CCNet
Center for Complex Networks

Shahid Beheshti University
Department of Physics

Advanced Computational Physics

Problem Set 4

(Due Date : 1404/09/04)

Problems

1. **Using the Monte Carlo method**

   (a) Solve the following integral for $\theta = [\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi]$:

   $$\int_0^\theta \sin(x)dx \tag{1}$$

   (b) Solve the following integration problem:

   $$\langle v_z^2 \rangle = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z v_z^2 p_v(\vec{v}) \tag{2}$$

   here $p_v(\vec{v}) = (\frac{\beta m}{2\pi})^{\frac{3}{2}} \exp(\frac{-\beta m \vec{v}^2}{2})$. Suppose that $\beta m = 2$. Could you estimate your result before doing integration?

   (c) Compute the value of π. Check your algorithm for various values of sampling, N.

2. **Random number generators**

   (a) According to Box-Muller method, generate Gaussian random numbers with $\sigma_0^2 = 3$ and $\langle x \rangle = 2$.

   (b) Plot the histogram of generated numbers.

3. **Importance sampling**

   (a) What does importance sampling mean?

   (b) Choose $p(x) = A\exp(-\lambda x)$ and evaluate the integral

   $$\int_0^\pi \frac{1}{x^2 + \cos^2(x)} dx \tag{3}$$

   Determine the value of λ that minimizes the variance of the integrand.

4. **Using Metropolis algorithm**

   (a) Generate random numbers based on Pareto distribution $p(x) = \frac{\alpha}{x^{\alpha+1}}$ for $1 \leqslant x < \infty$, $\alpha = 3$. Verify your results by plotting the histogram.