Friction and Statics

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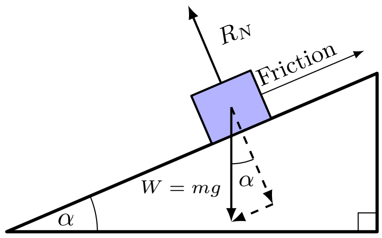
1 Introduction

This paper mainly covers the concepts of friction, statics and how they interact with each other. While we do gloss over resolving vectors, the paper in no way, shape or form addresses this fully.

2 Friction

What is friction? Friction is a force exerted by a *rough* surface on a body in a direction that restricts the potential of motion.[1] The magnitude of friction depends on two things: the normal contact force acting on the body and the coefficient of friction.



The normal contact force is a force exerted on the body by a surface with its magnitude equal to the component of weight perpendicular to the surface. As the name suggests, the normal contact force is perpendicular to the surface.

The coefficient of friction, denoted by μ, is a number between and including 0 and 1 that quantifies how rough some surface is. If $\mu = 0$ then the surface has no roughness and is called *smooth*.

[2]Before linking friction, the coefficient of friction and the normal contact force together, we must be aware that the force of friction actually varies. To be more precise, an object in equilibrium on a rough surface can experience friction of value 0 till some maximum value. But why is that? Qualitatively, for the static block on the plane in our diagram, the vector sum of the normal contact force and friction force is equal to the weight vector in magnitude and opposite in direction. The normal contact force is equal in magnitude to $W \cos \alpha$, and the friction force is equal in magnitude to $W \sin \alpha$. While R_N can take any value as long as the inclined plane doesn't break (i.e depends on the strength of the plane itself) the friction force has a maximum value which depends on the roughness of the plane. This means that the value of F varies according to $W \sin \alpha$ until it reaches its maximum which is bottle-necked by the roughness. Let the maximum value of friction be $F_{\max}$.

The maximum friction a surface can exert on a body is $F_{\max}$, where

$$F_{\max} = \mu R_N.$$

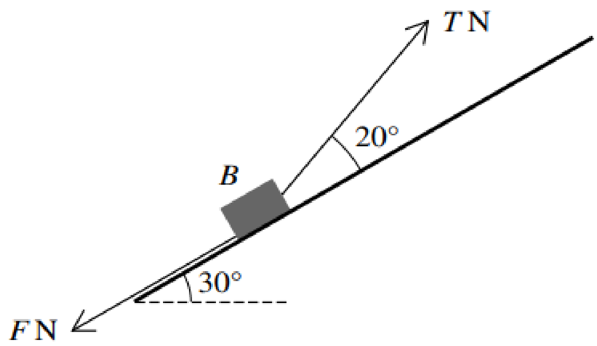
If the friction force, F, is equal to the maximum friction, $F_{\max}$, then the object is on the point of sliding and is said to be in *limiting equilibrium*.

[1]will be more clear later

[2]This whole paragraph is for those interested in a deep understanding. I highly recommend you read and understand this.

Before attempting a question, it is important to identify the direction of friction. Friction must always be opposite to the direction a body is is tending to move to. In other words, the direction of friction would be opposite to the direction of movement on a smooth plane; if we didn't have friction, which way would this body move? Friction would act in the direction opposite to your answer.

Example 1: A block B, of mass 2 kg, lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to B. The tension in the rope is T N. There is a friction force of F N acting on B (see diagram). The coefficient of friction between B and the plane is μ.



a) It is given that $F = 5$ and that the acceleration of B up the plane is $1.2\,\text{ms}^{-2}$

i) Find the value of T.

ii) Find the value of μ.

b) It is given instead that $\mu = 0.8$ and $T = 15$.
Determine whether B will move up the plane.

Solution:

a) **i)** Since the acceleration of B is up the plane, then the resultant force acting on B parallel to the plane must be the product of its mass and acceleration up or down the plane; that is,

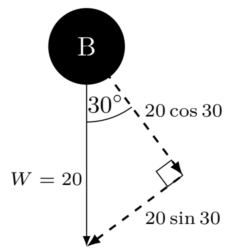
$$\vec{F} = m\vec{a}$$
$$T\cos(20) - F - 2 \times 10 \times \sin 30 = m\vec{a}$$
$$T\cos(20) - 5 - 10 = 2 \times 1.2$$
$$T = 18.52$$

ii) We know that

$$F = \mu R_N.$$

To find R_N, we must find the force the block exerts on the plane perpendicular to it. Hence,

$$R_N = 2 \times 10 \times \cos(30) - T\sin(20)$$
$$R_N = 2 \times 10 \times \cos(30) - 18.52\sin(20)$$
$$R_N = 10.99$$



To find μ

$$\begin{aligned}5 &= 10.99\mu\\ \mu &= 0.455\end{aligned}$$

b) If the block was to move up the plane, then

$$T\cos 20 > 2\times 10\times \sin(30) + F$$

To find $F_{\max}$,

$$\begin{aligned}R_N &= 2\times 10\times\cos(30) - 15\sin 20\\ R_N &= 12.19\\ \therefore F &= 0.8\times 12.19\\ F_{\max} &= 9.752\end{aligned}$$

Now, using our previous inequality

$$\begin{aligned}15\cos(20) &\overset{?}{>} 2\times 10\times\sin(30) + 9.752\\ 14.1 &\overset{?}{>} 19.752\end{aligned}$$

Hence, the block won't slide up.

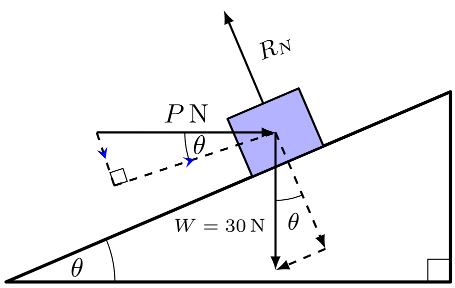
3 Statics

Statics is an area in mechanics that deals with rigid bodies in equilibrium: rigid bodies are bodies assumed to be completely inelastic and bodies in equilibrium are bodies that are *static* (not moving) with 0 resultant force acting on the body. It is important to be fully capable of resolving vectors in relevant directions. We usually resolve vectors, if necessary in one of two ways: in the x and y direction, or parallel and perpendicular to the plane. In reality, we have been dealing with statics already, so this shouldn't be anything new. We will only make do with 2 examples from.

Example 1: A box of mass 3 kg is held in limiting equilibrium on a rough plane angled at θ above the horizontal where $\tan\theta = \frac{3}{4}$ by a horizontal force of magnitude P N acting into the plane. Find the maximum possible value of P. It is given that $\mu = 0.6$

Solution:

Like any mechanics question, we start with a good sketch. We will use our previous sketch.



Note: we resolved P parallel and perpendicular to the plane; you can't resolve in the x and y directions as a horizontal force doesn't have a vertical component

Notice how we didn't include friction? The mentions that the box is in limiting equilibrium due to P. However, P can either be preventing the box from sliding down, or is about to make it slide up.

Let's test both scenarios to see where P is greatest.

- If P is about to slide down, $F_{\max}$ acts up the plane.

$$P\cos(\theta) = W\sin(\theta) - F_{\max}$$

- If P is about to slide up, $F_{\max}$ acts up the plane.

$$P\cos(\theta) = W\sin(\theta) + F_{\max}$$

Therefore, P is greatest when the block is about to slide up. Knowing that, we have find $P_{\max}$. Let's first find $F_{\max}$)

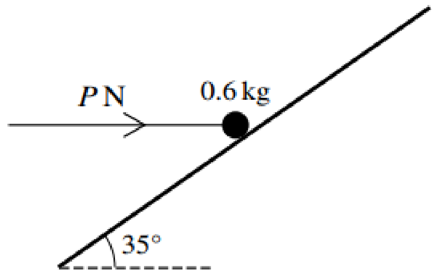
$$\begin{aligned}
R_N &= W\cos(\theta) + P\sin(\theta) \\
&= \frac{4}{5}W + \frac{3}{5}P \\
&= \frac{4}{5} \times 30 + \frac{3}{5}P \\
R_N &= 24 + \frac{3}{5}P \\
\therefore F_{\max} &= \frac{3}{5}\left(24 + \frac{3}{5}P\right) \\
F_{\max}) &= \frac{72}{5} + \frac{9}{25}P
\end{aligned}$$

Resolving parallel to the plane,

$$\begin{aligned}
P\cos(\theta) &= W\sin(\theta) + F_{\max} \\
\frac{4}{5}P &= \frac{3}{5} \times 30 + \frac{72}{5} + \frac{9}{25}P \\
\frac{11}{25}P &= \frac{162}{5} \\
P &= \frac{810}{11}
\end{aligned}$$

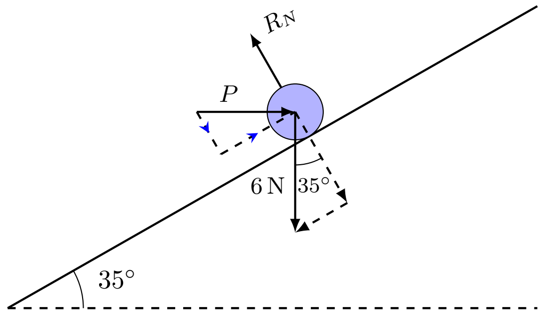
Example 2: A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P.



Solution:

Adding vectors to our picture



This is more or less the same as the previous question.

P will be minimum when the particle is about to slide down; friction will act upwards.

Finding friction,

$$R_N = P\sin(35) + 6\cos(35)$$

$$\therefore F_{\max} = 0.4(P\sin(35) + 6\cos(35))$$

$$F_{\max} = \frac{2}{5}P\sin(35) + \frac{12}{5}\cos(35)$$

Resolving up the plane

$$P\cos(35) = 6\sin(35) - \frac{2}{5}P\sin(35) - \frac{12}{5}\cos(35)$$

Solving for P,

$$P_{\min} = 1.41$$