Exercise II: Random walk

-Task I: implementation

- Generate a function that produces a random walk in one dimension *x*:
    - For step *N* = 0, Start at *x* = 0
    - Draw a number *r* from a uniform distribution between 0 and 1
    - If $r \geq 0.5$ add 1 to *x*, else add -1 to *x*
- Organize the output such that the full random walk is written out.

-Task II: simulation and evaluation

- Run *n* = 10000 random walks of *N* = 20000 steps each (that should only take ca. 2 minutes calculation time)
- Plot histograms of the distribution of x for all *n* = 10000 random walks for *N* = 100, 1000, 10000 and 20000.
- For *N* = 100, 1000, 10000 and 20000, check the convergence of all walks check in dependence of *n* for
    - the first moment, i.e. the mean $\langle x \rangle$,
    - the second moment, i.e. the variance $\langle (x - \langle x \rangle)^2 \rangle$ and
    - the third moment, giving the skewness $\langle (x - \langle x \rangle)^3 \rangle$.

-Task III: random walks and the diffusion equation (THEORY EXERCISE)

Usually, random walks are not rationalized in the form of number of steps *N*, but in form of the time increment ("step") Δt an individual step takes and the walk time $t = N\,\Delta t$.

In the limit of long length and time scales (i.e., a large number of steps), the evolution of random walks is described by the *diffusion equation*

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2}$$

with the evolving probability density $P(x,t)$ to find the "walker" at a *x* of choice at time *t* and the diffusion constant *D*.

Using the probability density of a random walk $P(x) = (2\pi N)^{-\frac{1}{2}} \exp\left(-\frac{x^2}{2N}\right)$ (see the lecture / the lecture script), show that a random walk is a solution to the diffusion equation with $D = x_0^2/2\Delta t$ with step size x_0^2. *Hint*: bear in mind that *P(x)* then turns into a time-dependent distribution *P(x,t)*.