وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog Integrated Systems Design

Lecture 02
Sampling

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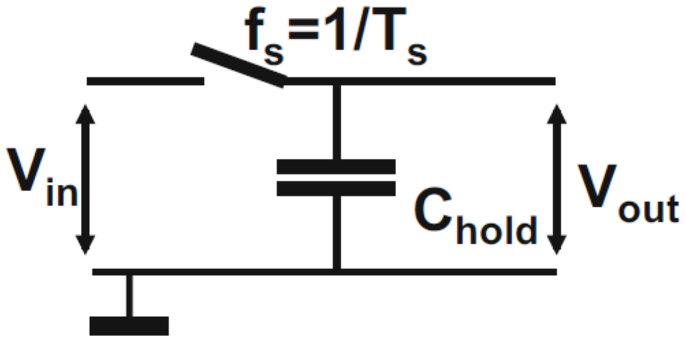
Sampling

- Sampling is time discretization
  - Converts a continuous time (CT) signal to a discrete time (DT) signal
  - The result is a sequence of samples
- The sampling instants are defined by a clock signal ($T_s = 1/f_s$)
  - The clock signal controls an electronic switch (e.g., MOS transistor)
- The sampled signal is stored as a voltage on a capacitor
- The circuit is called sample-and-hold (S/H) circuit



$$t = \frac{n}{f_s} = nT_s, \quad n = -\infty, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, \infty.$$

[M. Pelgrom, 2017]

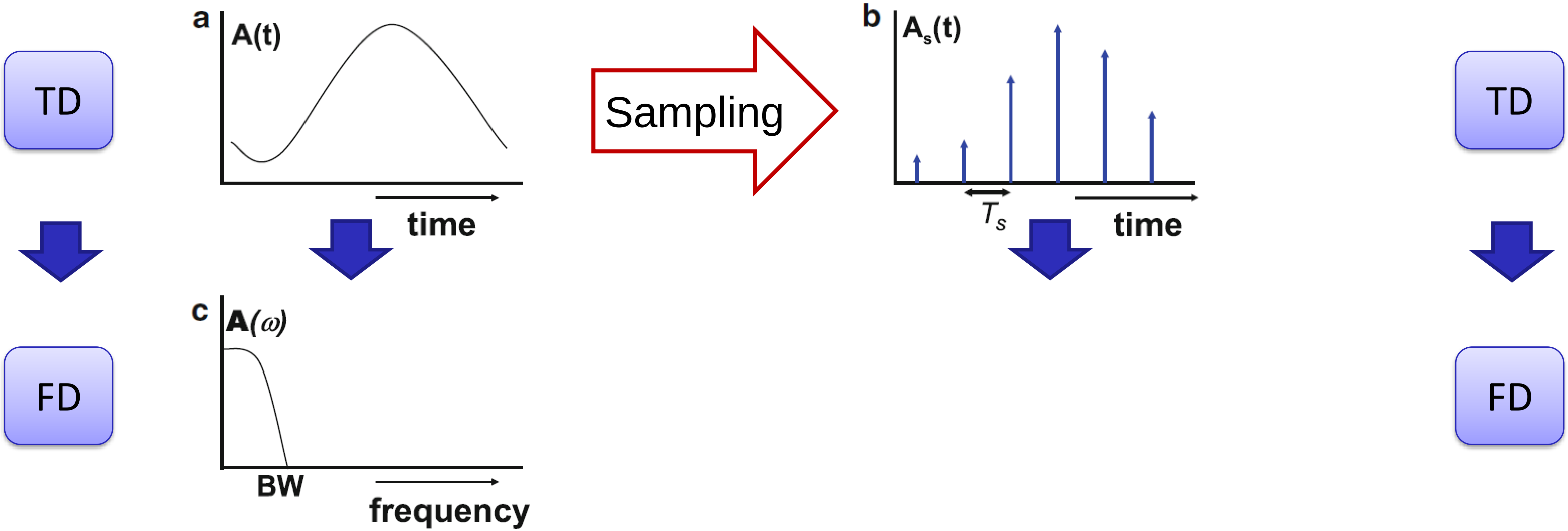
Time and Frequency Domains

$$TD\ step = \Delta t = \frac{1}{f_s} = \frac{1}{FD\ period}$$
$$FD\ step = \Delta f = \frac{1}{T_o} = \frac{1}{TD\ period}$$

Time domain		Technique	Frequency domain		Where in the chain?
CT/DT	Periodic	$\leftrightarrow$	C/D	Periodic	
CT	Yes	CT Fourier series (CTFS)	Discrete	No	-
CT	**No**	**CT Fourier transform (CTFT)**	**Continuous**	**No**	**Before S/H**
DT	**Yes**	**DT Fourier series (DTFS) → FFT**	**Discrete**	**Yes**	**After ADC**
DT	**No**	**DT Fourier transform (DTFT)**	**Continuous**	**Yes**	**After S/H**

Discrete ↔ Periodic

- Sampling causes "images" in the frequency domain
  - The sampled signal is folded around f_s and its multiples
  - The part from 0 to $f_s/2$ is the only part that has physical meaning



[M. Pelgrom, 2017]

Discrete ↔ Periodic

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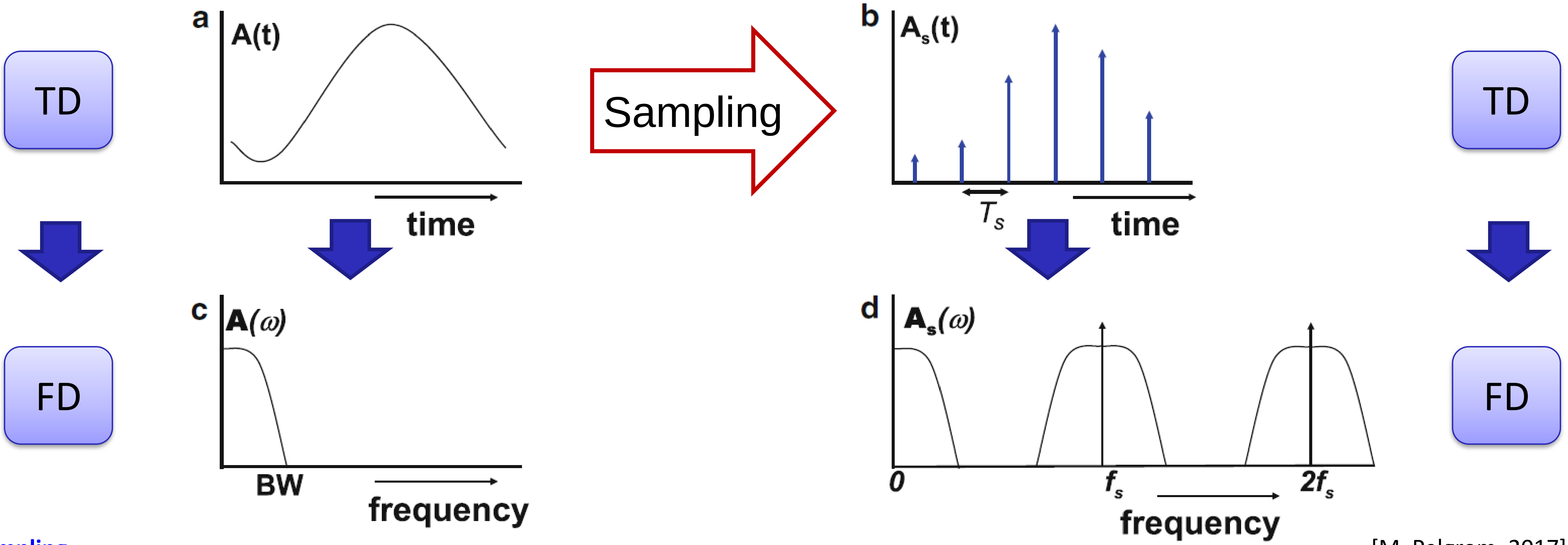


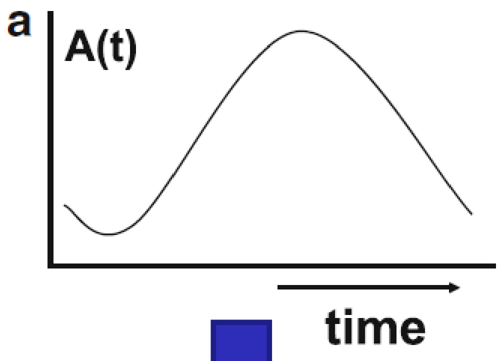
[M. Pelgrom, 2017]

What about Bandpass Signals?

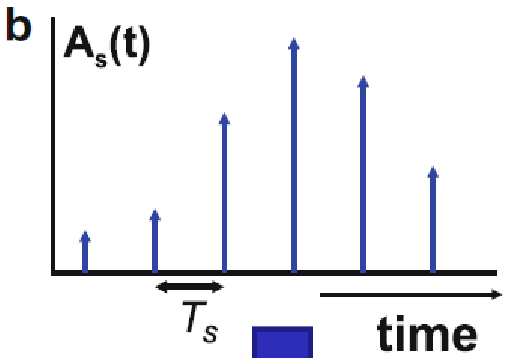
- Sampling causes "images" in the frequency domain
  - The sampled signal is folded around f_s and its multiples
  - The part from 0 to $f_s/2$ is the only part that has physical meaning

TD

FD



Sampling



TD

FD

[M. Pelgrom, 2017]

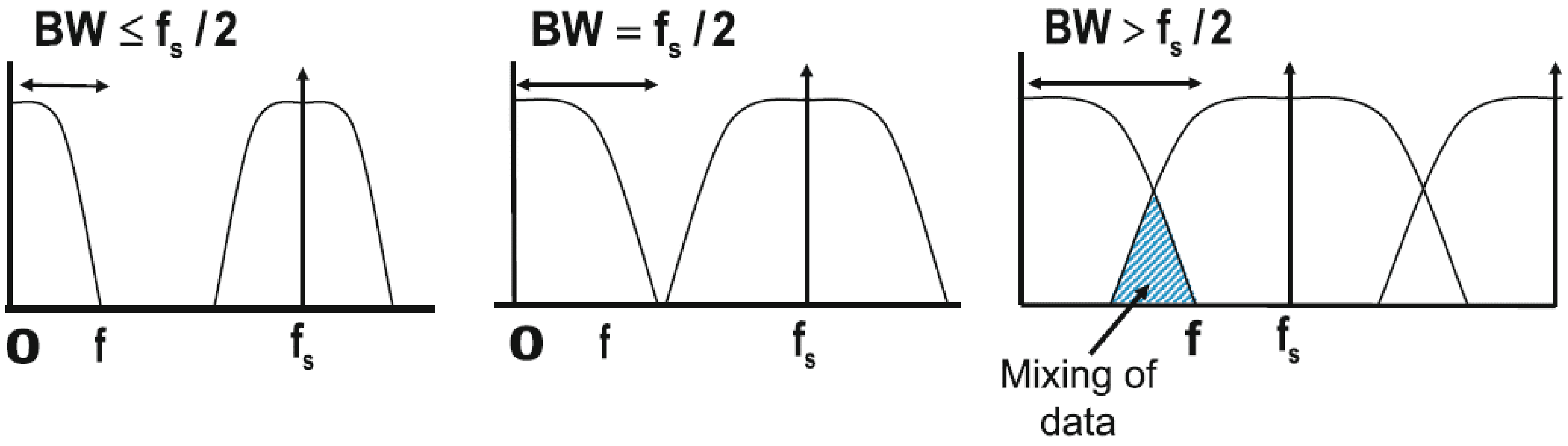
Aliasing and Nyquist Criterion

- Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled.
- Nyquist Criterion:

$$f_s > f_{Nyq} = 2 \times BW$$

NOT

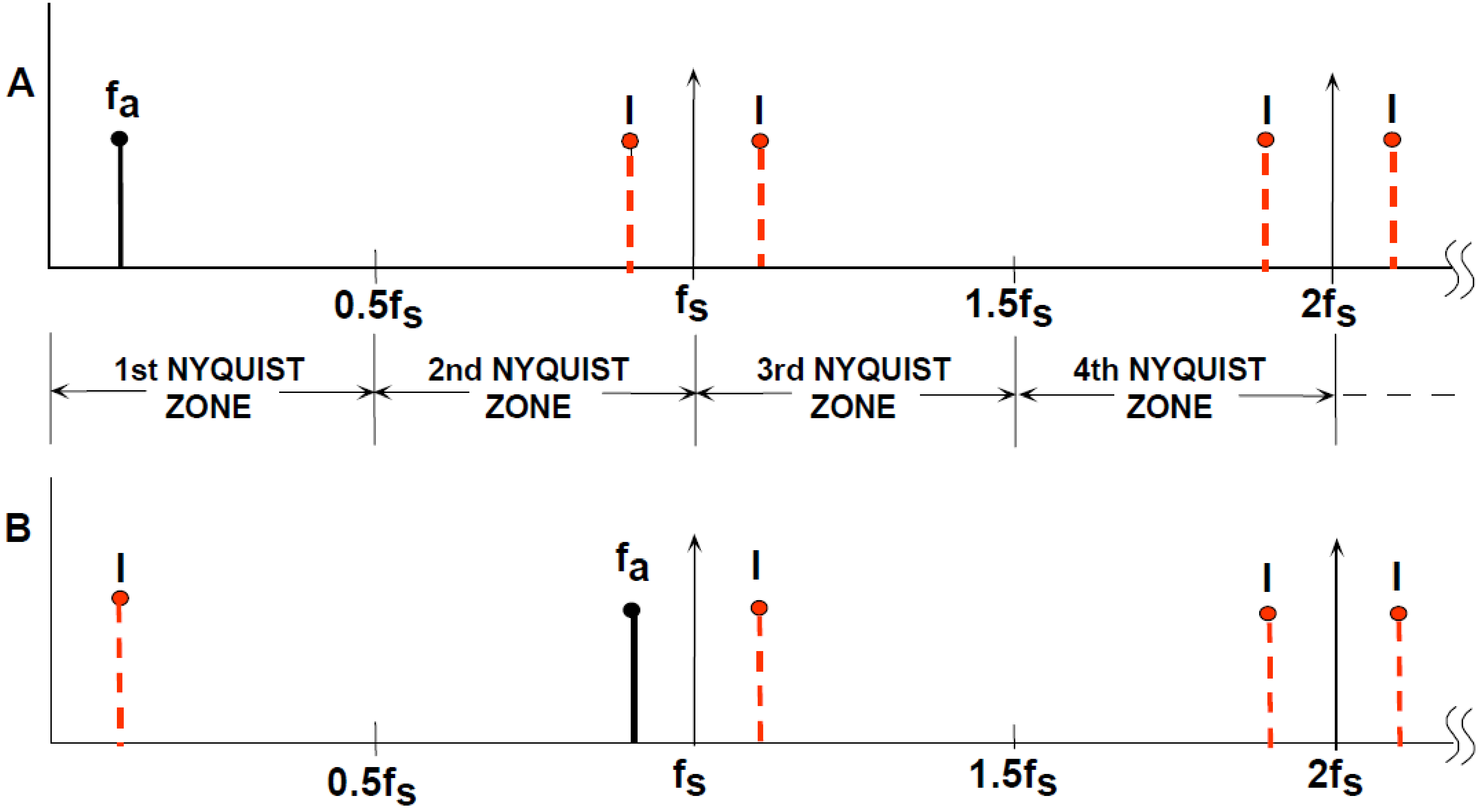
$$f_s > 2 \times f_{max}$$



[M. Pelgrom, 2017]

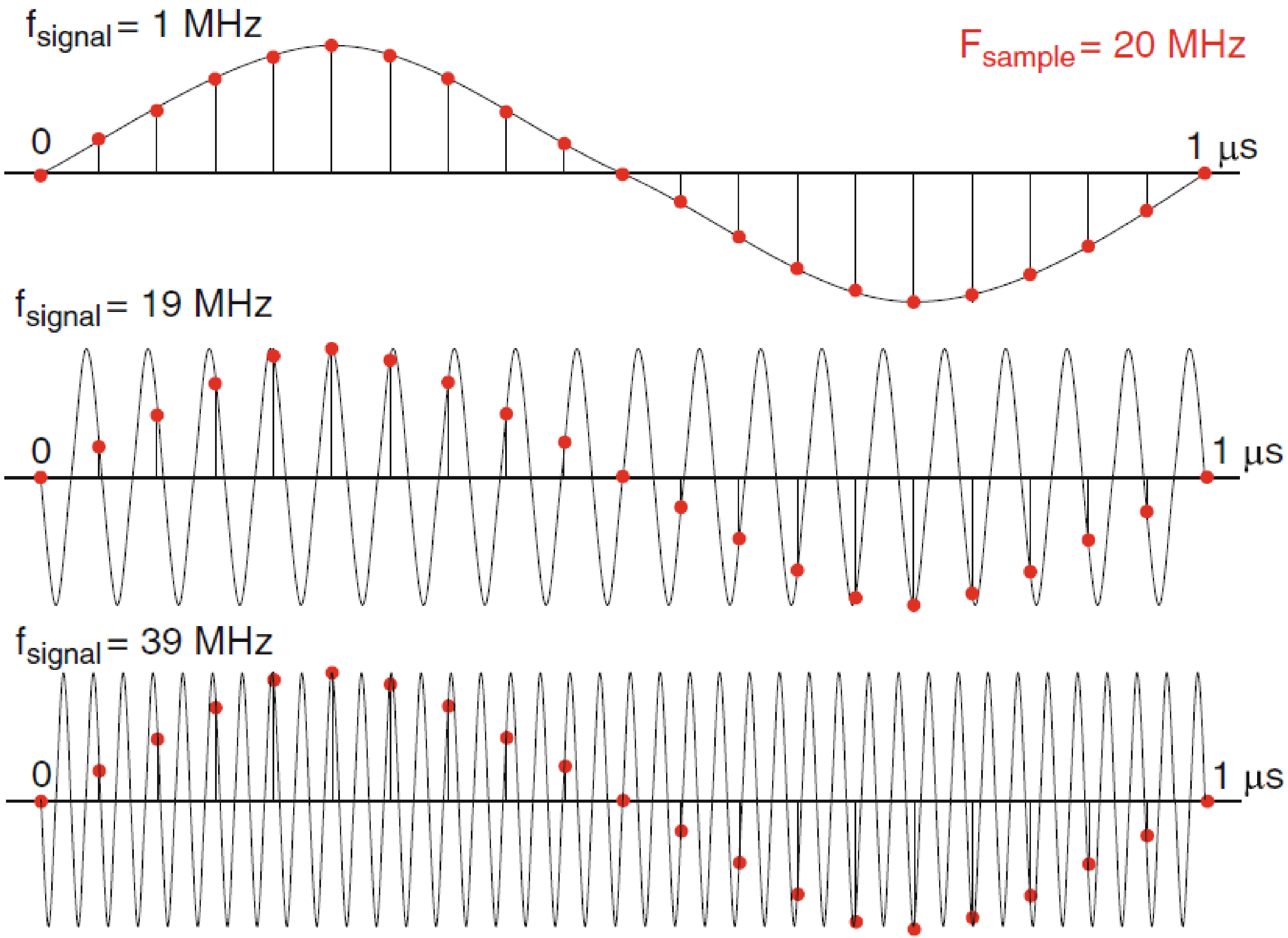
Aliasing/Subsampling in Frequency Domain

- The sampled signal does not have to be a baseband signal
  - Images appear at $|\pm k f_s \pm f_a|$, where $k = 0, 1, 2, \ldots$
- A.k.a. Under-sampling, Harmonic Sampling, Bandpass Sampling, IF Sampling, Direct IF-to-Digital Conversion



[W. Kester, 2005]

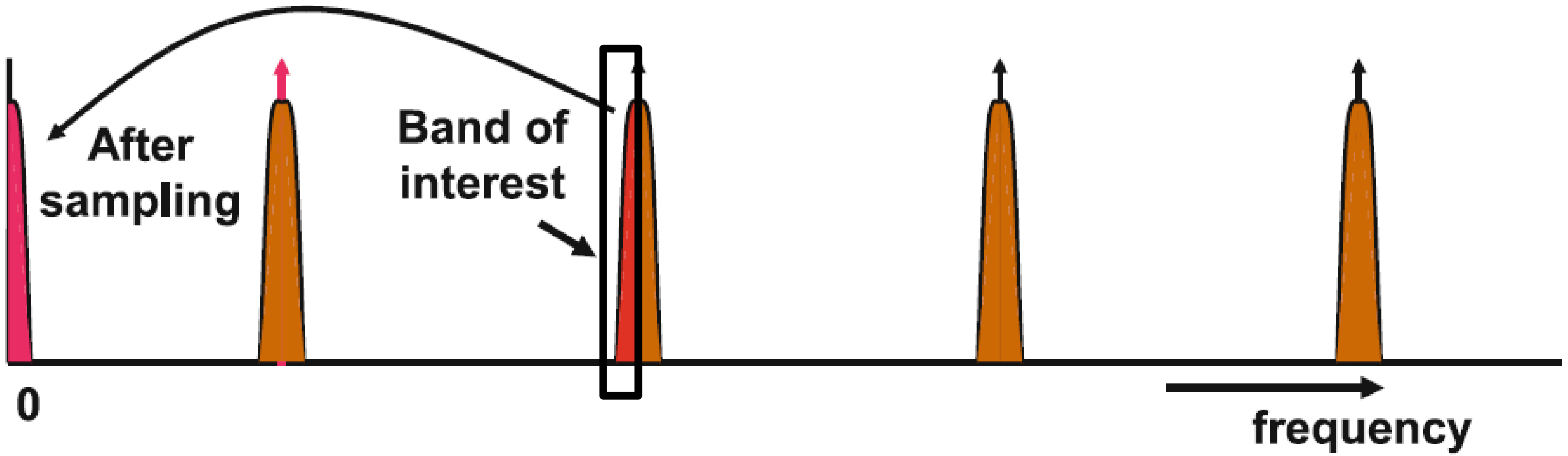
Aliasing/Subsampling in Time Domain



[W. Kester, 2005]

Subsampling

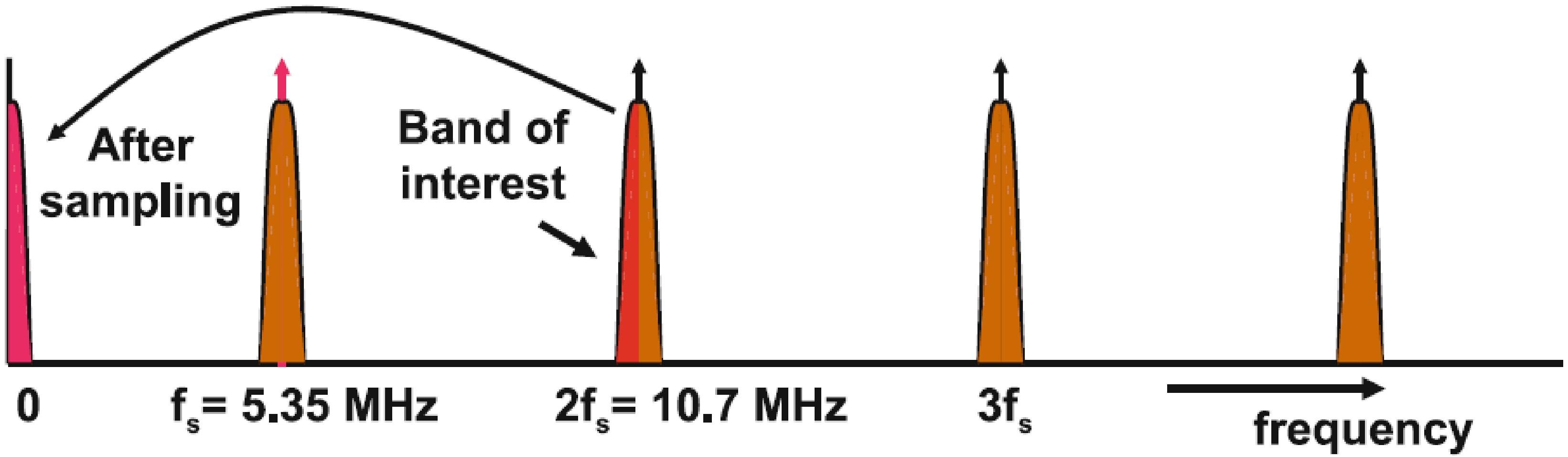
- The sampled signal does not have to be a baseband signal
- Subsampling can be used to demodulate (down-convert) an RF signal
- A.k.a. Under-sampling, Harmonic Sampling, Bandpass Sampling, IF Sampling, Direct IF-to-Digital Conversion
- Example: FM signal of 100kHz bandwidth at 10.7MHz
  - The signal is down-converted by a **?** MHz sampling clock



[M. Pelgrom, 2017]

Subsampling

- Subsampling can be used to demodulate (down-convert) an RF signal
- A.k.a. Under-sampling, Harmonic Sampling, Bandpass Sampling, IF Sampling, Direct IF-to-Digital Conversion
- Example: FM signal of 100kHz bandwidth at 10.7MHz
  - The signal is down-converted by a 5.35MHz sampling clock



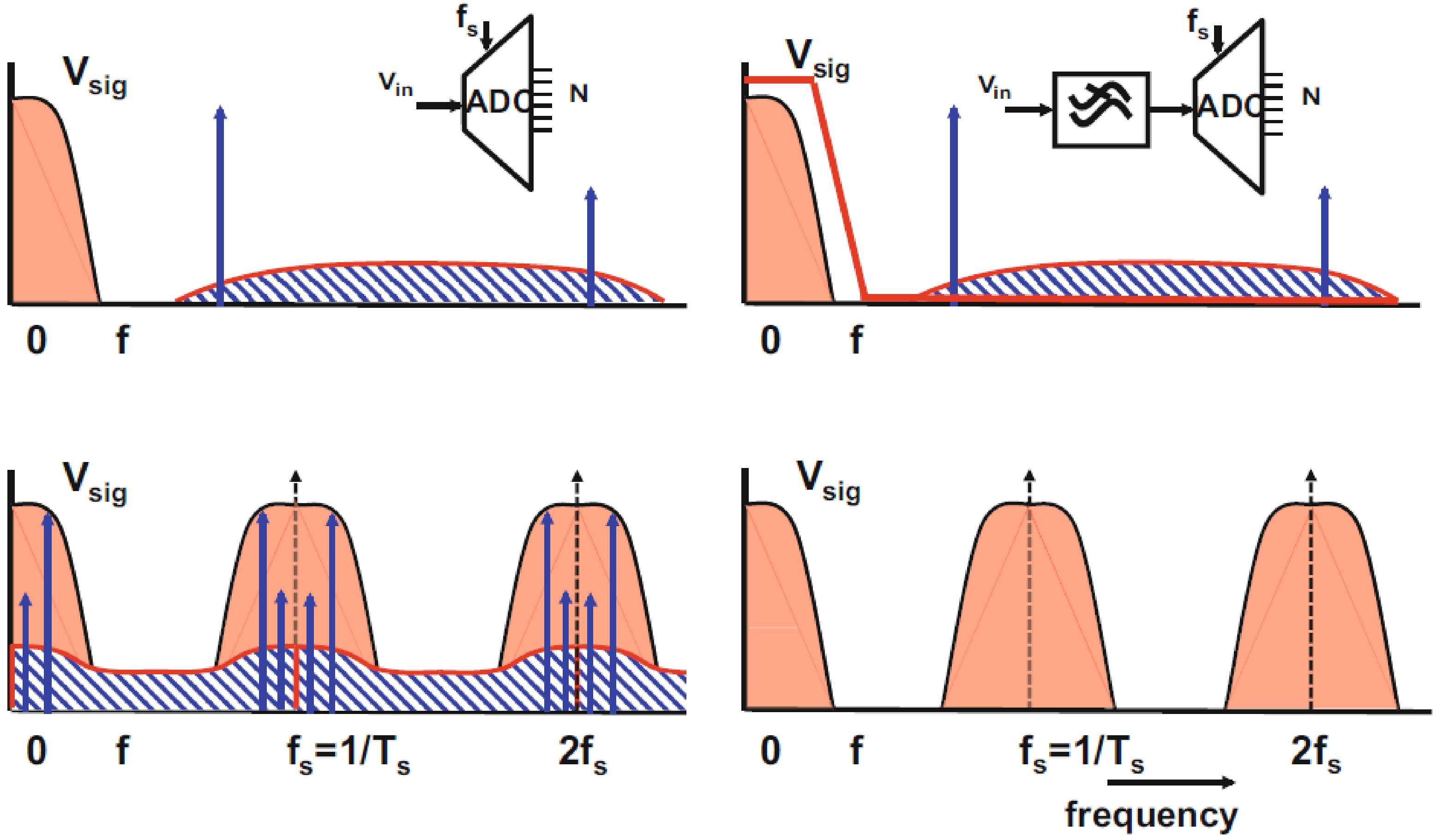
[M. Pelgrom, 2017]

Anti-Aliasing Filter (AAF)

- ❑ Anti-aliasing filters are active or passive (CT or DT?) filters.
  - The signal must be filtered before sampling (time discretization).

Before S/H

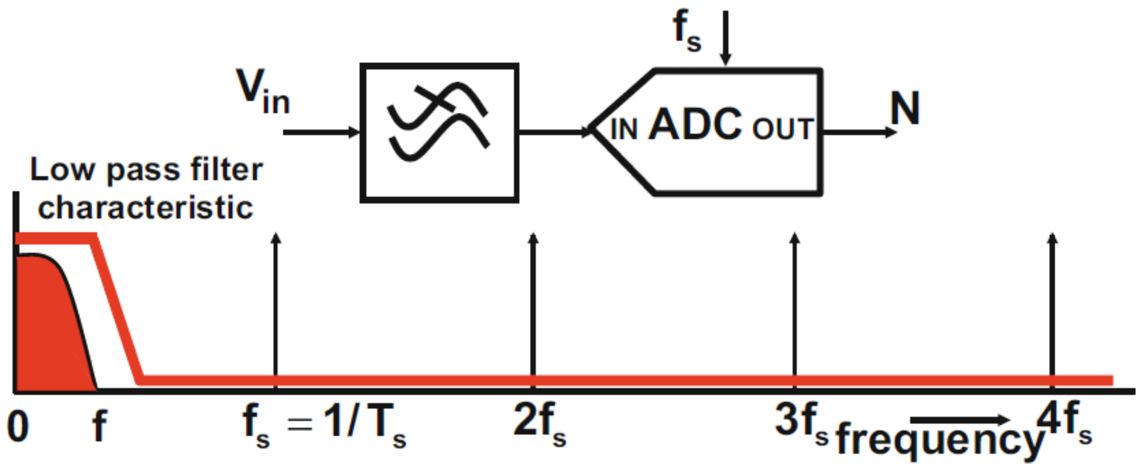
After S/H

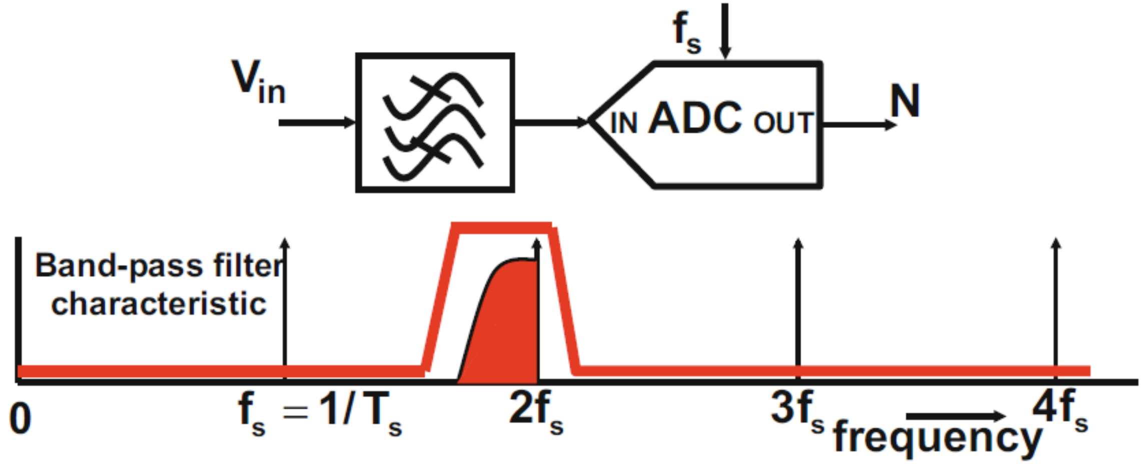


[M. Pelgrom, 2017]

Baseband vs Passband Signals

- Sampling of baseband signal: AAF is a (LPF,BPF,HPF)?
- Subsampling of RF (passband) signal: AAF is a (LPF,BPF,HPF)?

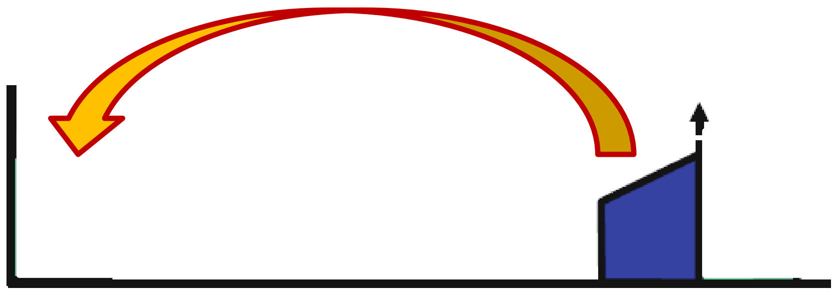




[M. Pelgrom, 2017]

Subsampling Example

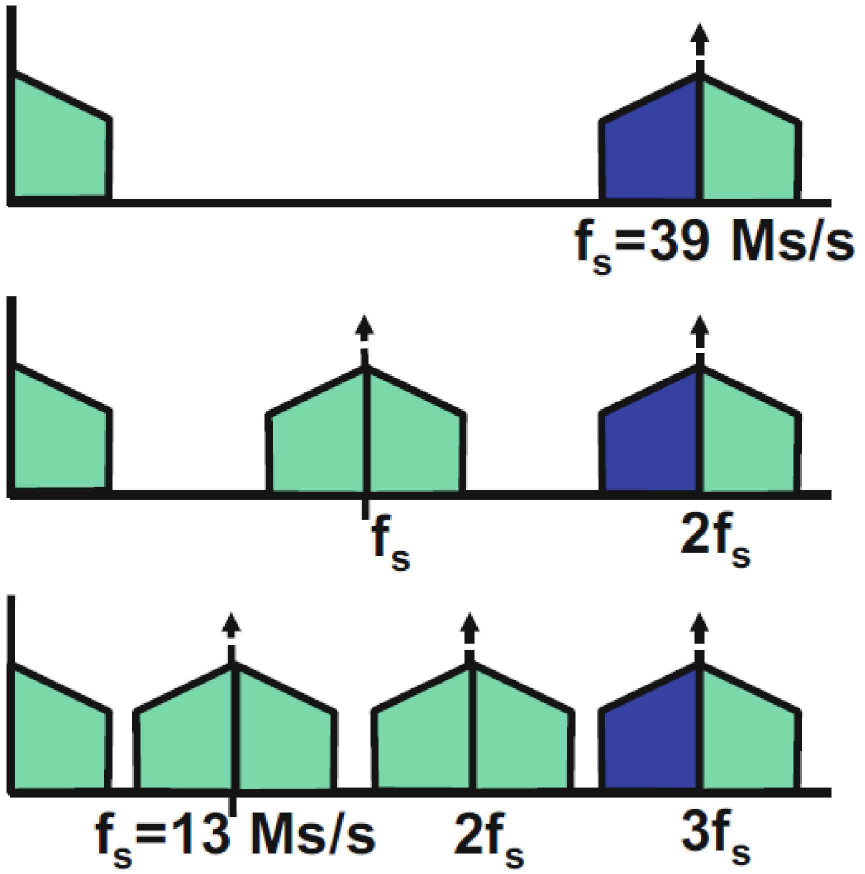
- Signal band: 33 MHz to 39MHz
- Which sampling rate to choose?
  - 78 MS/s
  - 39 MS/s
  - 19.5 MS/s
  - 13 MS/s



[M. Pelgrom, 2017]

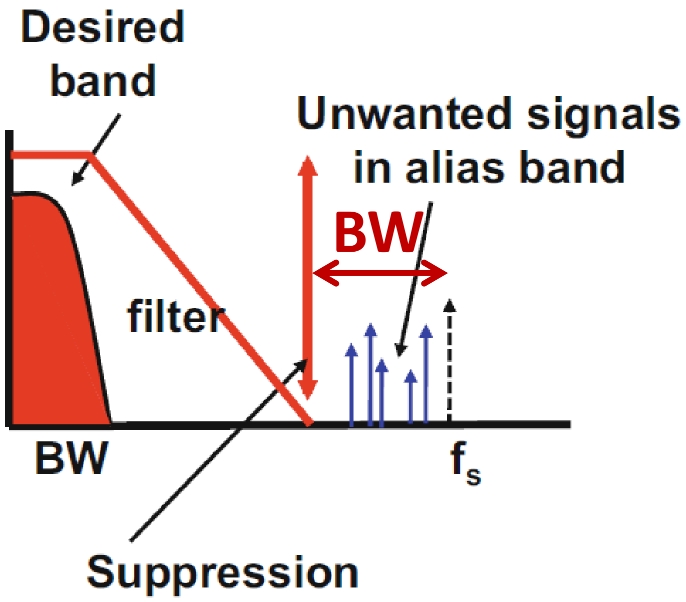
Subsampling Example

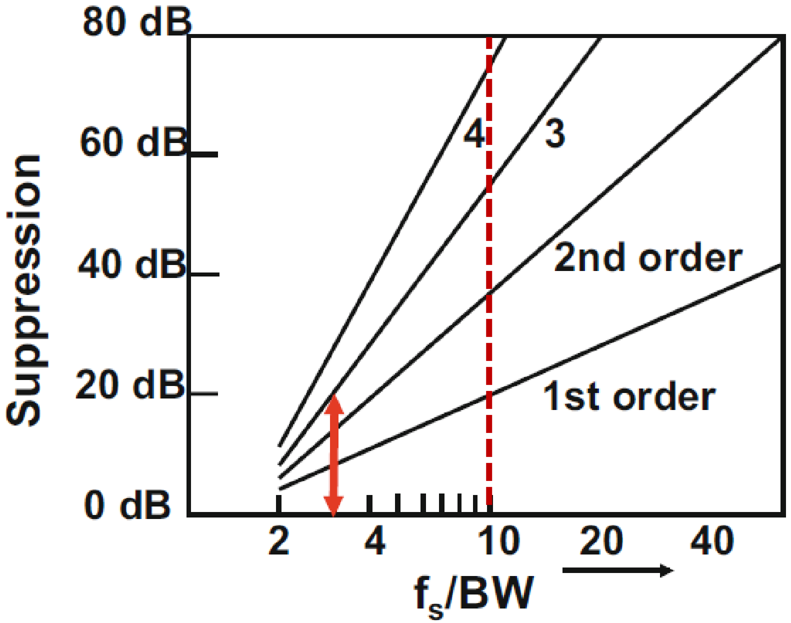
- Signal band: 33 MHz to 39MHz
- Which sampling rate to choose?
  - 78 MS/s
  - 39 MS/s
  - 19.5 MS/s
  - 13 MS/s
- What is the advantage/disadvantage of higher sampling frequency?



Alias Band Suppression

- Working at the limit of Nyquist criterion requires an ideal filter that does not exist.
- Signals in the alias band $(f_s - BW\ to\ f_s)$ will alias in the desired signal band after sampling.
  - Must be suppressed by AAF.
- Each pole gives a roll-off slope of 20 dB/decade = 6 dB/octave
- How much suppression @ $(f_s - BW)$ if $f_s/BW = 2$ for 4th order filter?

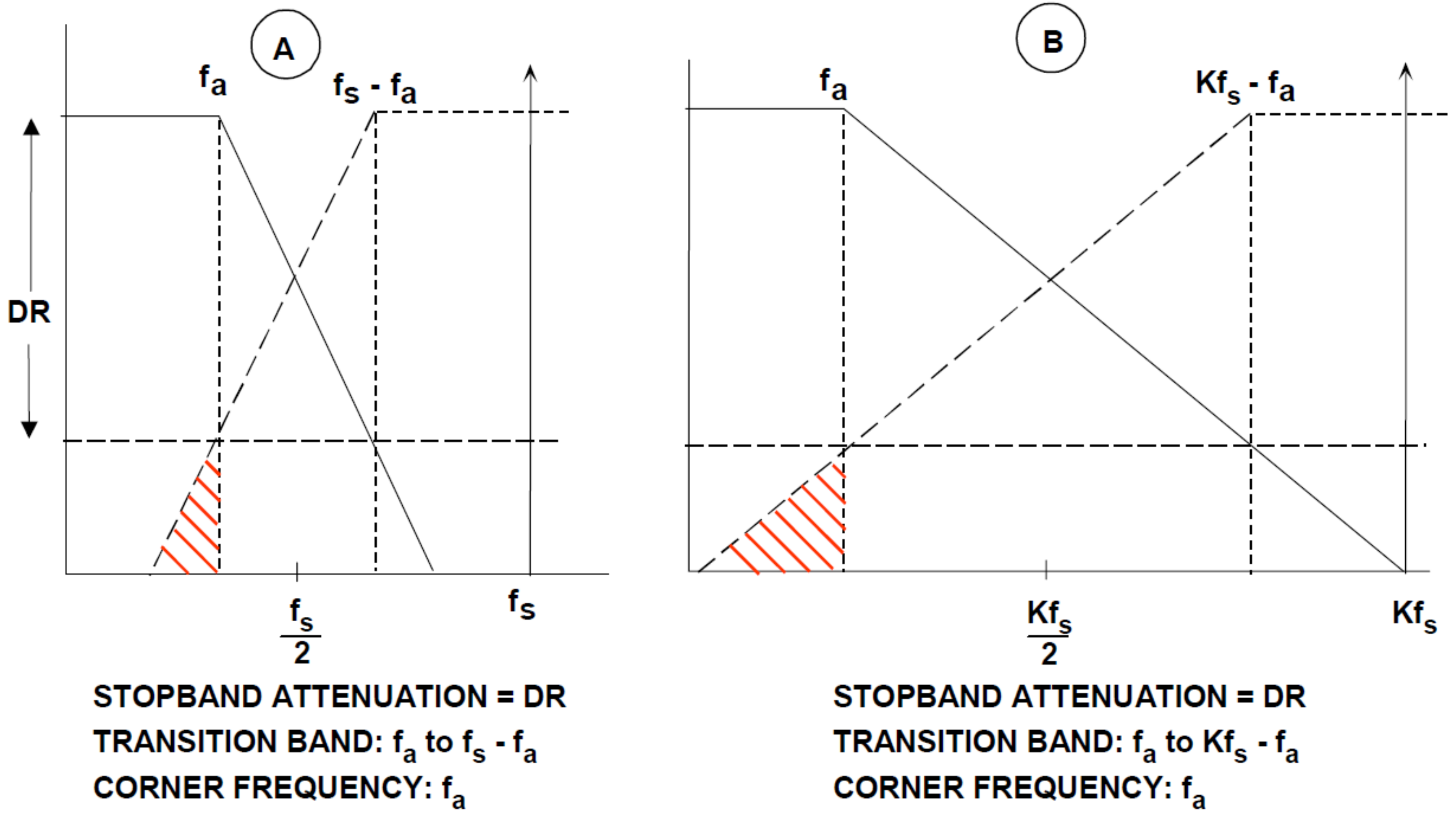




[M. Pelgrom, 2017]

Oversampling

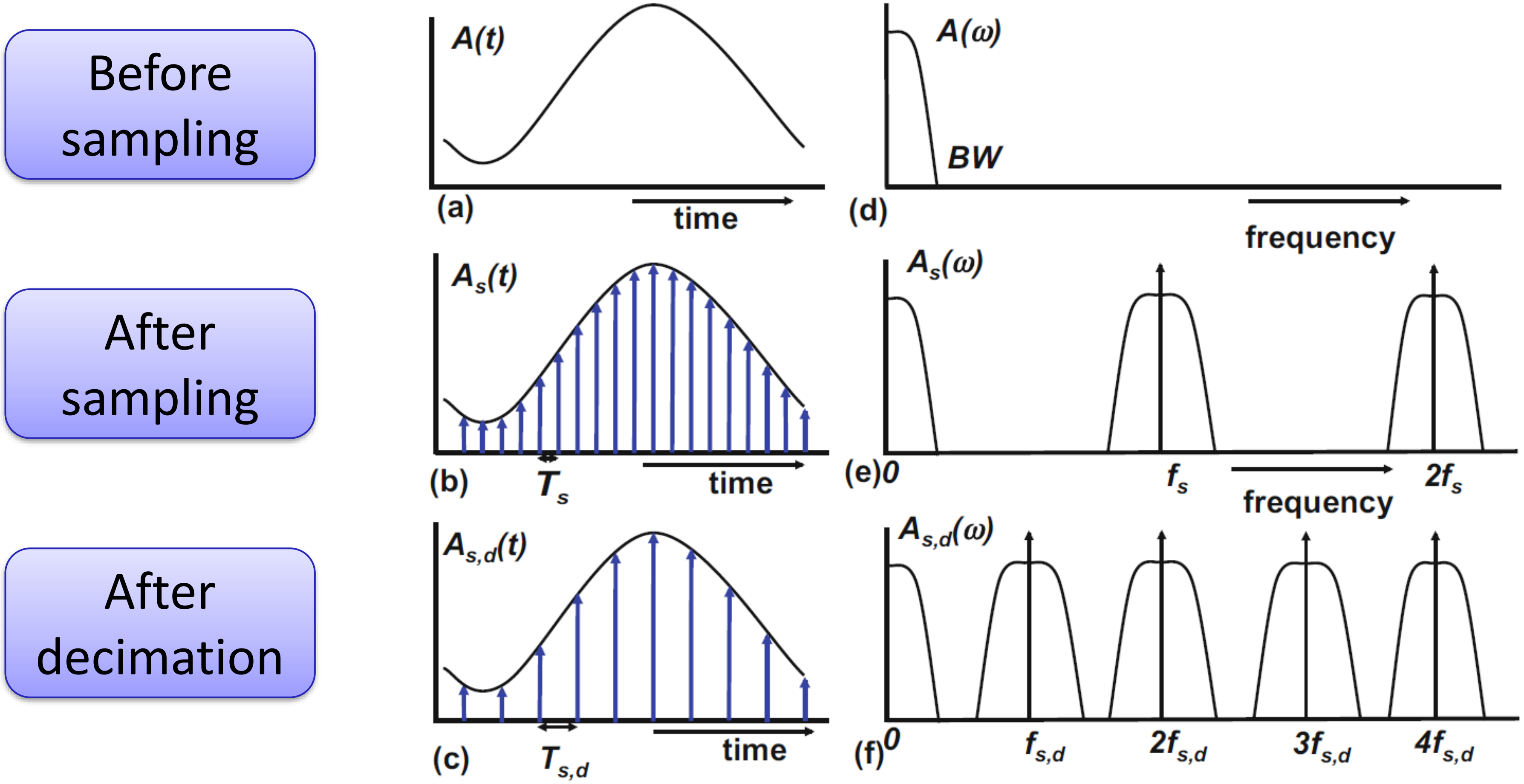
- Oversampling relaxes requirements on baseband antialiasing filter.



[W. Kester, 2005]

Decimation

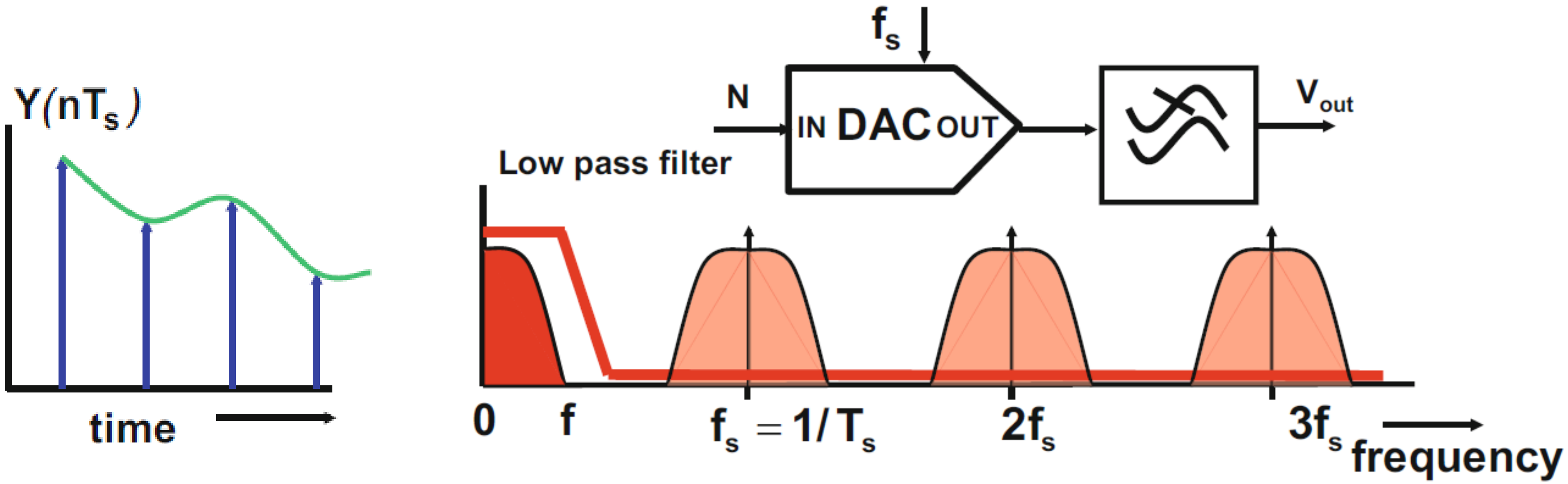
- Decimation is the process of reducing the sample rate of a signal.
- Unless the signal is already filtered and oversampled, digital filtering is necessary.



[M. Pelgrom, 2017]

Reconstruction Filter

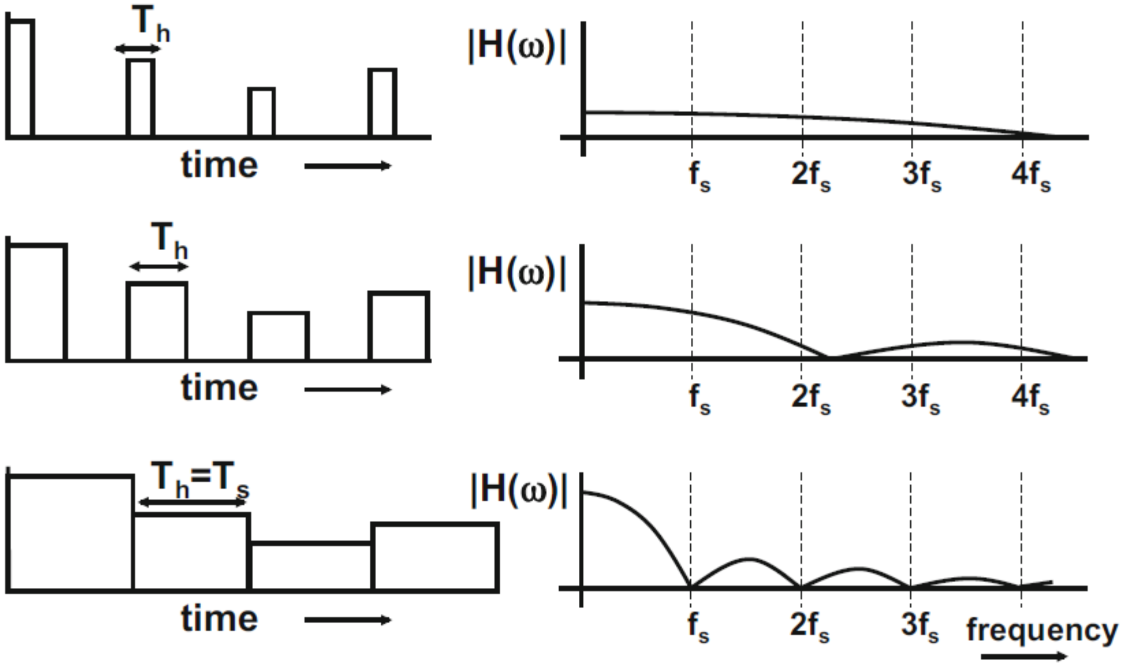
- ADC needs an anti-aliasing filter.
- DAC needs a reconstruction (smoothing) filter.
  - TD: The reconstruction filter "interpolates/restores/reconstructs" the signal.
  - FD: The reconstruction filter suppresses the "images".



[M. Pelgrom, 2017]

Zero-Order Hold (ZOH)

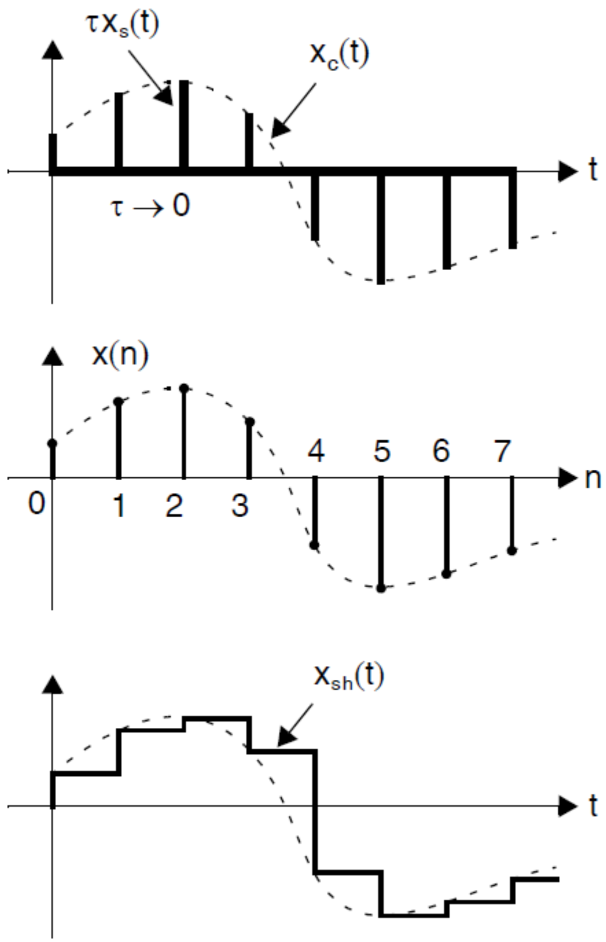
- Zero-order hold (ZOH) keeps the value of the signal at the sample moment.
- The Fourier transform of ZOH is a sinc function: sinc(x) = sin(x)/x
  - Nulls of sinc(x) at the inverse of hold time (pulse width)
- The zero-order hold (ZOH) performs inherent reconstruction (filtering out images).



[M. Pelgrom, 2017]

Zero-Order Hold (ZOH)

- The zero-order hold (ZOH) performs inherent reconstruction (filtering out images).
  - Succeeding reconstruction filter performs further interpolation (image suppression).



Time

Frequency

[Johns & Martin, 2012]

Zero-Order Hold (ZOH)

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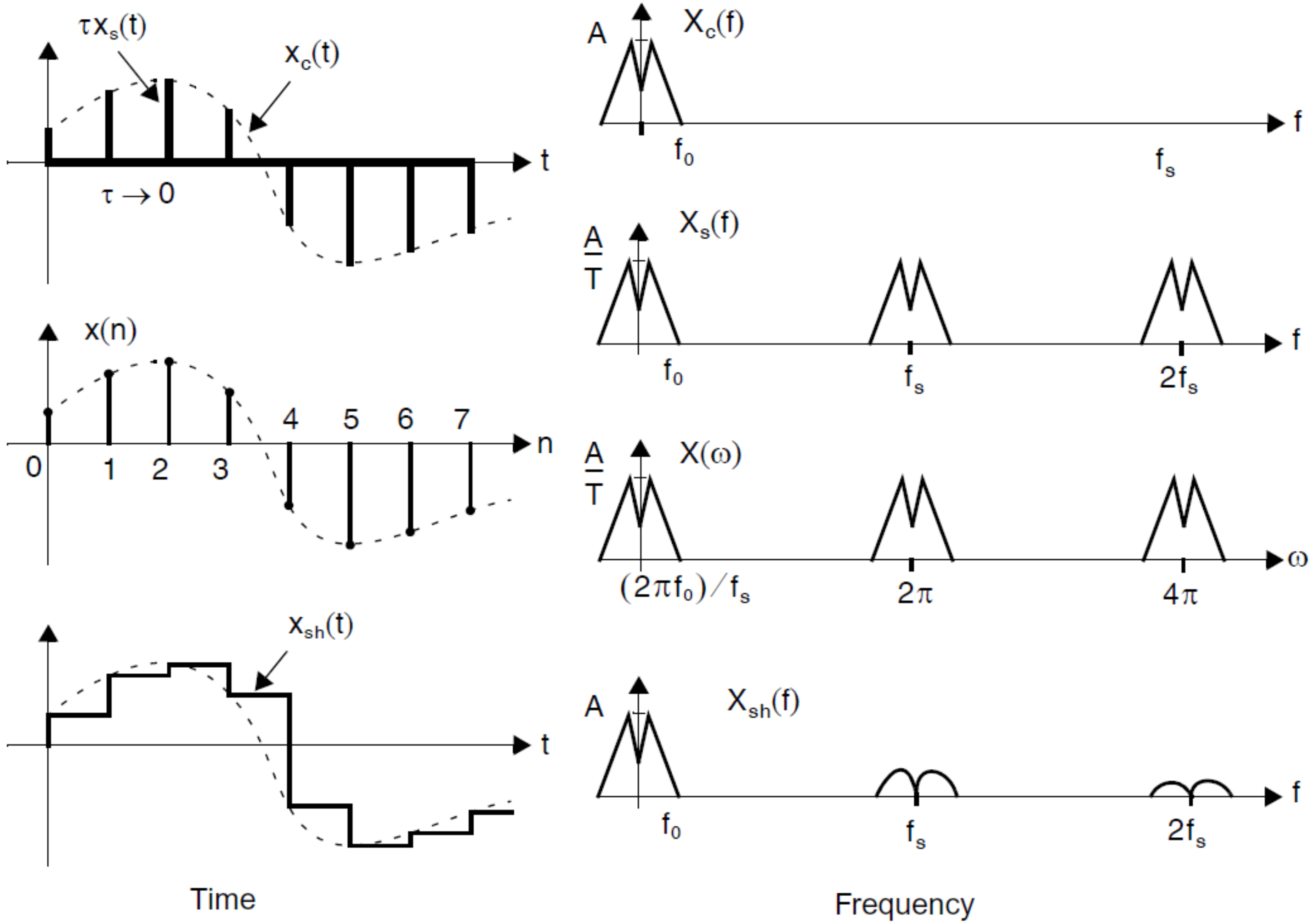
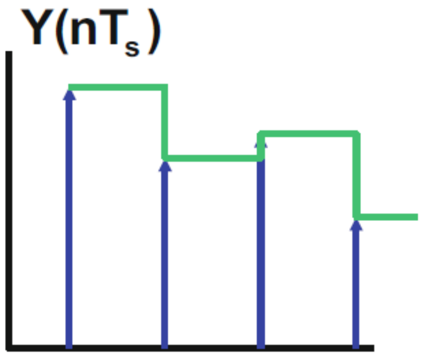
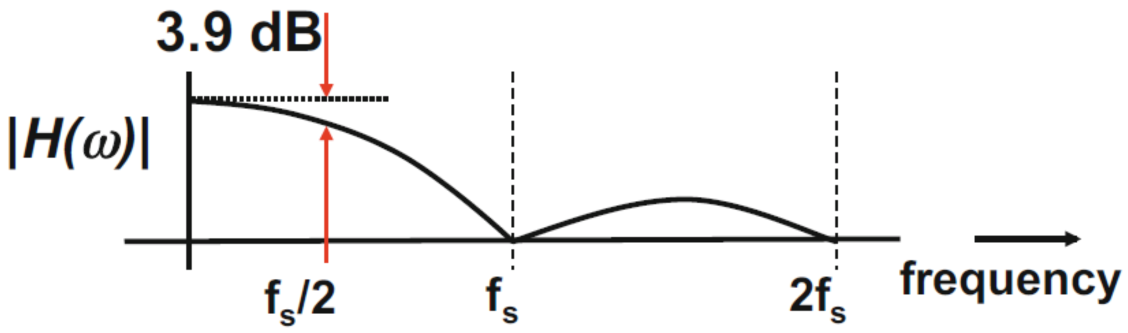


[Johns & Martin, 2012]

Passband Droop

- ZOH suppresses images but introduces amplitude distortion.
  - The passband distortion may be compensated by inverse-sinc response in the digital or analog domains.





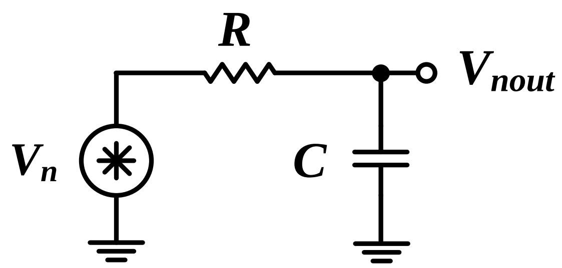
 [M. Pelgrom, 2017]

Noise in RC Circuit

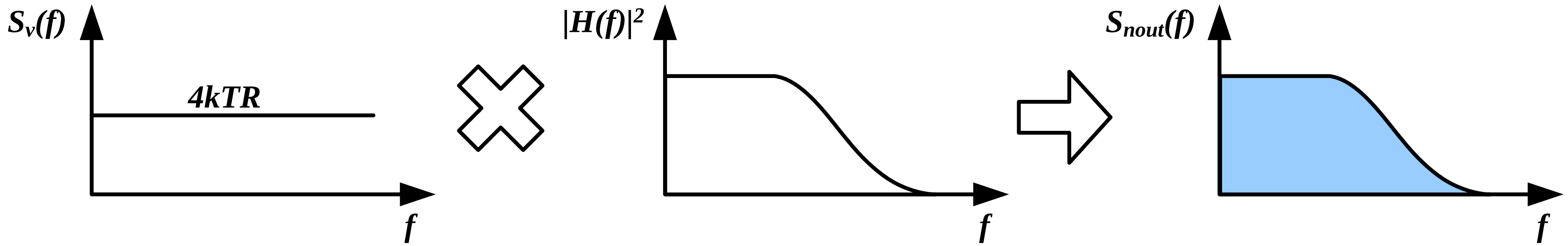
- Resistors generate white thermal noise.
  - But the BW is always limited by a cap.

$$S_{nout}(f) = S_v(f)\left|\frac{V_{nout}(j\omega)}{V_n(j\omega)}\right|^2$$

$$\overline{V_{nout}^2} = V_{noutrms}^2 = \int_{-\infty}^{\infty} S_{nout}(f)\,df$$



$$\boxed{\overline{V_{nout}^2} = \frac{kT}{C}}$$

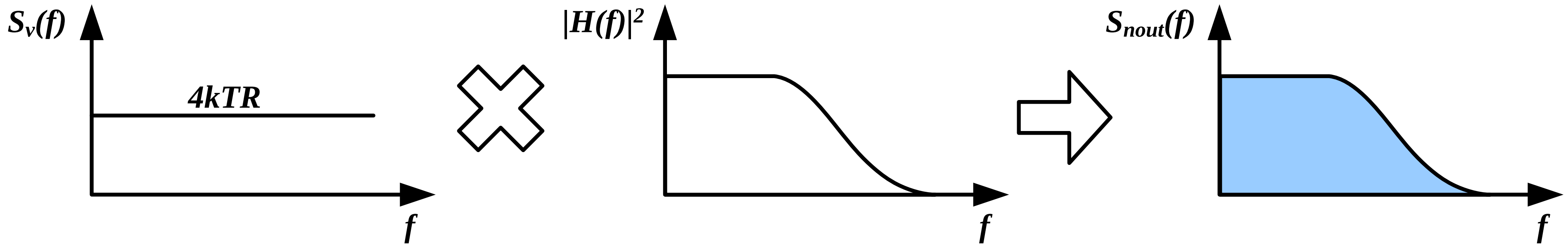


Noise in RC Circuit

$$\overline{V_{nout}^2} = \frac{kT}{C}$$

- RMS noise is independent of R! (why?)
- For $C = 1\,pF \rightarrow V_{nrms} \approx 64\ \mu Vrms$

$$V_{nrms} \approx \sqrt{\frac{1\,p}{C}} \times 64\ \mu Vrms$$

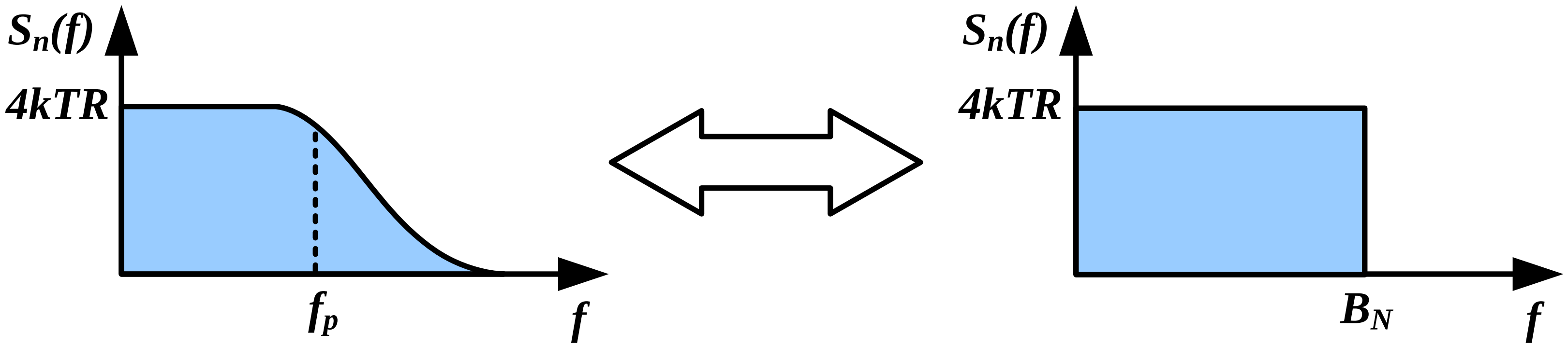


Equivalent Noise Bandwidth

- Define an equivalent noise BW (B_N) such that the area under a brick-wall response is the same area under the actual spectral density curve
- For a first order system

$$V_{nrms}^2 = \int_{-\infty}^{\infty} S_n(f)\,df = 4kTR \times B_N = \frac{kT}{C}$$

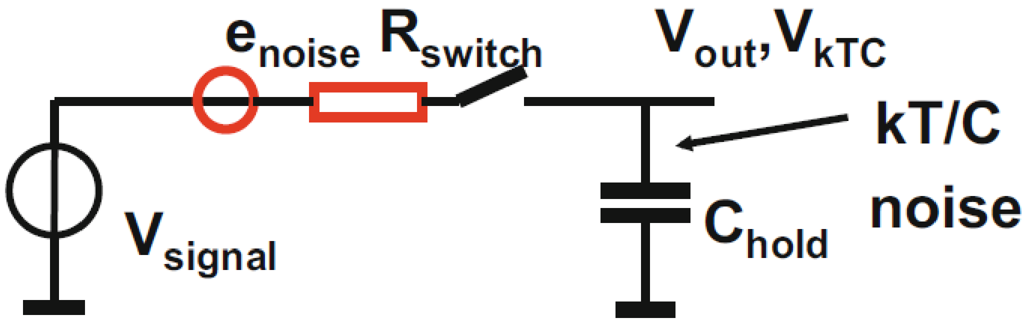
$$B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$$



Sampling Noise

- The sampling capacitor determines noise power ➔ SNR ➔ No. of ADC bits.

C_{hold}	$V_{nrms} = \sqrt{\frac{kT}{C}}$ at $T = 300\ K$	SNR (assume $V_{sigrms} = 1\ Vrms$)	No. of bits (see next lecture)
100 fF	203 μVrms	74 dB	12-bit
1 pF	64 μVrms	84 dB	13.7-bit
10 pF	20.3 μVrms	94 dB	15.4-bit



[M. Pelgrom, 2017]

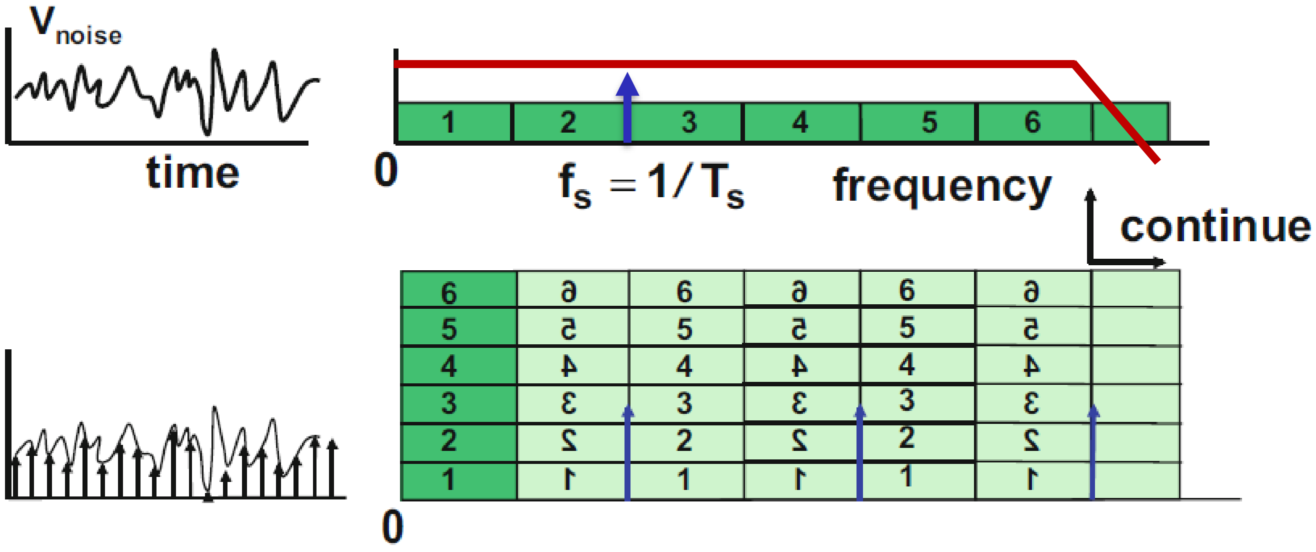
Noise Folding

- Before sampling: $P_n = kT/C = S_n(f) \times B_N$
- After sampling P_n is unchanged: $P_n = kT/C = S_{n,sampled}(f) \cdot \frac{f_s}{2}$

$$S_{n,sampled}(f) = \frac{kT}{C} \times \frac{2}{f_s} = S_n(f) \times B_N \times \frac{2}{f_s}$$

$$S_{n,sampled}(f) = S_n(f) \times \frac{2B_N}{f_s} = S_n(f) \times \frac{\pi BW}{f_s}$$

- Noise power is unchanged, but noise density increases (noise folding).



[M. Pelgrom, 2017]

References

- M. Pelgrom, Analog-to-Digital Conversion, Springer, 3rd ed., 2017.
- W. Kester, The Data Conversion Handbook, ADI, Newnes, 2005.
- B. Boser and H. Khorramabadi, EECS 247 (previously EECS 240), Berkeley.
- B. Murmann, EE 315, Stanford.
- Y. Chiu, EECT 7327, UTD.

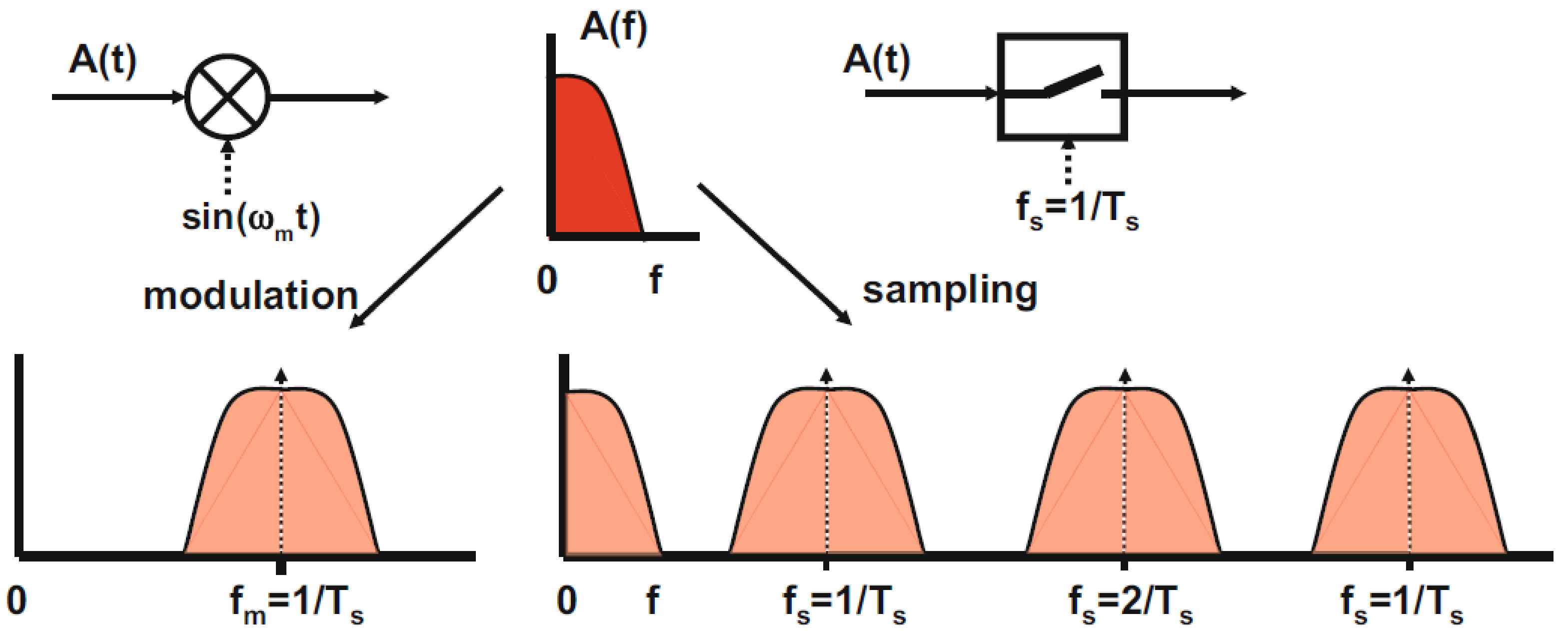
Thank you!

Sampling vs Modulation

- Two important dualities between TD and FD

  Discrete <-> Periodic & **Multiplication <-> Convolution**

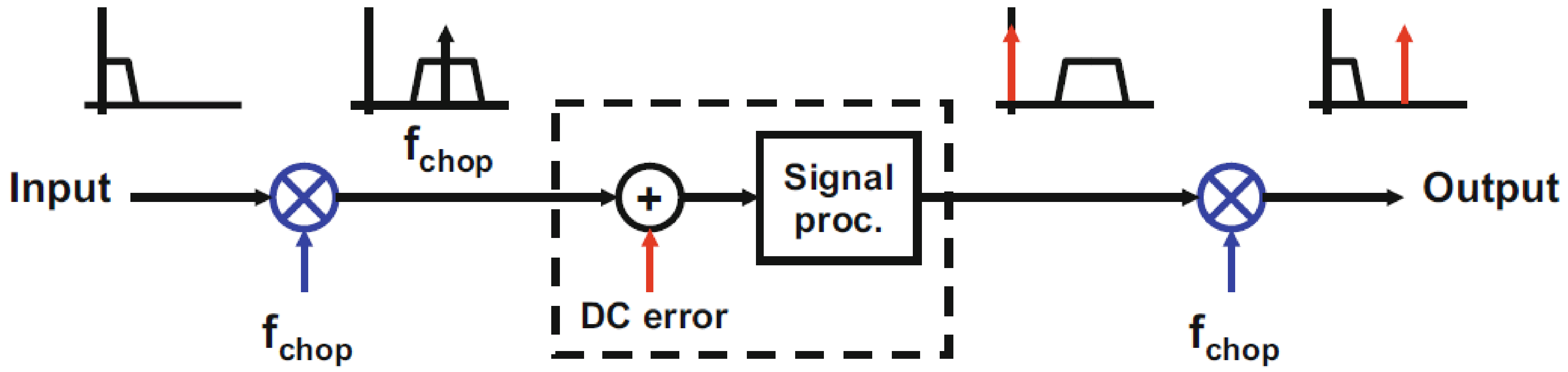
- A stream of impulses with period T_s in TD is a stream of impulses with period f_s in FD
  - Sampling can be viewed as a summation of modulations



[M. Pelgrom, 2017]

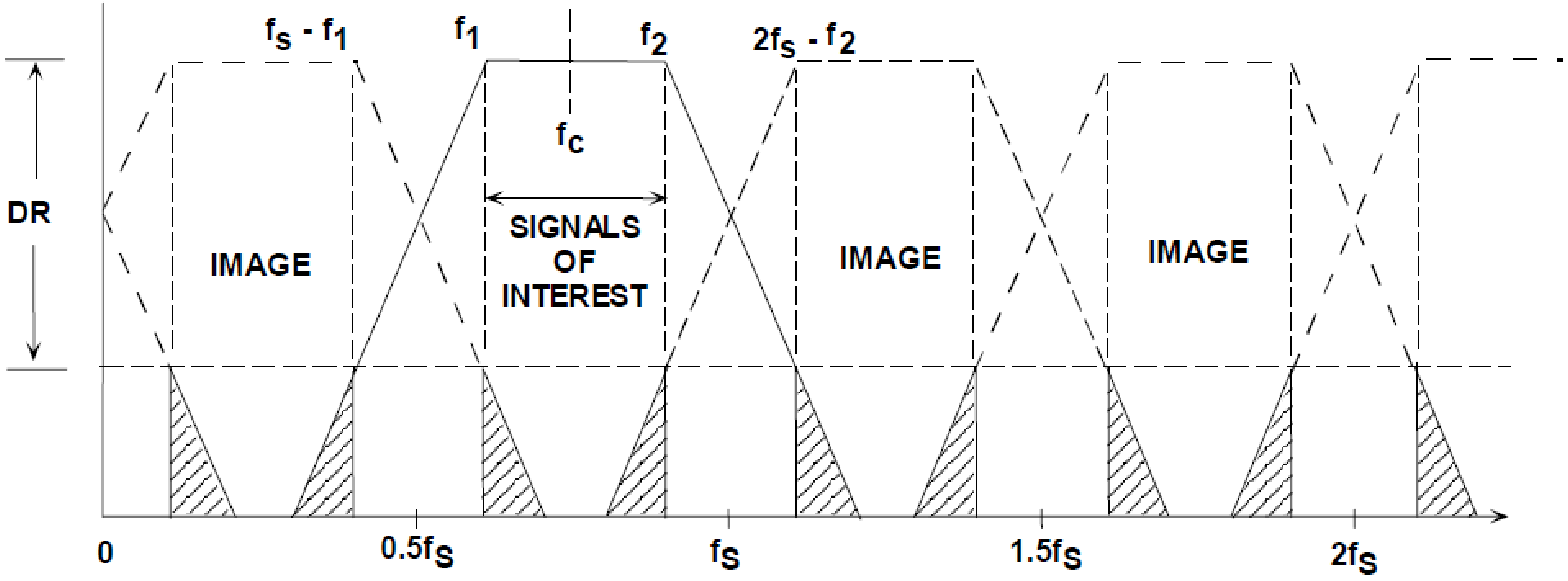
Chopping

- Chopping is a technique used for improving accuracy.
  - Sensitive signals are modulated to frequency bands where the signal processing is free of errors.
  - Mitigates the effect of DC offsets, flicker noise, etc.
- In differential circuits, chopping is implemented easily by alternating between the differential branches.



[M. Pelgrom, 2017]

Subsampling Anti-Aliasing Filter



BANDPASS FILTER SPECIFICATIONS:

- STOPBAND ATTENUATION = DR
- TRANSITION BAND: f_2 TO $2f_s - f_2$
  f_1 TO $f_s - f_1$
- CORNER FREQUENCIES: f_1, f_2

[W. Kester, 2005]