CCNet
Center for Complex Networks

Shahid Beheshti University
Department of Physics

Advanced Computational Physics

Problem Set 8

(Due Date : 1404/10/21)

Problems

1. **Wave transform**

   (a) Show that the wavelet
   $$u(t) = e^{-t^2}\,(1 - 2t^2)$$
   satisfies the admissibility condition. Apply this wavelet in a continuous wavelet transform to a sequence
   $$f_i = f(t_i)$$
   generated by a uniform random-number generator, assuming that the sampling points t_i are uniformly spaced in the interval $[0, 1]$. Plot the corresponding scalogram and discuss whether it exhibits any significant structures.

2. **Optimization of a fully connected Ising model**
   Consider an all-to-all (fully connected) Ising model consisting of N spins
   $$s_i \in \{-1, +1\}, \qquad i = 1, \ldots, N.$$
   The Hamiltonian of the system is defined as
   $$\mathcal{H}(\mathbf{s}) = -\sum_{i<j} J_{ij} s_i s_j, \tag{1}$$
   where the coupling coefficients J_{ij} are independent random variables drawn from a uniform distribution,
   $$J_{ij} \sim \mathcal{U}(-1, 1), \qquad J_{ij} = J_{ji}, \qquad J_{ii} = 0. \tag{2}$$
   The objective is to find the spin configuration $\mathbf{s}^*$ that minimizes the energy,
   $$\mathbf{s}^* = \arg \min_{\mathbf{s} \in \{-1,+1\}^N} \mathcal{H}(\mathbf{s}), \tag{3}$$
   which corresponds to the ground state of the system. Suppose $N = 100$.

   (a) Implement a **Genetic Algorithm (GA)** to search for low-energy spin configurations. Your implementation should specify:
   - the encoding of individuals,
   - the fitness function,
   - the selection, crossover, and mutation operators,
   - termination criteria.

(b) Implement a **Simulated Annealing (SA)** algorithm for the same system.

(c) Compare the performance of GA and SA in terms of:

- the minimum energy reached,
- convergence speed,
- robustness with respect to different realizations of J_{ij}.