Advanced Computational Physics

Problem Set 6

(Due Date : 1404/09/25)

Problems

1. **Ising model**

   (a) Write an MCMC program to compute and plot $\langle E \rangle$, $\langle |m| \rangle$, χ, and C_v as functions of temperature for the 2D Ising model with Hamiltonian

$$\mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j. \tag{1}$$

   Consider a system of size 32×32 with $k_B = J = 1$, and temperatures $T \in [0, 4]$ in steps of $\Delta T = 0.1$.

   (b) Now include a uniform external magnetic field $H_{\text{ext}} = 1$ and recompute $\langle E \rangle$, $\langle |m| \rangle$, χ, and C_v as functions of T.

2. **Finite-size analysis**

   (a) Compute the Binder cumulant

$$U_L = 1 - \frac{\langle m^4 \rangle_L}{3 \langle m^2 \rangle_L^2} \tag{2}$$

   for system sizes $L \in \{16, 32, 64, 128\}$ of the 2D Ising model. Here m is the magnetization (order parameter). Plot U_L for the different system sizes as a function of T and find T_c.