Center for Complex Networks

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Advanced Computational Physics

Problem Set 3

(Due Date : 1404/08/23)

Problems

1. **Derivation and implementation of the fourth-order Runge–Kutta method**

   (a) Derive the fourth-order Runge–Kutta (RK4) algorithm for numerically solving the first-order ordinary differential equation

   $$\frac{dy}{dt} = g(y, t) \tag{1}$$

   with a given initial condition.

   (b) Discuss the options in the selection of the parameters involved.

   (c) Comment on the trade-offs between accuracy, stability, and computational effort compared to lower-order methods (Euler).

   (d) Implement the RK4 algorithm for following equation

   $$\frac{dy}{dt} = -\lambda y,\ y(0) = 1 \tag{2}$$

   where $\lambda > 0$.

   (e) Compare your RK4 solution with the analytical result $y(t) = e^{-\lambda t}$ for step sizes $\Delta t = 0.1, 0.5, 1$.

   (f) Compute and plot the global error ($|y_{rk4} - y_{exact}|$) at a fixed time t_f as a function of Δt on a log-log scale. Verify the expected fourth-order convergence (error $\sim \Delta t^4$)

2. **Using Euler and RK4 methods**

   (a) Solve the following initial value problem:

   $$y''(t) + ay'(t) + \omega y(t) = \cos(\omega t) \tag{3}$$

   with $y(0) = A,\ y'(0) = 0$ and take any arbitrary values for other free parameters. Plot the phase diagram, namely $y'(t)$ as a function of $y(t)$.

   (b) Compare your results with that determined by one of the following: SciPy, Diffrax packages or Mathematica.

3. **Gaussian elimination**

   (a) For following linear equations, use Gaussian elimination to determine the solution vector $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)^{\mathrm{T}}$.

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 + x_4 - 2x_5 &= 5, \\ x_1 + 2x_2 - x_3 + 3x_4 + 4x_5 &= 12, \\ 3x_1 - x_2 + 2x_3 - 5x_4 + x_5 &= -3, \\ 4x_1 + x_2 - 3x_3 + 2x_4 - 4x_5 &= 10, \\ -2x_1 + 5x_2 + x_3 - 3x_4 + 2x_5 &= 7. \end{aligned}$$

   (b) Compare your result with `numpy.linalg.solve`.