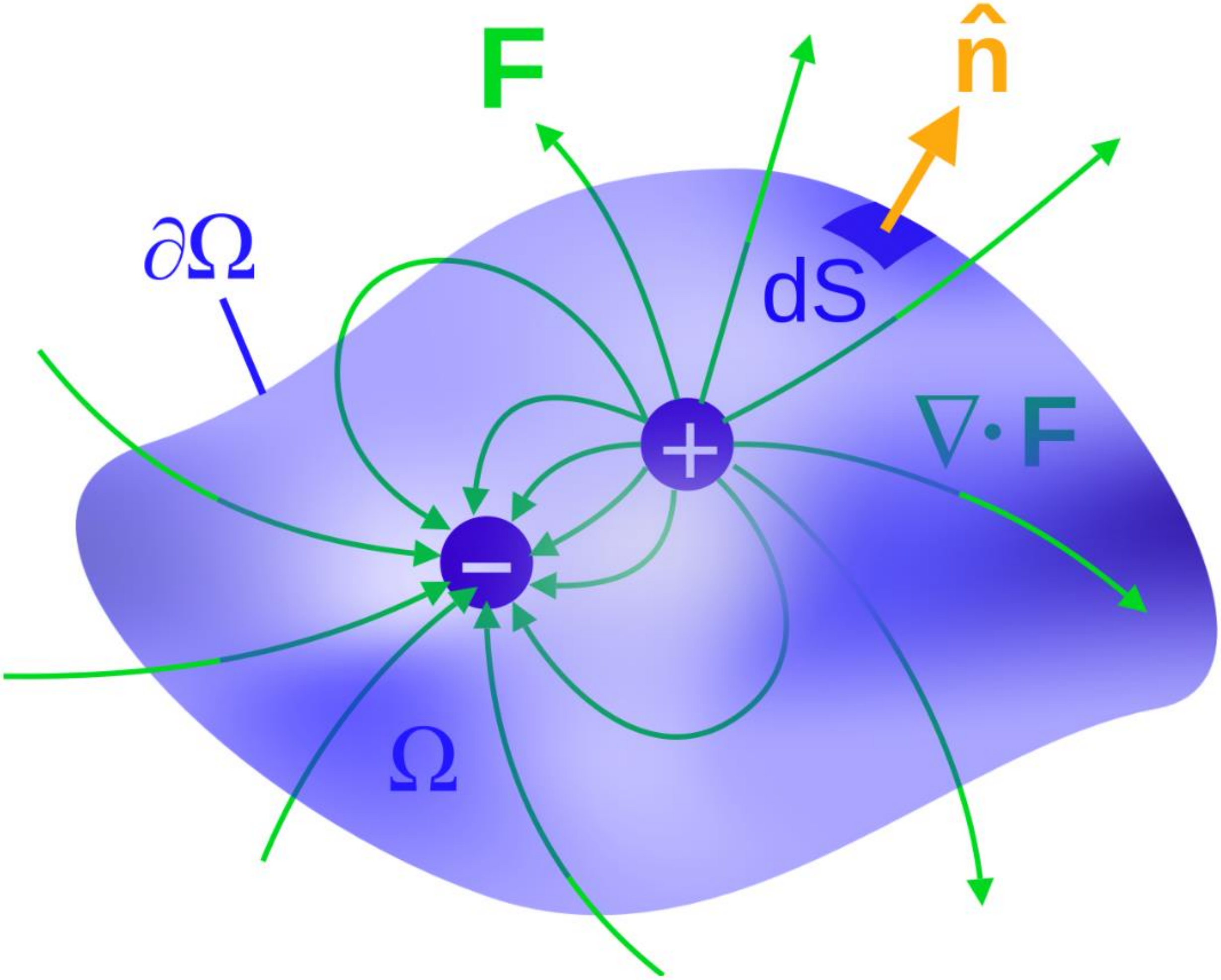
Numerical Solution to Laplace's Equation

Carleton University, Department of Electronics

ELEC 3105 Laboratory Exercise 1

Updated ANSYS Lab



PRE-LABORATORY EXERCISE

You need to complete the pre-lab and have the TA sign off your pre-lab work before starting the computer laboratory exercise.

Pre-1: Numerical solution to Poisson's and Laplace's equation

Please refer to the course lecture slides related to Poisson's and Laplace's equations for additional details on the technique. A summary is provided here. The starting equation is:

$$\overrightarrow{\nabla^2} V = -\rho/\varepsilon \tag{P-1}$$

which is known as Poisson's equation. It is a point function which implies that the "second derivative" (Gradient squared here) of the potential function at a particular point in space must equal the negative ratio of the charge density at the point divided by the dielectric constant at that same point. Should the charge density be zero then the equation simplifies to Laplace's form:

$$\overrightarrow{\nabla^2} V = 0 \tag{P-2}$$

A considerable amount of effort goes into solving this equation. For instance, once you solve for the potential you can determine the magnitude and direction of the electric field through:

$$\vec{E} = -\vec{\nabla} V \tag{P-3}$$

Once you know the electric potential and electric field you can pretty well calculate anything else related to electrostatic. The pre-lab will examine solving Laplace's equation using two different techniques. The first is a direct approach solving the second order differential equation. The second involves a numerical solution using a finite difference approach. Both techniques are discussed in detail in class.

Pre-1: Solving the differential equation

Laplace's equation is a second order differential equation. In Cartesian coordinates it is:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{P-4}$$

The same function V is subjected to derivatives with respect to (x, y, z) and when the second derivatives are formed and then summed, the resultant must be zero. Only then can the original function V be a valid solution to the equation. Under normal circumstances finding the function V that satisfies (P-4) can be difficult and when this occurs, other approaches are used to solve the equation (such as numerical indicated below). For this pre-lab we will consider a simple solution to (P-4).

Consider the parallel plate capacitor shown in figure Pre-1. The lower plate is at 0 volts, and resides in the (x, y) plane. The upper plate is at 100 volts, also resides in the (x, y) plane and intersects the z axis a distance *d* from the origin. We will treat *d* (capacitor plate separation) as small, such that we may approximate the capacitor plates as infinite in extent in the (x, y) planes. As a result, the potential function is independent of the x and y coordinates. This statement has to do with the translational symmetry that is present with regards to the x and y coordinates. As you move about in the (x, y) plane KEEPING **z** CONSTANT the environment always looks the same. Thus, in equation (P-4) the derivatives with respect to x and y are zero as (for this geometry) the potential is independent of x and y. The potential does vary in moving along the z direction. The potential is 0 volts at z=0 and is 100 volts at z=d.

Question Pre-1.1: Solve the differential equation (P-4) for the parallel plate capacitor of figure Pre-1. It is a second order differential equation so the general solution will have two constants. Determine these constants by making use of the know voltage values at z=0 and z=d. Take d = 1 mm. Plot several equipotential lines and from these draw in the electric field lines. What is the numerical value (magnitude and direction) of the electric field? **1 mark**

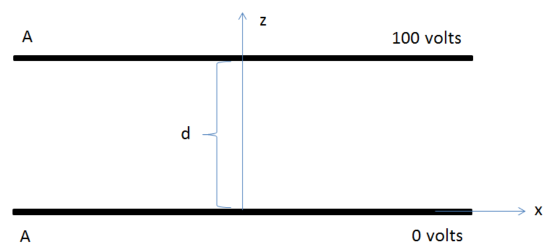


Figure Pre-1: Parallel plate capacitor geometry

Question Pre-1.2: Two concentric metal shells are shown in figure Pre-2: The inner shell has a radius of 1 cm and is at 100 volts, the outer shell has a radius of 2 cm and is at 200 volts. The region between the metal surfaces is charge free and air. Express Laplace's equation in spherical coordinates. Indicate which derivatives of the potential function will be zero and why they are zero. Solve the remaining differential equation and plot several equipotential lines for the region between the metal shells. Draw the electric field lines. **1 mark**

Question Pre-1.3: What approach would you use to solve the second order differential equation if the geometry of the capacitor plates does not conform to the unit vector directions of a coordinate system? **1 mark**

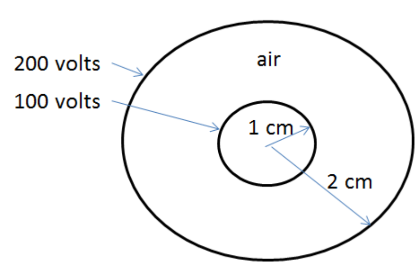


Figure Pre-2: Concentric metal shells geometry

Pre-2: Finite difference solution to Laplace's equation in 1-D

At this time, it is a good idea to review the course lecture slides related to the numerical solution to Poisson's and Laplace's equation. A review of the numerical technique is presented here for a geometry which results in a 1-D variation in the potential function. The parallel plate capacitor geometry shown in figure Pre-1 is such a geometry. The potential varies only the z direction and is constant in the (x, y) plane. Now, consider the parallel plate capacitor geometry redrawn in figure Pre-3. The z axis between the capacitor plates has been segmented and each point the z axis is assigned an index (i). The spacing between grid points is uniform and equal to h. The capacitor plate separation is d.

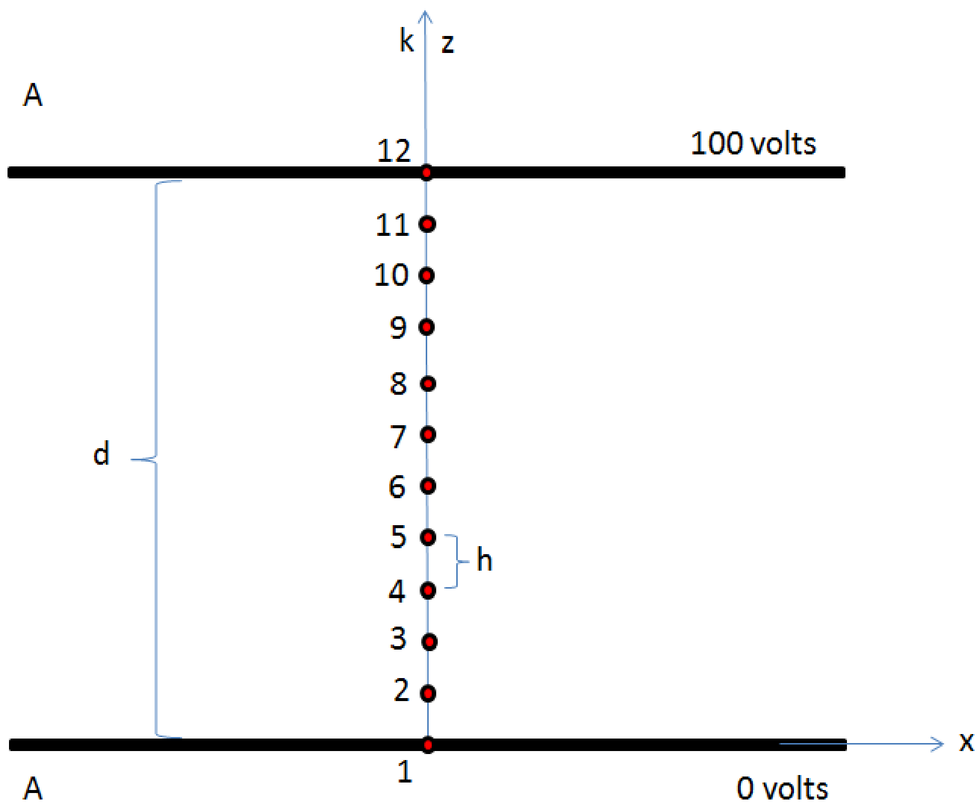


Figure Pre-3: Parallel plate capacitor geometry for numerical technique

Consider now any two adjacent grid points say points 4 and 5. The difference in voltage between these two points is $\Delta V_{5-4} = V_5 - V_4$. The separation along the z axis between these points is $\Delta z = h$. By definition, the first derivative of the potential with respect to the z axis is:

$$\frac{\partial V}{\partial z} = \lim_{h \to 0} \frac{V(z+h) - V(z)}{h} \qquad \text{(P-5)}$$

If at the moment we ignore the lim as h→0, we see that $V(z+h) - V(z)$ is the difference in voltage between adjacent grid points separated by $\Delta z = h$. Thus, an approximation to the first derivative can be obtained by $\frac{\Delta V}{\Delta z} \approx \frac{\partial V}{\partial z}$. So now, we have a way to calculate the first derivative by examining voltage values of adjacent point. But actually, Laplace's equation is made up of second derivatives. A second derivative is nothing more than the derivative of the derivative. So, let's first obtain the derivative between each grid point pair as shown in figure Pre-4. Note that the derivative points are offset from the potential points by h/2. We can now obtain the derivative of the derivative using the green grid points. $\frac{\partial\left(\frac{\partial V}{\partial z}\right)}{\partial z} = \frac{\partial^2 V}{\partial z^2} = \frac{\frac{\partial V(z+h)}{\partial z} - \frac{\partial V(z)}{\partial z}}{\partial z}$. The derivative of the derivative is also offset by h/2 in grid point location. This brings the second derivative grid point location back on top of the original grid point location. We are almost there, but we will start all over again. Let's get the derivative between points 4 and 5 and also between points 5 and 6:

$$\frac{\partial V_{5-4}}{\partial z} = \frac{V_5 - V_4}{h} \qquad \text{and} \qquad \frac{\partial V_{6-5}}{\partial z} = \frac{V_6 - V_5}{h} \tag{P-6}$$

Let's get the derivative of the derivative between points 4, 5 and 6:

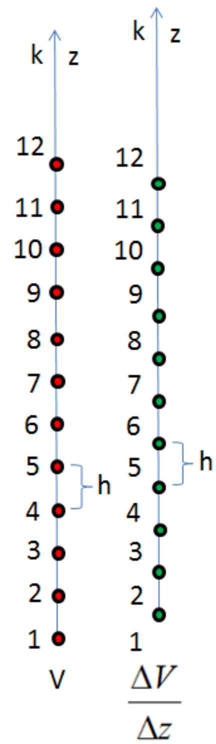
$$\frac{\partial^2 V}{\partial z^2} = \frac{\frac{\partial V_{6-5}}{\partial z} - \frac{\partial V_{5-4}}{\partial z}}{\partial z} = \frac{\frac{V_6 - V_5}{h} - \frac{V_5 - V_4}{h}}{h} = \frac{V_6 + V_4 - 2V_5}{h^2} \tag{P-7}$$

For the parallel plate capacitor problem there are no variations in the potential with respect to x and y and the region between the plates is charge free. Thus $\frac{\partial^2 V}{\partial z^2} = 0$ which when using (P-7) gives:

$$\frac{V_6 + V_4 - 2V_5}{h^2} = 0 \qquad \text{after rearranging} \qquad \frac{V_6 + V_4}{2} = V_5 \tag{P-8}$$

This expression indicates that the voltage at grid point 5 is the average value of the voltage one grid point up and grid point down. This expression can be turned into a numerical technique through the following algorithm:

- Divide the space into an equal number of grid points. Make certain that grid points are assigned to surfaces that are at fixed voltages (like the plates of the capacitors, see figure Pre-3)
- Assign an arbitrary voltage to each grid point that is not fixed. Try to select voltage values in the range of the fixed values.
- Update the voltage on each grid point by forming the average of its nearest neighbours.
- Using the updated values for the voltages, update them again by forming the average of nearest neighbours.
- Repeat the updating process until the voltage values at each grid point no longer change. Usually you will specify the number of decimal points for the accuracy and once the required number of decimal points are resolved the updating process is stopped.
- The final voltage values are the voltage values at the grid points.



Pre-4: Potential, first derivative and second derivative

Question Pre-2.1: For the parallel plate capacitor given in figure Pre-3, use the numerical technique to obtain the voltages at the grid points accurate to 1 decimal place. Make a good starting guess to the voltages. Take d = 1 mm. **1 mark**

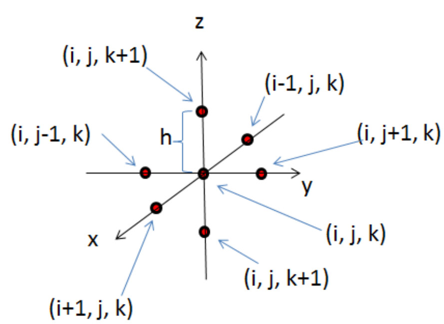
Question Pre-2.2: Develop an XL spread sheet to solve the parallel plate capacitor numerically to 3 decimal places. (If you wish you may write a MATLAB program instead). **1 mark**

Question Pre-2.3: Instead of using 12 grid points use 102 grid points. Modify your program to solve numerically Laplace's equation for the parallel plate capacitor to 5 decimal places. **1 mark**

Question Pre-2-4: Any numerical technique utilized requires an estimate of its accuracy. Examine the course lecture slides, text books on numerical techniques, … and obtain an estimate for the error involved in using this approach to solving Laplace's equation. **1 mark**

Pre-3: Finite difference solution to Laplace's equation in 2-D and 3-D

The numerical approach presented above can be easily extended into 2-D and 3-D. We need to develop the finite difference approximations to each of the second order derivatives in equation (P-4). We have already worked out the derivative part for the z direction. We imposed a grid along the z axis and formed the first and second derivative. Now in 3-D, we need to establish grid points along the other two axes. We thus end up with a volume of grid points with each grid point identified by the indices (i, j, k). We then form the second derivatives for each additional direction. Figure Pre-5 shows one of the grid points extracted (point i, j, k) and its six nearest neighbours.



Pre-5: 3-D grid points about center (i, j, k) point

The resultant combination of the three second order derivatives of equation (P-4) results in the following expression:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2} == \frac{V_{i+1,j,k}+V_{i-1,j,k}-2V_{i,j,k}}{h^2}+\frac{V_{i,j+1,k}+V_{i,j-1,k}-2V_{i,j,k}}{h^2}+\frac{V_{i,j,k+1}+V_{i,j,k-1}-2V_{i,j,k}}{h^2} \quad \text{(P-9)}$$

When dealing with Laplace's equation the above equation is equal to zero and thus can be simplified and rearranged to yield an expression for the voltage at point (i, j, k) as the average of its nearest neighbours (3-D Grid):

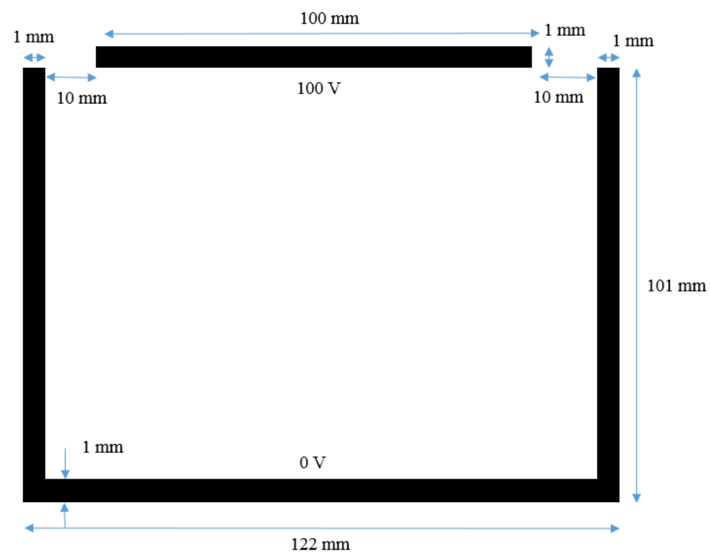
$$\frac{V_{i+1,j,k}+V_{i-1,j,k}+V_{i,j+1,k}+V_{i,j-1,k}+V_{i,j,k+1}+V_{i,j,k-1}}{6}=V_{i,j,k} \quad \text{(P-10)}$$

In the situation where the geometry can be analysed in 2-D, say x and y, the averaging would involve only 4 nearest neighbours with the grid using indices i and j.

$$\frac{V_{i+1,j}+V_{i-1,j}+V_{i,j+1}+V_{i,j-1}}{4}=V_{i,j} \quad \text{(P-11)}$$

The same numerical algorithm presented above can be applied to the 2-D and 3-D grid. The difficulty in using this approach in 2-D and 3-D comes from the bookkeeping required to keep all the grid point averaging correctly linked.

Question Pre-3.1: For the structure shown in figure Pre-6 use a 2-D numerical grid approach to obtain a mapping of the potential inside the electrode region. To keep the problem manageable, use a grid with a 10 mm spacing. Obtain the voltages on the grid points accurate to 1 decimal place and use either XL or MATLAB to solve. **1 mark**



Pre-6: Potential well electrode structure

Question Pre-3.2: From the potential values determined above draw in the electric field vectors. **1 mark**

Lab 1: Numerical Solution of Laplace's Equation

ELEC 3105

Updated ANSYS Lab

1. Before You Start

- This lab and all relevant files can be found at the course website.
- You will need to obtain an account on the network if you do not already have one from another course.
- Write your name in the sign in sheet when you arrive for the lab.
- You can work alone or with a partner.
- One lab write-up per person.
- Show units in all calculations, all graphs require a legend.

2. Objectives

The objective of this lab is to illustrate the use of a powerful numerical technique known as the **finite element method** to solve Laplace's equation for selected problems. The lab will run in the Department of Electronics undergraduate laboratory, room **AP340**. The software package we will use is **ANSYS Electronics Desktop – Maxwell 2D/3D Solver** from Ansys Corporation. This software will enable you to visualize the electric field vectors and the voltage equipotential in cross sections of structures consisting of conductors and insulators.

3. Background

The finite element method (FEM) is a numerical technique for finding approximate solutions to partial differential equations [1]. Consider the example of a 2-D solution and its corresponding mesh shown in Figure 1. The lines represent the direction and magnitude of flux density simulated using FEM in the solution image and the triangles (or sub regions) represent a single calculated solution in the mesh image. As an analogy, compare a jpeg file with large pixels, making the image blurry and a jpeg file with smaller pixels, allowing the image to become sharper. Therefore, the smaller the sub region, the more accurate the entire solution. A numerical solution is always an approximation of an analytical solution, which is based on mathematical theory.

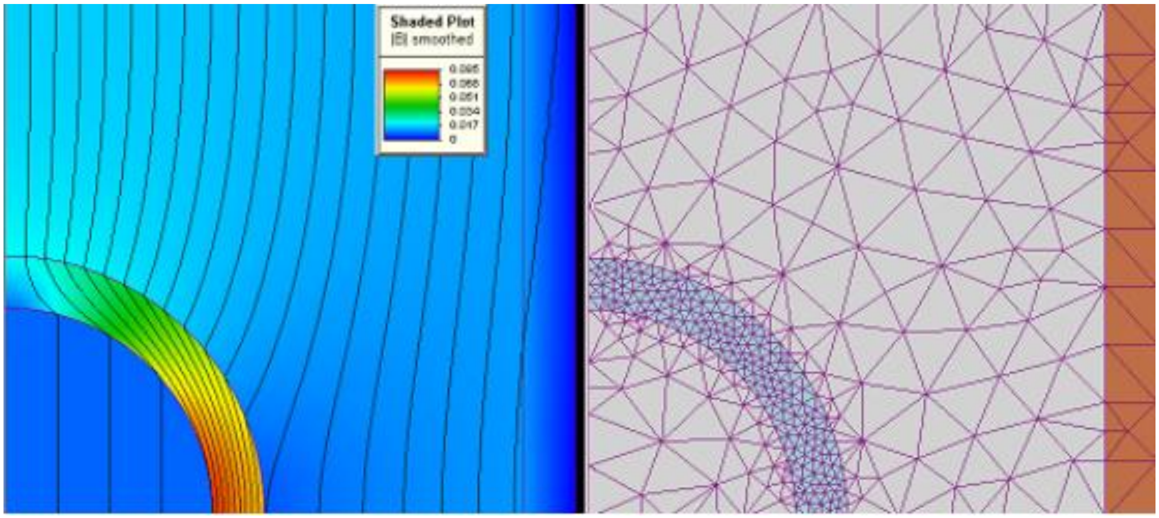


Figure 1: The 2-D solution (left) and mesh (right) [1]

Consider Laplace's equation describing the potential *V* in a 2-D region:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{1}$$

A solution can be found using FEM by approximating the size of dV. Smaller triangles are used where the potential V(x, y) is rapidly varying, and larger triangles are used where the potential is varying slowly. The potential is approximated within each triangle as a polynomial expansion in x and y. A numerical algorithm is used to solve for the coefficients of the polynomial in each triangle such that the nodes of adjacent triangles have the same potential. Conducting surfaces are constant potential surfaces - the user initially sets the value of the potential at the conductor.

Electric energy is stored in the electric field. The energy stored is given by the expression (units Joules).

$$W_E = \frac{1}{2}\int \vec{D} \bullet \vec{E} dV \tag{2}$$

where $\vec{D} = \varepsilon \vec{E}$ is the electric flux density (*C/m*^{2}), $\vec{E}$ is the electric field intensity (*V/m*), and the dot product is used in the integrand. The energy stored in a capacitor C is given by (units J, Joules):

$$W_E = \frac{1}{2} C(\Delta V)^2 \tag{3}$$

where ΔV is the potential difference between the conductors of the capacitor. The capacitance of a structure can be evaluated as (units F, Farads):

$$C = \frac{2W_E}{\Delta V^2} \tag{4}$$

ANSYS Maxwell 2D/3D can calculate the energy W_E over the 2-D cross-section and then calculate the approximate value of the capacitance C per unit length (F/m) of the structure using a capacitance matrix. You will be analyzing five different structures:

Problem 1 - Field at a sharp or raised point

Problem 2 - Field in a hollow

Problem 3 - Parallel wire transmission line

Problem 4 - Parallel wire transmission line with plastic coating

Problem 5- Rectangular potential well

You will be asked to plot the voltage and electric field vectors for these structures. The relation between electric field and voltage is found by using the relation below (units J/C or V). [2] (pg.60)

$$\Delta V_{AB} = \frac{W}{Q_{unit}} = -\int_A^B E \bullet dl = -\int_A^B |E||dl| \cos\theta \tag{5}$$

which describes the potential, V, of point A with respect to point B, defined as the work done, W, in moving a unit charge Q_{unit}, from A to B. The electric field and the potential are perpendicular. In the case of the structures in this lab, equation 5 can be simplified by choosing a path integral such that cos(θ) = 1. If the electric field is constant in the region of integration, then all that is left to calculate is the integral with respect to the displacement l. Based on these special circumstances, the resulting equation is

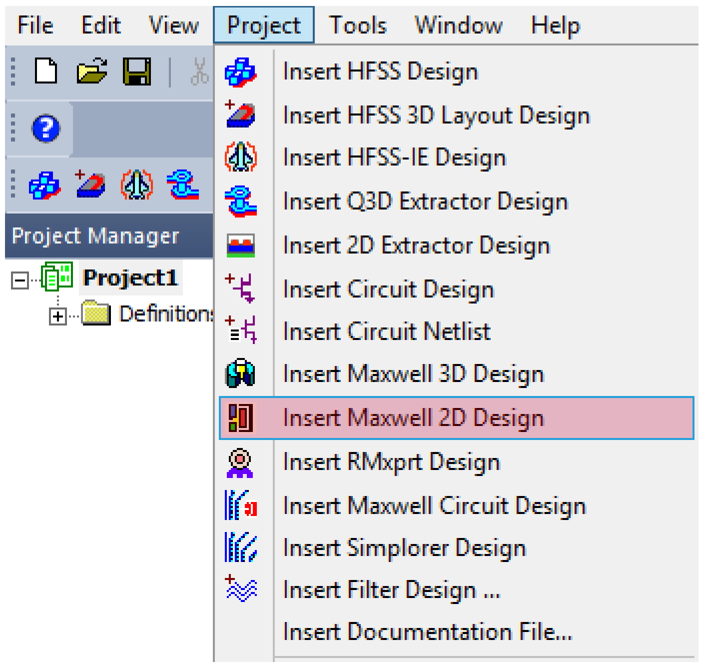
$$E = \frac{\Delta V}{\Delta l} \tag{6}$$

where ΔV is the difference in potential between two points and Δl is the distance between the points. The structures in this lab have pre-defined voltages. Keep track of their values as you go through the lab.

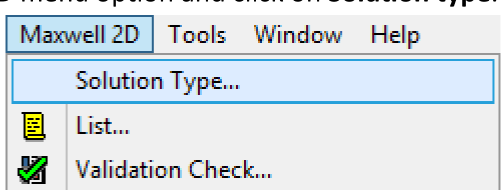
4. Running ANSYS Maxwell 2D

Note: It is always a good idea to regularly save your projects to prevent losing progress. If the instructions below are not clear for you, research what you are trying to accomplish on the internet to try and find a solution. Assume a plate thickness of 1mm unless otherwise stated.

1. Start the **ANSYS Electronics Desktop** program and select **Project**, then **Insert Maxwell 2D Design**.

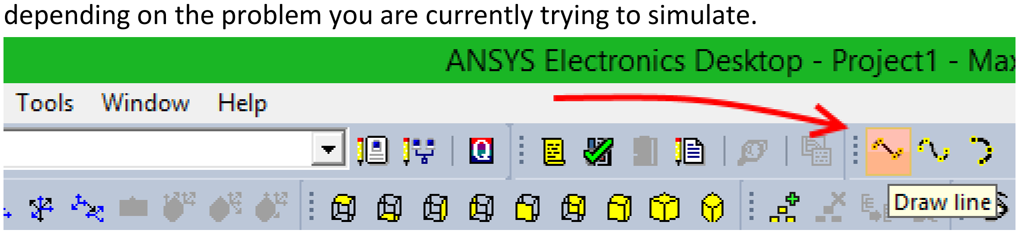


Now, select the **Maxwell 2D** menu option and click on **Solution type**.



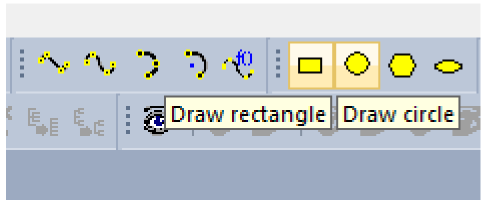
In the window the opens up, **select the required solution type**, for **lab #1 it is Electrostatic**, for **lab #2 it is Magnetostatic**. **Click OK** once you selected the correct option for you.

2. Click on the **Draw line** button shown below in order to draw the required structure geometry, depending on the problem you are currently trying to simulate.

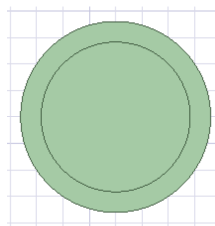


Click somewhere on the **white grid** located across the center of the interface in order to place a point, **click again** to place another point and a line will be automatically drawn between them. Play around with this function until you are comfortable using it to draw different shapes.

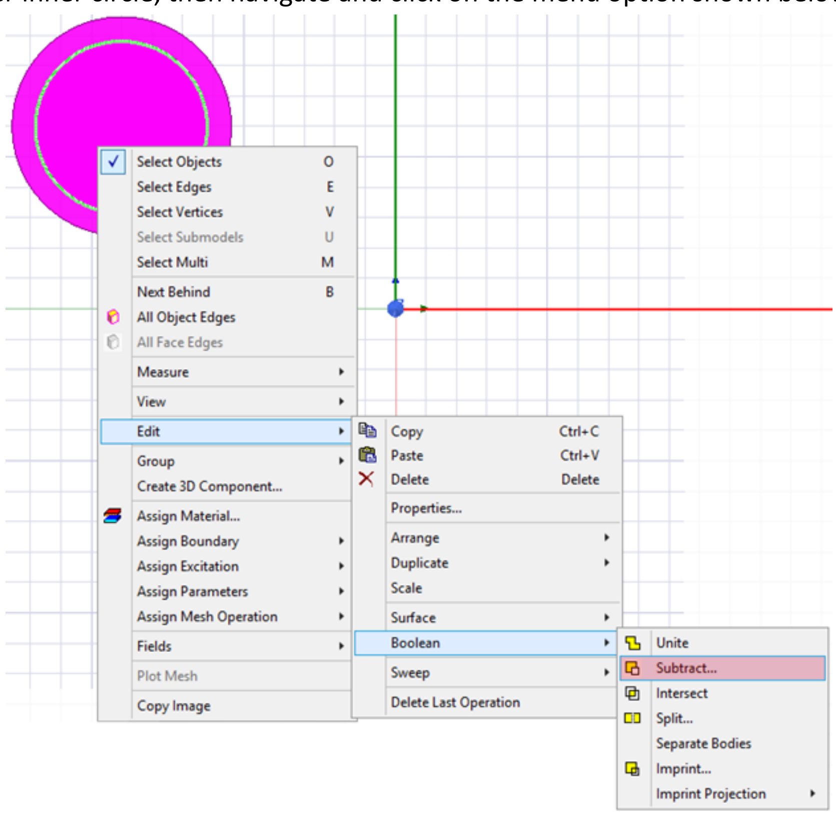
It is also possible to use the **rectangle** and **circle** functions in order to draw shapes. Familiarize yourself with how these functions work as well. You can zoom in and out by **holding the control key and scrolling** with your mouse, this may help in the case where you need a finer grid size. (Zoom in for a finer grid spacing).



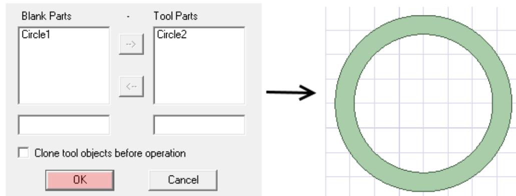
If a ring geometry is required, **draw one circle** with a radius equal to the radius of the ring's outside edge. Then **draw another circle** inside the first circle with a radius equal to the ring's inner edge as shown below.



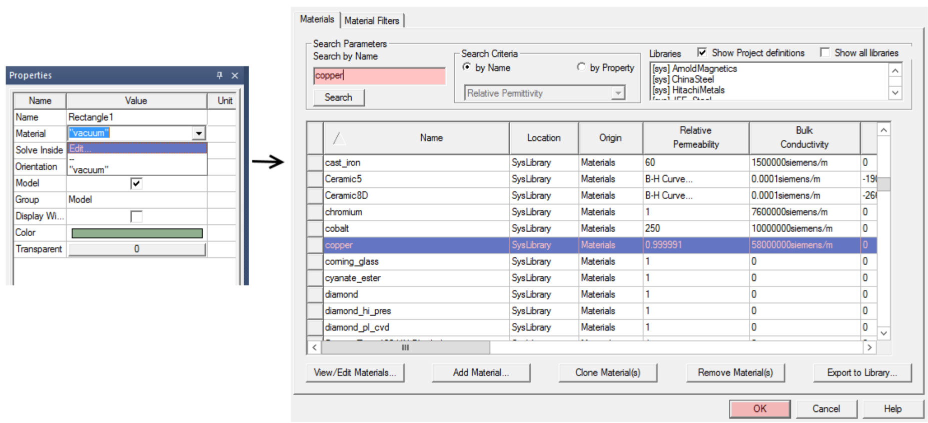
Next select both circles. You may do this by **holding the control key**, selecting the larger circle, then select the smaller inner circle, and **releasing the control key**. Once this is done, **right click** on the smaller inner circle, then navigate and click on the menu option shown below.



Click the **OK button** on the window the opens. You should be left with a ring as shown below.



3. Once the required shapes are drawn according to the problem, **click on a shape** in order to select it. Once it is selected it will change color and its properties will be displayed in the properties pane on the left side of the interface. Select the property box containing the material value and **click on Edit** and a window will open. In the "Search by name" box, **type in the name of the material** you would like to simulate, **select it from the list below** the box, and **click OK**.

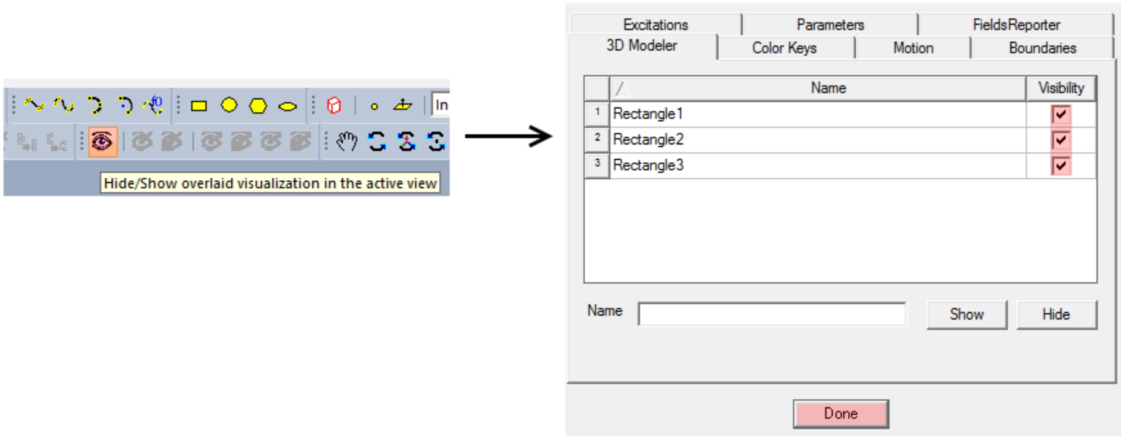


Do this for each different shape using the required material.

4. Once the geometries are drawn and have had materials assigned to them, **draw a large rectangle around them** and **set the material of this rectangle to air**. You may also change the transparency of this rectangle by changing the "Transparent" value located in the Properties pane.

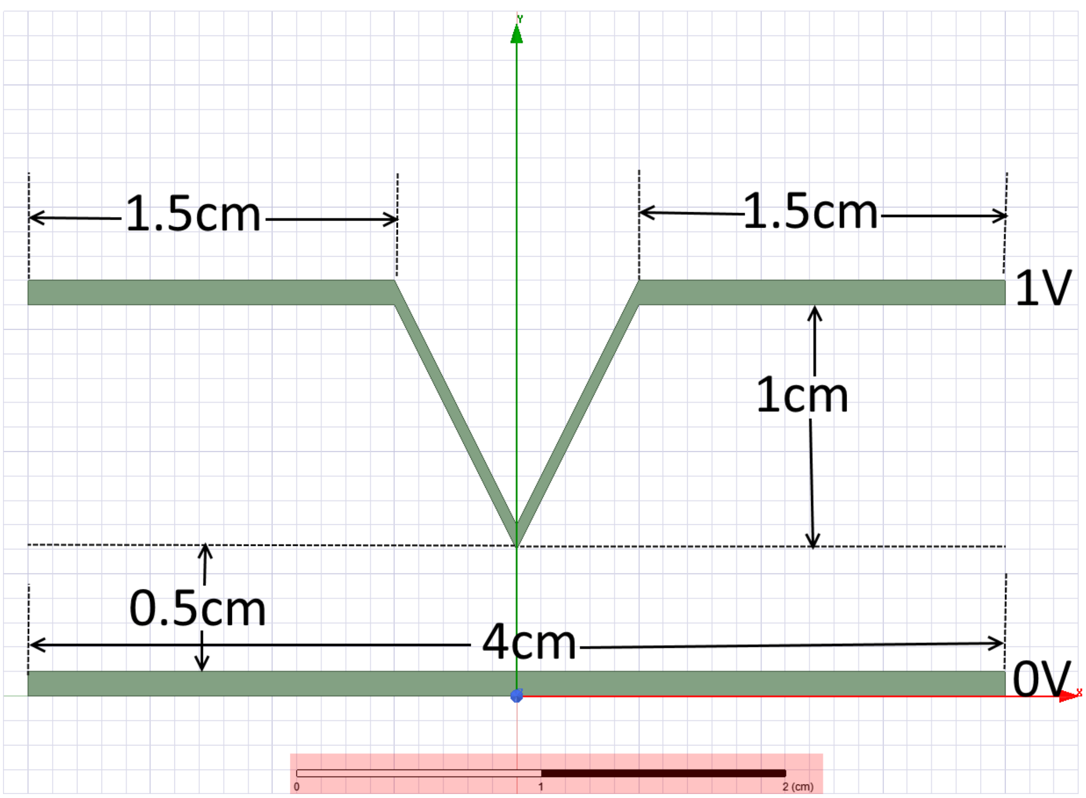
   Once this rectangle is drawn, **press the E key**, and then **while holding the control key**, **select all four edges** of the large rectangle you just drew that encapsulates the rest of the shapes. Now **right click anywhere on the white** grid and from the right click menu, **navigate to Assign Boundary (Balloon)**, and **click on Charge** (if you get a boundary error try carefully repeating this process). Set the Balloon type as **Charge** in the window that opens up, and **click OK**. You may now **press the O key** to return to object selection mode (as opposed to edge selection mode).

   It is also possible to hide or show the different geometries that you have drawn by **clicking on the eye icon** in the upper toolbar and selecting which shapes you want to remain visible in the window that opens. This is shown in the figure below. Note that if a shape is not visible, it will still be included within the simulation.

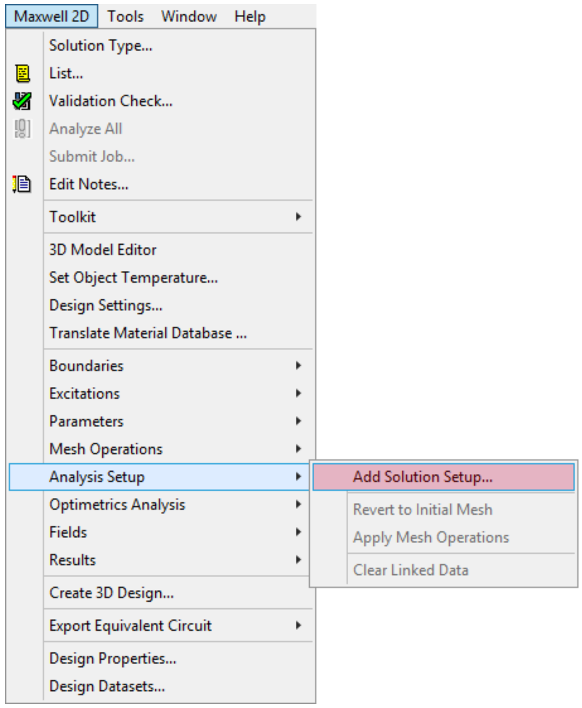


5. In order to assign voltages to materials, **right click** on the material that you wish to assign an excitation (for example a voltage), **navigate to Assign Excitation** in the right click menu, and **click on Voltage**. **Set the voltage value** required and **click ok**.

6. Verify that the shapes drawn have the correct size ratios. You should now change the scale by navigating to the top **Modeler** menu and clicking on it, then select **Units** and click it. On the window that appears check the **Rescale to new units** box and select **cm** or the proper unit for your drawing. Play around with this until your units are correct, you may verify if they are correct by using the scale located at the bottom of the white grid workspace interface shown below in the example for problem 1. (The copper plate thickness is set to 1mm in this figure.)

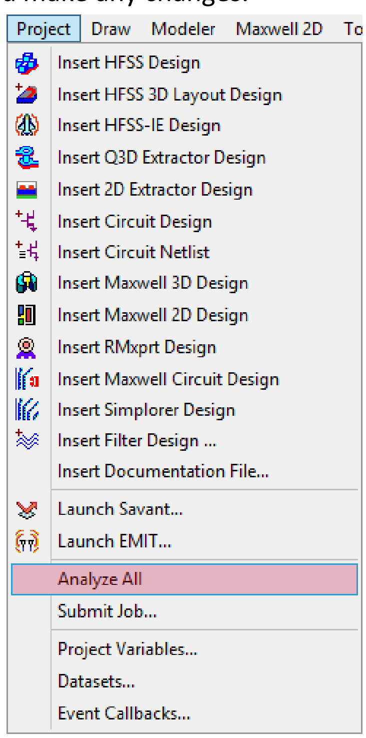


7. In order to simulate this project you must first add a solution setup by **navigating to the following menu shown below** and clicking on **Add Solution Setup**.

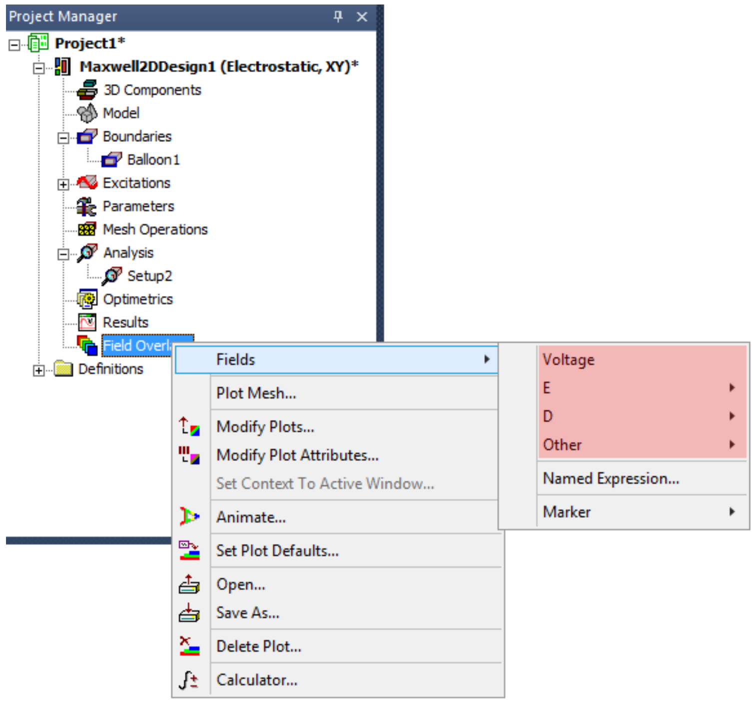


Click OK on the window that opens and then **save your project** by pressing down on the **control key and S key** at the same time. Choose a suitable location to save your project if required.

8. In order to simulate the project, navigate to the **Project** menu and click on **Analyze All**, as shown below. You must repeat this if you make any changes.



9. To plot the results, select the large air rectangle and right click on "**Field Overlays**" within the **Project Manager pane** on the left side of the interface as shown below. Select the type of result you would like to plot and a window will open. Click **Done** on the window and your plot will be visible.



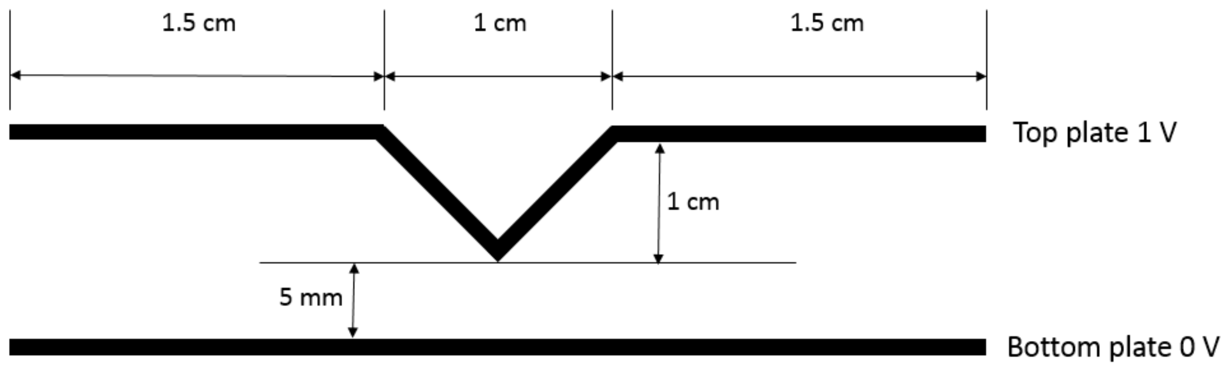
10. To find the capacitance, before analyzing the problem navigate to the project manager pane on the left side. Right click on **Parameters** and assign a matrix. Select one signal line and one signal ground (one for each source). Click OK then run the simulation. Once the simulation is completed, expand the **Parameters** tree, right click on **Matrix1** or the matrix you created, and choose **View Solution**. The capacitance units are shown near the top-right of the window and the value is shown in the large display box in the bottom half of the window.

11. Explore the **Project Manager** pane as it contains lots of useful information. You can modify field overlays by **right clicking on Field Overlays** and clicking on **Modify Plot Attributes**. This is useful for changing the resolution, color, and scale of the legend. You may also see your excitations and boundaries along with other parameters. Explore the **Properties** pane for useful settings too. You can rename shapes that you've created to custom names for easier identification if needed.

This brief tutorial on using the ANSYS Maxwell Solver should be enough to get you started, there is plenty of documentation available on the internet as well as built into the program itself.

5. Problem 1: Field at a Raised Point

This problem models a parallel plate capacitor in which one plate is dented toward the other as shown below. The top plate is at **1 V** and the bottom plate is at **0 V**. The material of both plates is **copper**. The material around the plates is **air**.



Answer the following questions for **Problem 1**.

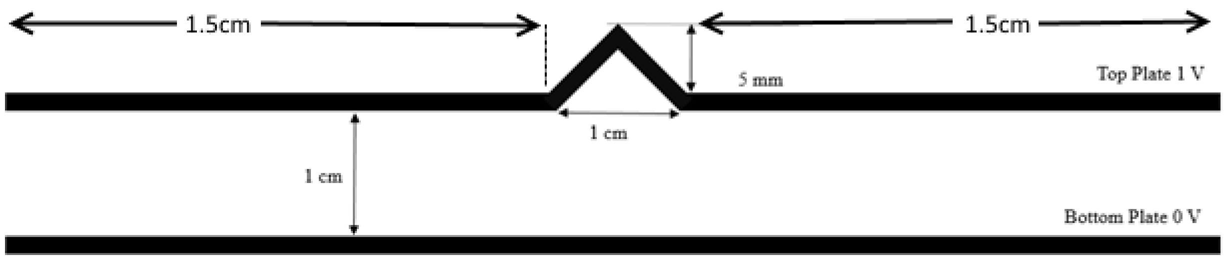
(a) Plot the equipotential voltages and the electric field lines of your structure together in one printout, or individually. Modify the scale of the plot to have 10 divisions (Instead of the default 15). Don't forget to clearly include the legends. **2 marks**

(b) Where is the location of the maximum electric field strength? What is the value of the maximum field strength? Use the coloured electric field intensity plot and the accompanied legend. Don't forget units. **2 marks**

(c) Insulating materials will break down or become conducting if the electric field strength exceeds the breakdown strength of the material. For air, the breakdown strength is about 3 x 10^6 V/m. If the gap is reduced to 1 mm, estimate the maximum voltage that could be applied to the top plate. Answer this question using theory and include units. You may use the simulator to check the calculation (Note: The simulator doesn't actually simulate the dielectric breakdown). **1 mark**

6. Problem 2: Field in a Hollow

This problem models a parallel plate capacitor with one plate dented away from the other as shown below. The top plate is at **1 V** and the bottom plate is at **0 V** source. The material of both plates is **copper** and the dielectric is **air**.



Answer the following questions for **Problem 2**.

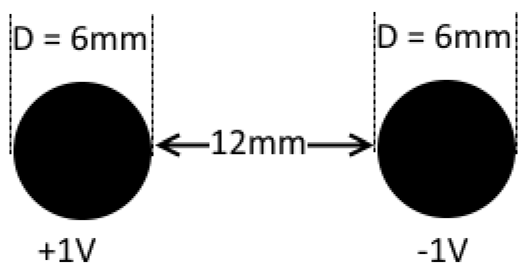
a) Plot the equipotential voltages and electric field lines of your structure as in Problem 1. **2 marks**
b) Consider the region between the two plates. Why is the electric field different in the hollow? **2 marks**

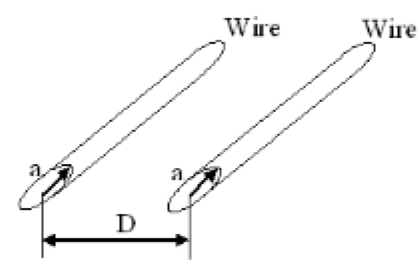
7. Problem 3: Parallel Wire Transmission Line

VHF and UHF antennas are usually connected to TV sets by transmission lines consisting of two parallel wires of fixed separation, as shown below. To design the transmission line, we need to find the capacitance per unit length between the wires. The capacitance per unit length is given analytically by (units F/m)

$$C = \frac{\pi\varepsilon}{\cosh^{-1}(\frac{\Delta D}{2a})} \tag{7}$$

where ΔV is the difference in potential between the two wires, **(verify this equation)** ε is the dielectric constant of the homogeneous material surrounding the wires, D is the center to center wire spacing, and *a* is the radius of the wires, as shown below. The dielectric constant of air is ε_0 = 8.854×10^{-12} F/m. For other materials, we multiply this value by the relative dielectric constant ε_r of the material (that is $\varepsilon = \varepsilon_0\varepsilon_r$). The function $\cosh^{-1}$ is found using the **hyp** button on any scientific calculator. The object of problem 3 is to **find the capacitance numerically and compare with the theoretical value.** We will assume that the radius of the wire is always 1 mm, but will allow for different spacing between the wires.





The wires have a diameter of **D = 6 mm**. The material of both wires is **copper**, one wire is at **1 V** while the other is at **-1 V**. If we assume that the parallel wires can be estimated by two parallel plates, then the capacitance, neglecting fringing, can also be written as (units F), [2] (pg. 96)

$$C = \frac{\varepsilon_0\varepsilon_r A}{l} = \frac{Q}{V} = \frac{Q}{El} \tag{8}$$

where A is the area of the plates, Q is the charge on the plates, *l* is the distance between the plates, ε_0 is the dielectric constant of air, and ε_r is the relative dielectric constant of the material between the plates. This relation indicates that the electric field is related to the dielectric properties of the material in between the plates.

Answer the following questions for **Problem 3.**

a) Plot the equipotential voltages and electric field lines of your structure. **2 marks**

b) What do you notice about the direction of the electric field at any point in relation to the equipotential lines? **1 mark**

c) Specify the region at which the electric field is maximum and state the maximum value. Use the legend to guide you. Theoretically you will find that the maximum should not be one point, but several points. **3 marks**

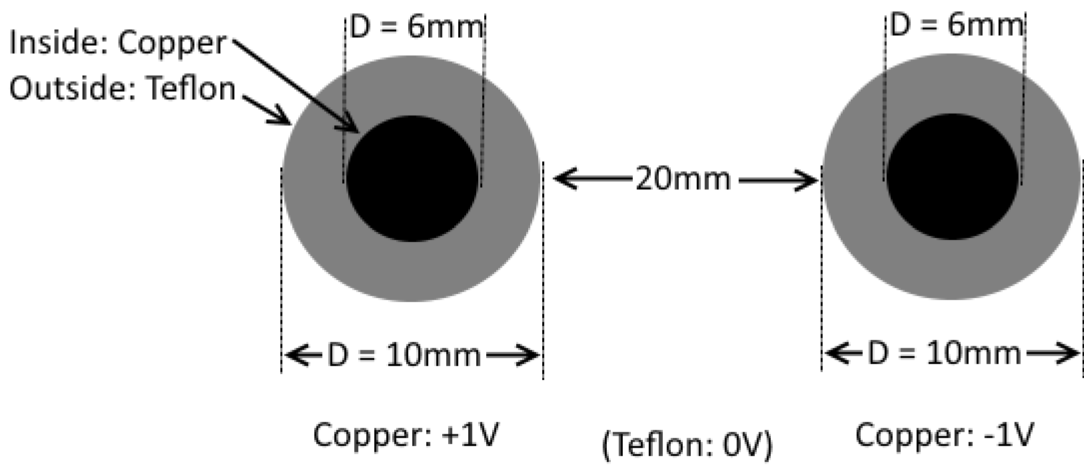
d) Estimate the capacitance per unit length of the transmission line using the software.

In our case, we are using two wires with ΔV = (1 V − (−1 V)) = 2 V. Therefore, $C = \frac{2U}{V^2}$ where V = 1 V becomes $C = \frac{2U}{\Delta V^2}$ where ΔV = 2 V. If you follow the instructions exactly, you must take the $\frac{1}{4}$ fraction into account in your final result. **3 marks**

e) Calculate the theoretical value of the capacitance per unit length as explained in the introduction to **Problem 3**. Compare to the estimated value of d) and explain any discrepancy. Remember that you are comparing 2 different methods of solving for capacitance: numerical and analytical. **3 marks**

8. Problem 4: Transmission Line with Plastic Coating

Now modify the structure in Problem 3 so that the wires are coated with a plastic (dielectric) layer of radius 2.0 mm. The plastic material is **Teflon** and when drawing, the center of the plastic should be the same as the center of the copper wire. Read the previous section for how to draw a ring.



Answer the following questions for **Problem 4.**

a) Plot the equipotential voltages and electric field lines of your structure. **2 marks**

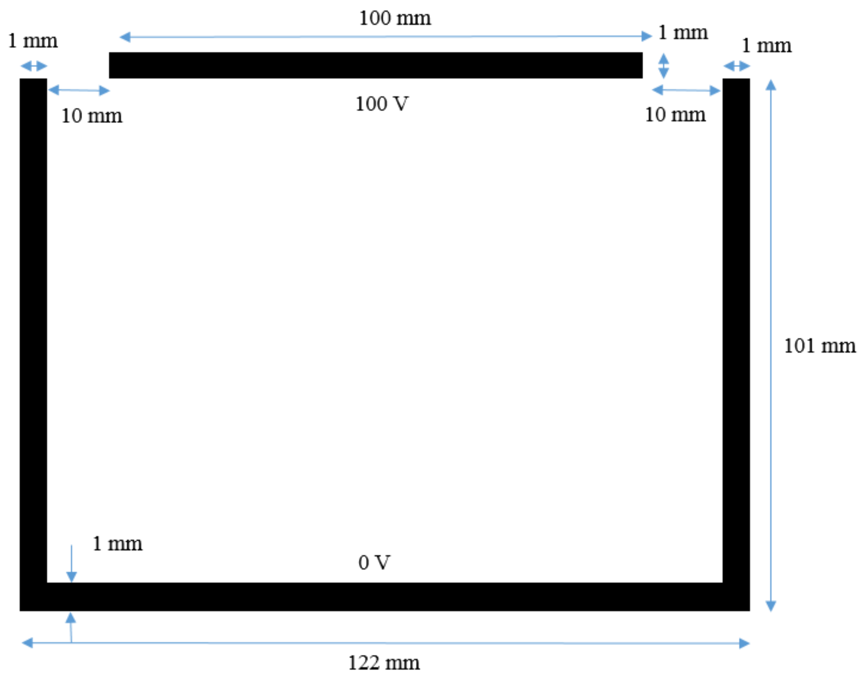
b) State the maximum value of the electric field and state why it is greater or less than the maximum values found in Question 3. **2 marks**

c) Estimate the capacitance per unit length of the transmission line using the simulation software. **2 marks**

d) Is the capacitance greater or less than the one estimated in Problem 3? Explain. **3 marks**

9. Problem 5: Rectangular potential well

The side plates and bottom plate are connected and all at **0V**. The top plate is at **100V**. The material around the plates is **air**.



Answer the following questions for **Problem 5.**

a) Plot the equipotential and electric field lines of your structure. **2 marks**
b) Compare results obtained here with those calculated in the pre-lab section. **2 marks**

References

[1] http://en.wikipedia.org/wiki/Finite_element_analysis, accessed September 2008.

[2] Edminister, J.A., Schaums Outlines: Electromagnetics, second edition, 1993.