Force diagrams and pulleys

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Introduction

This paper covers the concept of internal forces, free body diagrams, connected bodies and pulleys. In order in mechanics, you have to follow one general advice: sketch. Sketching in mechanics is basically solving half of the question: the math involved is very elementary, and a sketch ensures you are analyzing the situation of the question correctly. I cannot stress how important sketching is.

1 Tension

What is tension force? Tension force is a pulling force transmitted axially by the means of a string, a rope, chain, or similar object, or by each end of a rod, truss member, or similar. It is a contact force that acts in the opposite direction to the stretching force of the object. This means that tension force *is not* a force that acts on the string or connecting body.

2 Internal forces

Internal forces are forces within an object or structure that cancel each other out within the object but maintain equilibrium with the system. This may seem very abstract and intangible: it is. We will try elucidating this concept with and example.

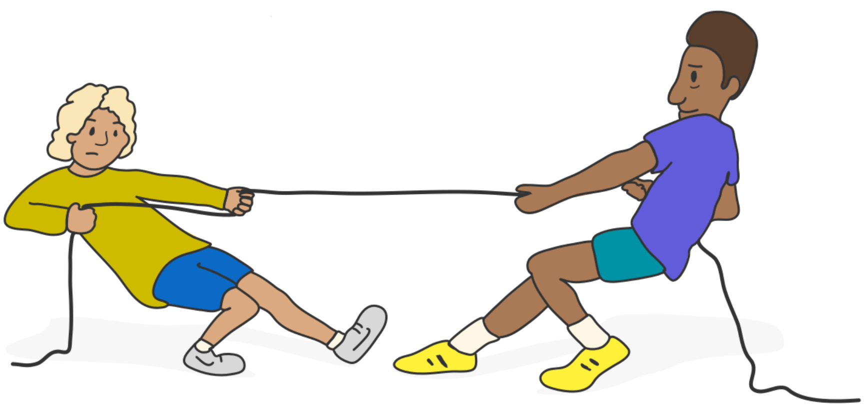


Figure 1: Tug of war

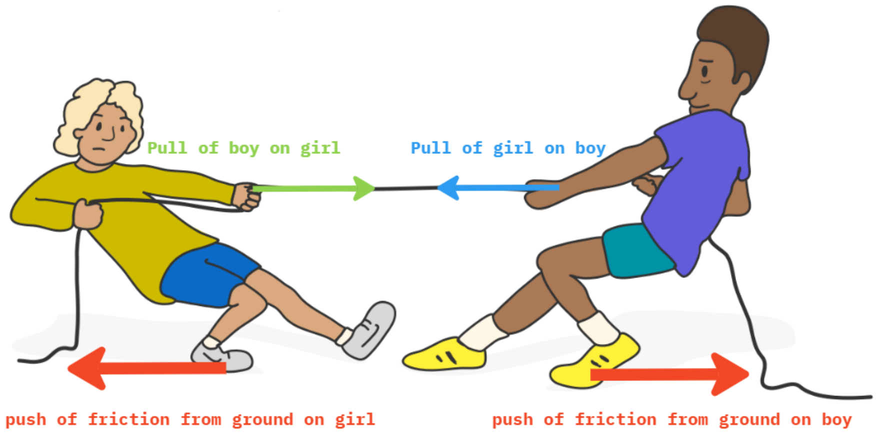


Figure 2: Tug of war (labelled)

Suppose you and your friend are playing tug of war and have reached a stalemate (i.e. you both are not moving and in place pulling your hardest); what does this mean? This means that the forces are balancing out! A more physics oriented explanation would be that you and your friend are pulling the rope with the same force. The rope pulls you with the same force; this force is called tension force.

We know that the friction and pull acting on both bodies is the same as the resultant force acting on each body is 0, and the tension force acting on each person is also the same.

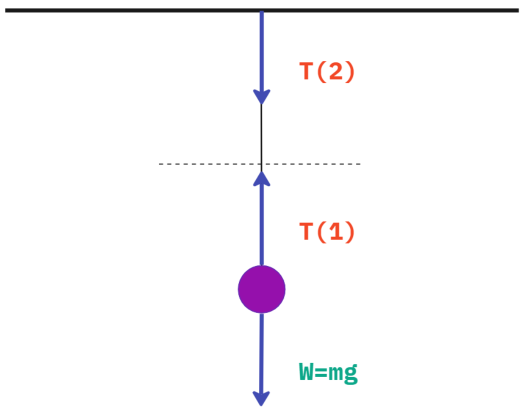
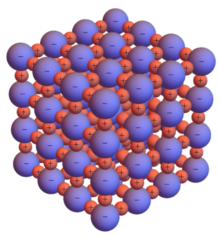


Figure 3: Suspended ball

We can clearly see that if we look at the whole system, tension forces cancel out; we call tension an internal force. However, we know that the tension force acting on the ball is equal to the weight of the ball since the ball is just hanging there. If we take the forces acting on the ball *only* however, we can see that tension does not cancel out and is thus no longer an internal force! One last general example.



This lattice structure that makes up our ball in Figure 3 has lots of electrostatic forces acting within in. We do not consider them however as we are looking at the whole ball, not a part of it, and the electrostatic forces cancel when we view the ball as a whole object.

The previous discussion shows that sometimes, we don't consider certain forces depending on how we look at the system. This leads us to a very important concept that we will use everywhere called "System Boundaries". Let's see some definitions

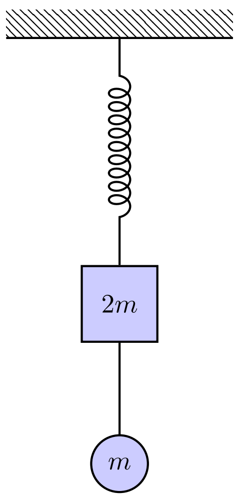
- An internal force: a force that acts between two parts of a system. It is a force that is exerted by one part of the system on another part. Internal forces are caused by the interaction of the particles or bodies that make up the system.
- An external force: a force that acts on a system from outside the system. It is a force that is exerted by something outside the system on one of the parts of the system. External forces can be either contact forces or non-contact forces.

But wait! What is a system??? Well, that's up for us to define.

Going back to tension, it, like any force, can either be internal or external. What determines if a force is internal or external is the definition of the system boundaries. The type of force is irrelevant. For example, if we consider the ball in Figure 3 as our system, then tension will be a relevant, external force. Using Newton's third law, we define an internal force as a force where the action and reaction are part of the same system.[1]

3 Free body diagrams

A free body force diagram is a diagram that isolates one object in a system and the forces acting on it. Indeed, we are considering that object as the system and the forces acting on it as external forces. It is important you know how to draw a free body force diagram as it can simplify many questions.

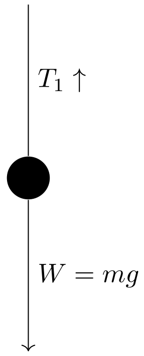


The figure on the right shows a block of mass $2m$ connected to a roof by a spring. The block is also connected to a ball of mass m by an inelastic string. Suppose the system is in equilibrium. How would we find the tensions in both the spring and the string? The answer is simple: free body diagrams.

We will start this question by drawing two free body diagrams: one for the block and one for the ball. Remember that a free body force diagram only includes the object and forces directly acting upon it.

We will start with the ball, defining it as our system.

[1] This is a more formal definition. It is okay if you don't understand this because you don't know Newton's third law

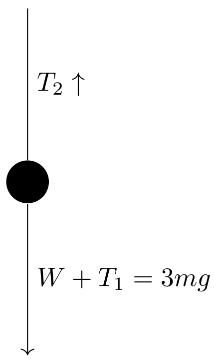


As we can see, the only two forces directly acting on the ball are its weight and the tension from the string.

Resolving in the upward direction ($R(\uparrow)$):

$$T_1 = mg$$

Drawing the free body diagram of the block, we get



Hence,

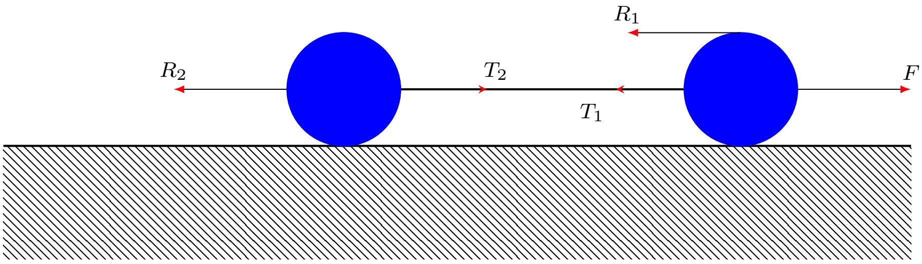
$$T_2 = 3mg$$

4 Connected particles

Suppose we have train compartment that it connected to and moved by the main compartment. The main compartment contains the engine that generates the power behind the supplied force of F N. The other compartment is connected to the train by a coupling. The main compartment experiences a total resistive force of R_a N, while the other compartment experiences a total resistive force of R_b N How would we model this situation?

Figure 4: Train model

However, we obviously cannot sketch this so we opt for the simpler and, albeit, more useful diagram.

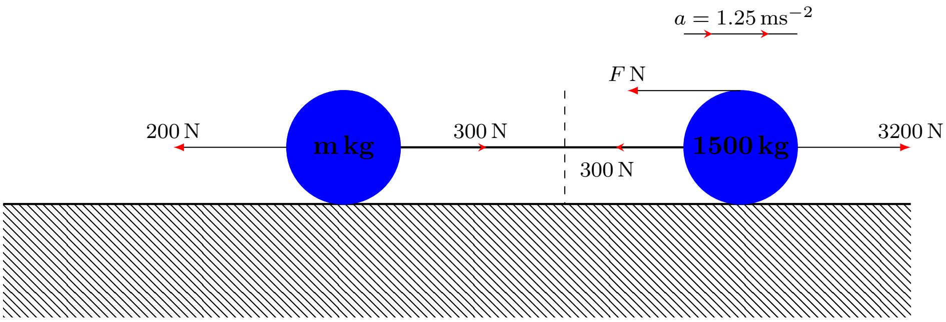


Let's try an example.

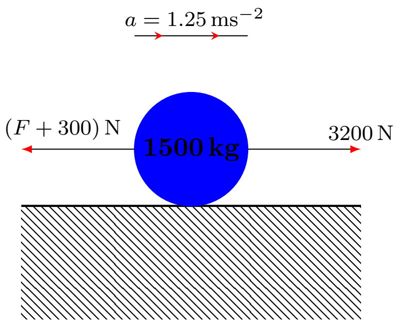
A car of mass 1500 kg is towing a trailer of mass m kg along a straight horizontal road. The car and the trailer are connected by a tow-bar which is horizontal, light and rigid. There is a resistance force of F N on the car and a resistance force of 200 N on the trailer. The driving force of the car's engine is 3200 N, the acceleration of the car is $1.25\,\mathrm{ms}^{-2}$ and the tension in the tow-bar is 300 N

Solution:

Let's start with a good sketch.



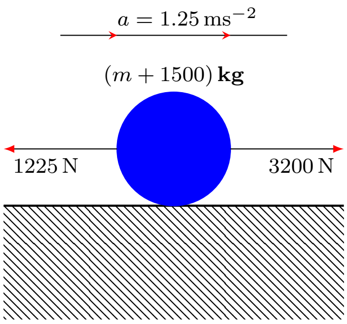
First, we will take the free body diagram of the car.



Using Newton's (II) law,

$$3200 - 300 - F = 1500 \times 1.25$$
$$F = 1025$$

Let us take the car, trailer and tow-bar as our system. Hence, we have $F = 1025$, the driving force and the resistive forces as our external forces acting on our system. Refining our sketch using our established system boundaries, we get the following.

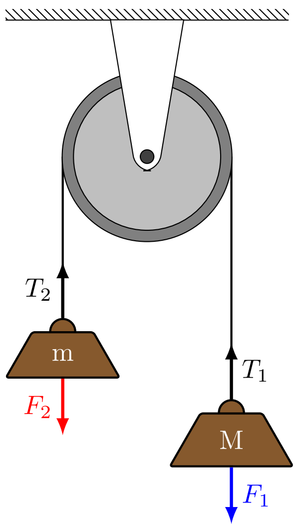


Using Newton's (II) law

$$3200 - 1225 = (m + 1500) \times 1.25$$
$$m = 80$$

5 Pulleys

At last, we reached the final idea. Well, it really is just an extension of the previous section but with a fancier scenario. A pulley is a simple machine that consists of a wheel with a grooved rim, along which a rope or cable is threaded. The wheel is attached to an axle, which allows it to rotate freely. Since this is A level Mechanics and not real life, our pulleys are always frictionless. Usually, the best course of action is to label the forces acting on both of our masses, and use their corresponding free body diagrams to extract the relevant equations. With pulley questions, it is always important to be organized, knowing what to do step by step. Pulley questions come in many forms, most of which are blended with other topics-most notably friction.

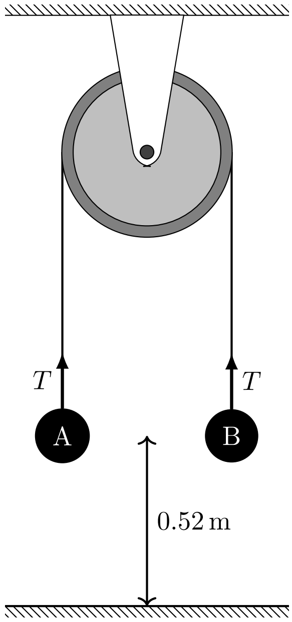


We will tackle two questions, one of which involves friction. If you have not yet covered friction, you can skip over that question.

Before that, we must mention some certain features of a pulley system.

- If the system is not in equilibrium, both particles must be accelerating at the same rate; that is, $a_A = a_B$.
- Since we are dealing with a single string, tension is equal.
- If one particle stops (by hitting the ground for example) the only force acting on the other particle in air is its weight.

Example 1: Particles A and B, of masses 0.3 kg and 0.7 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley. A is held at rest and B hangs freely, with both straight parts of the string vertical and both particles at a height of 0.52 m above the floor (see diagram). A is released and both particles start to move.



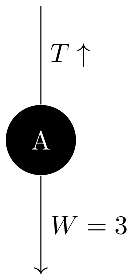
i) Find the tension in the string.

When both particles are moving with speed $1.6\,\mathrm{ms}^{-1}$ the string breaks

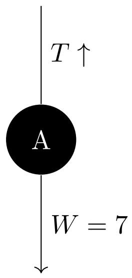
ii) Find the time taken, from the instant that the string breaks, for A to reach the floor

Solution:

i) Drawing the free body diagram of both A and B, we can find relevant equation.



$$T - 3 = 0.3a$$



$$7 - T = 0.7a$$

Adding both equations to each other, we get

$$a = 4\,\text{ms}^{-2}$$

Hence,

$$T = 0.3 \times 4 + 3$$
$$T = 4.2\,\text{N}$$

ii) First, we must find the height above ground for particle A once both particles reach $1.6\,\text{ms}^{-1}$. We will use $v^2 = u^2 + 2as$.

$$(1.6)^2 = 0^2 + 2 \times 4 \times s$$
$$s = 0.32$$

Note that $0.32\,\text{m}$ is the displacement of A from its initial position; therefore, the height of A once the string breaks is $0.84\,\text{m}$, travelling with an upward velocity of $1.6\,\text{ms}^{-1}$. Now, we must find the time for A to hit the ground. A hits the ground when its displacement is equal to $-0.84\,\text{m}$ taking up as positive. The acceleration of particle A is $-10\,ms^{-2}$. Using $s = ut + \frac{1}{2}at^2$

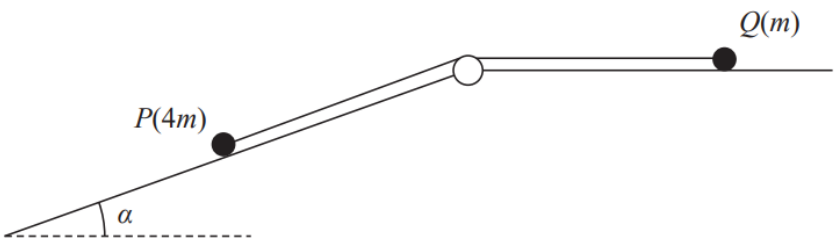
$$-0.84 = 1.6t - \frac{1}{2} \times 10t^2$$

Solving the quadratic with your preferred method, we get that

$$t = 0.6\,\text{s}$$

That was a bare bones example. Let's try a more interesting one.

Example 2: A particle P of mass 4m lies on the surface of a fixed rough inclined plane. The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$. The particle P is attached to one end of a light inextensible string. The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane. The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane. The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane. The coefficient of friction between P and the plane is $\frac{1}{4}$.



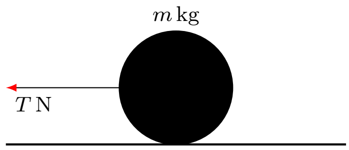
For the motion before Q reaches the pulley

a) find, in terms of m and g, the tension in the string,

b) find the magnitude of the force exerted on the pulley by the string.

Solution:

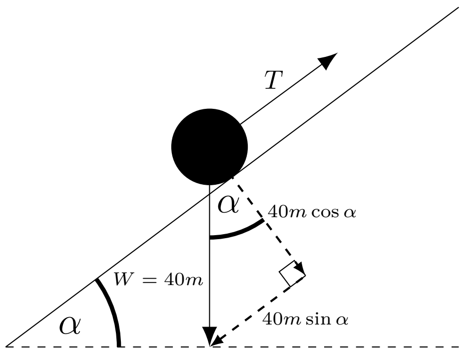
a) Let's take the free body diagram of Q



We know that

$$T = ma$$

Labelling P, we also have the following



The normal contact force acting on P is

$$40m \cos \alpha$$

Hence, the frictional force acting on P,F, is

$$F = 40m \cos \alpha \times \frac{1}{4}$$
$$F = 10m \cos \alpha$$

Using basic trigonometry, we know that

$$\sin \alpha = \frac{3}{5}$$

and

$$\cos \alpha - \frac{4}{5}$$

Which leads to

$$F = 8m$$

Using Newton's (II) law,

$$4ma = 40m \sin \alpha - 8m - T$$
$$4ma = 24m - 8m - T$$
$$4ma = 16m - T$$

but $T = ma$, so

$$4ma = 16m - ma$$
$$5a = 16$$
$$a = \frac{16}{5}$$
$$\therefore T = \frac{16}{5}m = \frac{8}{25}mg$$

b) We must resolve vertically and horizontally to find the net force acting on the pulley. Resolving horizontally, we get that

$$\begin{aligned}|F_x| &= |T - T\cos\alpha| \\ &= \left|\frac{16}{25}m\right| \\ |F_x| &= \frac{16}{25}m\end{aligned}$$

Resolving vertically, we get that

$$\begin{aligned}|F_y| &= |T\sin\alpha| \\ &= \left|\frac{16}{5}m \times \frac{3}{5}\right| \\ &= \left|\frac{48}{25}m\right| \\ |F_y| &= \frac{48}{25}m\end{aligned}$$

Therefore,

$$\begin{aligned}F_{net} &= \sqrt{\left(\frac{48}{25}m\right)^2 + \left(\frac{16}{25}m\right)^2} \\ F_{net} &= 2.02m\end{aligned}$$