Lab 1: Numerical Solution to Lapace's Equation

ELEC 3105 A – A5

Name: Youssef Ibrahim

1.0 INTRODUCTION

The purpose of this lab is to use Finite Element Method (FEM) to solve Laplace's equations. ANSYS Maxwell 2D/3D software was used to simulate five different structures and visualize electric field vectors and voltage equipotential in cross sections of structures consisting of conductors and insulators.

2.0 PROBLEM 1

<u>2.1 Problem 1.a</u> Plot the equipotential voltages and the electric field lines of your structure together in one printout, or individually. Modify the scale of the plot to have 10 divisions (Instead of the default 15). Don't forget to clearly include the legends.

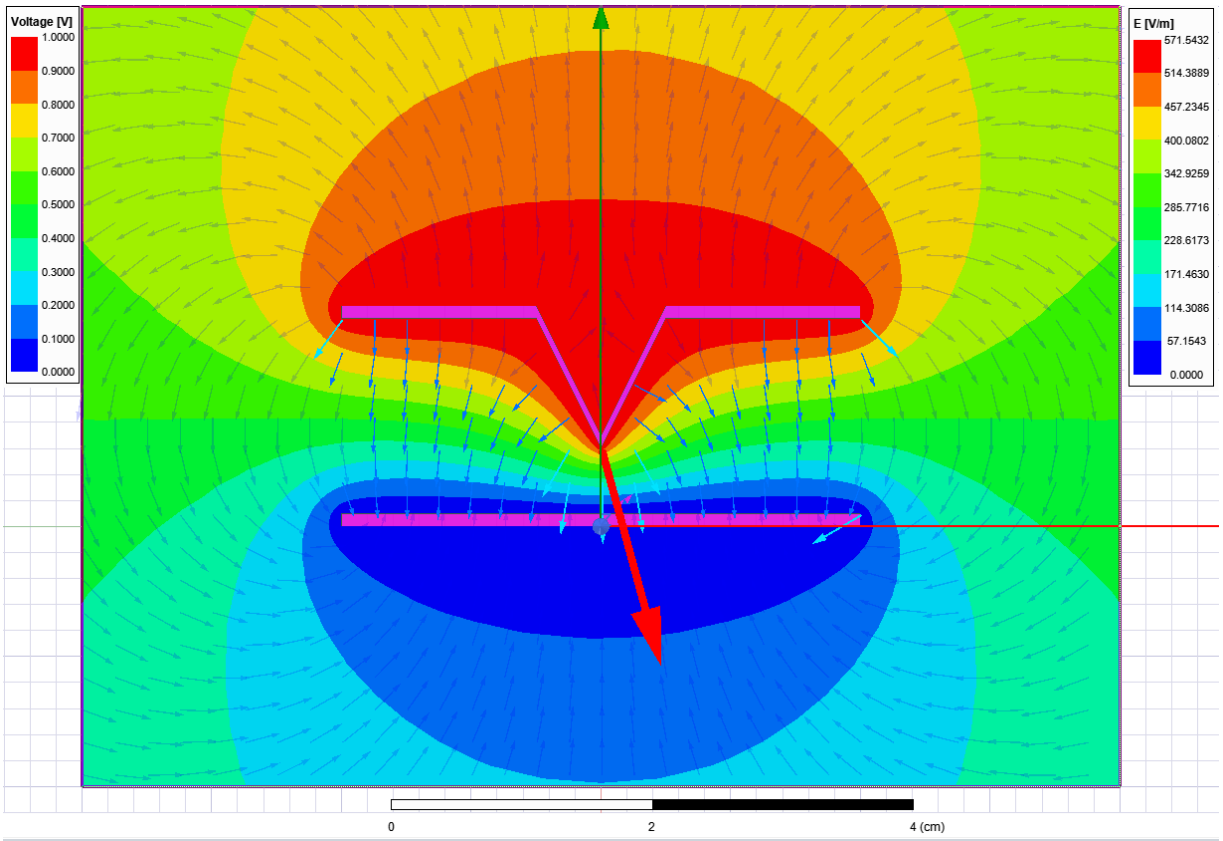


Figure 2.1: Structure 1 Equipotential Voltages and Electric Field Lines

Figure 2.1 above shows the equipotential voltages and electric field lines for structure 1.

2.2 Problem 1.b Where is the location of the maximum electric field strength? What is the value of the maximum field strength? Use the coloured electric field intensity plot and the accompanied legend. Don't forget units.

Figure 2.1 above shows that the location of the maximum electric field strength is at the tip of the V-shaped portion of the structure. The value of the maximum field strength is 5.7154 x 10^2 V/m, which is shown in Figure 2.1 above by a red arrow below the tip of the V-shaped portion.

2.3 Problem 1.c Insulating materials will break down or become conducting if the electric field strength exceeds the breakdown strength of the material. For air, the breakdown strength is about 3 x 106 V/m. If the gap is reduced to 1 mm, estimate the maximum voltage that could be applied to the top plate. Answer this question using theory and include units. You may use the simulator to check the calculation (Note: The simulator doesn't actually simulate the dielectric breakdown).

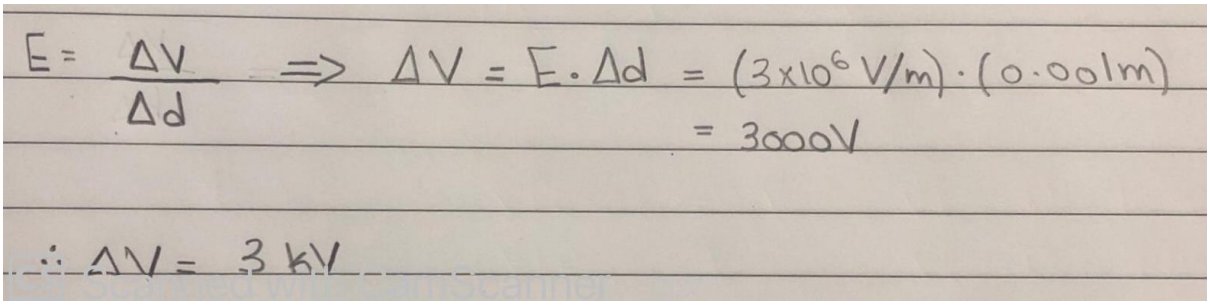


Figure 2.2: Maximum Voltage Calculation

Figure 2.2 above shows that the maximum voltage that could be applied to the top plate is 3Kv.

3.0 PROBLEM 2

3.1 Problem 2.a Plot the equipotential voltages and electric field lines of your structure as in Problem 1.

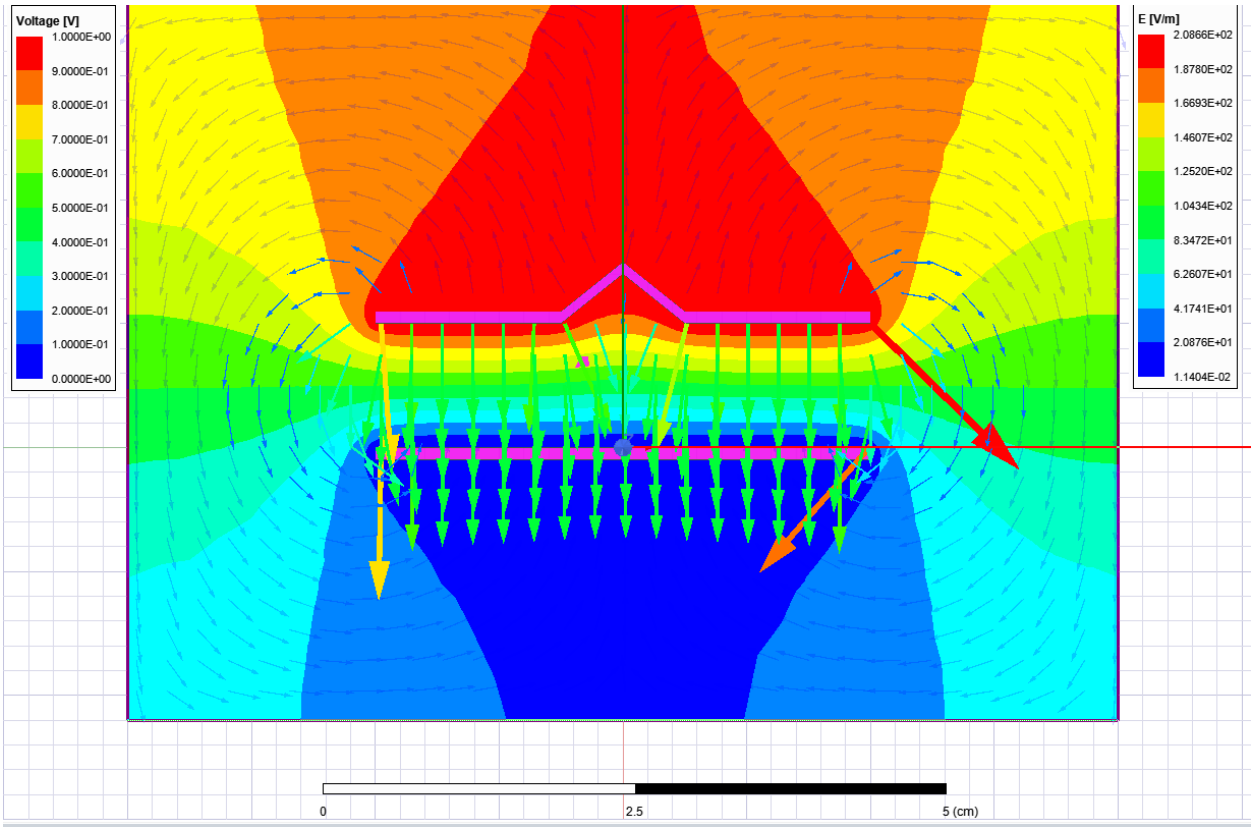


Figure 3.1: Structure 2 Equipotential Voltages and Electric Field Lines

Figure 3.1 above shows the equipotential voltages and electric field lines for structure 2.

3.2 Problem 2.b Consider the region between the two plates. Why is the electric field different in the hollow?

The x component of the electric field cancels each other since they point towards each other, the electric field is mostly noticeable at the negative y direction. This is due that the hollow points inwards.

4.0 PROBLEM 3

4.1 Problem 3.a Plot the equipotential voltages and electric field lines of your structure.

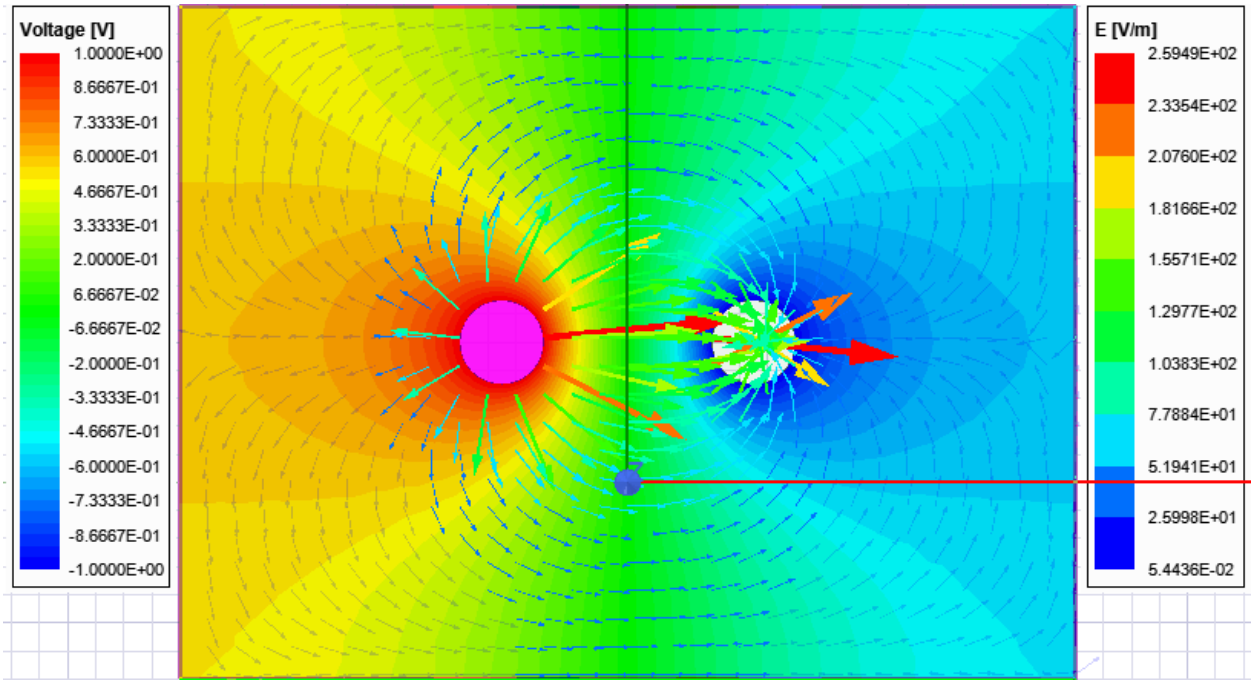


Figure 4.1: Structure 3 Equipotential Voltages and Electric Field Lines

Figure 4.1 above shows the equipotential voltages and electric field lines for structure 3.

4.2 Problem 3.b What do you notice about the direction of the electric field at any point in relation to the equipotential lines?

Figure 4.1 above shows that the electric field lines are perpendicular to the equipotential lines.

4.3 Problem 3.c Specify the region at which the electric field is maximum and state the maximum value. Use the legend to guide you. Theoretically you will find that the maximum should not be one point, but several points.

The region at which the electric field is maximum is in between the two wires, at the exit point of the left wire and the entrance of the right wire.

4.4 Problem 3.d Estimate the capacitance per unit length of the transmission line using the software.

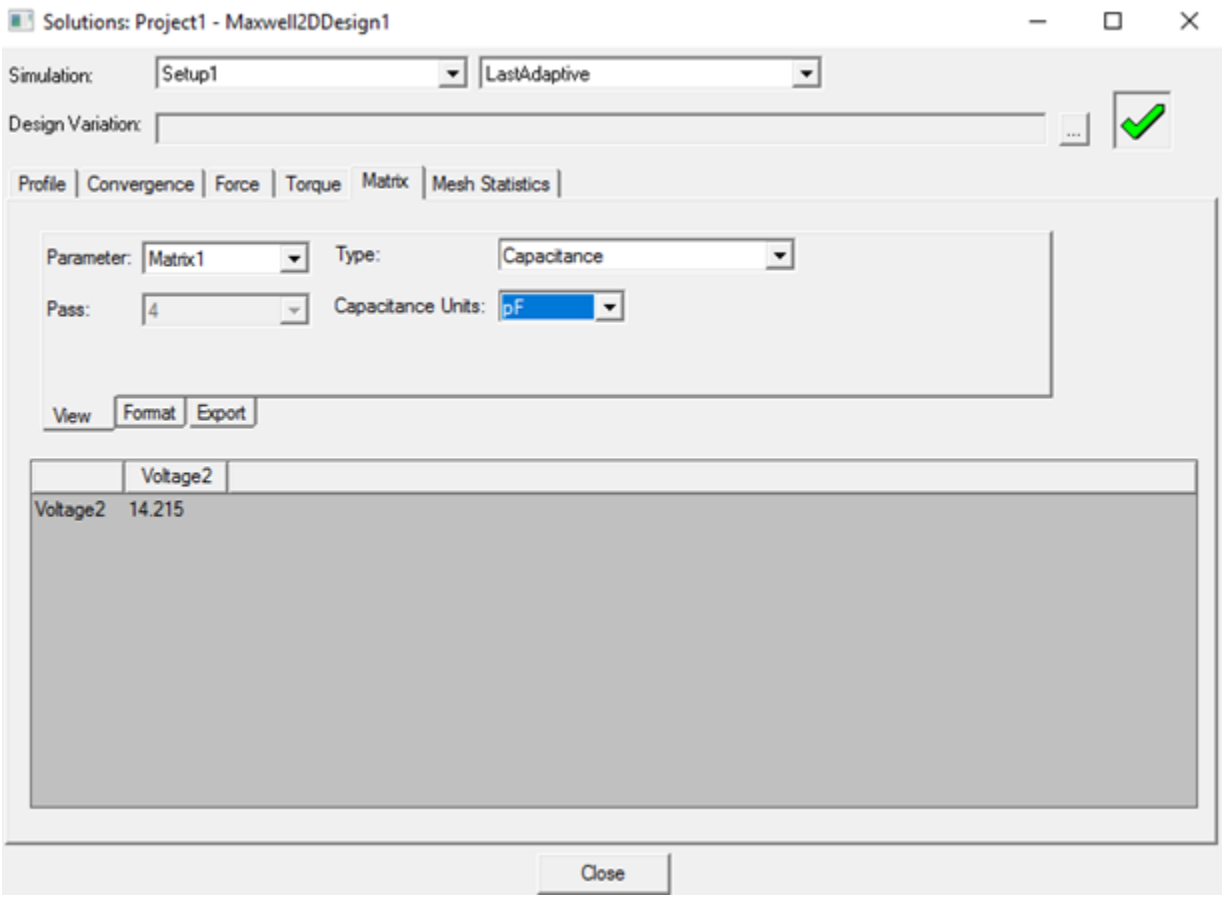


Figure 4.2: Capacitance Simulation for Structure 3

Figure 4.2 above shows that using ANSYS software the estimate of the capacitance per unit length of the transmission line is 14.215 pF.

<u>4.5 Problem 3.e</u> Calculate the theoretical value of the capacitance per unit length as explained in the introduction to Problem 3. Compare to the estimated value of d) and explain any discrepancy.

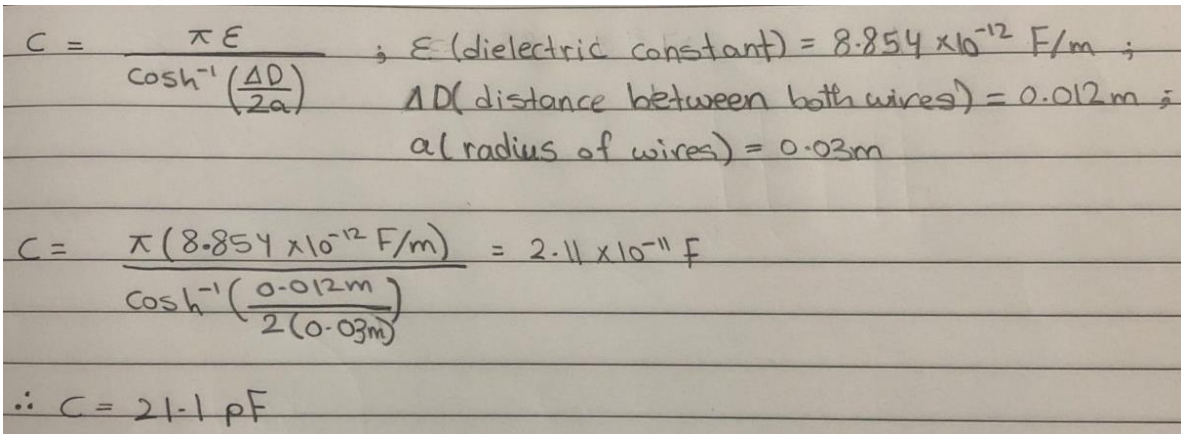


Figure 4.3: Capacitance Calculation

Figure 4.3 above shows that the theoretical value per unit length is 21.1 pF, which greater than the simulated value by 6.885 pF. The simulated value is more accurate than the theoretical value since the simulation includes many factors and equations that the theoretical method doesn't take into consideration.

5.0 PROBLEM 4

5.1 Problem 4.a Plot the equipotential voltages and electric field lines of your structure.

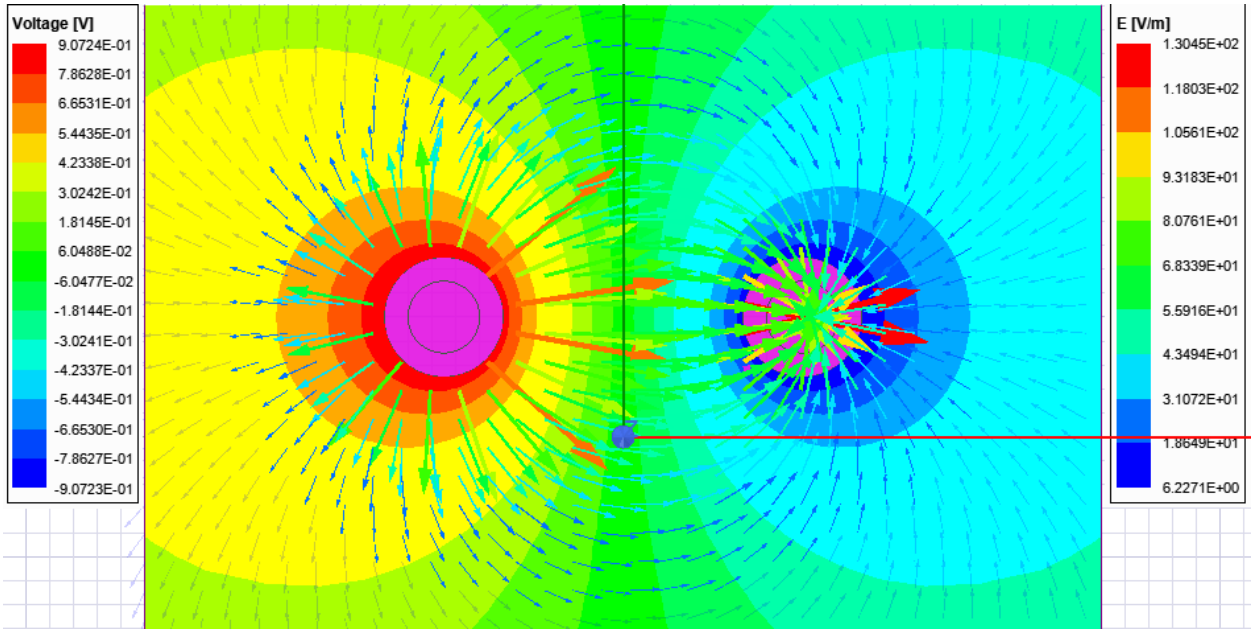


Figure 5.1: Structure 4 Equipotential Voltages and Electric Field Lines

Figure 5.1 above shows the equipotential voltages and electric field lines for structure 4.

5.2 Problem 4.b State the maximum value of the electric field and state why it is greater or less than the maximum values found in Question 3.

The maximum value of the electric field is 130.45 x 10^2 V/m, which is lower than the value obtained from problem 3 due to the presence of an insulator Teflon that decreases the electric field strength.

5.3 Problem 4.c Estimate the capacitance per unit length of the transmission line using the simulation software.

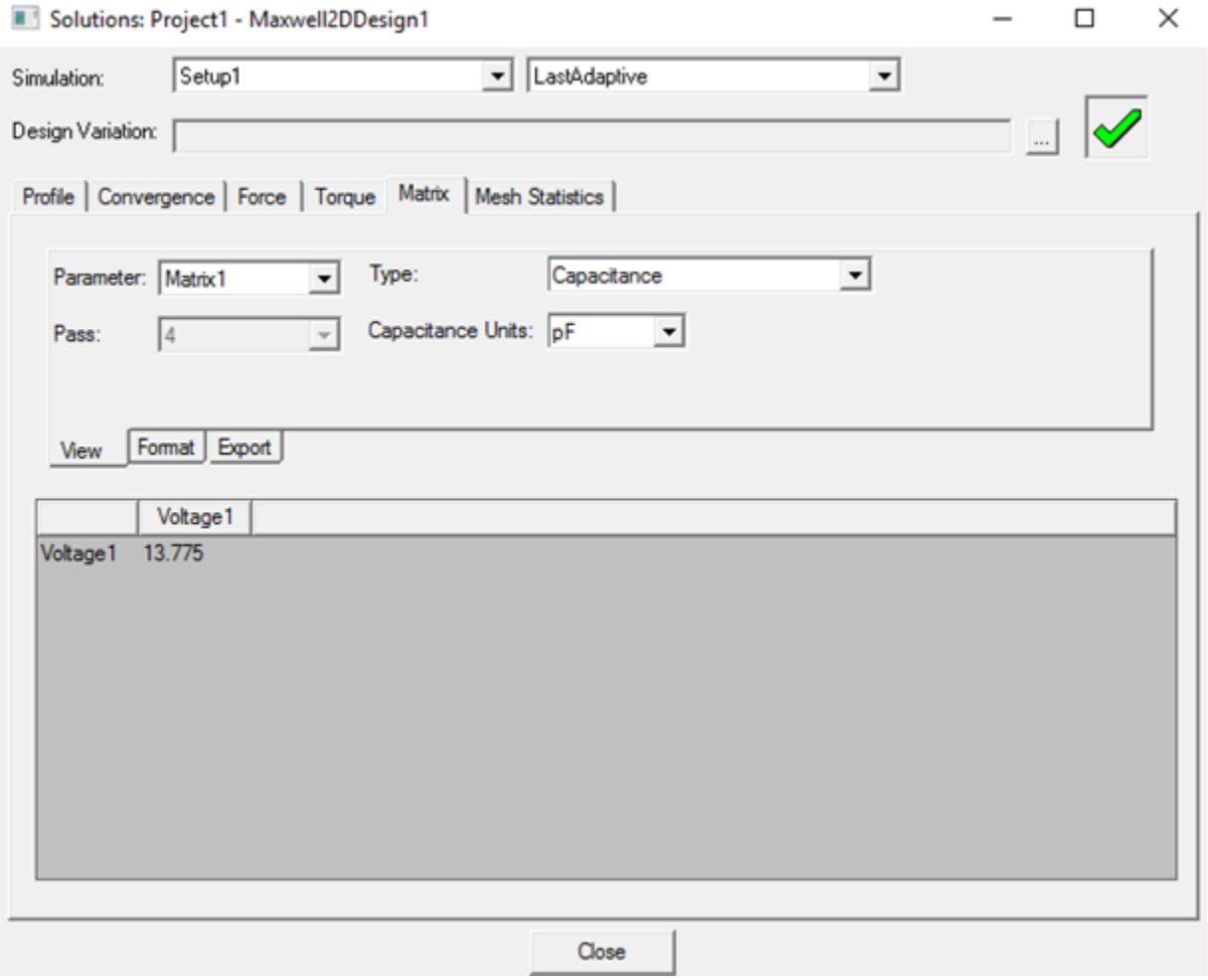


Figure 5.2: Capacitance Simulation for Structure 4

Figure 5.2 above shows that using ANSYS software the estimate of the capacitance per unit length of the transmission line is 13.775 pF.

5.4 Problem 4.d Is the capacitance greater or less than the one estimated in Problem 3? Explain.

The estimated capacitance is less than the estimated capacitance in problem 3. This can be explained using the equation showed below.

$$C = \frac{\varepsilon_0 \varepsilon_r A}{l} = \frac{Q}{V} = \frac{Q}{El}$$

It is shown in the equation above that the capacitance is inversely proportional to distance between the wires “l”, and electric field (E). Since the distance between wires was increased from 12 mm to 20 mm, this caused the capacitance to decrease. In problem 4, electric field strength was decreased due to the added insulator, which was supposed in return to increase capacitance, but the effect caused by increasing the distance between the wires was greater than the effect caused by decreasing the electric field strength.

6.0 PROBLEM 5

6.1 Problem 5.a Plot the equipotential and electric field lines of your structure.

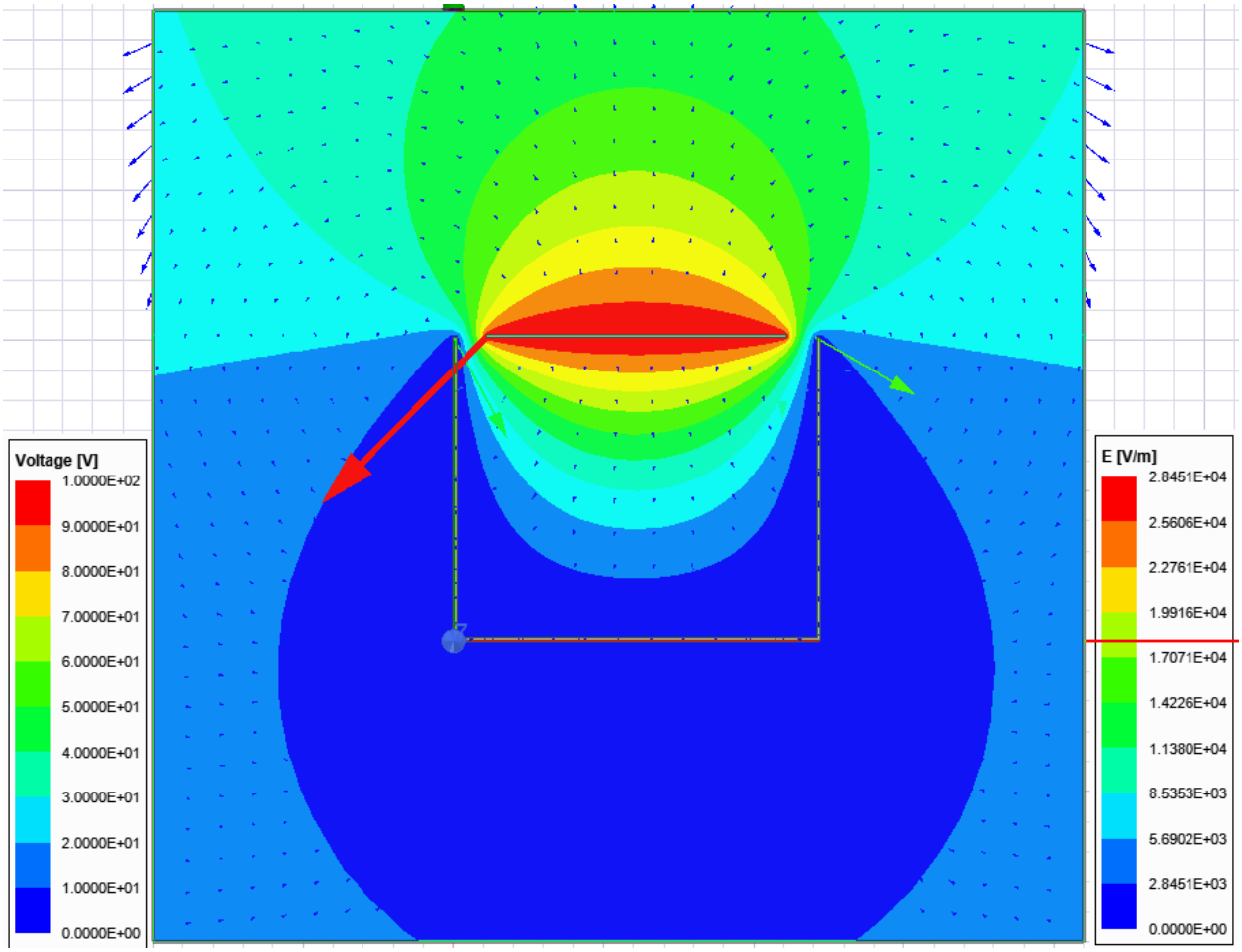


Figure 6.1: Structure 5 Equipotential Voltages and Electric Field Lines

Figure 6.1 above shows the equipotential voltages and electric field lines for structure 5.

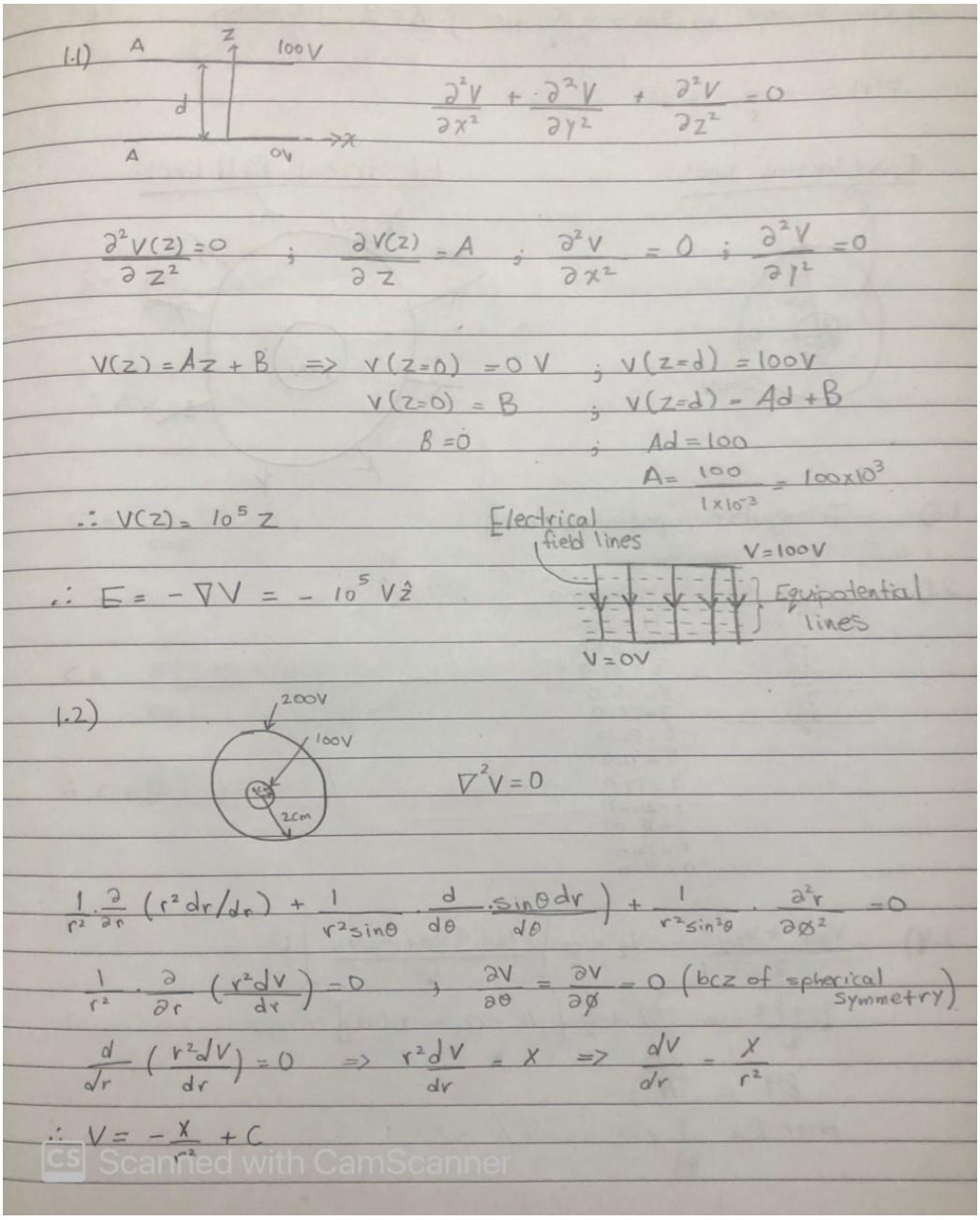
6.2 Problem 5.b Compare results obtained here with those calculated in the pre-lab section.

All in all, results obtained from this report was very close to the results obtained from the prelab

7.0 CONCLUSION

In conclusion, ANSYS software is a great way to solve, simulate, and plot Laplace's equations to provide a better visualization and understanding of concepts.

8.0 PRELAB



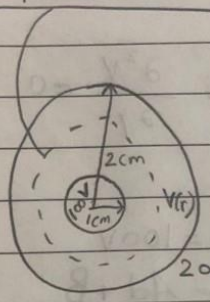
At 1cm $V = 100V \Rightarrow 100 = -\frac{X}{0.01} + C$

At 2cm $V = 200V \Rightarrow 200 = -\frac{X}{0.02} + C$

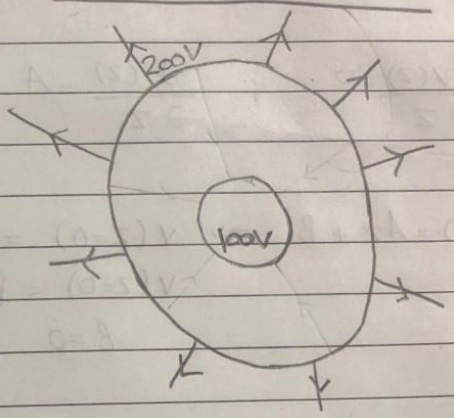
$\left.\begin{matrix} \\ \\ \end{matrix}\right\} X = 2 \; ; C = 300$

$\therefore \; V(r) = -\frac{2}{r} + 300$

Equipotential lines:

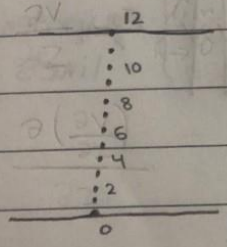


Electrical Field lines:



1.3) Separation of variables

2.1)



12 = 100.0
11 = 92.0
10 = 84.0
9 = 74.0
8 = 66.0
7 = 58.0
6 = 50.0
5 = 42.0
4 = 34.0
3 = 26.0
2 = 18.0
1 = 10.0
0 = 0.0

2.2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	71.000	24.000	22.000	13.625	15.063	10.156	12.578	8.727	11.406	7.996	10.768	7.597	10.381	7.388
3	48.000	44.000	27.250	30.125	20.313	25.156	17.453	22.813	15.992	21.535	15.193	20.763	14.775	20.256
4	17.000	30.500	38.250	27.000	35.250	24.750	33.047	23.258	31.664	22.391	30.758	21.954	30.130	21.800
5	13.000	32.500	26.750	40.375	29.188	40.938	29.063	40.516	28.789	39.980	28.714	39.498	28.825	39.093
6	48.000	23.000	42.500	31.375	46.625	33.375	47.984	34.320	48.297	35.037	48.238	35.697	48.056	36.336
7	33.000	52.500	36.000	52.875	37.563	55.031	39.578	56.078	41.285	56.496	42.681	56.614	43.847	56.594
8	57.000	49.000	63.250	43.750	63.438	45.781	64.172	48.250	64.695	50.324	64.989	51.997	65.131	53.355
9	65.000	74.000	51.500	74.000	54.000	73.313	56.922	73.313	59.363	73.482	61.313	73.649	62.864	73.766
10	91.000	54.000	84.750	64.250	83.188	68.063	82.453	70.477	82.270	72.301	82.309	73.731	82.402	74.865
11	43.000	95.500	77.000	92.375	82.125	91.594	84.031	91.227	85.238	91.135	86.150	91.154	86.866	91.201
12	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

2.3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	86.00000	9.50000	32.25000	5.50000	23.87500	6.68750	20.53125	7.87500	18.56641	8.64453	17.18457	9.06689	16.11011	9.25244
3	19.00000	64.50000	11.00000	47.75000	13.37500	41.06250	15.75000	37.13281	17.28906	34.36914	18.13379	32.22021	18.50488	30.44055
4	43.00000	12.50000	63.25000	21.25000	58.25000	24.81250	53.73438	26.70313	50.17188	27.62305	47.25586	27.94287	44.77100	27.89294
5	6.00000	62.00000	31.50000	68.75000	36.25000	66.40625	37.65625	63.21094	37.95703	60.14258	37.75195	57.32178	37.28101	54.74451
6	81.00000	50.50000	74.25000	51.25000	74.56250	50.50000	72.68750	49.21094	70.11328	47.88086	67.38770	46.61914	64.71802	45.43164
7	95.00000	86.50000	71.00000	80.37500	64.75000	78.96875	60.76563	77.01563	57.80469	74.63281	55.48633	72.11426	53.58228	69.62634
8	92.00000	91.50000	86.50000	78.25000	83.37500	71.03125	81.34375	66.39844	79.15234	63.09180	76.84082	60.54541	74.53467	58.47180
9	88.00000	86.50000	85.50000	86.37500	77.31250	83.71875	72.03125	81.28906	68.37891	79.04883	65.60449	76.95508	63.36133	74.98450
10	81.00000	79.50000	86.25000	76.37500	84.06250	73.03125	81.23438	70.35938	78.94531	68.11719	77.06934	66.17725	75.43433	64.46497
11	71.00000	86.00000	67.25000	81.75000	68.75000	78.75000	68.68750	76.60156	67.85547	75.08984	66.75000	73.91357	65.56860	72.89001
12	91.00000	55.00000	77.25000	61.12500	73.43750	64.34375	71.96875	65.35156	71.23438	65.38281	70.75781	64.95996	70.34570	64.30920
13	39.00000	68.50000	55.00000	65.12500	59.93750	65.18750	62.01563	65.86719	62.91016	66.42578	63.16992	66.77783	63.04980	66.94360
14	46.00000	55.00000	53.00000	58.75000	56.93750	59.68750	59.76563	60.46875	61.61719	60.95703	62.79785	61.13965	63.54150	61.07312
15	71.00000	37.50000	62.50000	48.75000	59.43750	54.34375	58.92188	57.36719	59.00391	59.16992	59.10938	60.30518	59.09644	61.04395
16	29.00000	70.00000	44.50000	60.12500	51.75000	58.15625	54.96875	57.53906	56.72266	57.26172	57.81250	57.05322	58.54639	56.83167
17	69.00000	51.50000	57.75000	54.75000	56.87500	55.59375	56.15625	56.07813	55.51953	56.45508	54.99707	56.78760	54.56689	57.09558
18	74.00000	45.50000	65.00000	53.62500	59.43750	54.15625	57.18750	53.50000	56.18750	52.73242	55.76270	52.08057	55.64478	51.56714
19	22.00000	78.50000	49.50000	64.12500	51.43750	58.78125	50.84375	56.29688	49.94531	55.07031	49.16406	54.50195	48.56738	54.30750
20	83.00000	53.50000	63.25000	49.25000	58.12500	47.53125	55.40625	46.39063	53.95313	45.59570	53.24121	45.05420	52.97021	44.70764
21	85.00000	48.00000	49.00000	52.12500	43.62500	52.03125	41.93750	51.60938	41.24609	51.41211	40.94434	51.43848	40.84790	51.61292
22	13.00000	44.50000	41.00000	38.00000	45.93750	36.34375	47.81250	36.10156	48.87109	36.29297	49.63574	36.64160	50.25562	37.06177
23	4.00000	34.00000	27.00000	39.75000	29.06250	43.59375	30.26563	46.13281	31.33984	47.85938	32.33887	49.07275	33.27563	49.94971
24	55.00000	9.50000	38.50000	20.12500	41.25000	24.18750	44.45313	26.57813	46.84766	28.38477	48.50977	29.90967	49.64380	31.25024
25	15.00000	43.00000	13.25000	42.75000	19.31250	45.31250	22.89063	47.56250	25.42969	49.16016	27.48047	50.21484	29.22485	50.88196
26	31.00000	17.00000	47.00000	18.50000	49.37500	21.59375	50.67188	24.28125	51.47266	26.57617	51.91992	28.54004	52.12012	30.22766
27	19.00000	51.00000	23.75000	56.00000	23.87500	56.03125	25.67188	55.38281	27.72266	54.67969	29.59961	54.02539	31.23047	53.43823
28	71.00000	30.50000	65.00000	29.25000	62.68750	29.75000	60.09375	31.16406	57.88672	32.62305	56.13086	33.92090	54.75635	35.03992
29	42.00000	79.00000	34.75000	69.37500	35.62500	64.15625	36.65625	60.39063	37.52344	57.58203	38.24219	55.48730	38.84937	53.91602
30	87.00000	39.00000	73.75000	42.00000	65.62500	43.56250	60.68750	43.88281	57.27734	43.86133	54.84375	43.77783	53.07568	43.70398
31	36.00000	68.50000	49.25000	61.87500	51.50000	57.21875	51.10938	54.16406	50.19922	52.10547	49.31348	50.66406	48.55859	49.62366
32	50.00000	59.50000	50.00000	61.00000	48.81250	58.65625	47.64063	56.51563	46.93359	54.76563	46.48438	53.33936	46.17163	52.16724
33	83.00000	31.50000	72.75000	35.75000	65.81250	38.06250	61.92188	39.70313	59.33203	40.86328	57.36523	41.67920	55.77588	42.26465
34	13.00000	86.00000	21.50000	70.62500	27.31250	65.18750	31.76563	62.14844	34.79297	59.96484	36.87402	58.21240	38.35767	56.74268
35	89.00000	11.50000	68.50000	18.87500	64.56250	25.46875	62.37500	29.88281	60.59766	32.88477	59.05957	35.03613	57.70947	36.65405
36	10.00000	51.00000	16.25000	58.50000	23.62500	59.56250	28.00000	59.04688	30.97656	58.15430	33.19824	57.20654	34.95044	56.29797
37	13.00000	21.00000	48.50000	28.37500	54.56250	30.53125	55.71875	32.07031	55.71094	33.51172	55.35352	34.86475	54.88647	36.10095
38	32.00000	46.00000	40.50000	50.62500	37.43750	51.87500	36.14063	52.37500	36.04688	52.55273	36.53125	52.56641	37.25146	52.49609
39	79.00000	60.00000	52.75000	46.50000	49.18750	41.75000	49.03125	40.02344	49.39453	39.55078	49.77930	39.63818	50.10571	39.98059
40	88.00000	59.50000	52.50000	47.75000	46.06250	46.18750	43.90625	46.41406	43.05469	47.00586	42.74512	47.64502	42.70972	48.24768
41	40.00000	45.00000	42.75000	45.62500	43.18750	46.06250	43.79688	46.08594	44.61719	45.93945	45.51074	45.78125	46.38965	45.67834
42	2.00000	26.00000	38.75000	38.62500	46.06250	41.40625	48.26563	42.82031	48.82422	44.01563	48.81738	45.13428	48.64697	46.17236
43	12.00000	32.50000	34.50000	46.50000	39.62500	50.46875	41.84375	51.56250	43.41406	51.69531	44.75781	51.51270	45.95508	51.25281
44	63.00000	43.00000	54.25000	40.62500	54.87500	42.28125	54.85938	44.00781	54.56641	45.50000	54.20801	46.77588	53.85864	47.87000
45	74.00000	76.00000	46.75000	63.25000	44.93750	59.25000	46.17188	57.57031	47.58594	56.72070	48.79395	56.20459	49.78491	55.83838
46	89.00000	50.50000	72.25000	49.25000	63.62500	50.06250	60.28125	51.16406	58.87500	52.08789	58.20117	52.79395	57.81812	53.32654
47	27.00000	68.50000	51.75000	64.00000	55.18750	61.31250	56.15625	60.17969	56.58984	59.68164	56.79395	59.43164	56.86816	59.27722
48	48.00000	53.00000	55.75000	61.12500	59.00000	62.25000	60.07813	62.01563	60.48828	61.50000	60.66211	60.94238	60.73633	60.40918
49	79.00000	43.00000	70.50000	54.00000	69.31250	58.84375	67.87500	60.79688	66.41016	61.64258	65.09082	62.04102	63.95020	62.23535
50	38.00000	88.00000	52.25000	77.50000	58.68750	73.50000	61.51563	70.80469	62.79688	68.68164	63.41992	66.95801	63.73438	65.54810

51	97.00000	61.50000	84.50000	63.37500	77.68750	64.18750	73.73438	64.79688	70.95313	65.19727	68.82520	65.42773	67.14600	65.52893
52	85.00000	81.00000	74.50000	77.87500	69.68750	73.96875	68.07813	71.10156	67.59766	68.96875	67.43555	67.33398	67.32349	66.05371
53	65.00000	87.50000	71.25000	76.00000	70.25000	71.96875	68.46875	70.39844	66.98438	69.67383	65.84277	69.21924	64.96143	68.83545
54	90.00000	61.50000	77.50000	62.62500	74.25000	62.96875	72.71875	62.86719	71.75000	62.71680	71.00293	62.58887	70.34741	62.48193
55	58.00000	67.50000	54.00000	72.50000	55.68750	73.46875	57.26563	73.10156	58.44922	72.33203	59.33496	71.47559	60.00244	70.63867
56	45.00000	46.50000	67.50000	48.75000	72.68750	51.56250	73.48438	54.03125	72.91406	55.95313	71.94824	57.41602	70.92993	58.52356
57	35.00000	67.50000	43.50000	72.87500	47.43750	73.50000	50.79688	72.72656	53.45703	71.56445	55.49707	70.38428	57.04468	69.30798
58	90.00000	40.50000	78.25000	46.12500	74.31250	50.03125	71.96875	52.88281	70.21484	55.04102	68.82031	56.67334	67.68604	57.90027
59	46.00000	89.00000	48.75000	75.75000	52.62500	70.43750	54.96875	67.70313	56.62500	66.07617	57.84961	64.98779	58.75586	64.19177
60	88.00000	57.00000	73.25000	59.12500	66.56250	59.90625	63.43750	60.36719	61.93750	60.65820	61.15527	60.83838	60.69751	60.93701
61	68.00000	57.50000	69.50000	57.37500	67.18750	56.43750	65.76563	56.17188	64.69141	56.23438	63.82715	56.40723	63.11816	56.59998
62	27.00000	82.00000	41.50000	75.25000	46.31250	71.62500	48.90625	69.01563	50.53125	66.99609	51.65918	65.39795	52.50244	64.12244
63	96.00000	25.50000	81.00000	35.25000	76.06250	41.37500	72.26563	44.89063	69.30078	47.08398	66.96875	48.59766	65.12671	49.72864
64	24.00000	80.00000	29.00000	76.87500	36.43750	72.90625	40.87500	69.58594	43.63672	66.94141	45.53613	64.85547	46.95483	63.20825
65	64.00000	32.50000	72.75000	37.62500	69.75000	40.37500	66.90625	42.38281	64.58203	43.98828	62.74219	45.31201	61.28979	46.42395
66	41.00000	65.50000	46.25000	62.62500	44.31250	60.90625	43.89063	59.57813	44.33984	58.54297	45.08789	57.72412	45.89307	57.06812
67	67.00000	60.00000	52.50000	51.00000	52.06250	47.40625	52.25000	46.29688	52.50391	46.18750	52.70605	46.47412	52.84644	46.90930
68	79.00000	39.50000	55.75000	41.50000	50.50000	43.59375	48.70313	45.42969	48.03516	46.86914	47.86035	47.96875	47.92554	48.81506
69	12.00000	51.50000	30.50000	50.00000	35.12500	50.00000	38.60938	49.77344	41.23438	49.53320	43.23145	49.37695	44.78369	49.31226
70	24.00000	21.50000	44.25000	28.75000	49.50000	33.62500	50.84375	37.03906	51.03125	39.59375	50.89355	41.59863	50.69897	43.22485
71	31.00000	37.00000	27.00000	49.00000	32.12500	51.68750	35.46875	52.28906	37.95313	52.25391	39.96582	52.02100	41.66602	51.75354
72	50.00000	32.50000	53.75000	35.50000	53.87500	37.31250	53.73438	38.86719	53.47656	40.33789	53.14844	41.73340	52.80811	43.03210
73	34.00000	70.50000	44.00000	58.75000	42.50000	55.78125	42.26563	54.66406	42.72266	54.04297	43.50098	53.59521	44.39819	53.23938
74	91.00000	55.50000	63.75000	49.50000	57.68750	47.21875	55.59375	46.57813	54.60938	46.66406	54.04199	47.06299	53.67065	47.57983
75	77.00000	57.00000	55.00000	56.62500	51.93750	55.40625	50.89063	54.55469	50.60547	54.04102	50.62500	53.74609	50.76147	53.58081
76	23.00000	54.50000	49.50000	54.37500	53.12500	54.56250	53.51563	54.63281	53.47266	54.58594	53.45020	54.45996	53.49097	54.29700
77	32.00000	42.00000	53.75000	49.62500	57.18750	51.62500	58.37500	52.39063	58.56641	52.85938	58.29492	53.23584	57.83252	53.55530
78	61.00000	53.00000	49.75000	60.00000	50.12500	62.18750	51.26563	62.50000	52.24609	62.00391	53.02148	61.20508	53.61963	60.33960
79	74.00000	57.50000	66.25000	50.62500	67.18750	50.90625	66.62500	52.10156	65.44141	53.18359	64.11523	54.00342	62.84668	54.59229
80	54.00000	79.50000	51.50000	74.37500	51.68750	71.06250	52.93750	68.38281	54.12109	66.22656	54.98535	64.48828	55.56494	63.07947
81	85.00000	45.50000	82.50000	52.75000	74.93750	54.96875	70.14063	56.14063	67.01172	56.78711	64.86133	57.12646	63.31226	57.28137
82	37.00000	85.50000	54.00000	75.50000	58.25000	69.21875	59.34375	65.64063	59.45313	63.49609	59.26758	62.13623	58.99780	61.22693
83	86.00000	62.50000	68.50000	63.75000	63.50000	63.71875	61.14063	62.76563	59.98047	61.74805	59.41113	60.86914	59.14160	60.13623
84	88.00000	51.50000	73.50000	51.50000	69.18750	53.06250	66.18750	54.32031	64.04297	55.32617	62.47070	56.14697	61.27466	56.82336
85	17.00000	84.50000	34.50000	74.62500	42.62500	68.65625	47.50000	65.32031	50.67188	63.19336	52.88281	61.68018	54.50513	60.51636
86	81.00000	17.50000	75.75000	33.75000	68.12500	41.93750	64.45313	47.02344	62.34375	50.43945	60.88965	52.86328	59.75806	54.65918
87	18.00000	67.00000	33.00000	61.62500	41.25000	60.25000	46.54688	59.36719	50.20703	58.58594	52.84375	57.83594	54.81323	57.11926
88	53.00000	48.50000	47.50000	48.75000	52.37500	51.15625	54.28125	53.39063	54.82813	55.24805	54.78223	56.76318	54.48047	57.99646
89	79.00000	28.00000	64.50000	43.12500	61.06250	48.31250	60.23438	50.28906	60.28906	50.97852	60.68262	51.12500	61.17969	51.04700
90	3.00000	80.50000	38.75000	73.37500	44.25000	69.31250	46.29688	67.18750	47.12891	66.11719	47.46777	65.59619	47.61353	65.36987
91	82.00000	49.50000	82.25000	45.37500	77.56250	44.28125	74.14063	43.96875	71.94531	43.95703	70.50977	44.10205	69.56006	44.34656
92	96.00000	84.00000	52.00000	81.75000	44.31250	78.96875	41.64063	76.70313	40.78516	74.90234	40.73633	73.52393	41.07959	72.50452
93	86.00000	54.50000	81.25000	43.25000	80.37500	39.00000	79.26563	37.60156	77.85938	37.51563	76.53809	38.05713	75.44897	38.90601
94	13.00000	78.50000	34.50000	79.00000	33.68750	79.56250	33.56250	79.01563	34.24609	78.17383	35.37793	77.37402	36.73242	76.71753
95	71.00000	14.50000	76.75000	24.12500	78.75000	28.12500	78.76563	30.89063	78.48828	33.24023	78.20996	35.40771	77.98608	37.46228
96	16.00000	75.00000	13.75000	78.50000	22.56250	77.96875	28.21875	77.96094	32.23438	78.24609	35.43750	78.59814	38.19214	78.93237
97	79.00000	13.00000	80.25000	21.00000	77.18750	28.31250	77.15625	33.57813	78.00391	37.63477	78.98633	40.97656	79.87866	43.85938
98	10.00000	85.50000	28.25000	75.87500	34.06250	76.34375	38.93750	78.04688	43.03516	79.72656	46.51563	81.15918	49.52661	82.32922
99	92.00000	43.50000	71.50000	47.12500	75.50000	49.56250	78.93750	52.49219	81.44922	55.39648	83.33203	58.07666	84.77979	60.48718
100	77.00000	57.50000	66.00000	75.12500	65.06250	81.53125	66.04688	84.85156	67.75781	86.93750	69.63770	88.40039	71.44775	89.48999
101	23.00000	88.50000	78.75000	83.00000	87.56250	82.53125	90.76563	83.02344	92.42578	83.87891	93.46875	84.81885	94.20020	85.72388
102	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000

2.4) $\%\ \text{error} = \frac{|\text{Actual} - \text{Expected}|}{\text{Expected}} \times 100 = \frac{74 - 73.13}{73.13} \times 100$

$= 1.19\%$

3.2)