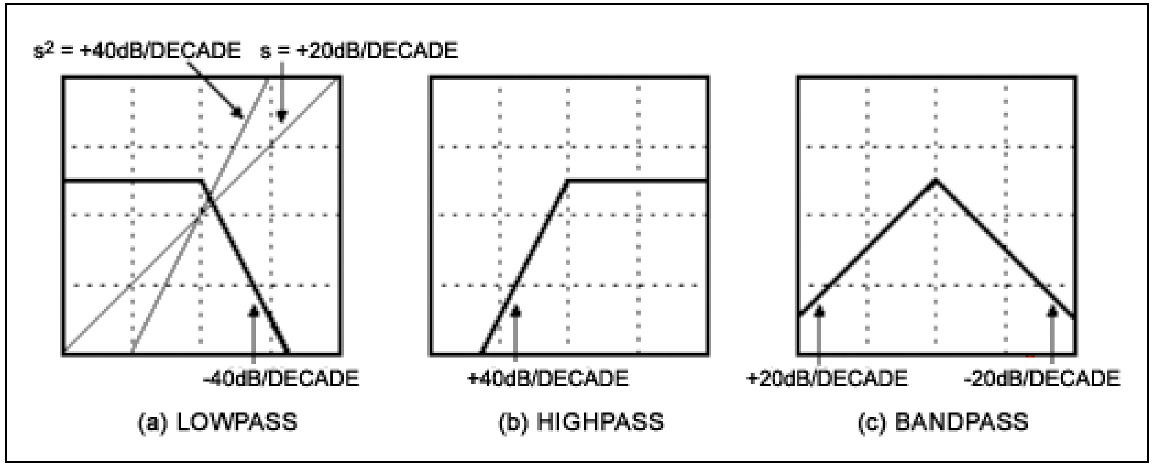


[Maxim Integrated App Note 733]

Second-Order Passive LC HPF

$$H(s) = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

- We can transform a low-pass function into a high-pass one by changing the numerator and leaving the denominator alone → adding zeros to the transfer function
- Multiplying by $\left(\frac{s}{\omega_o}\right)^2$ adds a +40dB/decade slope to this function
- The additional slope provides a low-frequency roll-off below the cutoff frequency; above cutoff it gives a flat response by canceling the original -40dB/decade slope



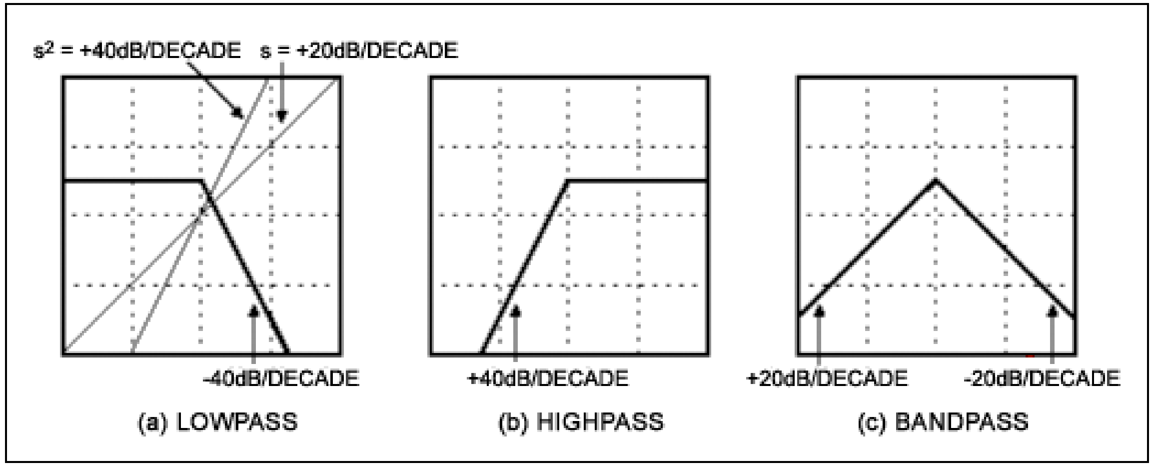
[Maxim Integrated App Note 733]

Second-Order Passive LC BPF

- We can use the same idea to generate a band-pass filter
  - Multiply the low-pass responses by s
  - Adds a +20dB/decade slope
  - The net response is then +20dB/decade below the cutoff and -20dB/decade above

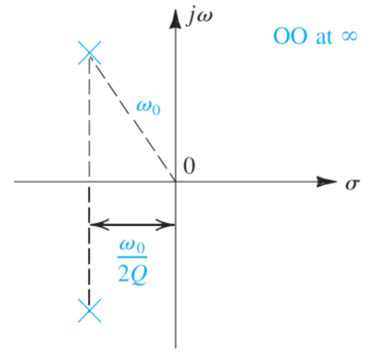
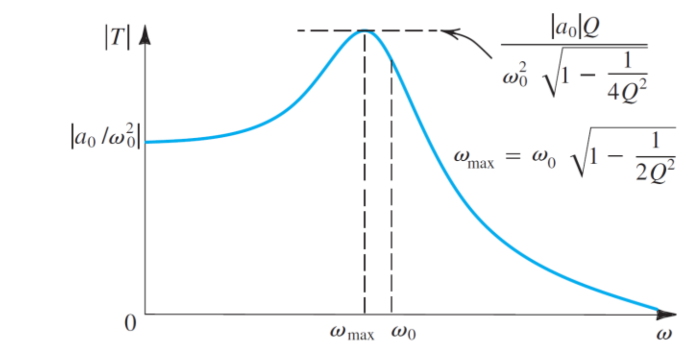
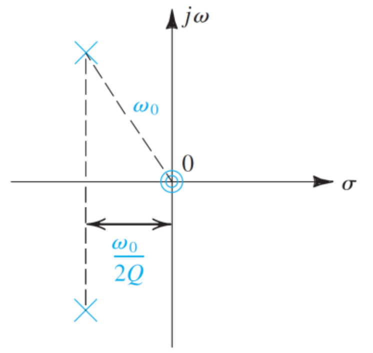
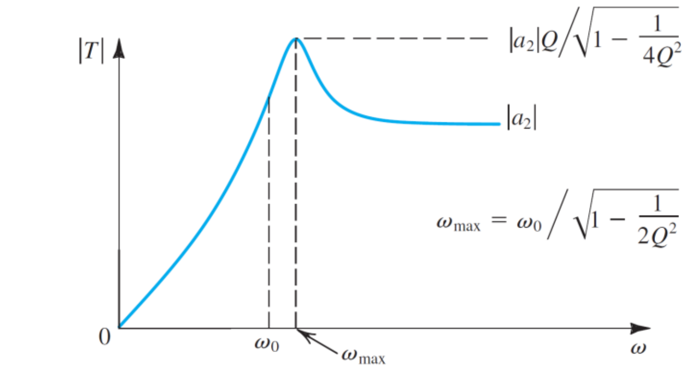
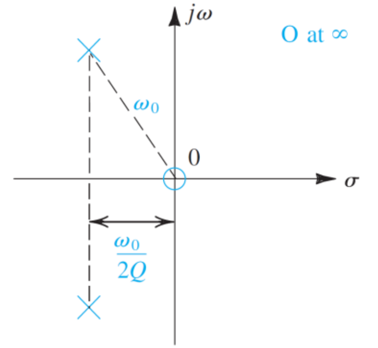
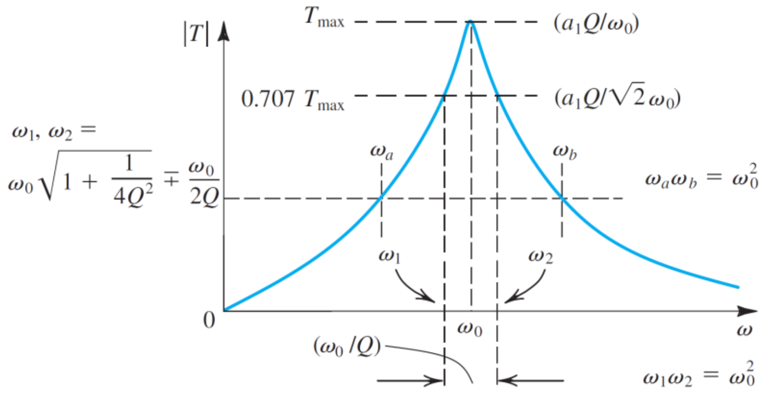
$$H(s) = \frac{\omega_o s}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2}$$

- The rate of cutoff in a second-order band-pass filter is half that of the other types
  - The available 40dB/decade slope is shared



[Maxim Integrated App Note 733]

Second-Order Filters

Filter Type and $T(s)$	s-Plane Singularities	$\|T\|$
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

[Sedra/Smith, 2015]

Higher-Order Filters

- A polynomial in s of any length can be factored into a series of quadratic terms (and a single first-order term for odd polynomial)
- Example: A fifth-order low-pass filter

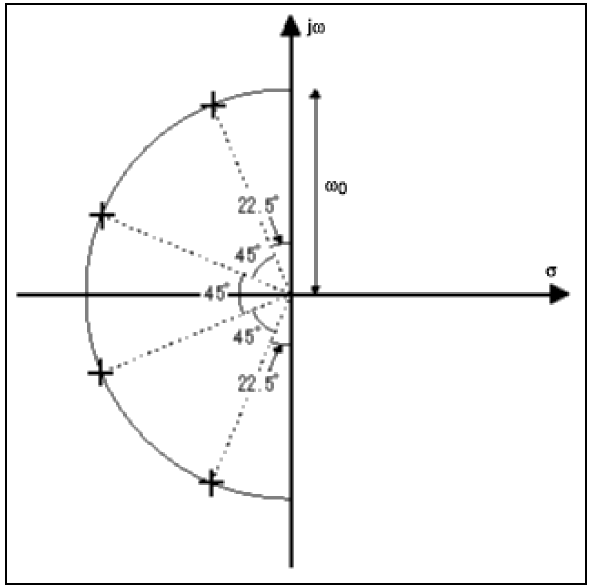
$$H(s) = \frac{1}{s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

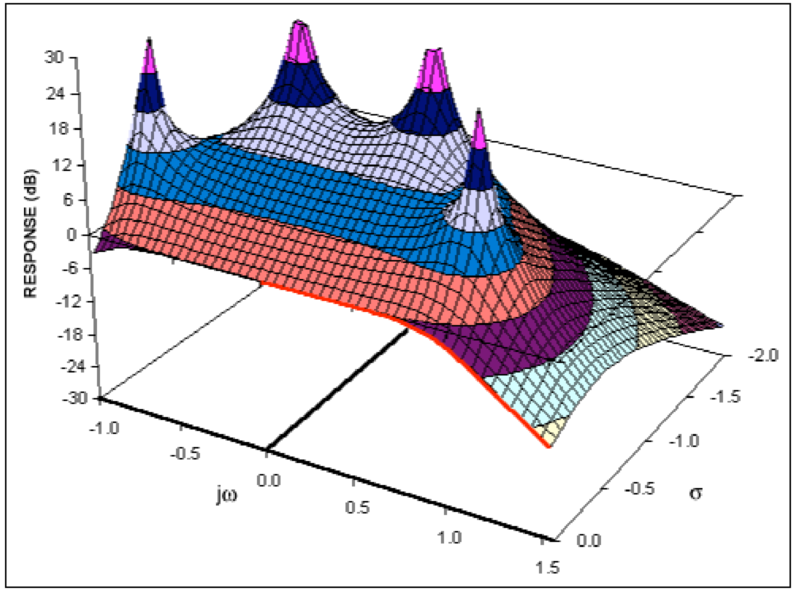
$$H(s) = \frac{1}{\left(s^2 + \frac{s\omega_1}{Q_1} + \omega_1{}^2\right)\left(s^2 + \frac{s\omega_2}{Q_2} + \omega_2{}^2\right)(s + \omega_3)}$$

  - Two second-order and one first-order sections in cascade
  - Each second-order term (**biquad**) contributes one complex-conjugate pole pair
  - The first-order term contributes a pole on the negative-real axis
- If the transfer function has a higher-order polynomial in the numerator, it can be factored as well
  - The second-order sections will be something other than low-pass sections

The Butterworth Filter

- Offers a response that is flat in the passband and cuts off as sharply as possible afterwards
  - Achieved by arranging the poles of a low-pass filter with equal spacing around a semicircular locus
  - The poles have different Q values, but all have the same ω_0
- Has moderate phase distortion
- HPF and BFP: Start by LPF and then apply transformations

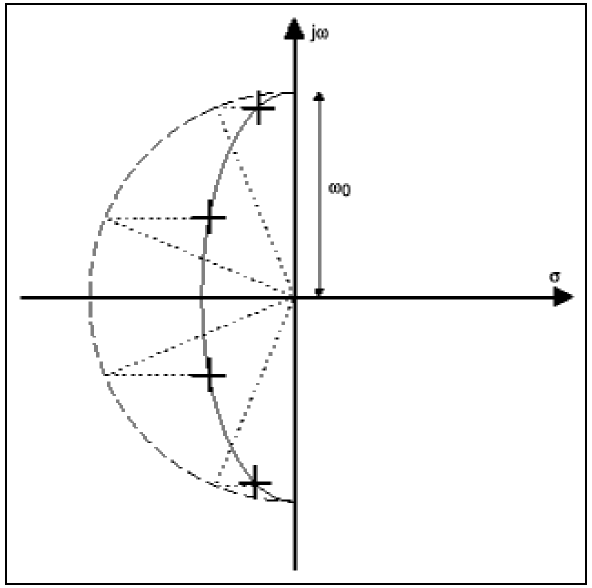


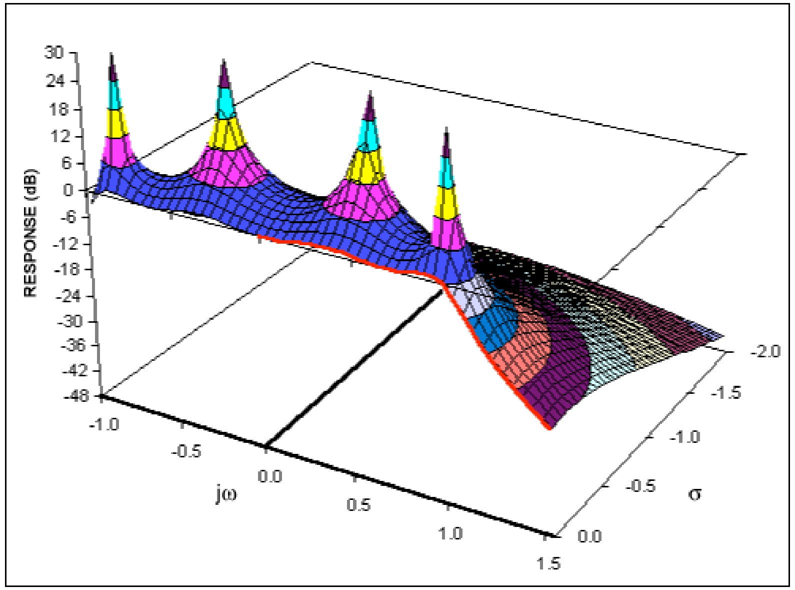


[Maxim Integrated App Note 733]

The Chebychev Filter

- By bringing poles closer to the jω axis (increasing their Qs), we can make the frequency cutoff steeper than that of a Butterworth
  - Sharper transition band
- The effects of each pole will be visible in the filter response, giving a variation in amplitude known as ripple in the passband
  - In a Chebychev filter poles are arranged to make variations equal (equiripple response)
- Has poor phase response

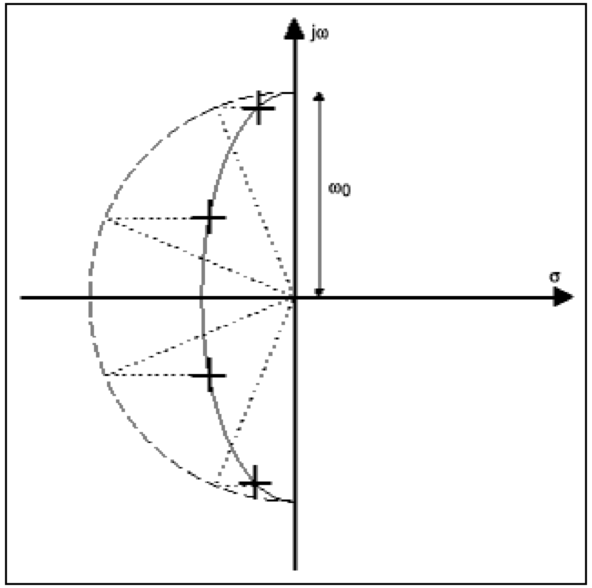


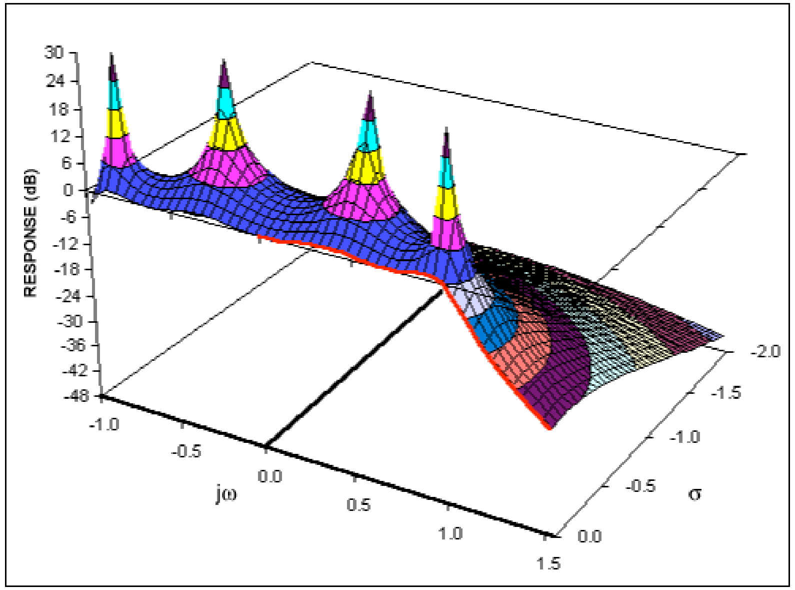


[Maxim Integrated App Note 733]

The Chebychev Filter

- Chebychev can be derived from a Butterworth by moving each pole closer to the jω axis in the same proportion
  - The poles lie on an ellipse
  - Each pole contributes one peak to the passband ripple
  - Moving the poles closer to the jω axis increases the passband ripple but provides a more abrupt cutoff in the stopband
    - Trade-off between passband ripple and stopband cutoff

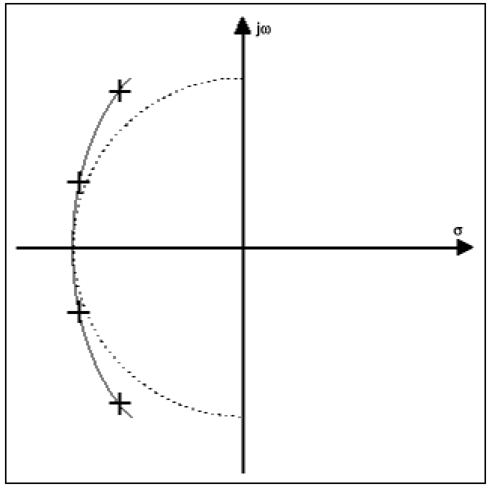




 [Maxim Integrated App Note 733]

The Bessel Filter

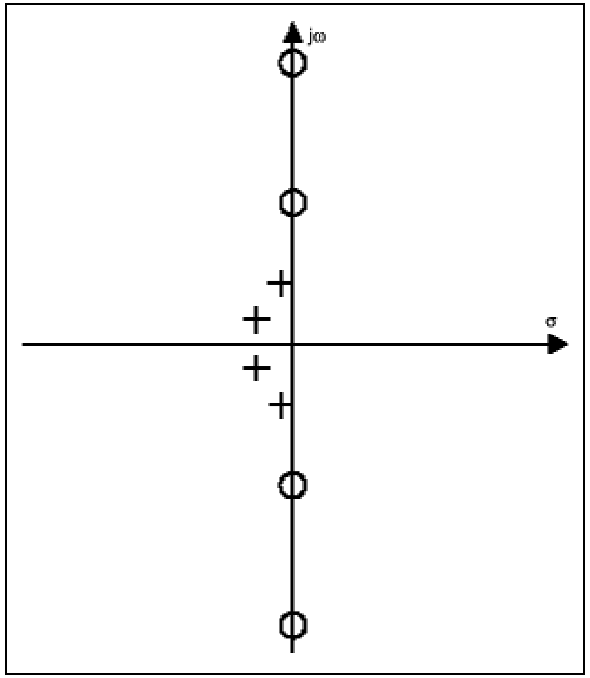
- Butterworth and Chebychev filters have sharp cutoffs
  - But poles close to the jω axis have high Q
  - The filter's transient response has overshoot and ringing
- The Bessel filter represents a trade-off in the opposite direction
  - Poles lie on a locus further from the jω axis
  - Transient response is improved, but at the expense of a less steep cutoff in the stopband
  - Has best phase response (maximally flat group delay)

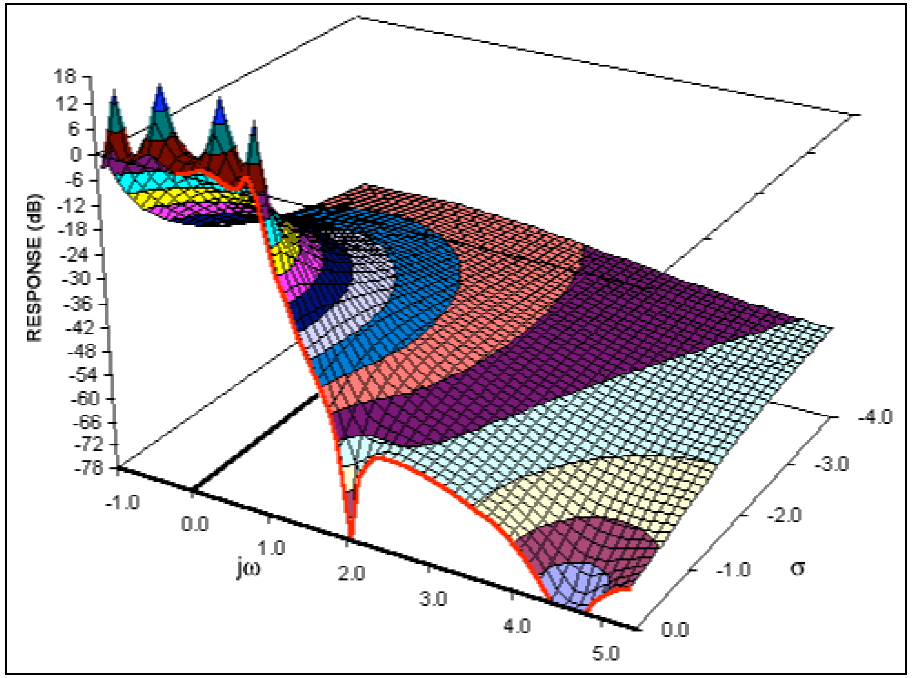


[Maxim Integrated App Note 733]

The Elliptic Filter

- By increasing the Q of poles nearest the passband edge, you can obtain a filter with sharper stopband cutoff than that of the Chebychev, without incurring more passband ripple
- Doing this alone would produce a gain peak
  - You can compensate for the peak by providing a zero at the bottom of the stopband

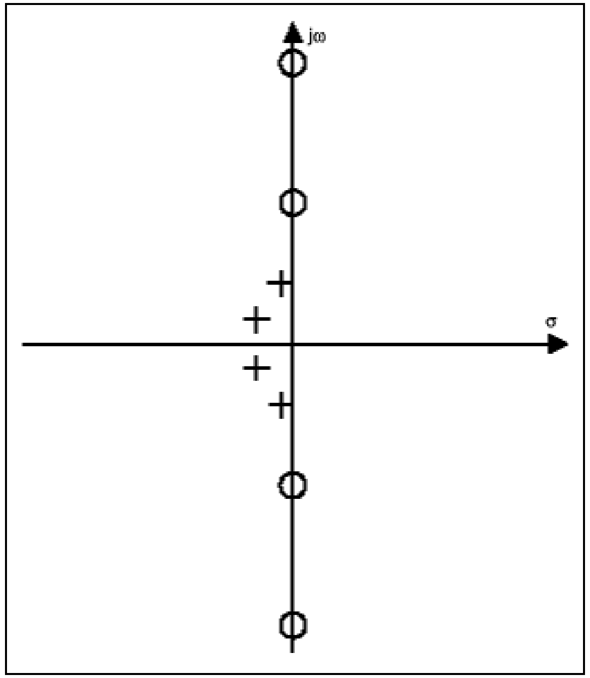


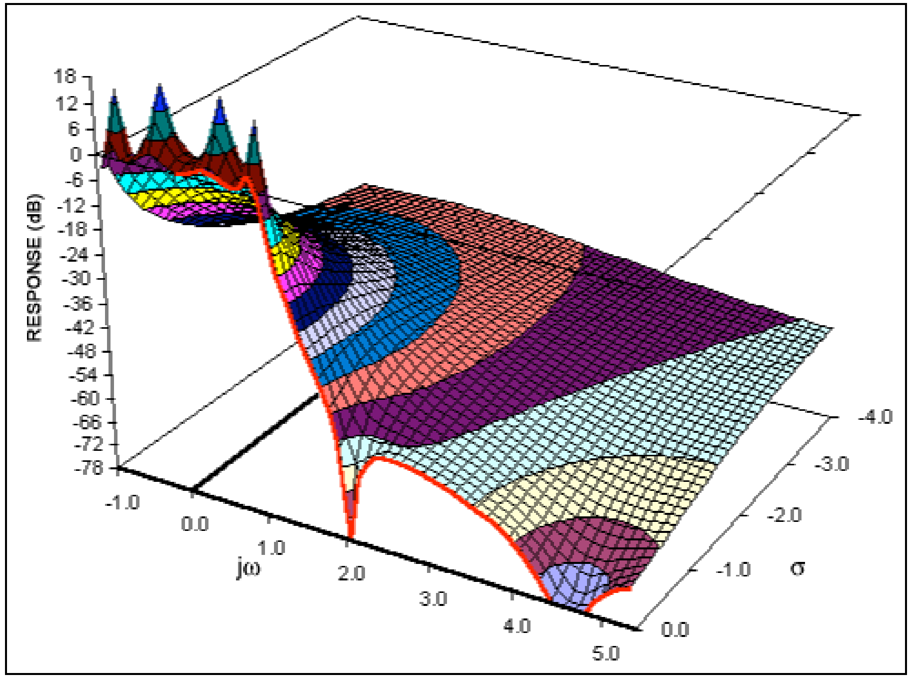


 [Maxim Integrated App Note 733]

The Elliptic Filter

- Additional zeroes must be spaced along the stopband to ensure that the filter response remains below the desired level of stopband attenuation
- The elliptic filter's high-Q poles produce a transient response that is even worse than that of the Chebychev
- Has poorest phase response

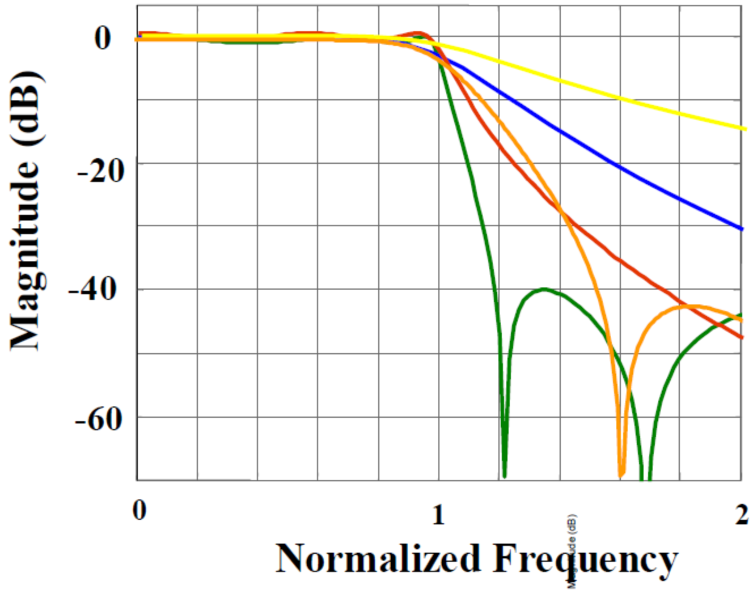




 [Maxim Integrated App Note 733]

Filters Comparison

- Filters with high attenuation per pole has poor phase response
- For higher attenuation while preserving constant group delay
  - In the case of passive filters: higher component count
  - For integrated active filters: higher chip area & power



Chebyshev II filter
–No ripple in passband
–Nulls or notches in stopband
–Sharper transition band compared to Butterworth
–Passband phase more linear compared to Chebyshev I

All 5th order filters with same corner freq.

Bessel
Butterworth
Chebyshev I
Chebyshev II
Elliptic

[H. K., Berkeley EECS 247, 2010]

Second-Order Filter Revisited

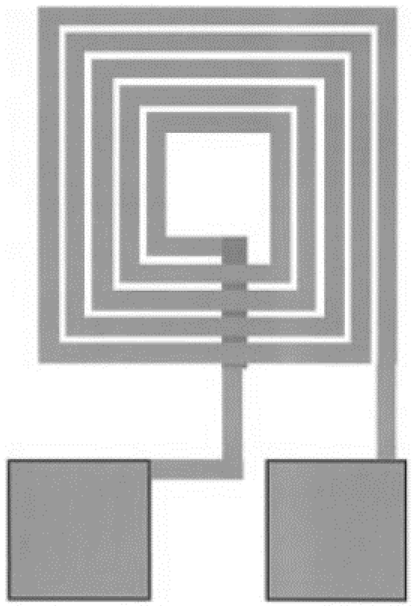
- A second-order filter has s^2 in the denominator and two poles in the complex plane
- General Form:

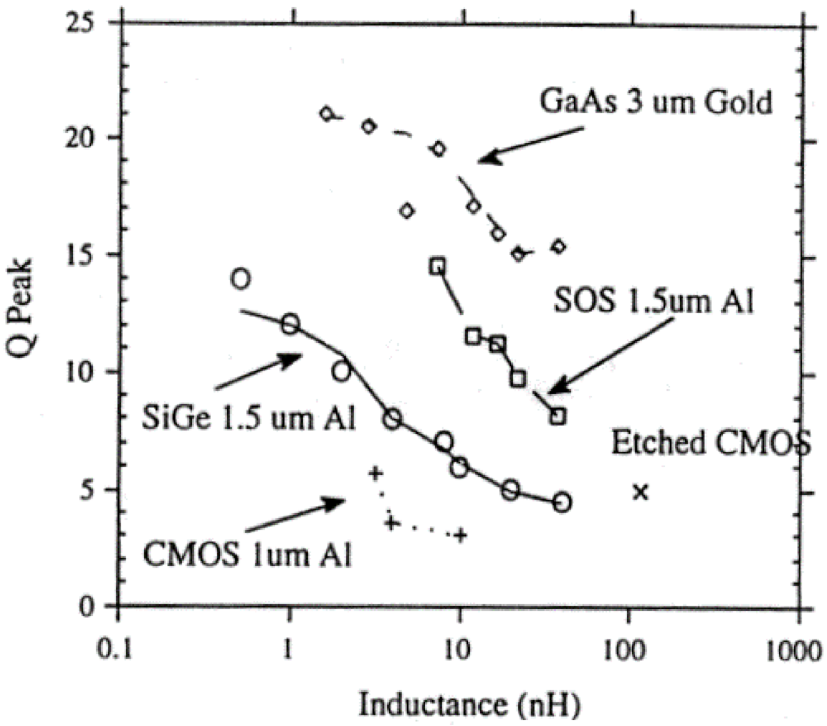
$$H(s) = \frac{K(s + z1)(s + z2)}{(s + p1)(s + p2)}$$

  - Also known as **biquad section**
  - Biquadratic means ratio of two quadratic functions
- Can be built by
  - Using inductance and capacitance in a passive circuit
    - No DC gain, difficult to integrate, coupled filter parameters, poor selectivity
  - An active circuit of resistors, capacitors, and amplifiers
    - The active circuitry replaces the inductor
- Filters constructed with Resistors and Capacitors **only** result in real poles
  - Don’t give interesting transfer functions

Monolithic Inductors

- Feasible monolithic inductor in CMOS technologies
  - L < 10 nH with Q < 10
- Also integrated C limited to 10s of pF
- Passive biquads not feasible for sub-100 MHz frequencies ($\omega_o^2 = \frac{1}{LC}$)

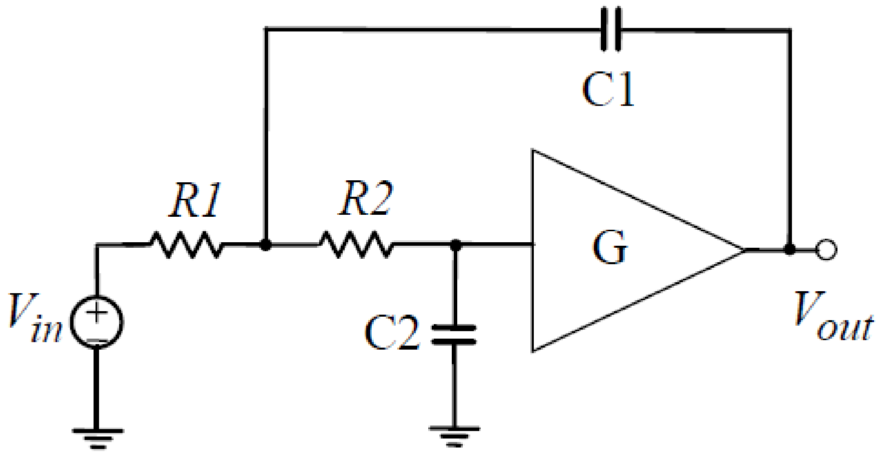




[H. K., Berkeley EECS 247, 2010]

Active Biquads

- Many topologies can be found in filter textbooks
  - Each has pros and cons: Sensitivity to variations and parasitics, no. of opamps, component spread, signal swing, independent parameter tuning, etc.
- Example: Single-opamp Sallen-Key biquad



$$H(s) = \frac{G}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$\omega_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

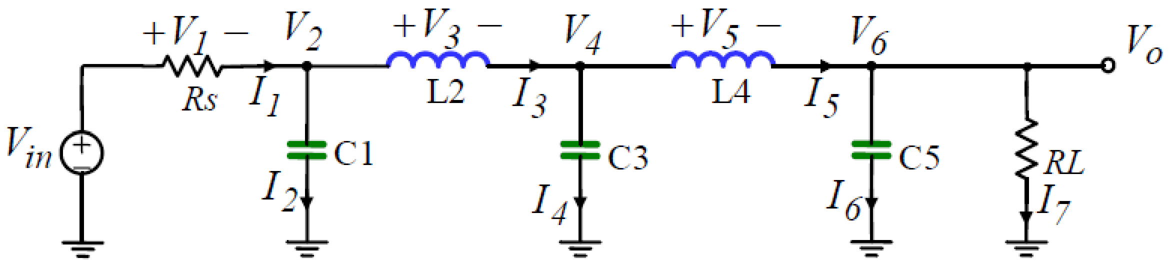
$$Q_P = \frac{\omega_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

- Cascade biquads to obtain any filter order!
  - But highly sensitive to component mismatch (not suitable for high-Q and high-order)
- Another better alternative: **Integrator-based ladder filters**

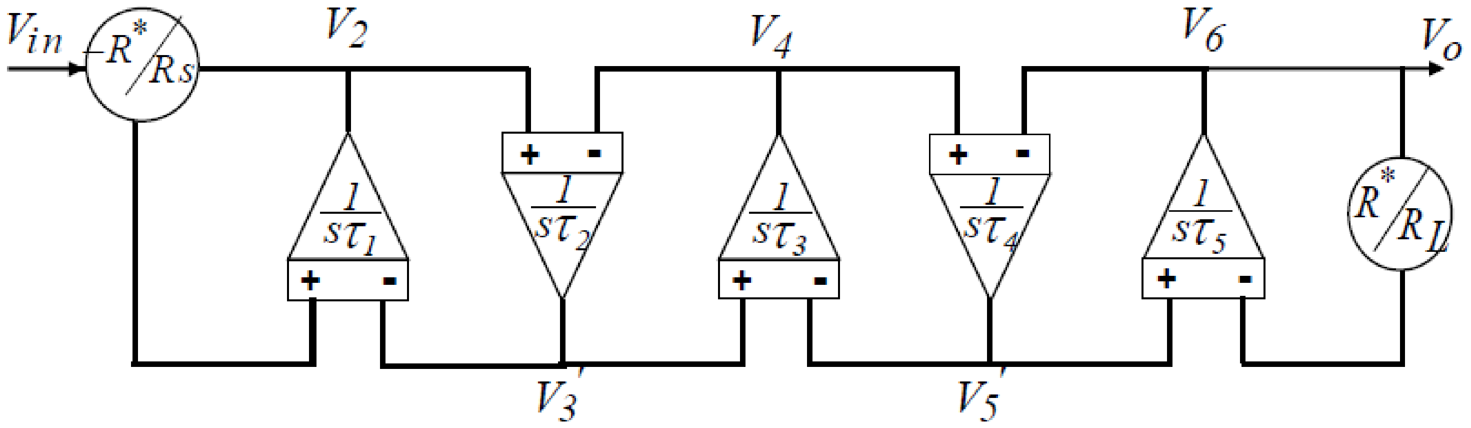
[H. K., Berkeley EECS 247, 2010]

Integrator-based Ladder Filters

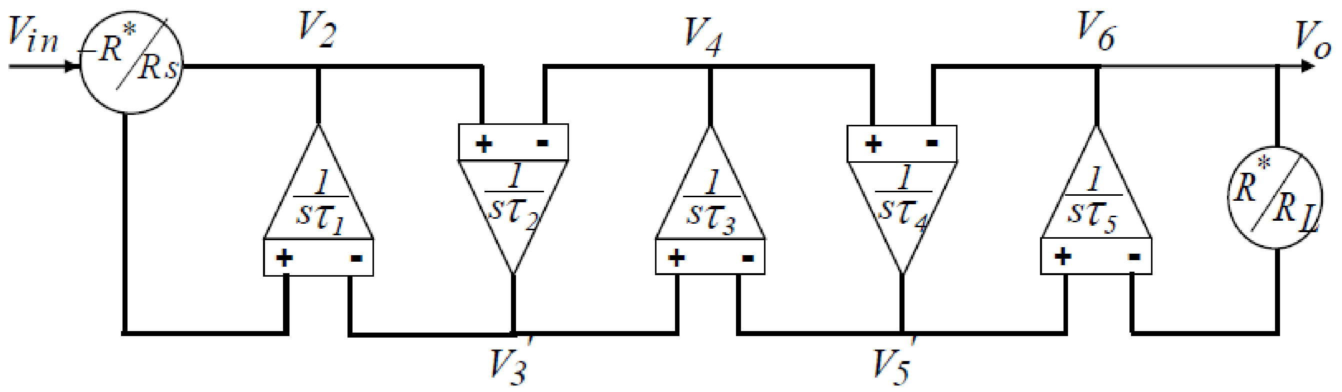
- Start with RLC ladder filter (Ex: 5th order LPF)



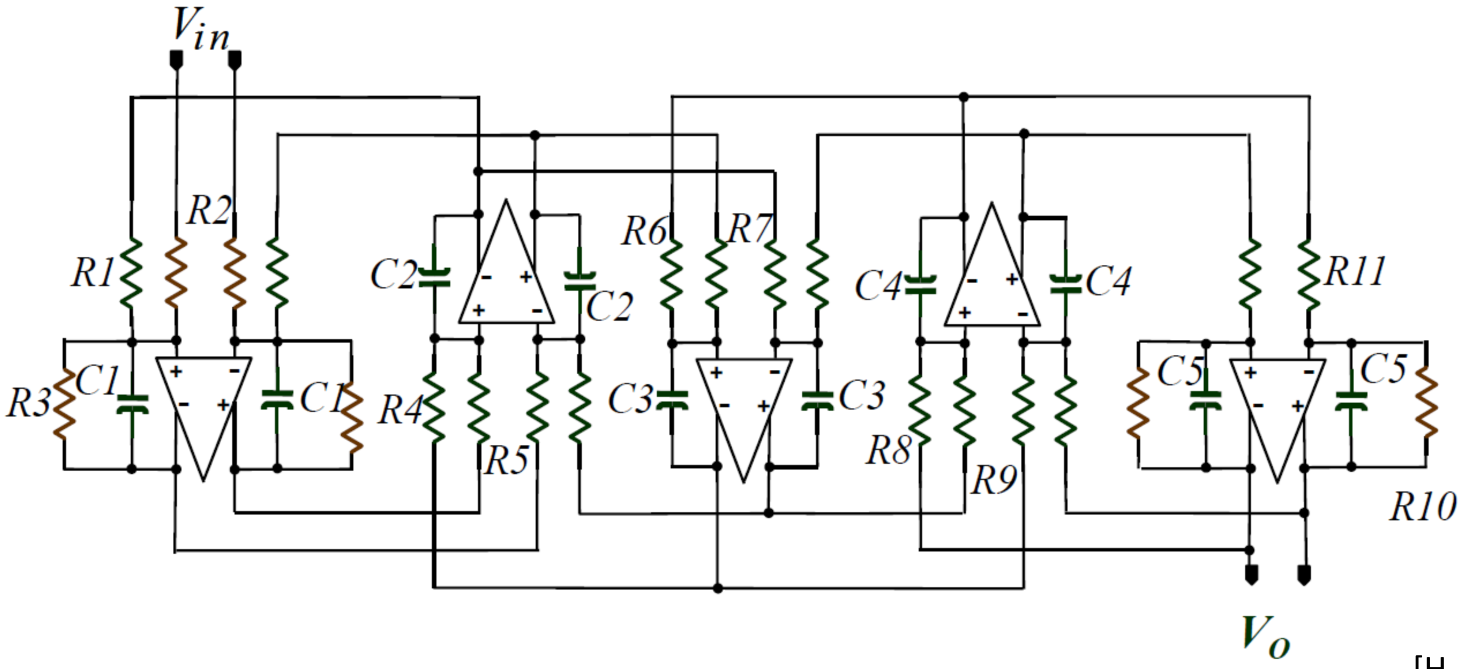
- Convert to integrator-based ladder filter using signal-flow graph (SFG) techniques



Integrator-based Ladder Filters



- Implement the integrators using opamps/OTAs



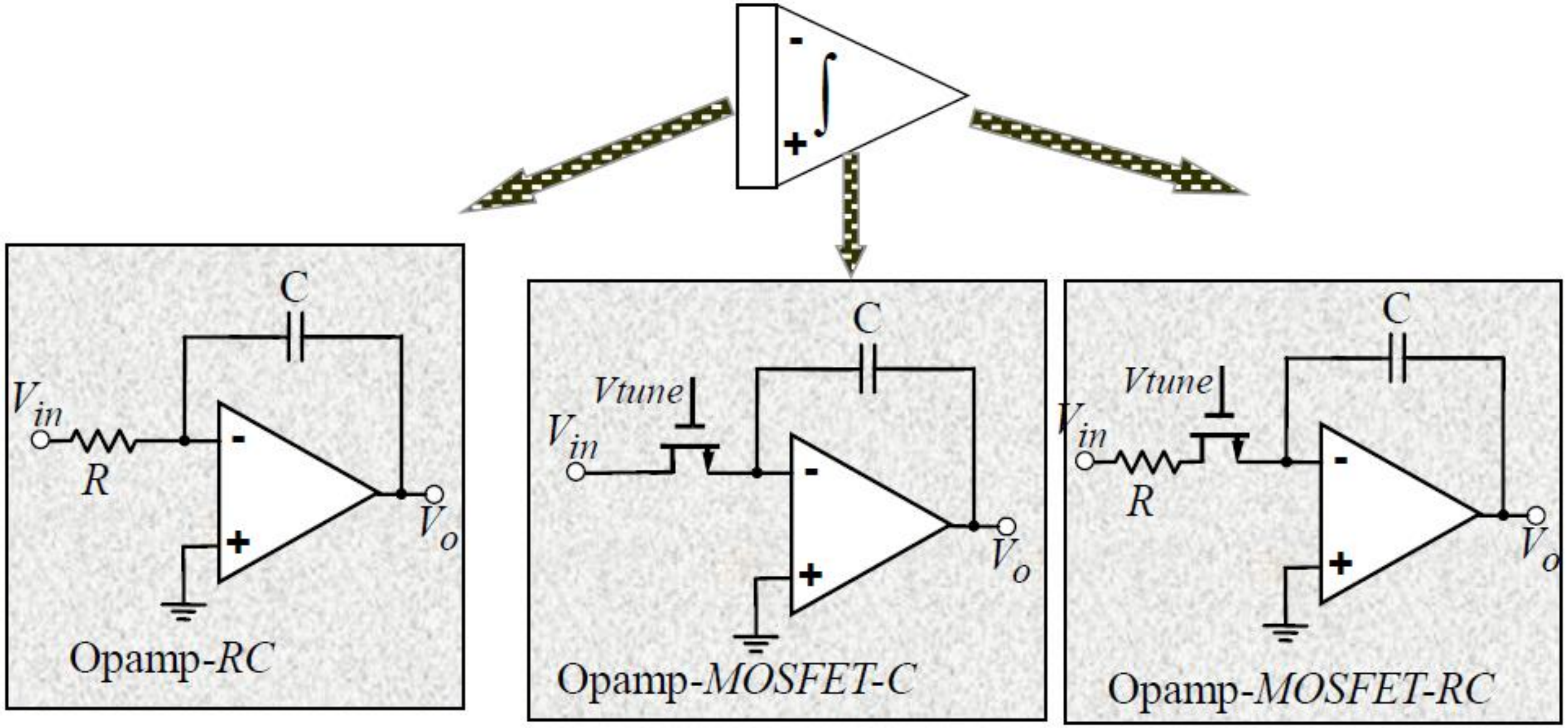
[H. K., Berkeley EECS 247, 2010]

Types of Integrator-based Filters

- Continuous Time (CT)
  - Resistive element based
    - Opamp-RC
    - Opamp-MOSFET-C
    - Opamp-MOSFET-RC
  - Transconductance (Gm) based
    - Gm-C
    - Gm-C-OTA (a.k.a. opamp-Gm-C)
- Discrete Time (DT) (Sampled Data)
  - Switched-capacitor integrator

[H. K., Berkeley EECS 247, 2010]

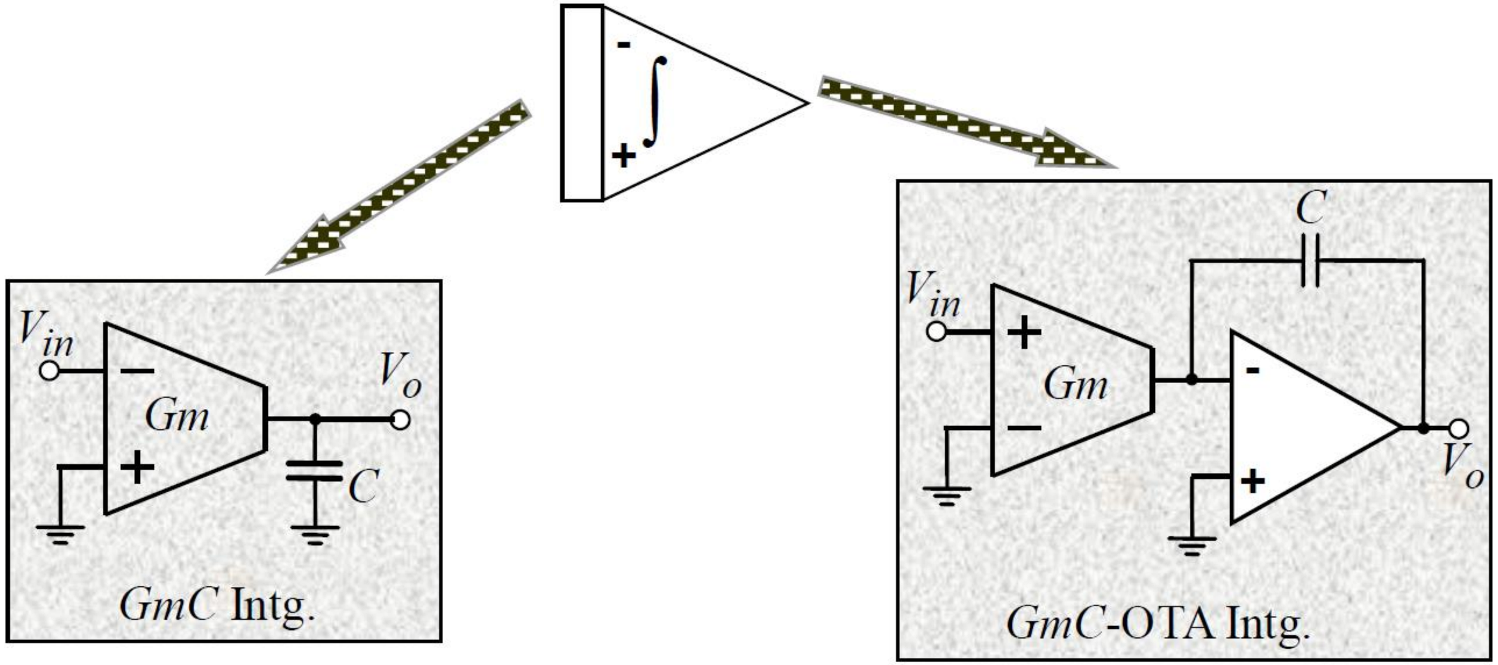
CT Resistor Based Integrator Filters



Ideal transfer function: $\frac{V_o}{V_{in}} = \frac{-\omega_o}{s}$ where $\omega_o = \frac{1}{R_{eq}C}$

CT Transconductor Integrator Filters

- The Gm-cell is similar to an OTA
  - But Gm should be linear and well-controlled (ex: use degeneration resistance)
  - The simplest form is a differential pair



Ideal transfer function: $$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{G_m}{C}$$

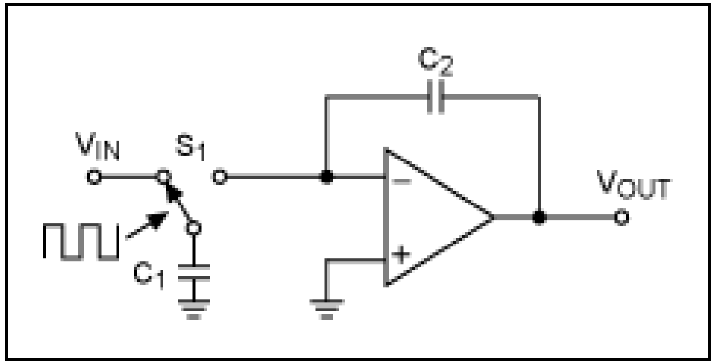
[H. K., Berkeley EECS 247, 2010]

CT Filters Summary

- Opamp RC filters
  - Good linearity ➔ High dynamic range (*60-90dB*)
  - Only discrete tuning possible
  - Medium usable signal bandwidth (<*10MHz*)
- Opamp MOSFET-C
  - Linearity compromised (typical dynamic range 40-60dB)
  - Continuous tuning possible
  - Low usable signal bandwidth *(<5MHz*)
- Opamp MOSFET-RC
  - Improved linearity compared to Opamp MOSFET-C (D.R. *50-90dB*)
  - Continuous tuning possible
  - Low usable signal bandwidth *(<5MHz*)
- Gm-C
  - Highest frequency performance *(up to 100s of MHz*)
  - Typically, dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (*40-70dB*)

DT Switched Capacitor Integrator

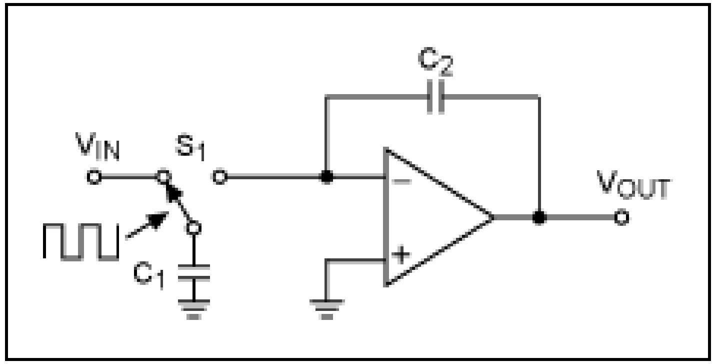
- The characteristics of CT filters depend on the accuracy of their RC time constants
- Variations in ABSOLUTE values of integrated resistors, capacitors, and gm can be ±20%
- But the RATIO of integrated matched capacitors chip can be accurately controlled (better than 0.1%)
- Switched-capacitor filters use these capacitor ratios to achieve precision without the need for precise external components
- Simply, the combination of C_1 and the switch emulates a resistor
  - $I = Q/T = Q * f_{CLK} = C_1 * V_{IN} \times f_{CLK}$
  - $R = V_{IN}/I = 1/(C_1 * f_{CLK})$



[Maxim Integrated App Note 733]

DT Switched Capacitor Integrator

- $R = 1/(C_1 * f_{CLK})$
- The integrator's $\omega_0 = 1/RC_2 = C_1 * f_{CLK}/C_2$
  - ω_0 is proportional to the ratio of the two capacitors
    - Can be controlled with great accuracy
  - ω_0 is proportional to the clock frequency
    - Vary the filter characteristics by changing fCLK
- But the switched capacitor is a sampled-data system!
  - Nyquist sampling criterion holds!
  - Must be preceded by a CT anti-aliasing filter!



[Maxim Integrated App Note 733]

DT Switched Capacitor Integrator

- Switched-capacitor filters have the advantage that they can handle low frequencies without using large values of R and C
  - Simply lower the clock frequency
- There may be small spikes at the step transitions caused by charge injected by the switches
  - May not be a problem if the system bandwidth following the filter is much lower than the clock frequency
  - Or add another filter at the output of the switch-capacitor filter to remove the clock ripple
- It is best to keep the ratio of clock-to-center frequency (f_{CLK}/f_{IN}) as large as possible
  - Typical ratios range from approximately 20:1 to 200:1
  - Power hungry OTAs!

Filter Design Tools

- ADI Analog Filter Wizard
  - http://www.analog.com/designtools/en/filterwizard/
- TI WEBENCH® Filter Designer
  - http://www.ti.com/design-tools/signal-chain-design/webench-filters.html
- http://sim.okawa-denshi.jp/en/Fkeisan.htm

References

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- Maxim Integrated App Note 733: A Filter Primer.
- T. C. Carusone, D. Johns, and K. W. Martin, “Analog Integrated Circuit Design,” 2nd ed., Wiley, 2012.
- A. Sedra and K. Smith, “Microelectronic circuits,” Oxford University Press, 7th ed., 2015.

Thank you!