

Analog Integrated Systems Design

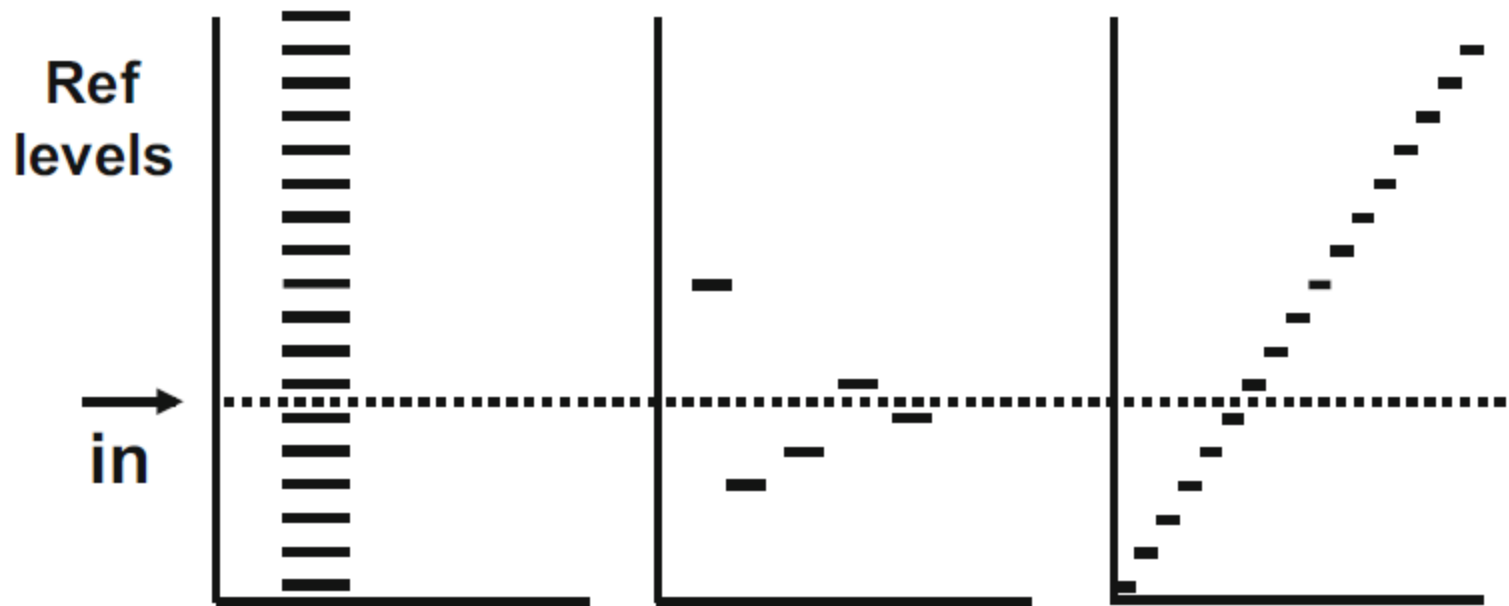
Lecture 12 Nyquist ADCs (1)

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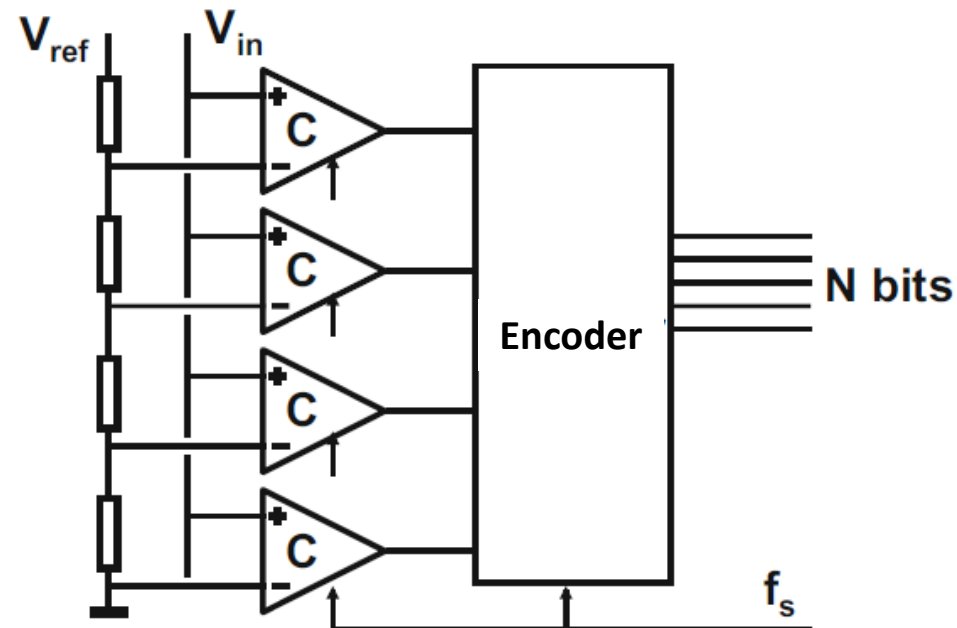
ADCs Classification

- ❑ Nyquist ADCs: signal BW close to Nyquist limit ($f_s/2$)
 1. Parallel search (e.g., flash ADC): $T_{conv} \sim T_{clk}$
 2. Sequential search (e.g., SAR ADC): $T_{conv} \sim N \times T_{clk}$
 3. Linear search (e.g., dual-slope ADC): $T_{conv} \sim 2^N \times T_{clk}$



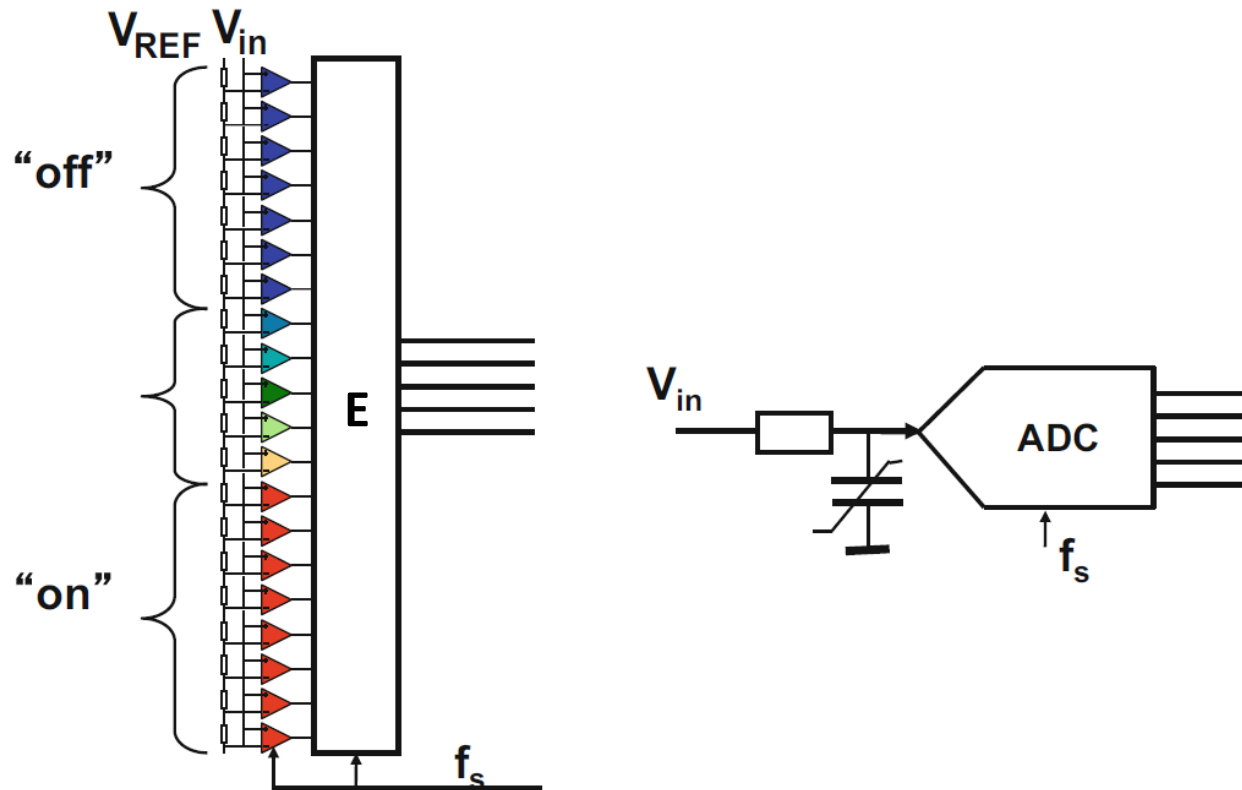
Flash ADC

- ❑ Resistor ladder DAC
- ❑ $(2^N - 1)$ comparators \rightarrow complexity, power, area, and input capacitance grow exponentially with no. of bits
- ❑ Parallel search \rightarrow very fast
 - Encoder converts thermometer code to straight binary code
- ❑ S/H inherent in the digital latches of the comparators



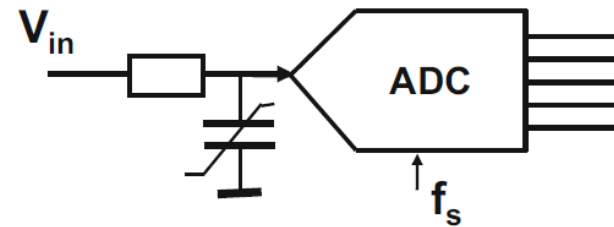
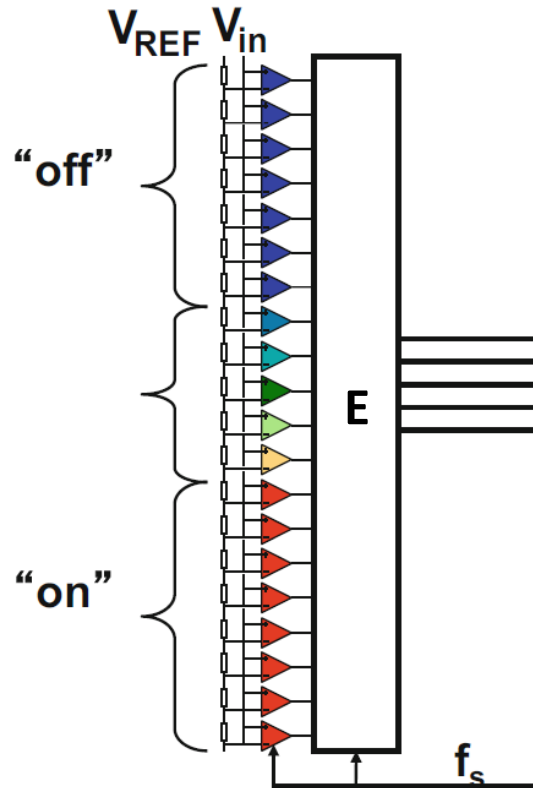
Nonlinear Input Impedance

- ❑ Input cap of comparator is the input cap of a diff pair
- ❑ Comparators below the decision level create a high capacitance
- ❑ Comparators above the decision level will show a low capacitance
- ❑ Non-linear input capacitance creates distortion



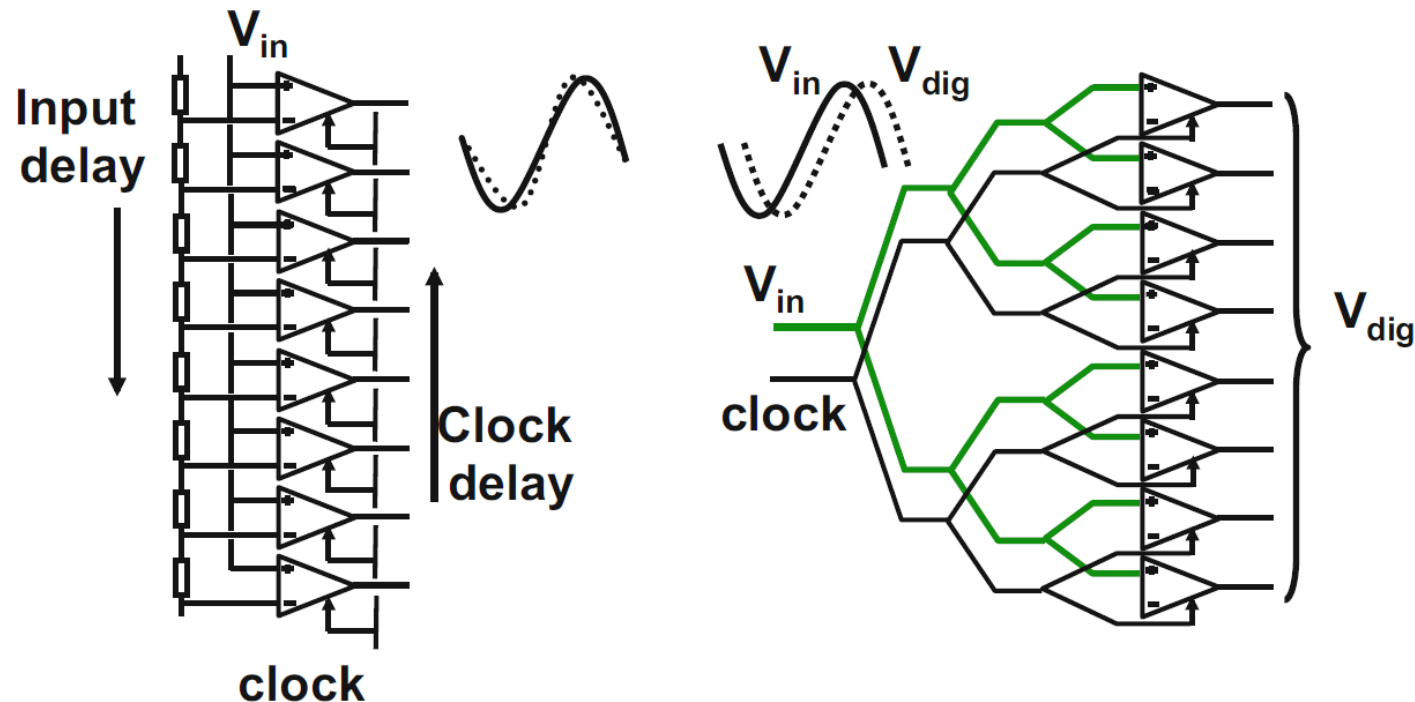
Nonlinear Input Impedance

- ❑ Analysis shows: $HD2 = 20 \log \frac{\omega_{sig} R_{sig} (C_{max} - C_{min})}{4}$
- ❑ Differential architecture cancels 2nd order distortion
 - But more current and area



Tree Layout

- ❑ The impedance of the wiring in combination with a string of load elements (e.g., inputs to the comparators) can easily lead to delay differences between individual comparators
 - Timing error translated to 2nd order distortion $\sim 0.5\omega_{sig}\Delta T$
- ❑ Tree-like layout is necessary to avoid delay differences



Flash ADC Example

□ Given:

- $V_{REF} = 1V, R_{tap} = 1\Omega, C_{tap} = 100fF, \text{ and } N = 7$

□ Time constant for resistor ladder

$$\tau = \frac{R_{tot}C_{tot}}{\pi^2} = \frac{R_{tap}C_{tap}2^{2N}}{\pi^2} = 0.16ns$$

- R_{tot} is total resistance and C_{tot} is total capacitance

□ For 0.5LSB error

$$f_s = \frac{1}{0.69(N+1)\tau} \approx 1.1GHz$$

□ $R_{tot} = 128\Omega \rightarrow I = \frac{V_{REF}}{R_{tot}} = 8mA$

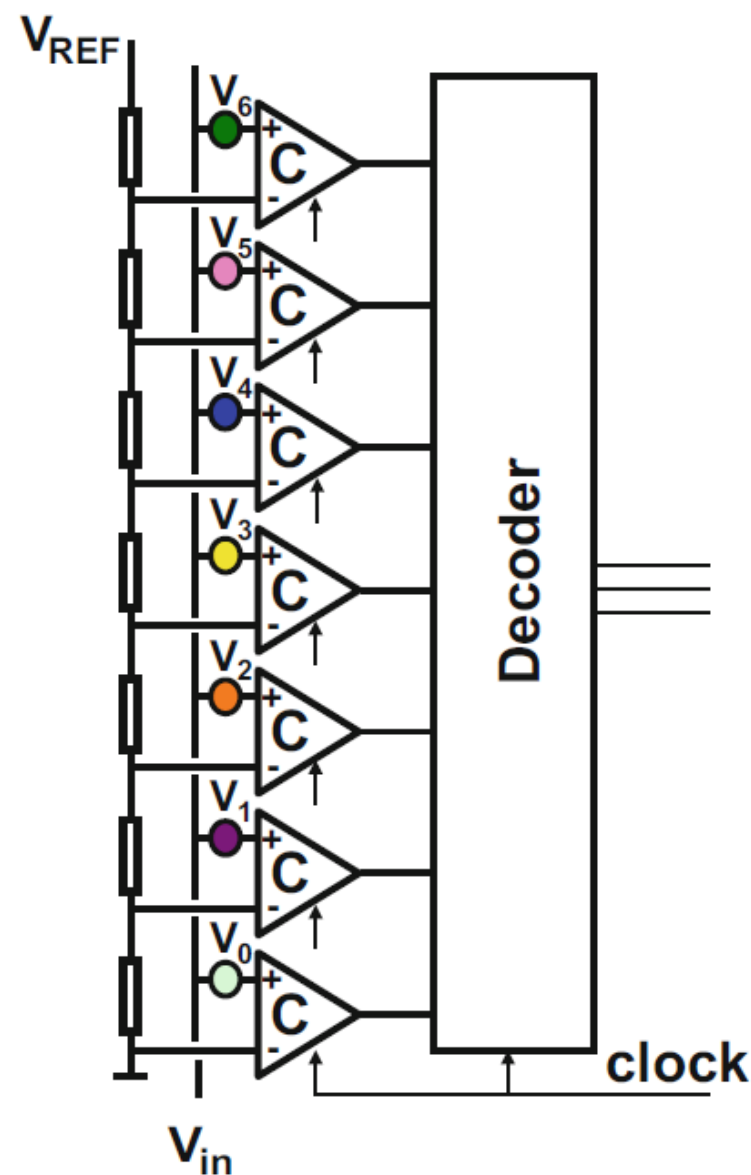
□ Low-ohmic ladder is required: Often constructed from metal layers

Flash Comparator

- ❑ Requirements:
 - Low capacitive load
 - No DC input current
 - Low kick-back noise
 - High switching speed (large bandwidth)
 - Low power
 - Low random offset
- ❑ All requirements need small devices except the offset requirement
 - 0.5 LSB must be $> 3\text{-sigma-offset}$
 - BJT has 10x lower offset compared to MOSFET
 - But it draws DC current → Use Darlington pair

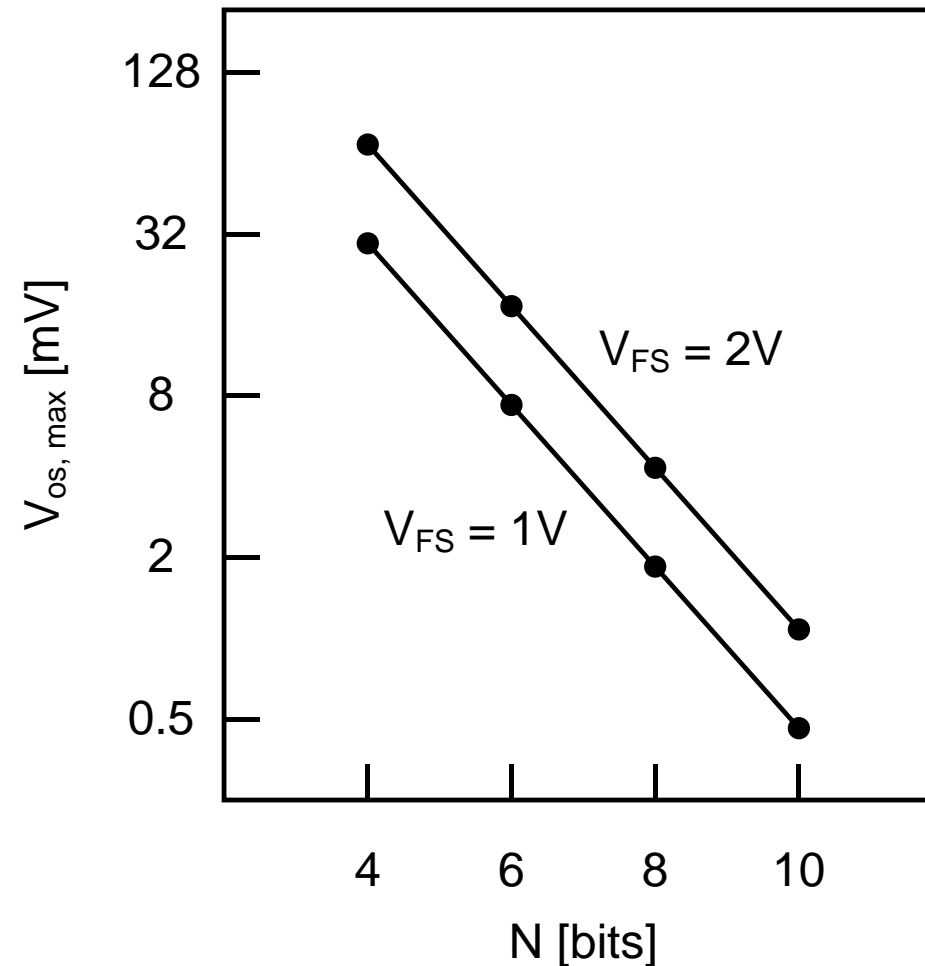
Comparator Offset

- ❑ Flash resolution limited by the input referred mismatch of the comparators
- ❑ For BJT $\sigma_{in,os} \approx 0.3mV$
- ❑ For MOSFET $\sigma_{in,os}$ is much higher
 - Ex: $\sigma_{in,os} \approx 5mV$
 - $0.5LSB > 3\sigma_{in,os}$
 - $LSB = 30mV$
 - Let $VFS = 1V$
 - $N = \log_2(VFS/LSB) = 5\text{-bit}$
- ❑ Input CM difference creates systematic mismatch between comparators



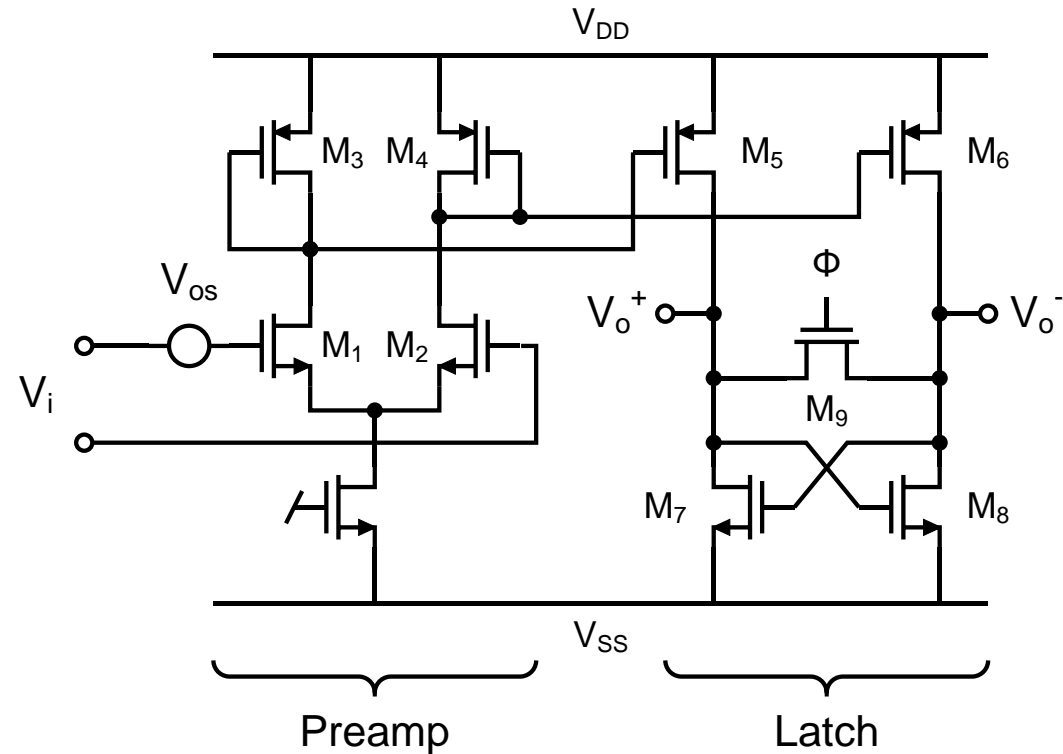
Offset-Limited Resolution

- ❑ Practically limited to resolutions of 4-8 bits



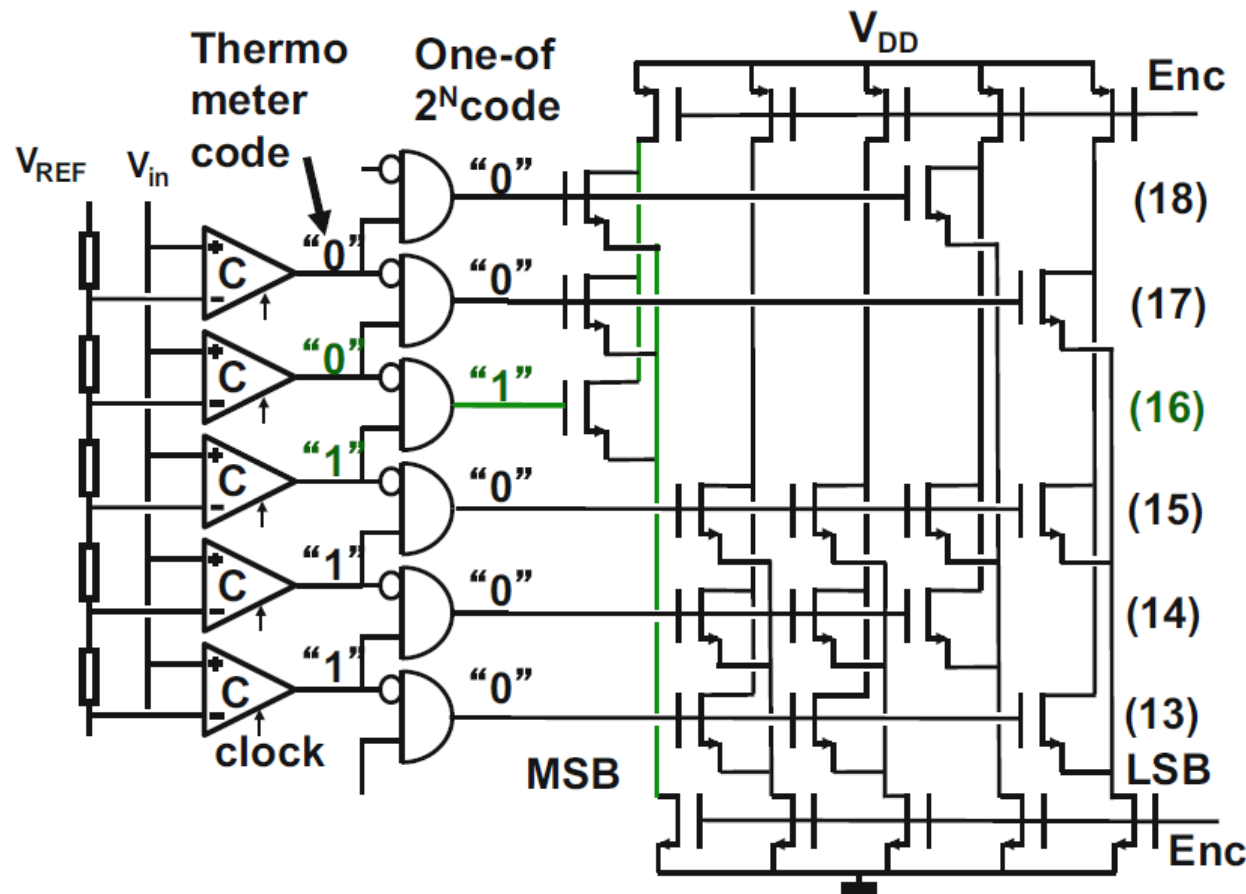
Comparator Offset Sources Revisited

- ❑ Preamplifier input pair mismatch (V_{th}, W, L)
- ❑ PMOS loads mismatch
- ❑ Latch mismatch
- ❑ CI / CF imbalance of reset switch (M_9)
- ❑ Clock routing
- ❑ Parasitics
- ❑ Typical flash comparator may use multiple preamplifiers



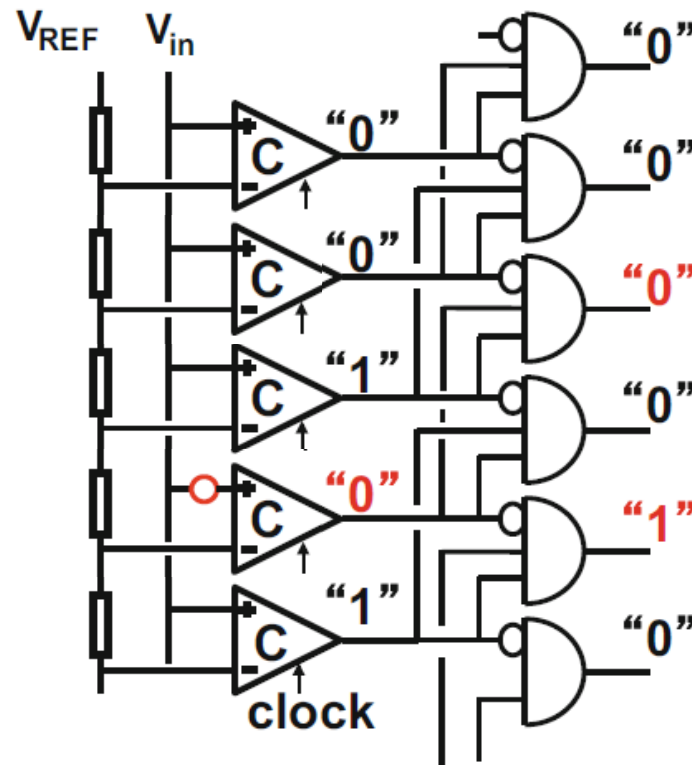
Thermometer Encoder

- ❑ Thermometer code converted to 1-of- 2^N code
- ❑ Encoder to straight binary similar to dynamic NOR gate (wired-NOR) → Matrix of transistors as a straight binary code



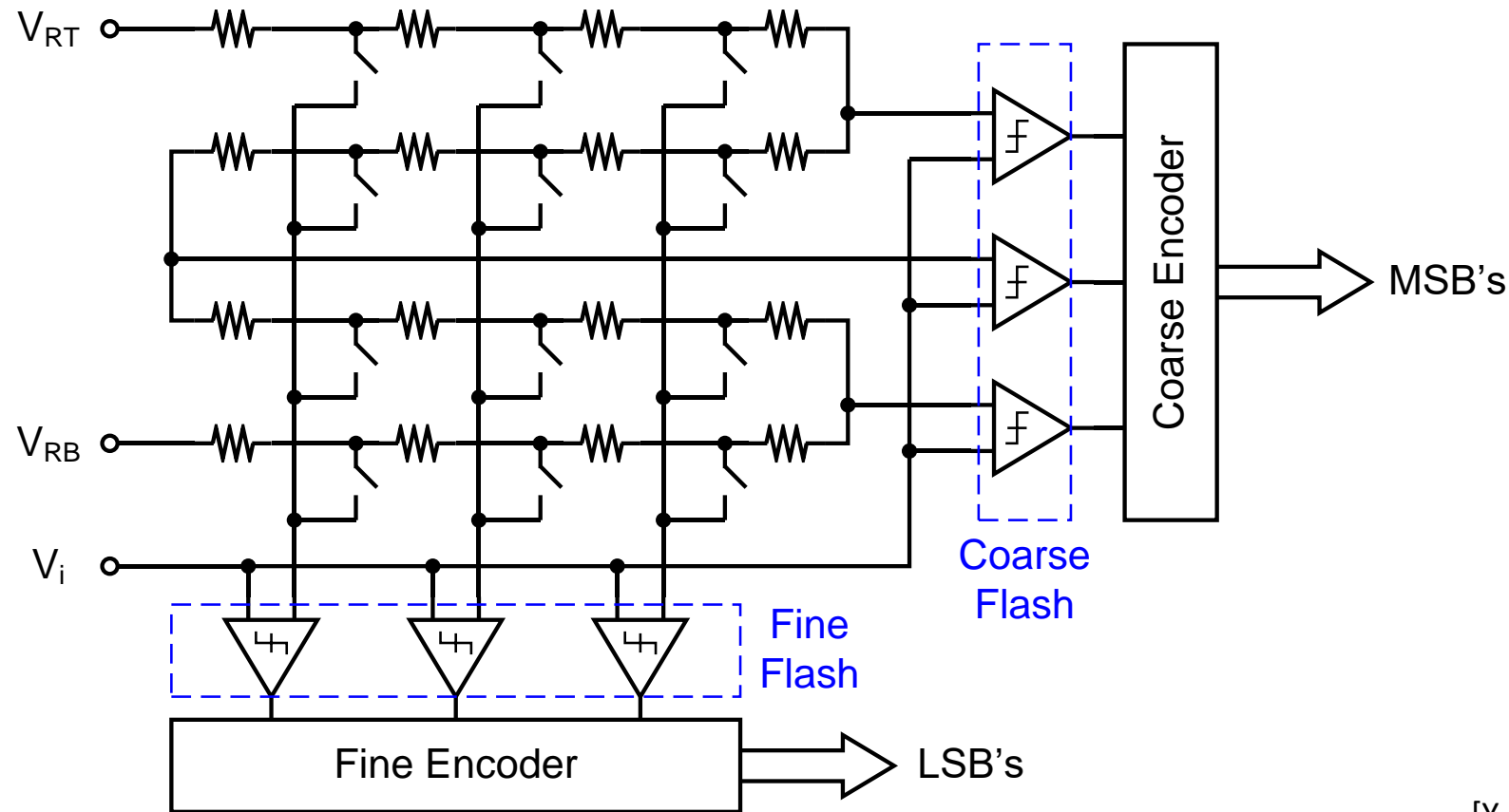
Sparkle Codes

- ❑ Mismatch, metastability, noise may make more than one Wired-NOR input active → Sparkle codes (a.k.a. bubble errors)
- ❑ Many bubble correction schemes were proposed (may provide “biased correction”)
 - Trade-off between robustness and speed



Subranging/Coarse-Fine ADC

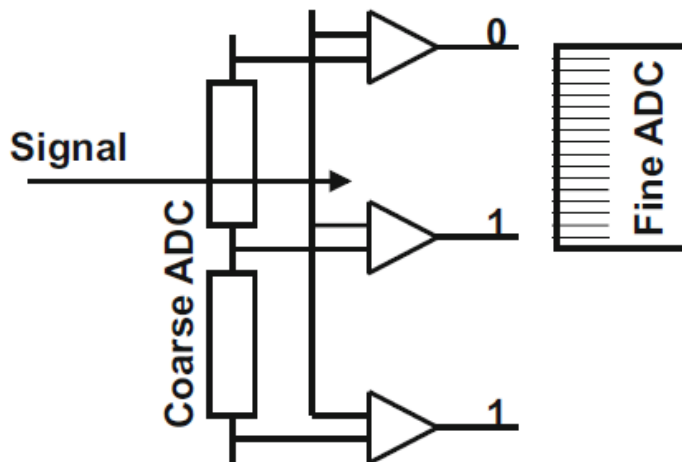
- ❑ Ruler analogy: coarse step gets “cm” and fine step gets “mm”
- ❑ Reduced complexity (relative to flash): $N = N_1 + N_2 \rightarrow (2^{N_1} + 2^{N_2} - 2)$ comparators
- ❑ Reduced C_{in} , area, and power consumption



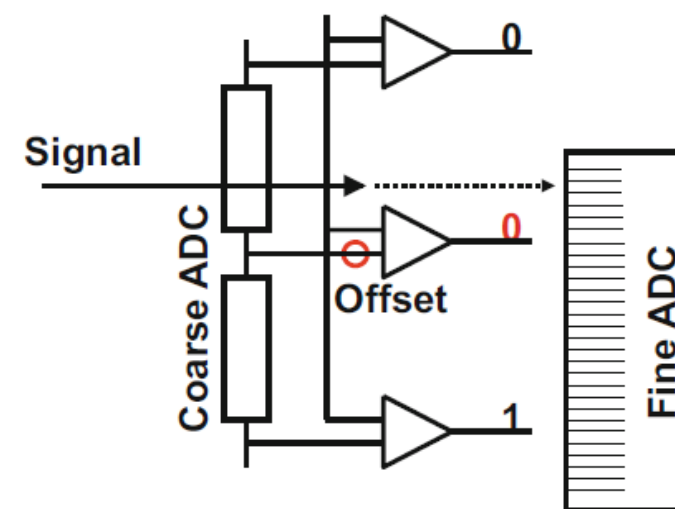
Over-range

- ❑ By adding additional levels on both sides of the fine ADC range, errors from the coarse ADC can be corrected
- ❑ One form of using redundancy to recover errors

The coarse flash converter takes a correct decision and the fine-converter receives a signal within its range

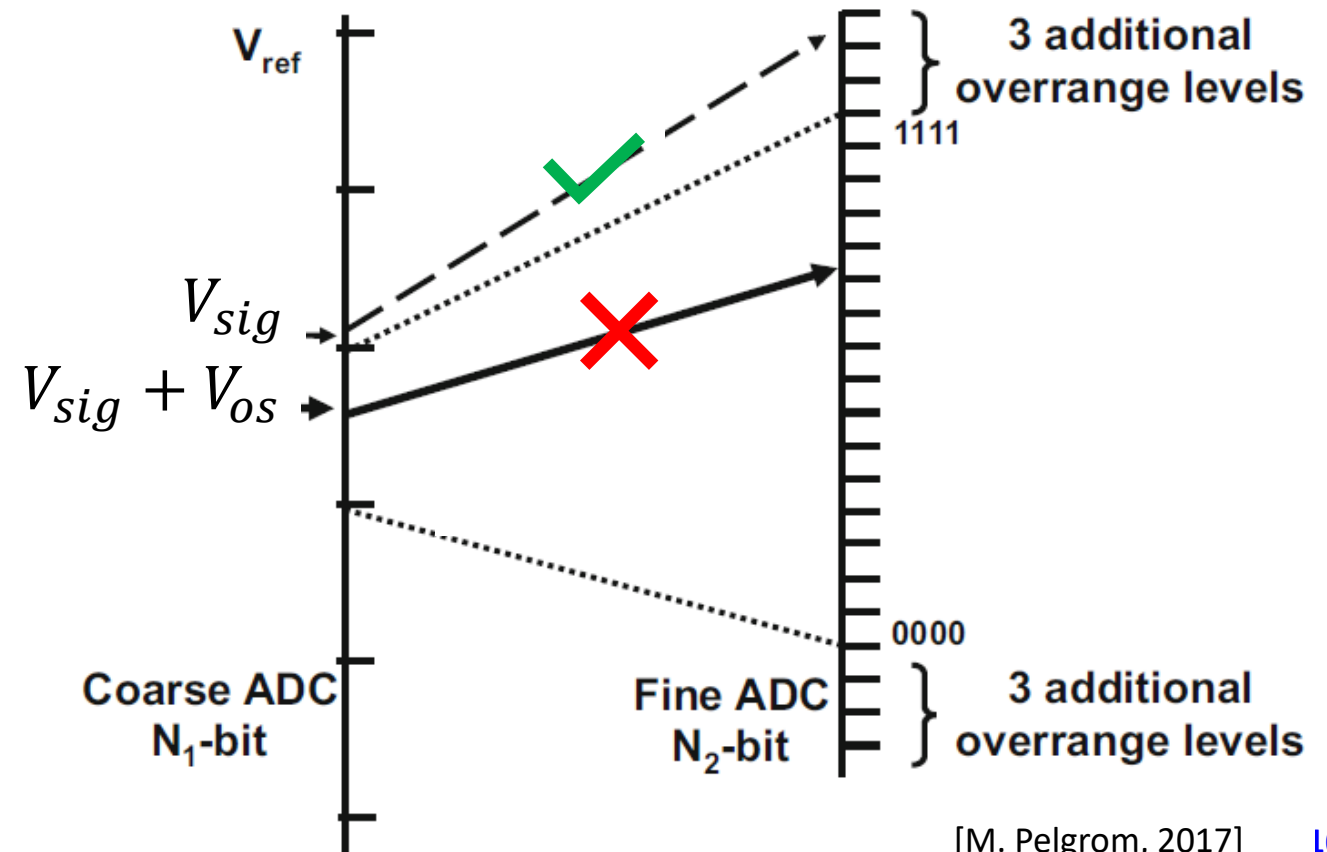


One comparator has offset and the coarse converter indicates the wrong range for the fine converter



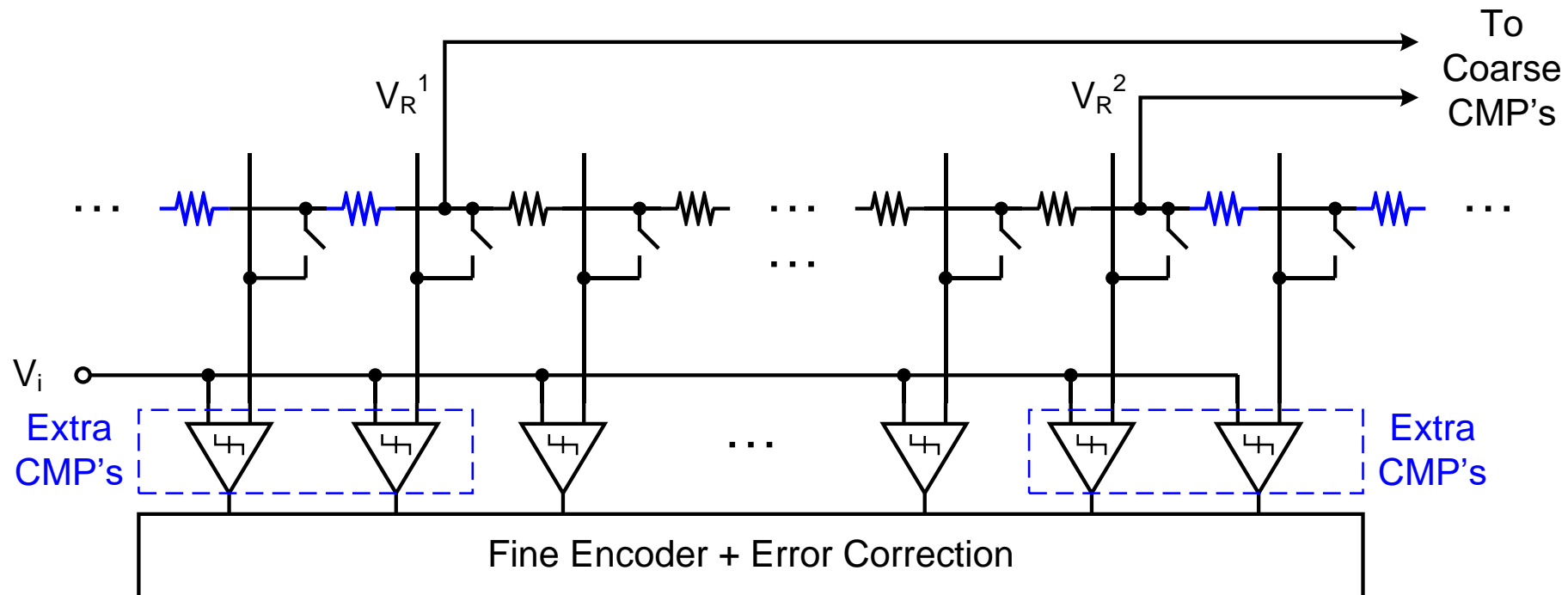
Over-range

- ❑ Correct coarse decision (dashed line)
- ❑ Wrong coarse decision (bold line): Over-range in the fine converter corrects this wrong decision
 - We added 6 more levels to fine
 - $N_{fine} = \log_2(16 + 6) = 4.46 \text{ bit}$



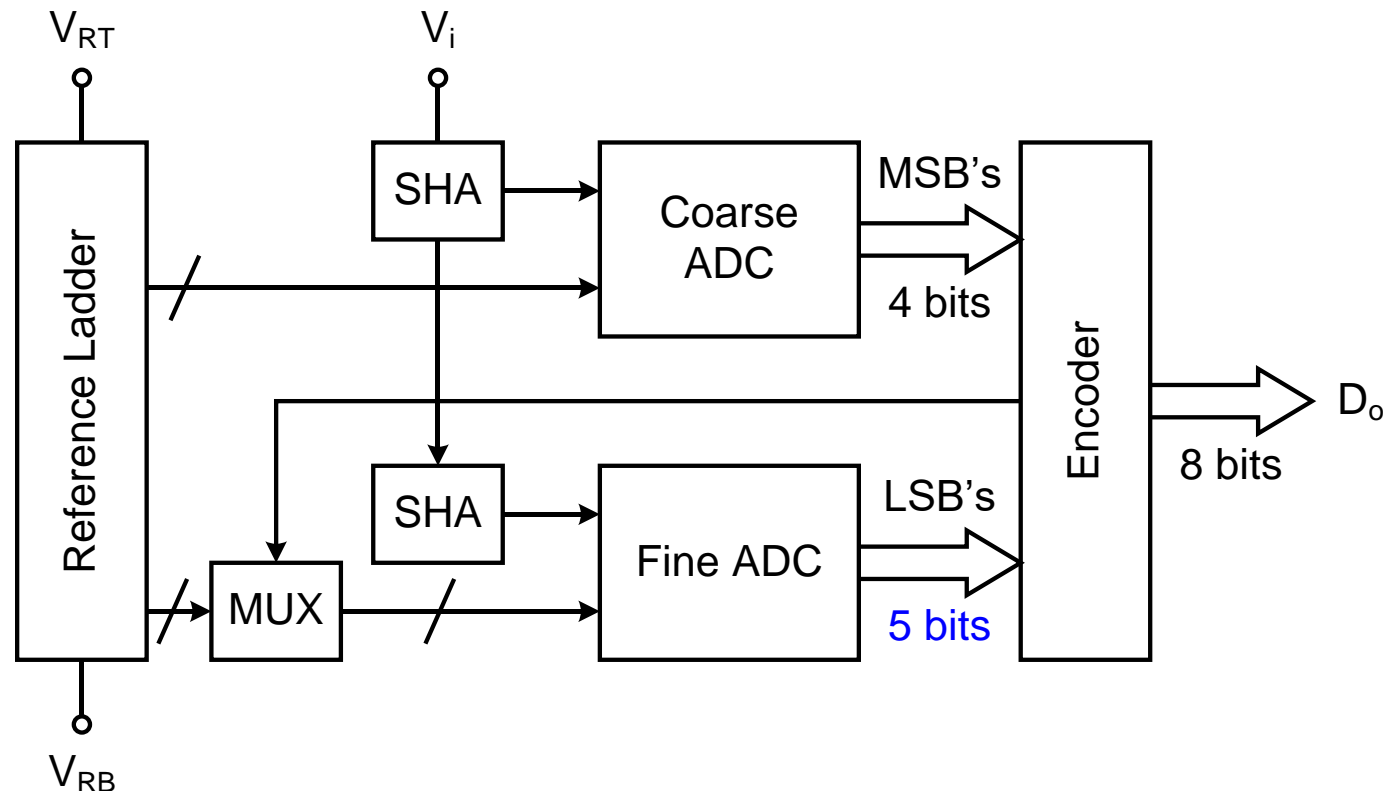
Digital Redundancy in Fine ADC

- ❑ The range of fine ADC search extended on both sides
- ❑ Offset tolerance on coarse comparators can be relaxed
- ❑ Fine comparator offset still must be controlled to N-bit level



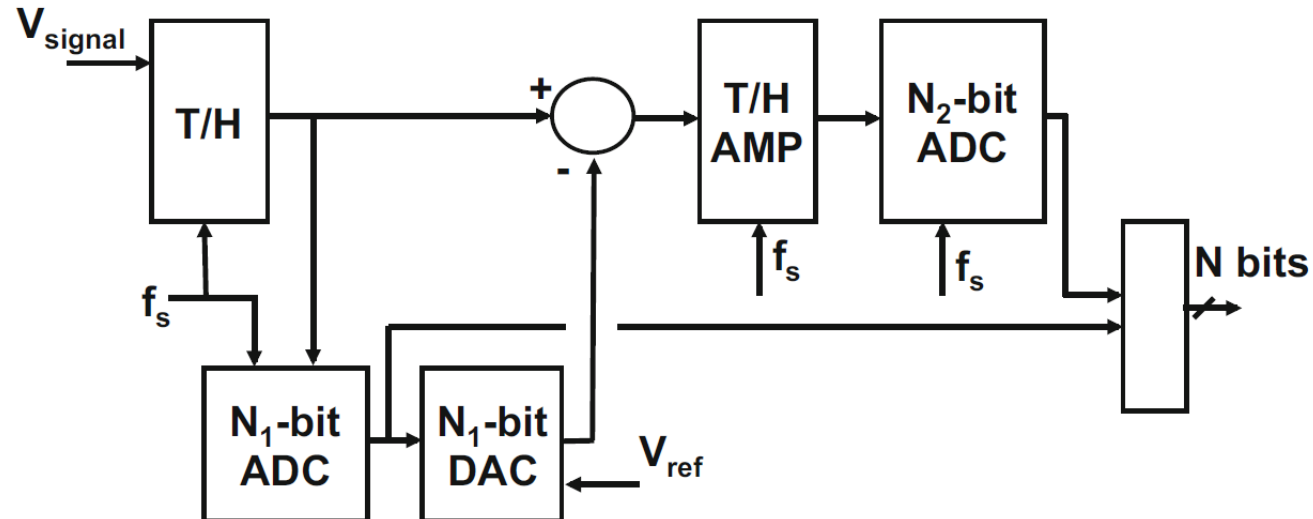
Subranging/Coarse-Fine ADC Block Diagram

- ❑ Typically two SHAs are required for the coarse and fine ADCs
- ❑ Typically 3 clock phases per conversion
 - Sample + Coarse comparison + Fine comparison
- ❑ No residue amplifier required (compare to next slide)



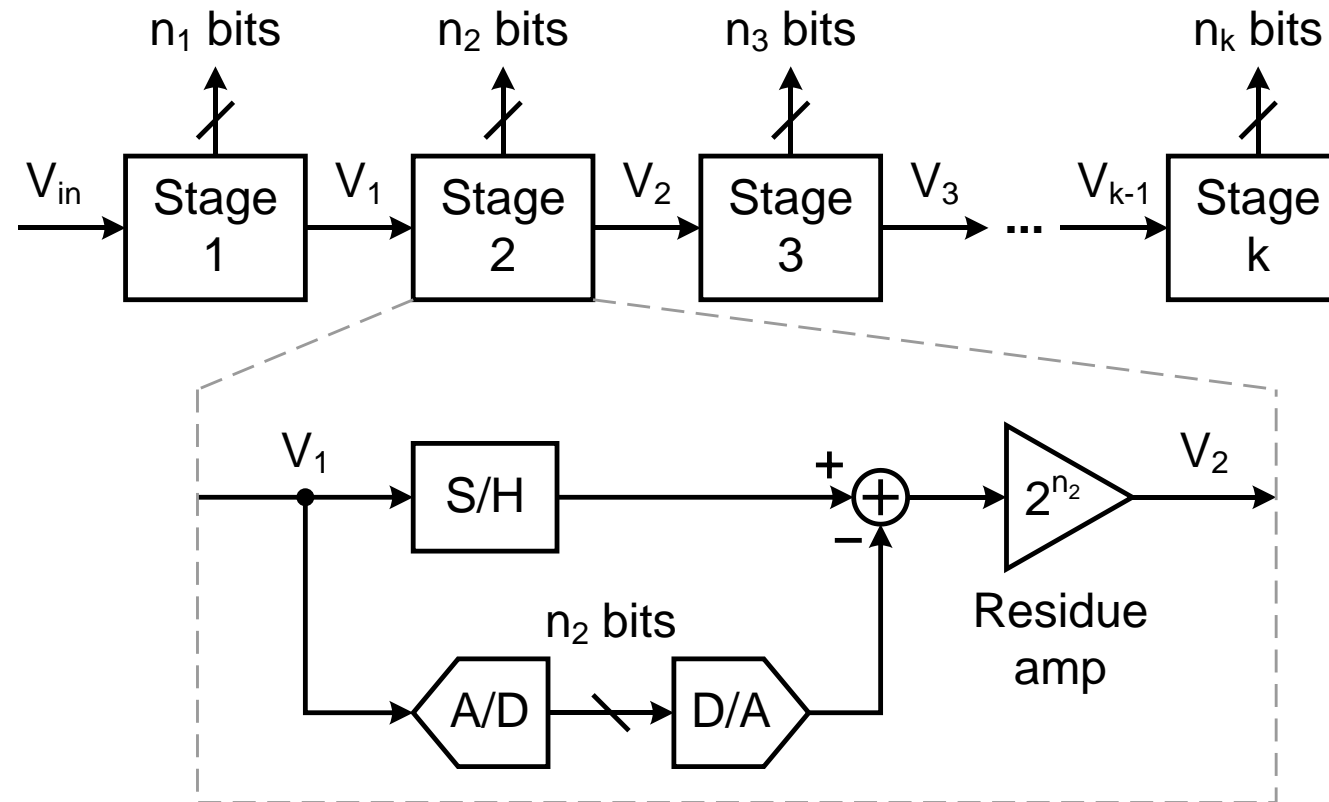
Two-Step Subranging/Pipelined ADC

- ❑ A coarse ADC estimates the input signal
 - Result fed to a DAC → Subtraction generates residue signal
- ❑ A fine ADC converts the residue signal
- ❑ Amplification may precede the fine ADC (residue amplifier)
 - Limited speed (GBW = const)
 - 2nd T/H allows pipelining → pipelined ADC
- ❑ $N = N_1 + N_2 \rightarrow (2^{N_1} + 2^{N_2} - 2)$ comparators



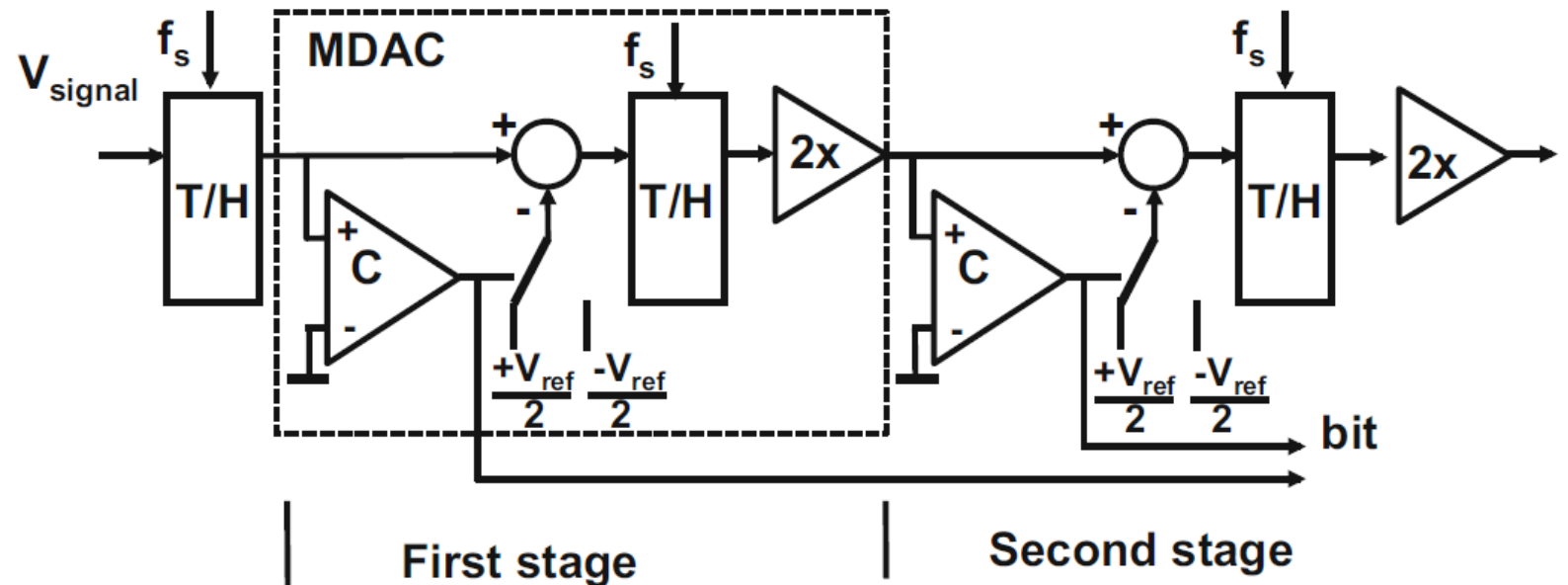
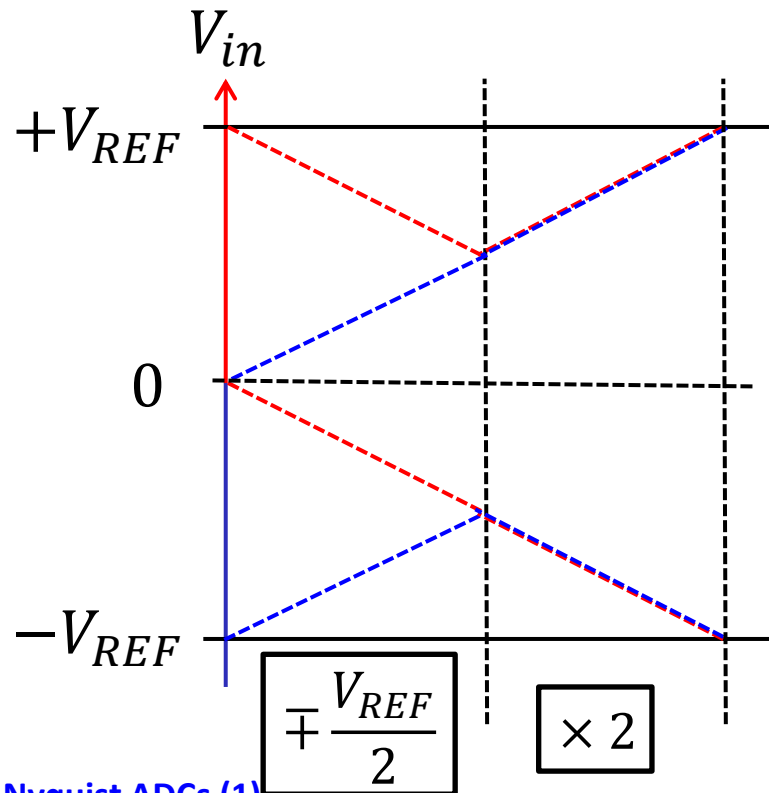
Pipelined ADC

- ❑ A generalization of subranging ADC: Cascade of k -bit stages
 - $N = n_1 + n_2 + \dots + n_k$
- ❑ Sample rate is high, but large latency (not suitable for FB loops)



1-bit/stage Pipelined ADC

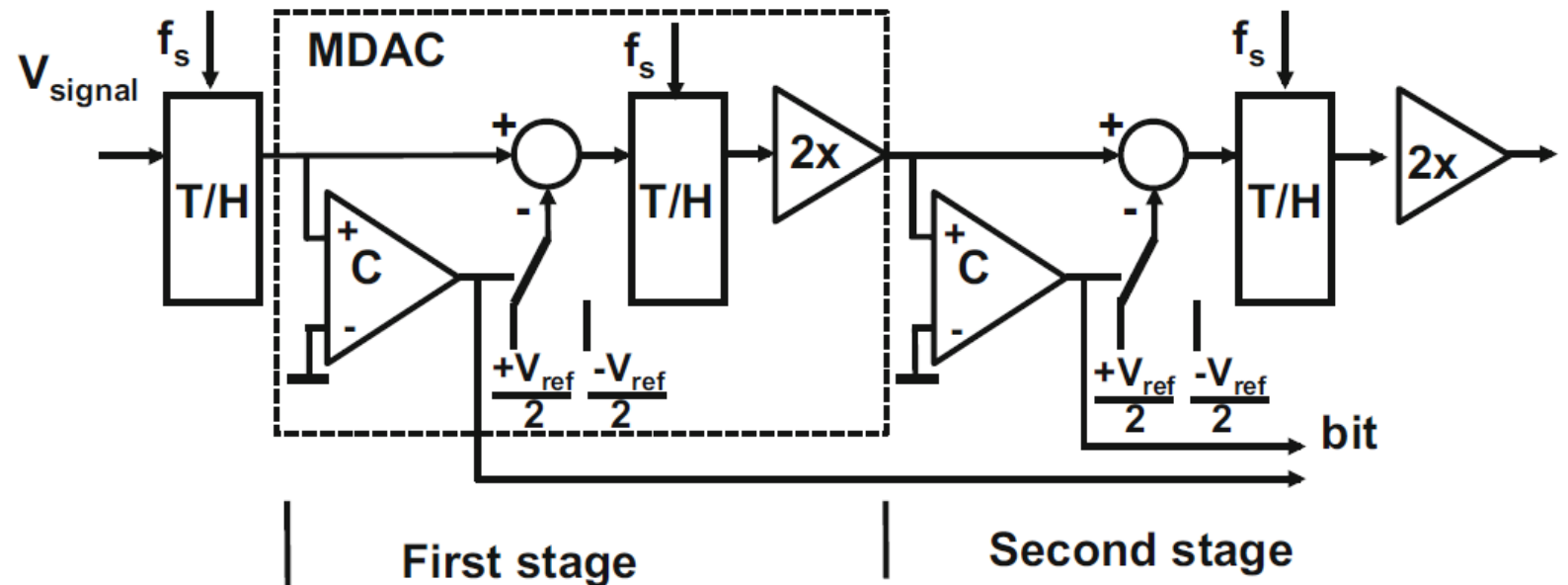
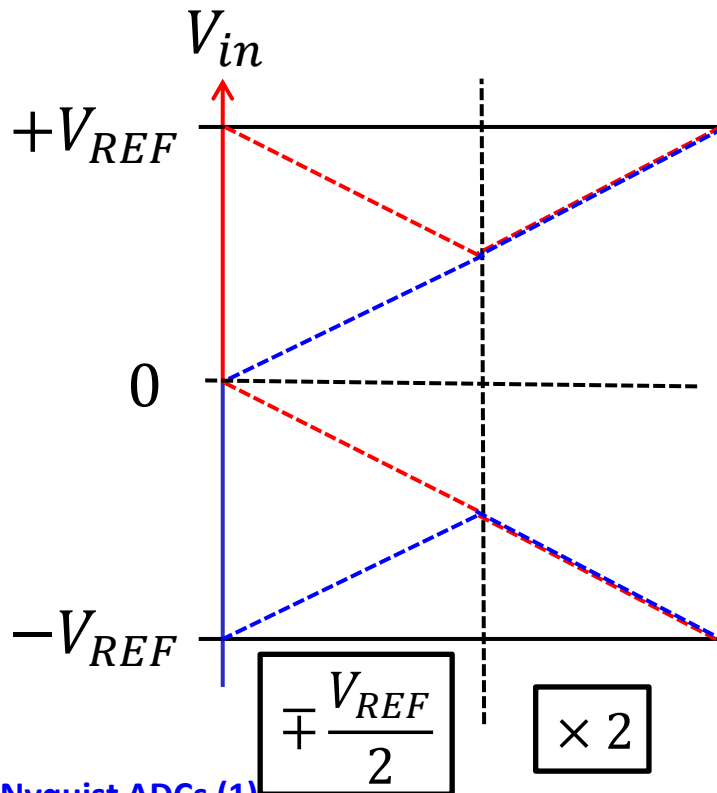
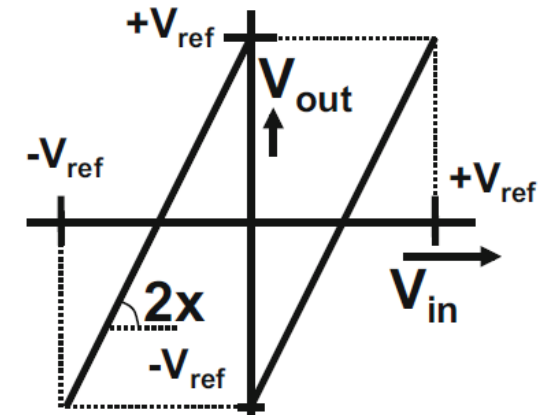
- ❑ The ADC is 1-bit: comparator
- ❑ The DAC is also 1-bit → perfectly linear by definition
- ❑ Low inter-stage gain (x2): high speed (GBW = const)
- ❑ No. of pipelined stages (a.k.a. MDACs) = no. of bits



MDAC Characteristics

$$V_{out} = 2 \left(V_{in} \mp \frac{V_{ref}}{2} \right)$$

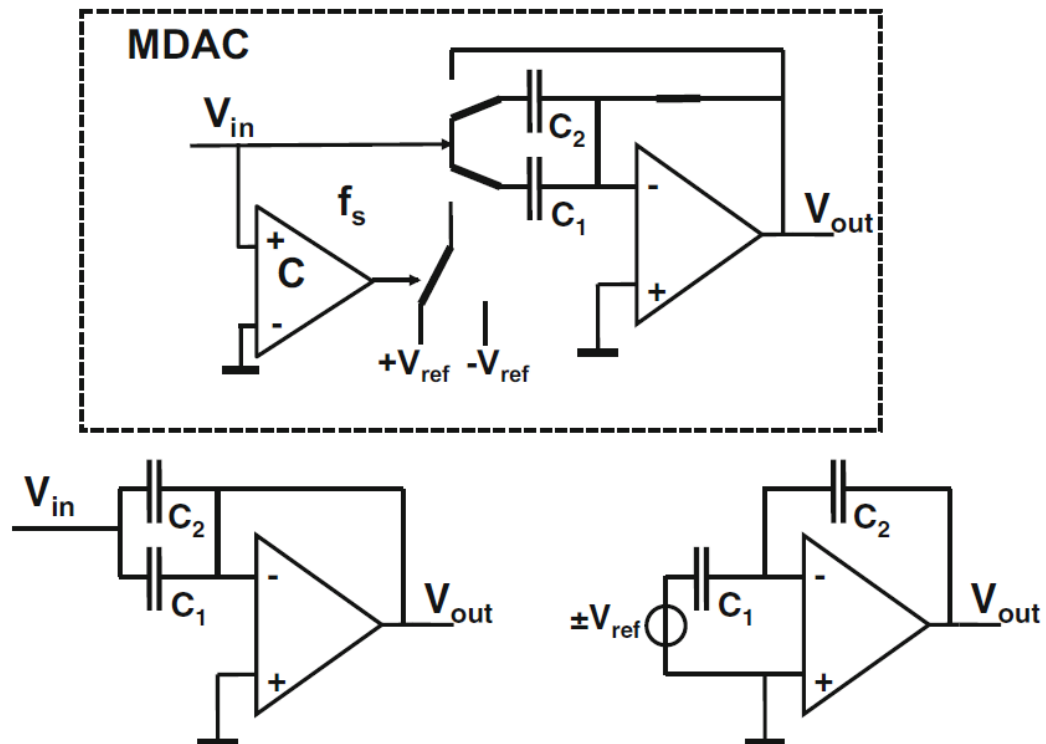
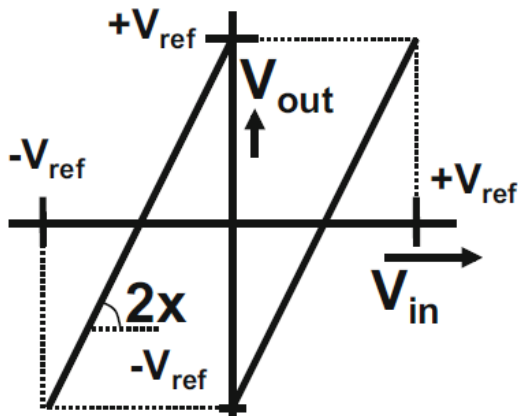
- ❑ Comparator threshold defines breakpoint
- ❑ DAC output (multiplied by $\frac{DAC\ gain}{V_{in}\ gain} = \frac{2}{2}$) defines x-intercept



MDAC Implementation

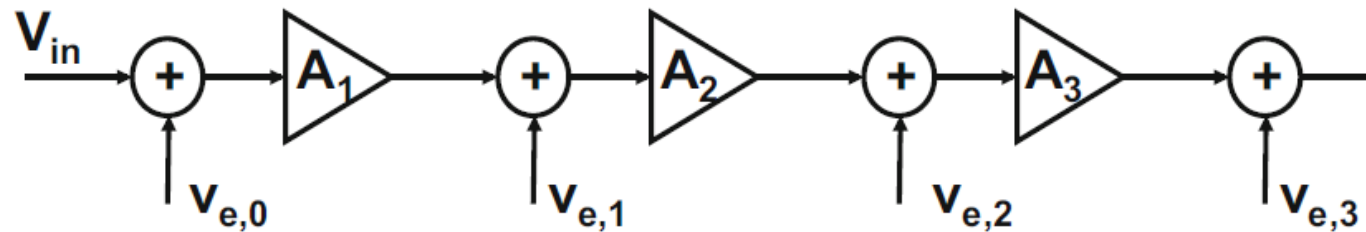
$$V_{out} = \frac{C_1 + C_2}{C_2} V_{in} \mp \frac{C_1}{C_2} V_{ref} = 2 \left(V_{in} \mp \frac{V_{ref}}{2} \right)$$

- ❑ Comparator threshold defines breakpoint
- ❑ DAC output (multiplied by $\frac{DAC \text{ gain}}{V_{in} \text{ gain}} = \frac{1}{2}$) defines x-intercept



Pipelined ADC Error Sources

- ❑ Comparator offset, residue amplifier offset, exact 2x amplification
- ❑ The offset and gain errors must together remain under 0.5LSB to obtain a correct transfer curve
 - With some offset compensation this condition can be reached for 10–12 bit accuracies
- ❑ Errors in early stages are more critical (later errors divided by gain)

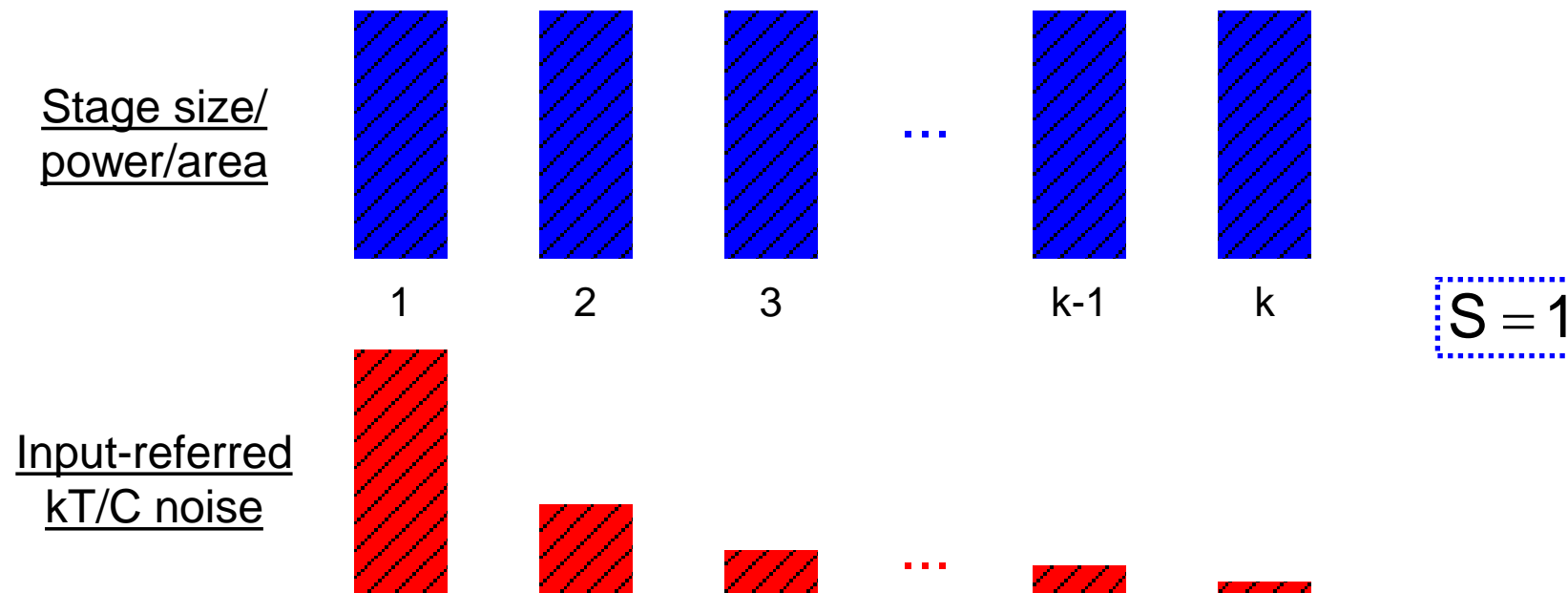


$$v_{in,det} = v_{e,0} + \frac{v_{e,1}}{A_1} + \frac{v_{e,2}}{A_1 A_2} + \frac{v_{e,3}}{A_1 A_2 A_3} + \dots$$

$$v_{in,random} = \sqrt{v_{e,0}^2 + \frac{v_{e,1}^2}{A_1^2} + \frac{v_{e,2}^2}{A_1^2 A_2^2} + \frac{v_{e,3}^2}{A_1^2 A_2^2 A_3^2} + \dots}$$

Scaling Pipelined Stages

- ❑ No Stage Scaling
 - All stages identically sized – same capacitors, op-amps, comparators $\rightarrow S = 1$
 - Later stages are clearly oversized due to inter-stage gains
- ❑ We can save a lot of power/area in later stages with minimal increase in noise

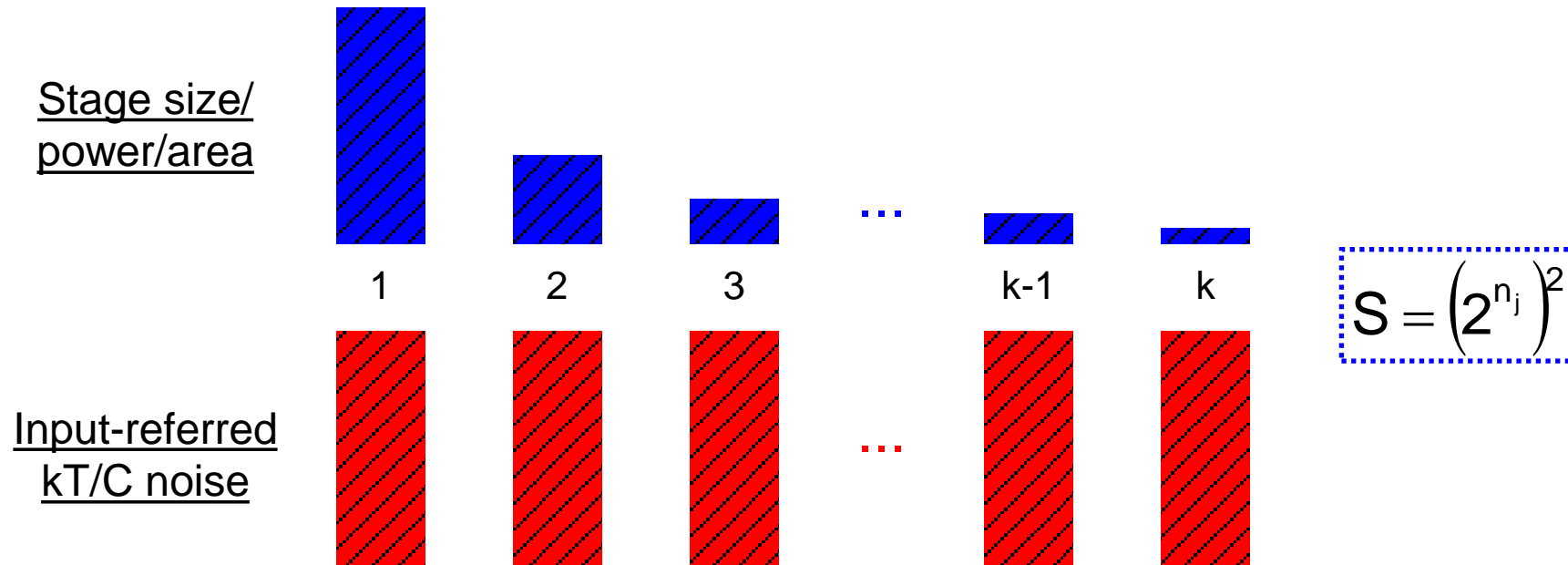


Scaling Pipelined Stages

❑ Aggressive Stage Scaling

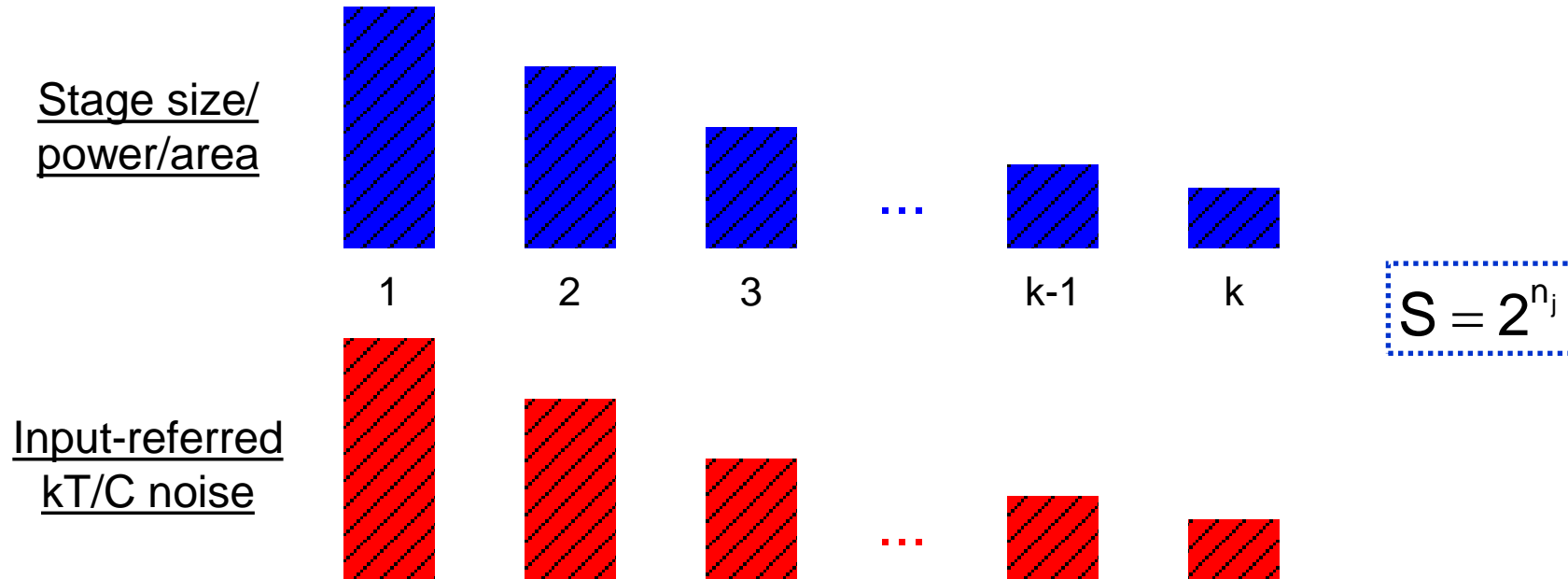
- Stages sized such that the input-referred noises are identical $\rightarrow S \approx (2^{n_j})^2$
- Later stages are clearly downsized too aggressively

❑ We can save a lot of noise in later stages with minimal increase in power/area



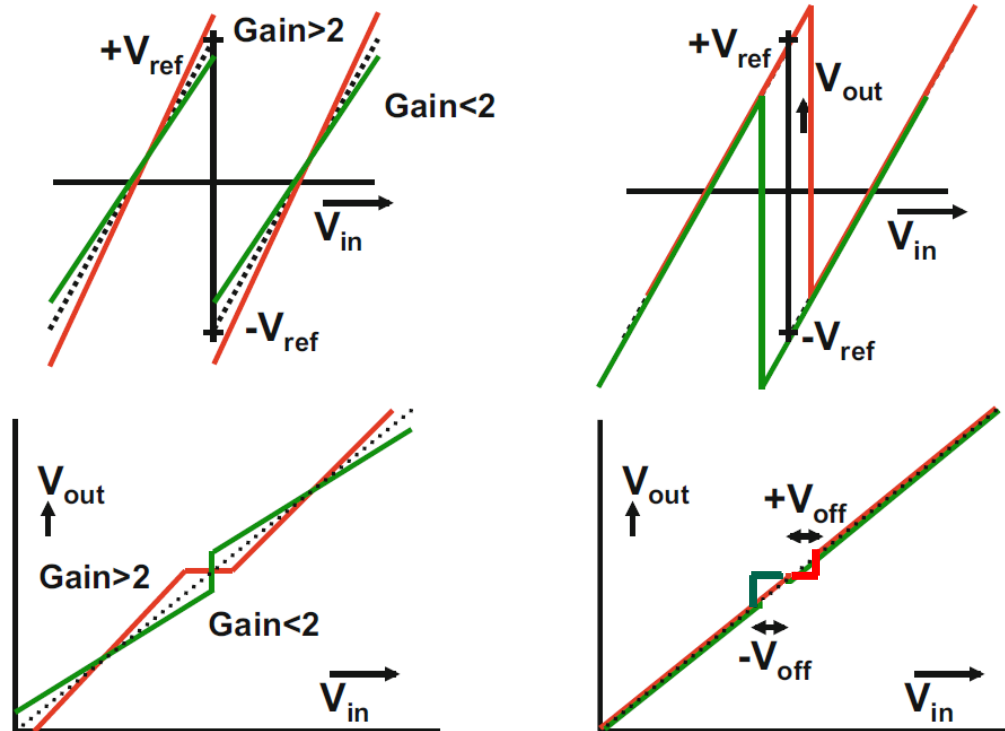
Scaling Pipelined Stages

- ❑ Optimum Stage Scaling
 - Optimum scaling lies in between the two extremes $\rightarrow S \approx 2^{n_j}$
- ❑ Area/power vs noise trade-off is balanced (gain vs loss is roughly the same)



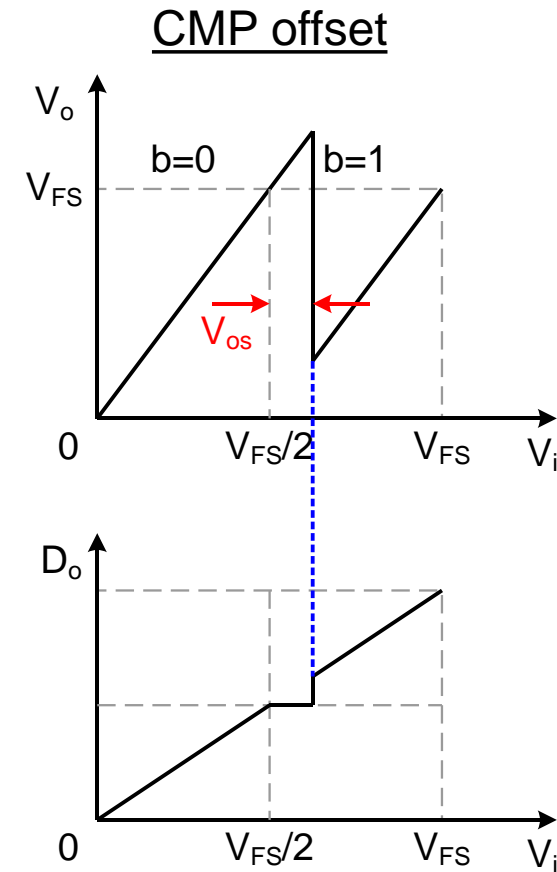
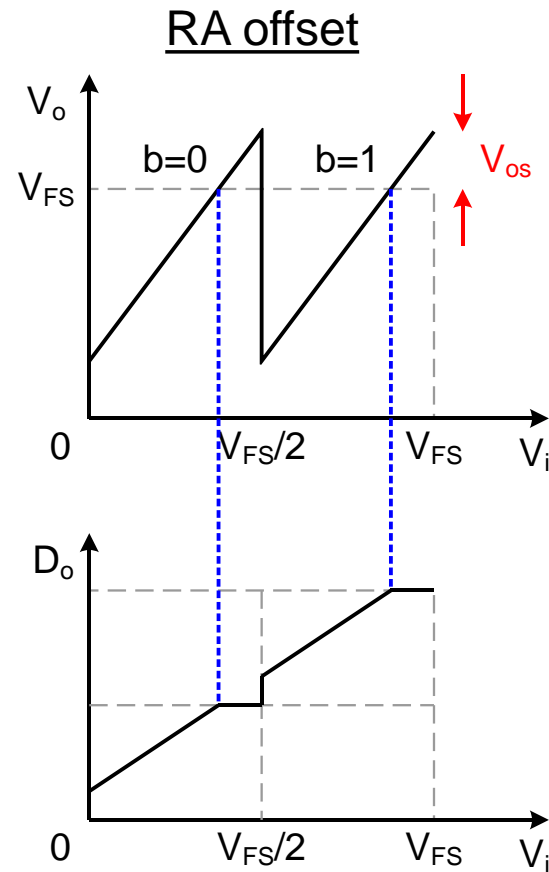
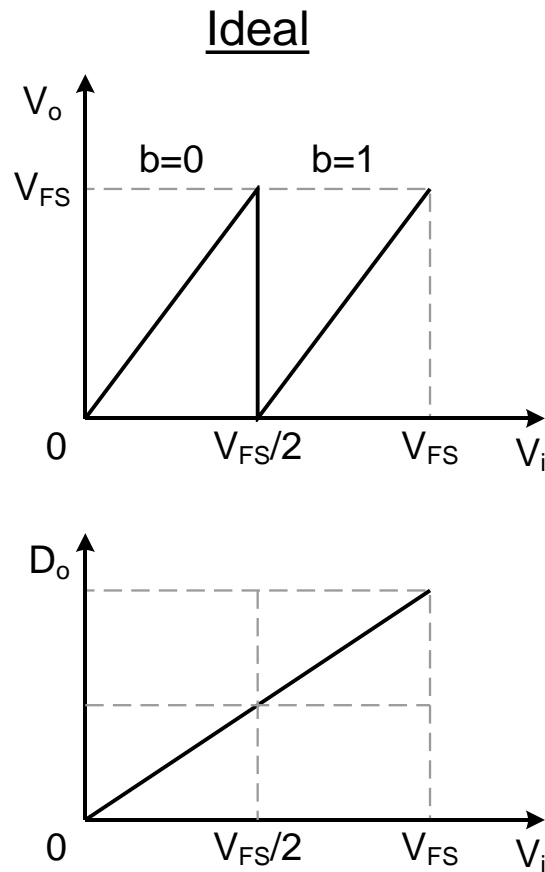
Pipelined ADC Errors

- ❑ Comparator threshold defines the breakpoint of MDAC transfer characteristics
- ❑ Offset/gain errors
 - Non-linearity in the overall transfer function
 - Offset errors can be solved by using 1.5-bit architecture



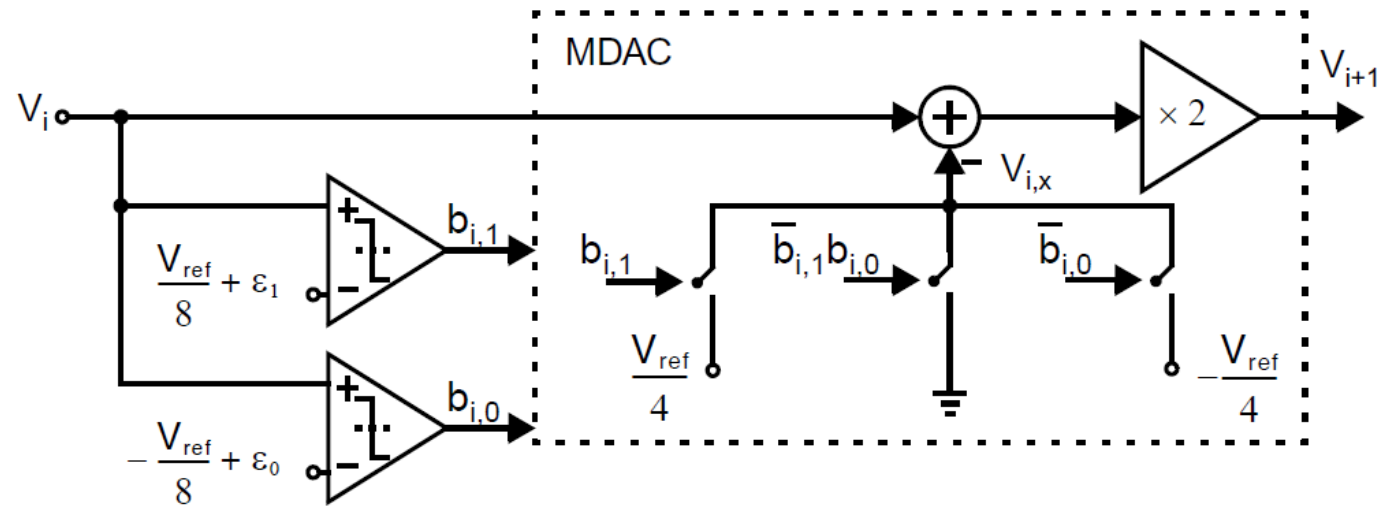
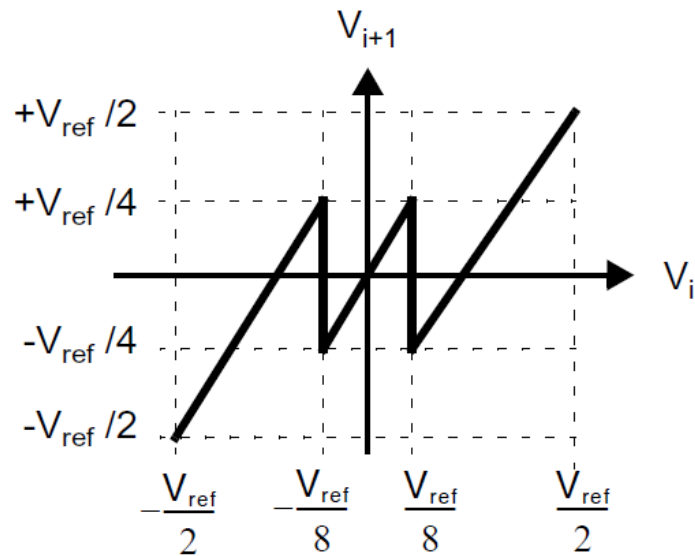
Offset Errors

- ❑ Reduce offset sensitivity by using 1.5-bit topology



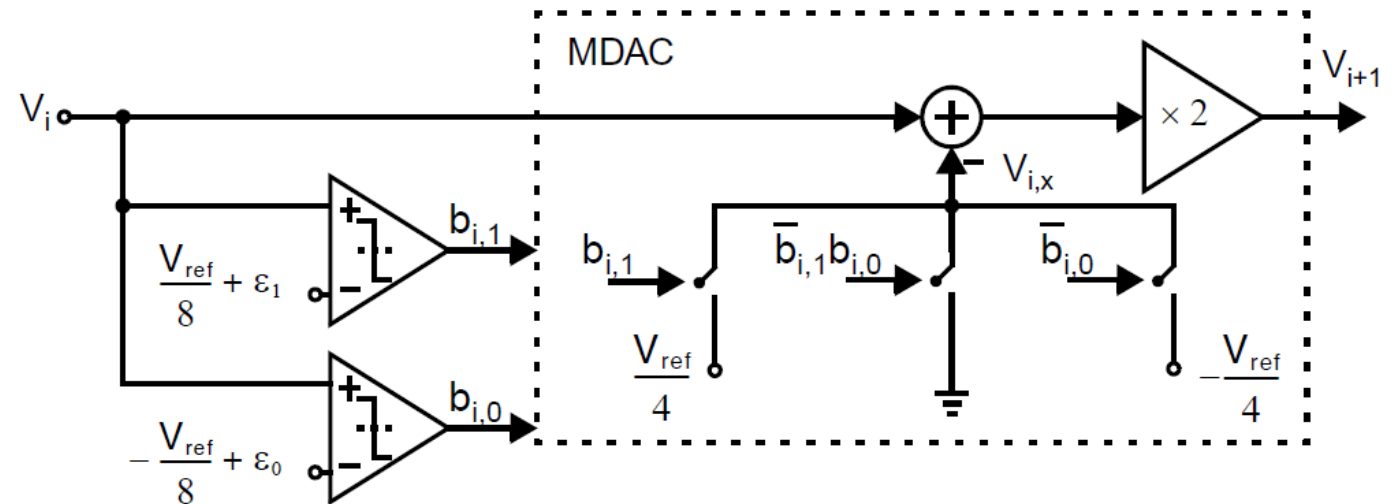
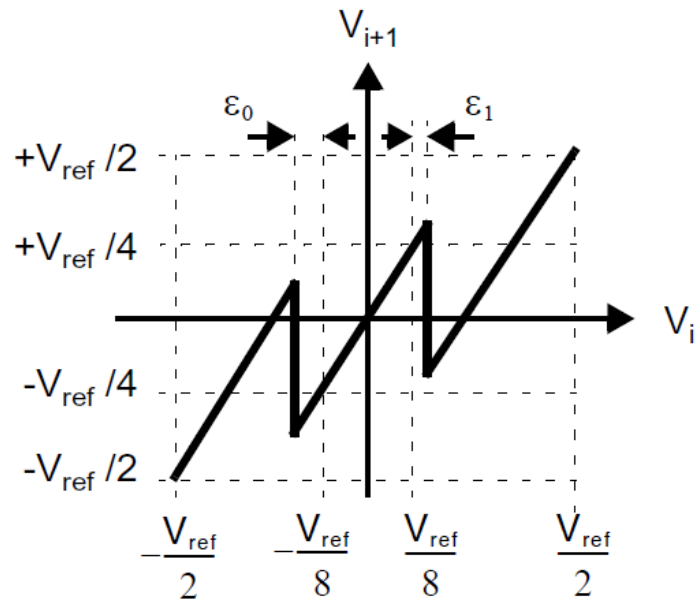
1.5-bit Concept

- ❑ 1-bit ADC: 2-levels, one threshold
- ❑ 2-bit ADC: 4-levels, three thresholds
- ❑ The use of two thresholds (three levels) is referred to as 1.5-bit ADC ($\log_2 3 \approx 1.58$)
 - Comparator and residue amplifier (RA) offsets can be corrected



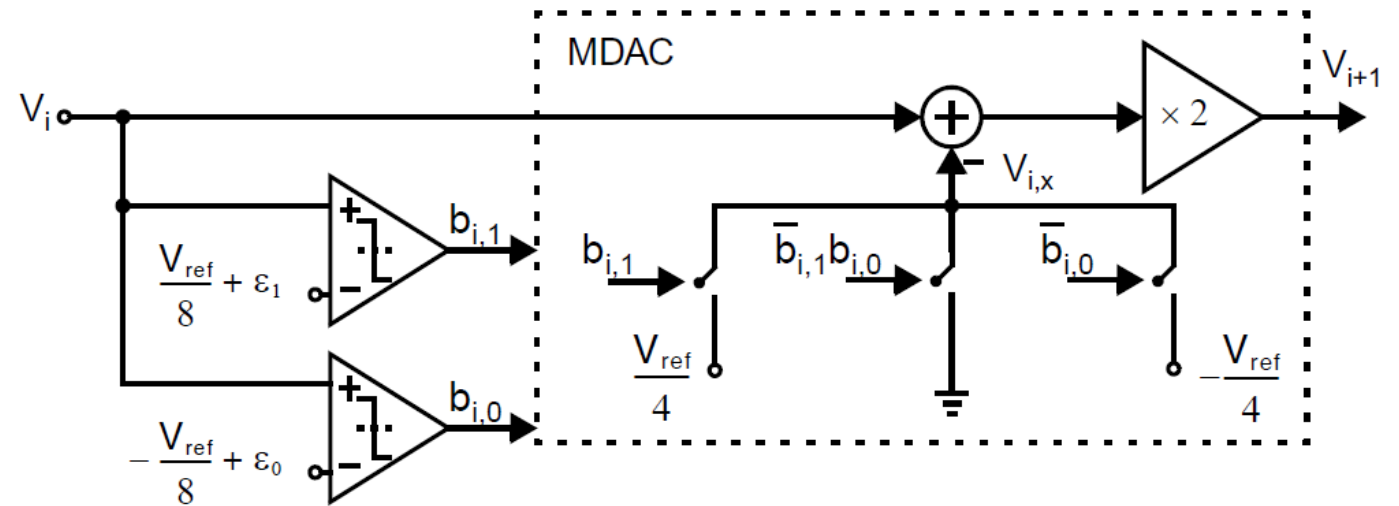
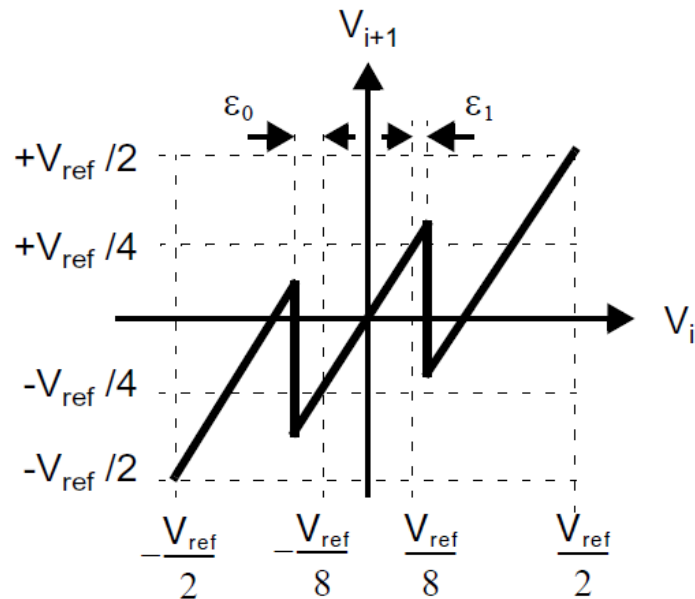
Digital Correction

- ❑ 1.5-bit digital outputs: 00 (left), 01 (middle), 11 (encoded as 10) (right)
- ❑ Left region \rightarrow certain 0. Right region \rightarrow certain 1,
- ❑ Middle region \rightarrow o/p **postponed (P)** to next stage
 - Output is amplified (x2) to be better resolved by next stage
- ❑ Errors in comparator thresholds are corrected by next stage



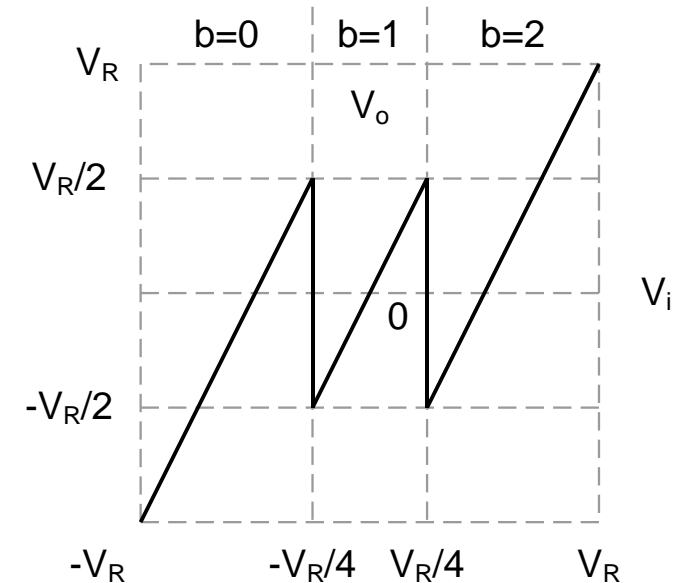
Digital Correction

- ❑ As long as the error is stored in the residue it can be corrected
 - The information is not yet lost
- ❑ Digital Correction corrects comparator and RA offset errors
 - It does not correct DAC or residue generator errors
- ❑ Digital adder required to combine digital output



1.5-bit Example

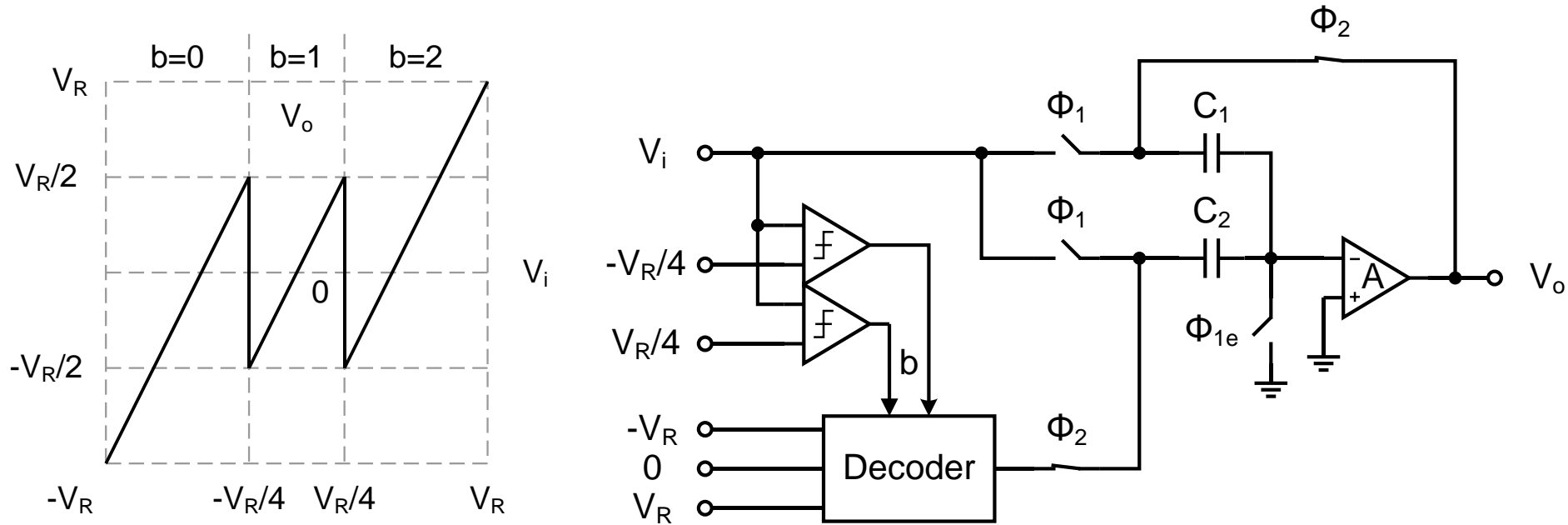
- Assume 6-bit pipelined ADC with 1.5-bit/stage
- Let $V_R = +1V$ and $V_{in} = -0.21$
- Note that $dec2bin\left(\frac{V_{in} - (V_{REF,LOW})}{\Delta V_{REF}} \times 2^N\right) = dec2bin(25.3) = 011001$



Stage Input (V)	MDAC operation	Decision	B5	B4	B3	B2	B1	B0
$V_{in} = -0.21$	$V_x \times 2$	P	0	1				
-0.42	$\left(V_x + \frac{V_{REF}}{2}\right) \times 2$	0		0	0			
0.16	$V_x \times 2$	P			0	1		
0.32	$\left(V_x - \frac{V_{REF}}{2}\right) \times 2$	1				1	0	
-0.36	$\left(V_x + \frac{V_{REF}}{2}\right) \times 2$	0					0	0
0.28	—	1						1
			0	1	1	0	0	1

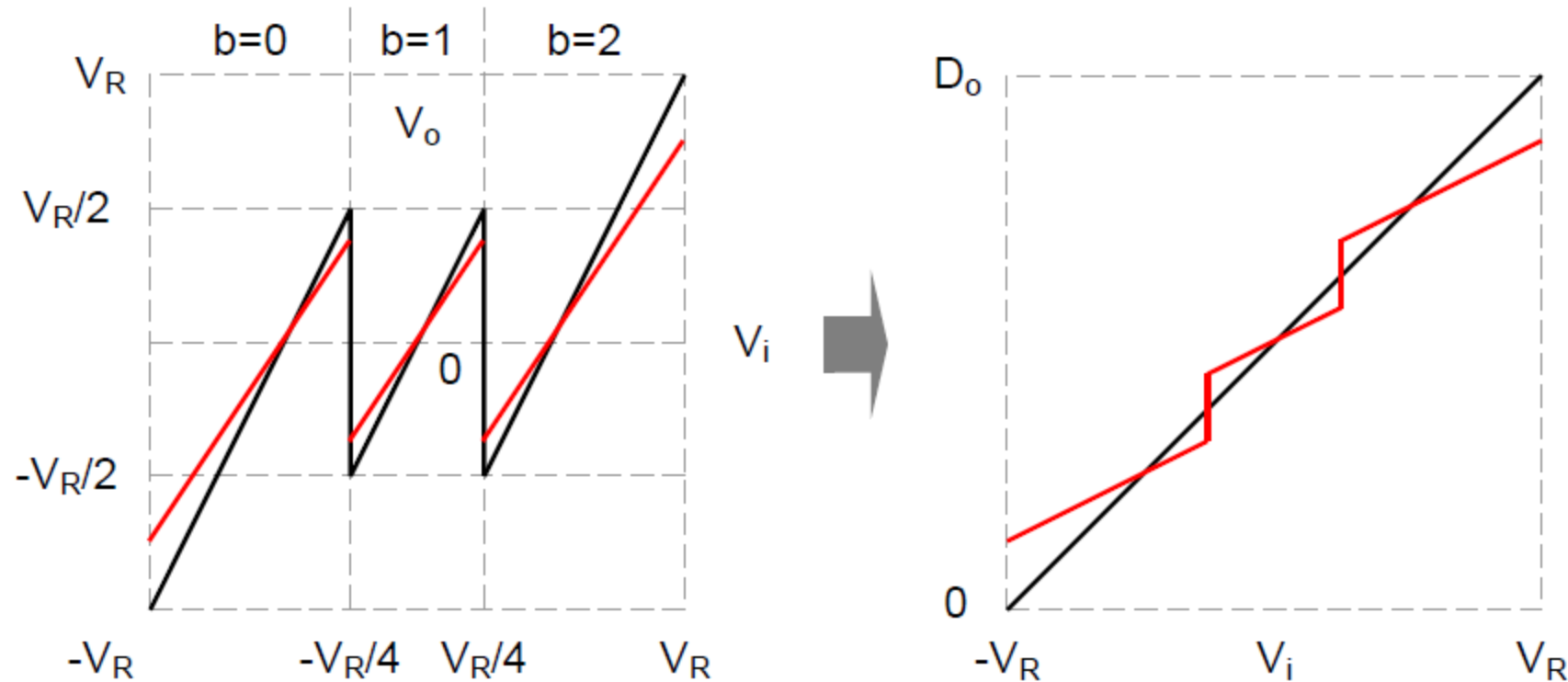
1.5-bit Stage

- ❑ 2X gain + 3-level DAC + subtraction all integrated
- ❑ Digital redundancy relaxes the tolerance on CMP/RA offsets



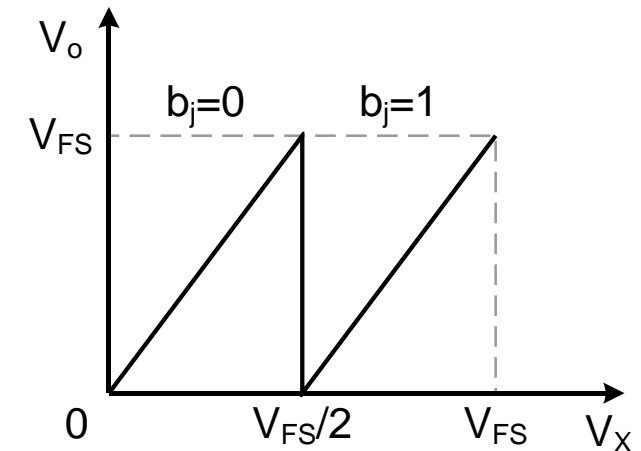
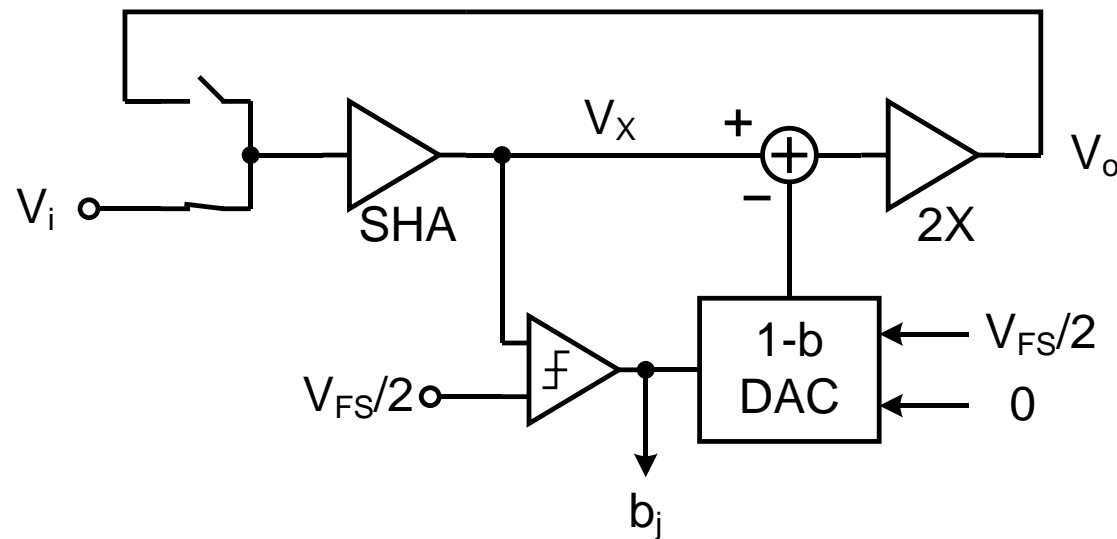
RA Gain Error and Non-linearity

- ❑ RA gain error and non-linearity are not solved by 1.5-bit MDAC.
- ❑ More complex digital calibration techniques are required for more than 10-12 bit resolution.



Algorithmic (Cyclic) ADC

- ❑ Similar to pipelined, but the hardware is reused for every bit
- ❑ The signal is compared to reference to generate the MSB bit
 - Based on this bit the signal is shifted (residue is calculated) and multiplied by 2
 - This residue is fed back to the sample and hold for the next run



References

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Thank you!