Lab #3: Control of an Inverted Pendulum

SYSC 3600 A

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PRELAB

$$H(s) = \frac{\Theta_o(s)}{\Theta_r(s)} = \frac{B}{s^2 + \frac{k_d}{M\ell}s + \left[\frac{k_p - (M+m)g}{M\ell}\right]}$$

k_p = 2× ζ× ω_n ×M×L

$k_d = M \times L \times \omega_n^2 + g(M+m)$

<u>Given:</u>

M = 1000 kg

m = 200 kg

l= 10 m

ω_n = 0.5 rad/sec

ζ = 0.7

g = 9.81 m/s2

$k_p = 14272$

$k_d = 7000$

INTRODUCTION

The purpose of this lab is to study and analyse the non-linear system of the inverted pendulum problem. It focuses on presenting and analyzing the proportional-plus-derivative (PD) controller using SIMULINK. It also investigates the validity of a linear approximation for a nonlinear system.

4.0 LAB

4.1 Inverted pendulum demo in SIMULINK

In balancing the inverted pendulum, an overall underdamped response is preferred over a critically damped or an overdamped response because the inverted pendulum will always be moved, meaning the pendulum will always balance itself and this happens when theta is positive. The inverted pendulum will be moving slowly and stop if theta changed from positive to negative.

4.2 Testing the pendulum

The SIMULINK model shown below was used to simulate the zero-input response of the inverted pendulum and the cart.

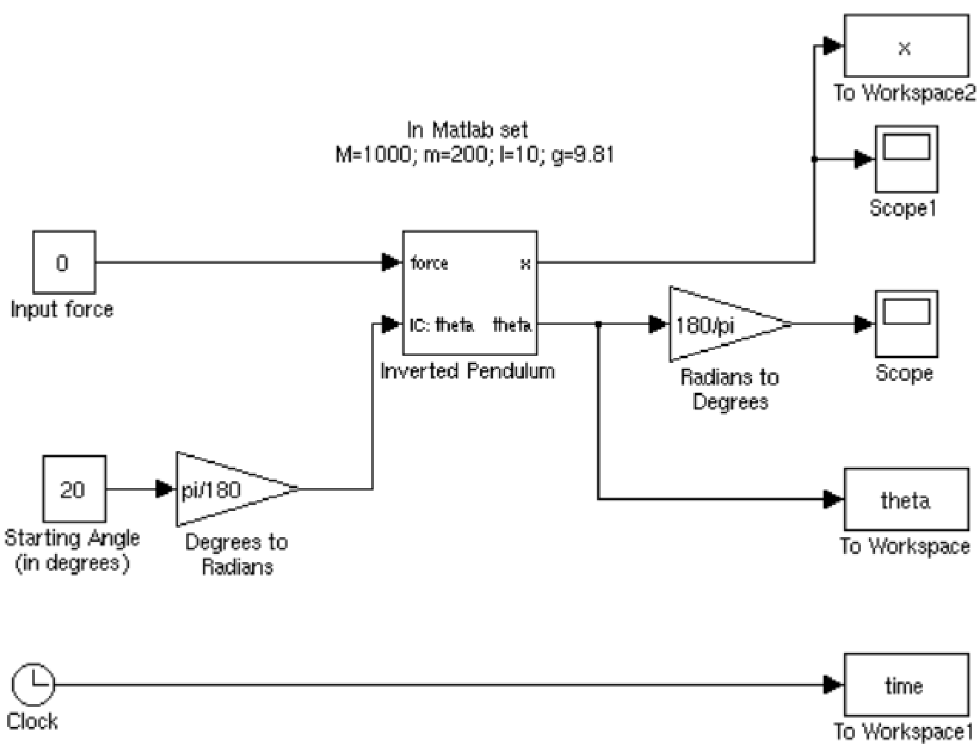

Figure 1: Simulation Diagram for the Inverted Pendulum

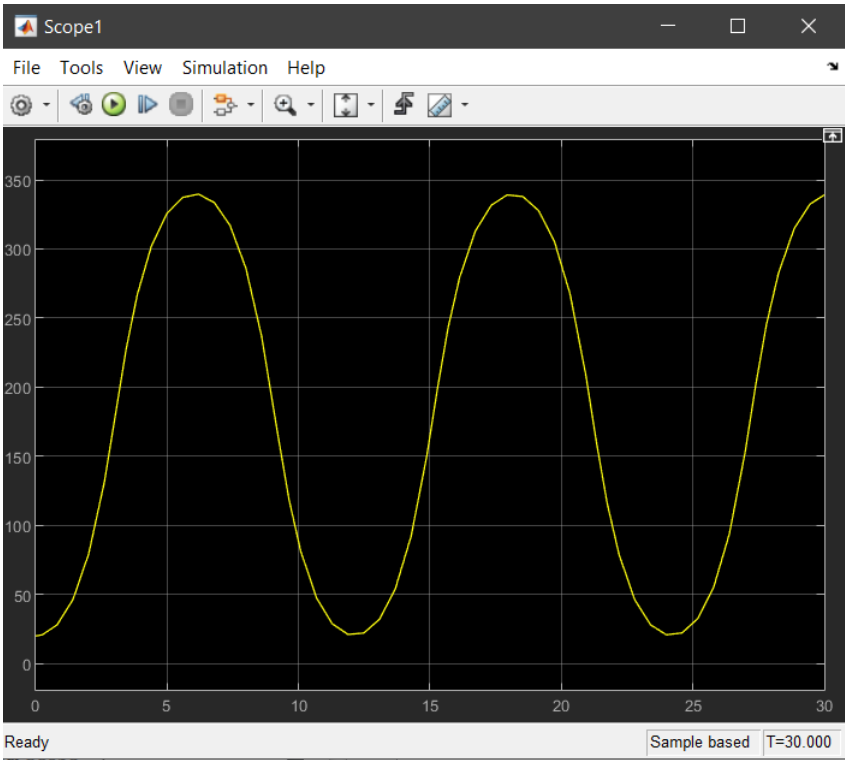

Figure 2: Inverted Pendulum Angle from t=0 to t=30

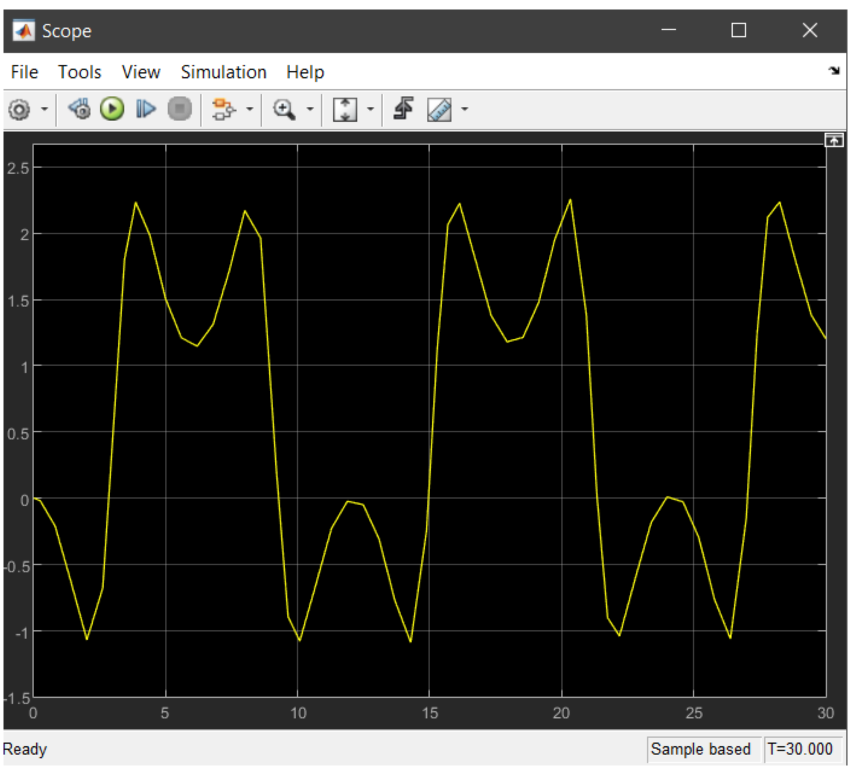

Figure 3: Position of Cart from t=0 to t=30

When the starting angle was set to 20 degrees along with a corresponding zero-input response, the pendulum was constantly rotating and it was observed that it was a sinusoid function from the output of the scope and the cart was moving back and forth. When the starting angle was increased above 20 degrees for example 30 degrees, it was observed that cart is moving forward and backward but this time further and faster than the previous case.

4.3 Simulation of PD controlled non-linear inverted pendulum

4.3.1 Problem of derivative with the PD controller

The equation of the lowpass filter with 3-dB cut off frequency 100 rads/s is:

$$H_{LP}(s) = \frac{1}{\left(\frac{s}{\omega_{cf}}\right)+1}$$
$$H_{LP}(s) = \frac{1}{\left(\frac{s}{100}\right)+1}$$
$$H_{LP}(s) = \frac{1}{0.01s+1}$$

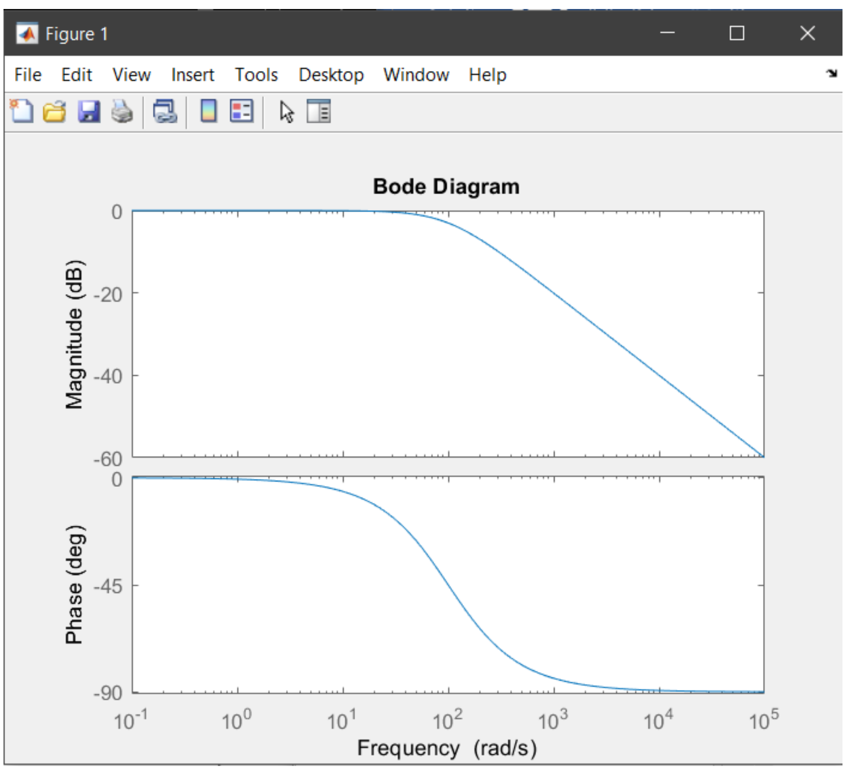

Figure 4: Bode Plot for Low Pass Filter

From the bode diagrams above one can see that the magnitude is decreasing at a rate of -20dB/decade. Also, while the magnitude is decreasing, the frequency is increasing. The lowpass filter has a cut off frequency at 3dB of 100 rad/sec.

```
num = [0, 0, 1];
den = [0, 0.01, 1];
```

```
>> sys = tf(num, den)
bode(sys, {0.1, 10^5})

sys =

      1
  ----------
  0.01 s + 1

Continuous-time transfer function.
```

Figure 5: Code used to Generate the Bode Diagram

4.3.3 Simulating the PD controlled inverted pendulum

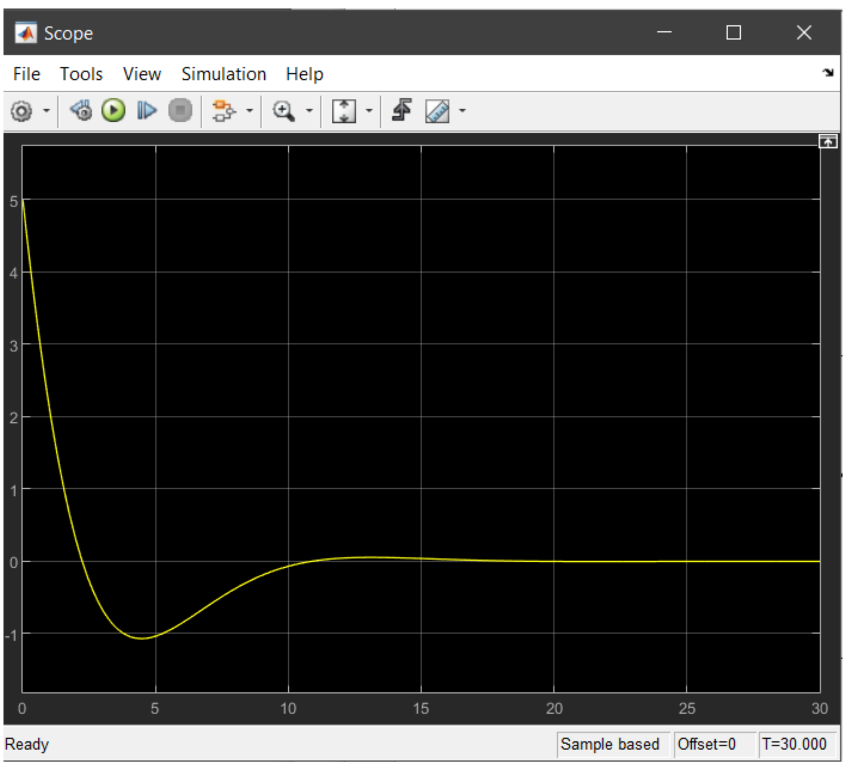

Figure 6: $\theta_o(t_o^-) = 5^\circ$

The PD controller was able to balance the pendulum.

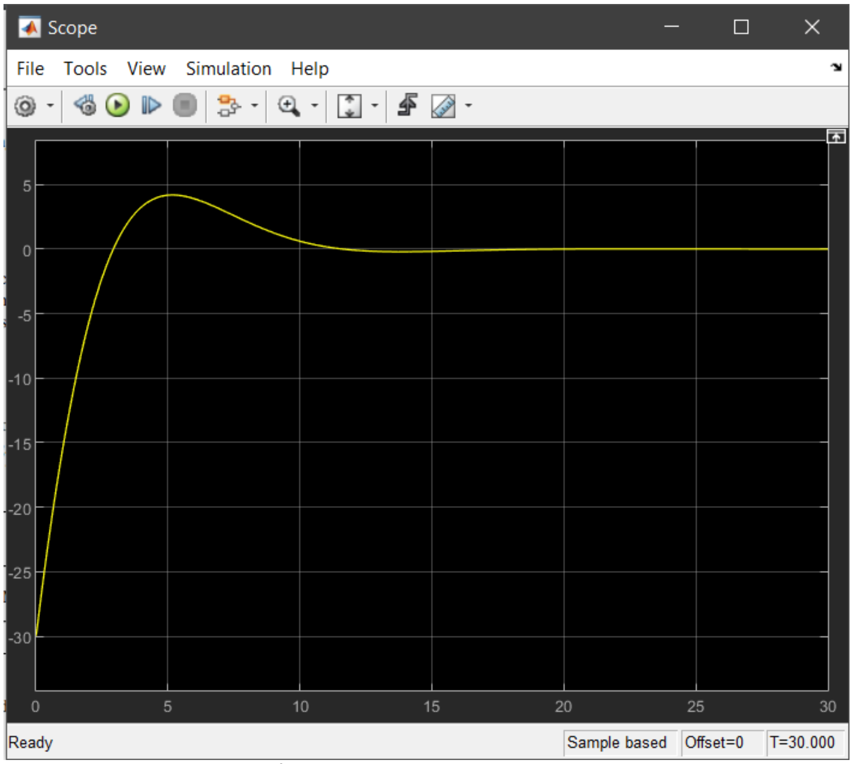

Figure 7: $\theta_o(t_o^-) = -30^{\circ}$

The PD controller was able to balance the pendulum.

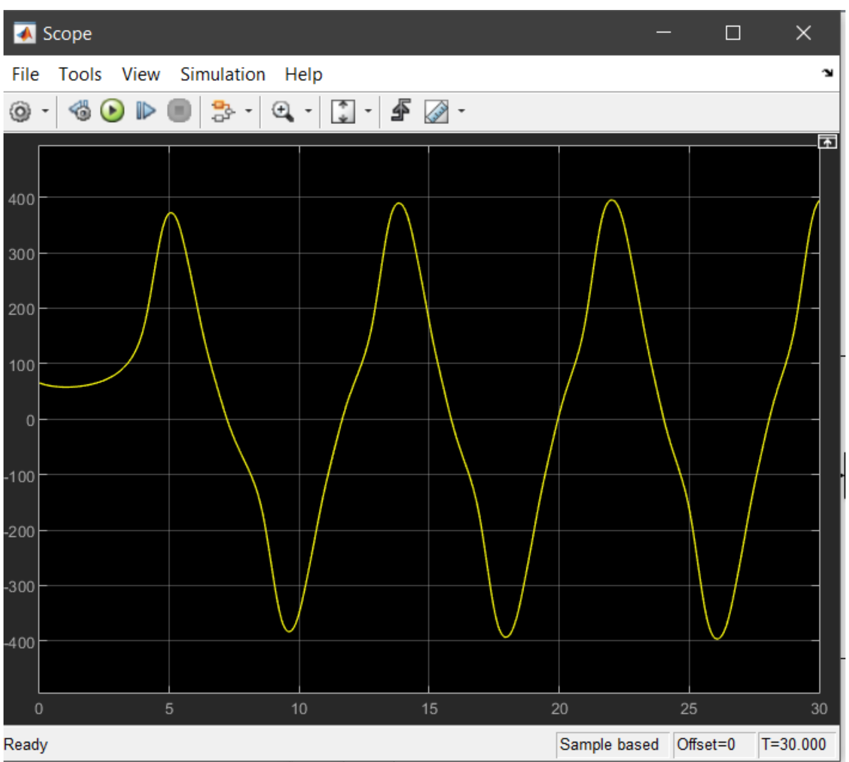

Figure 8: $\theta_o(t_o^-) = 65^{\circ}$

Pendulum kept rotating because of the values of k_p and k_d not allowing the system to generate enough force to balance the pendulum.

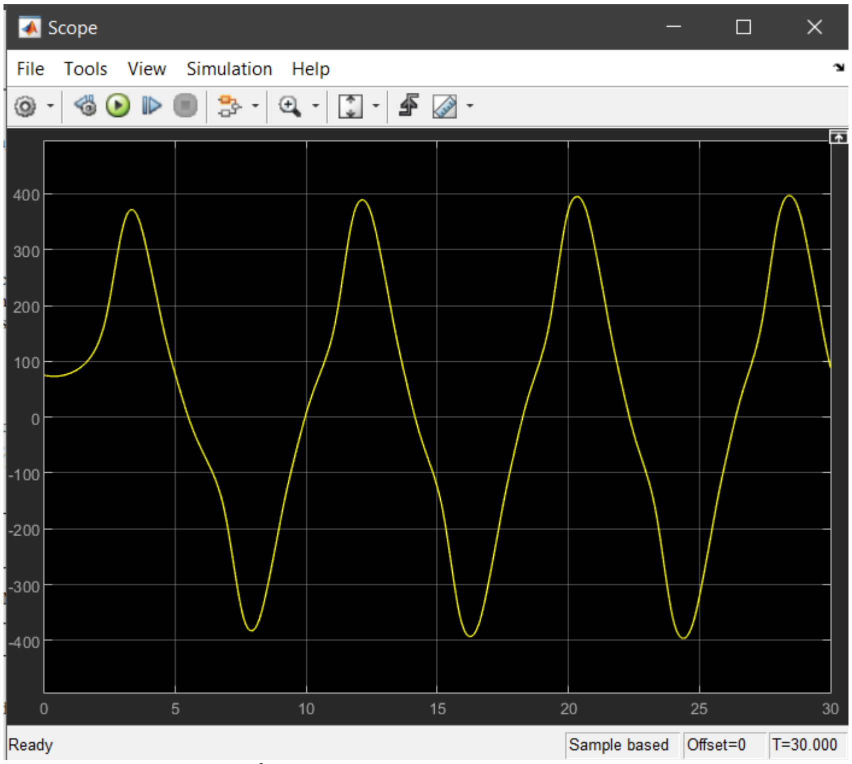

Figure 9: $\theta_o(t_o^-) = 75^\circ$

Pendulum kept rotating because of the values of k_p and k_d not allowing the system to generate enough force to balance the pendulum.

4.4 Additional questions

1. The closed-loop system will not keep the pendulum at an angle that is set to something other than zero and the reason for that is the system will always try to balance the pendulum to the reference angle. If the angle was set to a value that is very close to zero or zero then the system will keep the pendulum at that angle because when the system reaches the reference angle, it will stop attempting to balance the pendulum Also, if the angle was not zero then it will always oscillate because the mass of the pendulum will be pushed down and it will continue forever as an attempt by the system to stabilize the pendulum position.

2. There are two initial conditions for Eq. 13, the first one is the initial angular position θ_o of the pendulum which is the starting angle of the pendulum in Figure 10. The other one is the initial angular velocity $\dot{\theta}_o$ of the pendulum which is the initial angular velocity of the pendulum before the PD controller system begins to balance. In figure 10, the initial angular velocity is assumed to be zero in the implementations, and it should be considered.

CONCLUSION

This lab has provided a clear understanding of the of the inverted pendulum problem. It presented the main differences between the closed loop system and open loop system. The open loop system depends on the input while the closed loop system depends on the PD controller for reducing errors as mentioned in section 4.3.1. For this case, the lowpass filter was presented and applied before the PD controller to reduce the error signal.