Lab 2: Servo System Simulation

SYSC 3600 A

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1.0 INTRODUCTION

The purpose of this lab examines the dynamics of a position controller (servo system) for a motor and gain further experience in digital simulation using Simulink. The problem is to design a controller and test the resulting dynamics. Figure 1.0 below shows is an example servo system where an input dial is used to control the angular position of an output shaft.

The servo system demonstrated in Figure 1.0 below can be used as the manual azimuth control of a telescope or a directional antenna (i.e., rotate the telescope on its base according to the position of the input dial.).

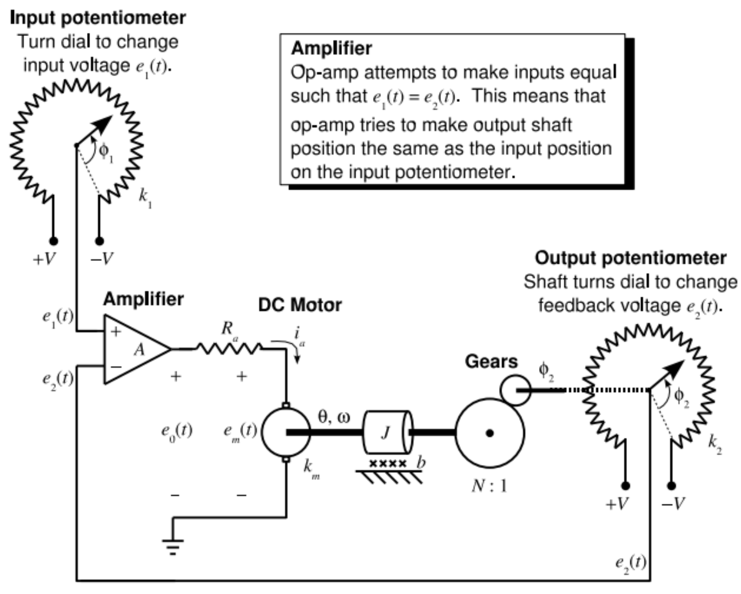

Figure 1.0: Servo System with Input Dial and Output Shaft

The lab includes two parts, the first part includes a servo system with a position feedback, and the second part includes the servo system with a rate feedback.

2.0 CONTROLLER DESIGN

Figure 2.0 below shows the block diagram of the servomechanism.

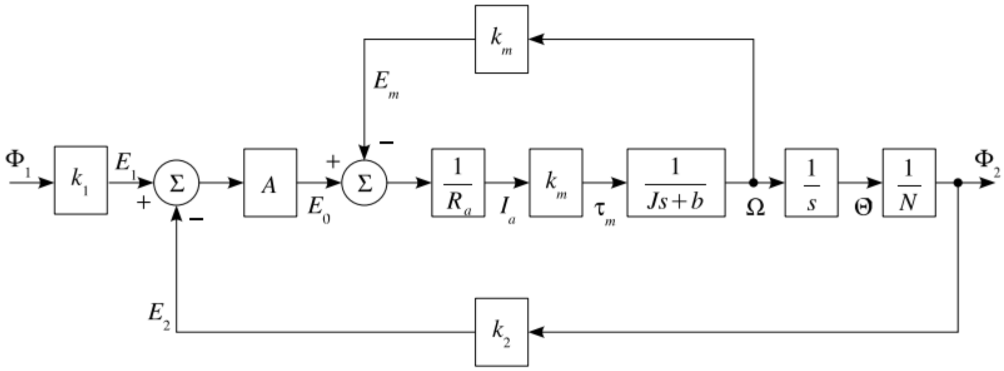

Figure 2.0: Block Diagram of the Servomechanism

From Figure 2.0 above we can generate the following Transfer function:

$$H(s) = \frac{\frac{Ak_1k_m}{R_aJN}}{s^2 + \frac{R_ab + k_m^2}{R_aJ}s + \frac{Ak_2k_m}{R_aJN}}$$

The transfer function shown above is a 2nd-order system and it fits the form shown below.

$$H(s) = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$H(s) = \frac{kw_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

To achieve an acceptable position controller several different feedback gains will be implemented which includes static gain (k), damping ratio (ζ), and undamped natural frequency (ω_n).

From this second order system we can obtain the equations for k which gives the steady-state gain of the system, ζ the damping ratio (factor) which indicates the behavior of the step response to different cases, and ω_n which is the undamped natural frequency in rads/s.

$k = \frac{k_1}{k_2} = 1$

$\omega_n = \sqrt{\frac{A\,K_2 K_m}{R_a\,JN}}$

$\zeta = \frac{R_a\,B + k_m^2}{2 * R_a J \omega_n}$

The transfer function without a rate feedback loop is shown below.

$$H_{rate}(s) = \frac{k_1 AB/N}{s^2 + (C + ABk_r)s + k_2 AB/N}$$

The equations to calculate the undamped natural frequency ω_n, the static gain k, and the damping ratio ζ.

$k = \frac{k_1}{k_2} = 1$

$B = \frac{k_m}{J * R_a}$

$\zeta = \frac{c + ABk_r}{2\omega_n}$

$\omega_n = \sqrt{\frac{Ak_2 k_m}{JR_a N}}$

If the damping ratio is generated and greater than 1, then the behavior of the system is over damped. If it was 1 then the system is considered critically damped with faster rise time in the step response and no overshoot/oscillations occur. If the damping ratio was greater than 0 but less than 1 then the system is considered underdamped with a faster step response such that it overshoots occurs causing response oscillations. If the damping ratio is equal to 0, then the behavior of the system is undamped, then the step response is a pure sinusoid with a frequency of ω_n. If the damping ratio is non-zero, then undamped natural frequency ω_n does not carry a physical meaning, but we can compute the damped natural frequency ω_d to be as shown below.

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

2.1 Block Diagram of Controller

Figure 2.1 below illustrates the block diagram of the controller. One can see the position feedback signal is supplied.

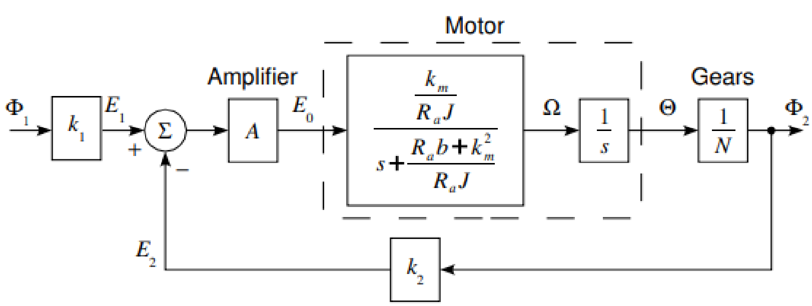

Figure 2.1: Servo's Simulation Diagram with Position Feedback

Figure 2.2 below illustrates the block diagram of the controller. One can see the rate feedback signal is supplied.

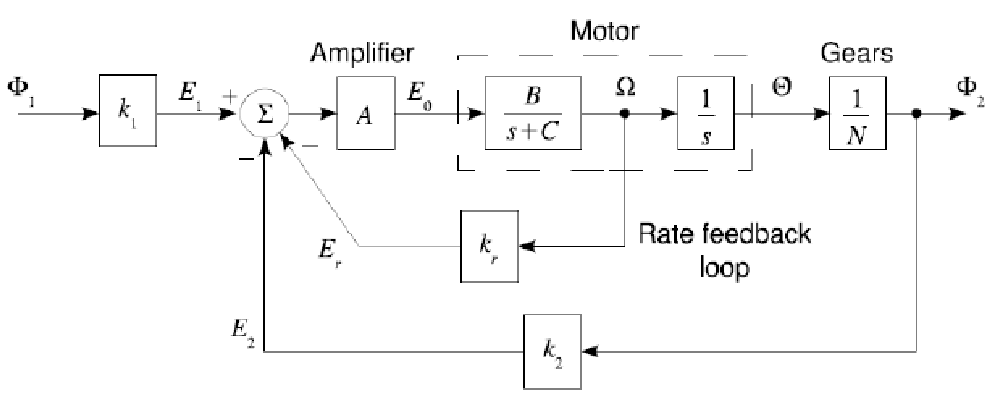

Figure 2.2: Servo's Simulation Diagram with Rate Feedback

3.0 RESULTS

3.1 Matlab/Simulink Implementation

3.1.1 Values Used for Servo Components

The following values are the parameters that was implemented into the Matlab script.

$$
\begin{aligned}
k_m &= 1.5275 \text{ kg} \cdot \text{m}^2/\text{sec}^2/\text{A} \quad (\text{Nm/A}) \\
J &= 100 \text{ kg} \cdot \text{m}^2 \\
b &= 100 \text{ kg} \cdot \text{m}^2/\text{sec} \\
R_a &= 1\ \Omega \\
N &= 12 \\
k_1 &= 12 \text{ V/rad} \\
k_2 &= k_1
\end{aligned}
$$

3.2 Results

3.2.1 Block Diagram Implementation

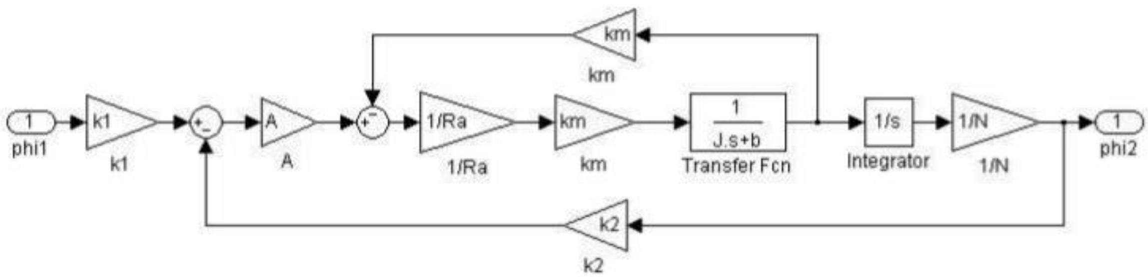

Figure 3.0: SIMULINK Subsystem Realizing the Servo with Position Feedback.

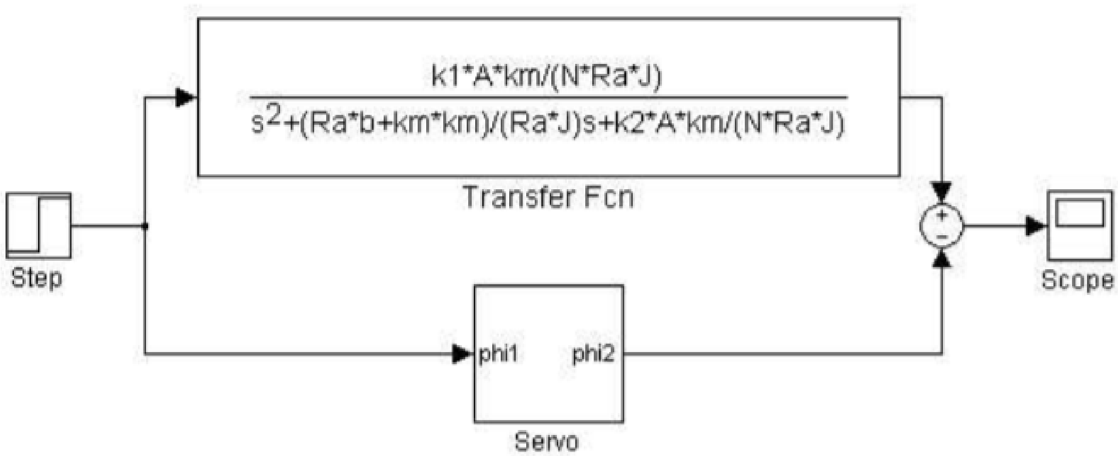

Figure 3.1: Simulink diagram used to compare the step response of the block diagram for the servo with position feedback subsystem and its transfer function

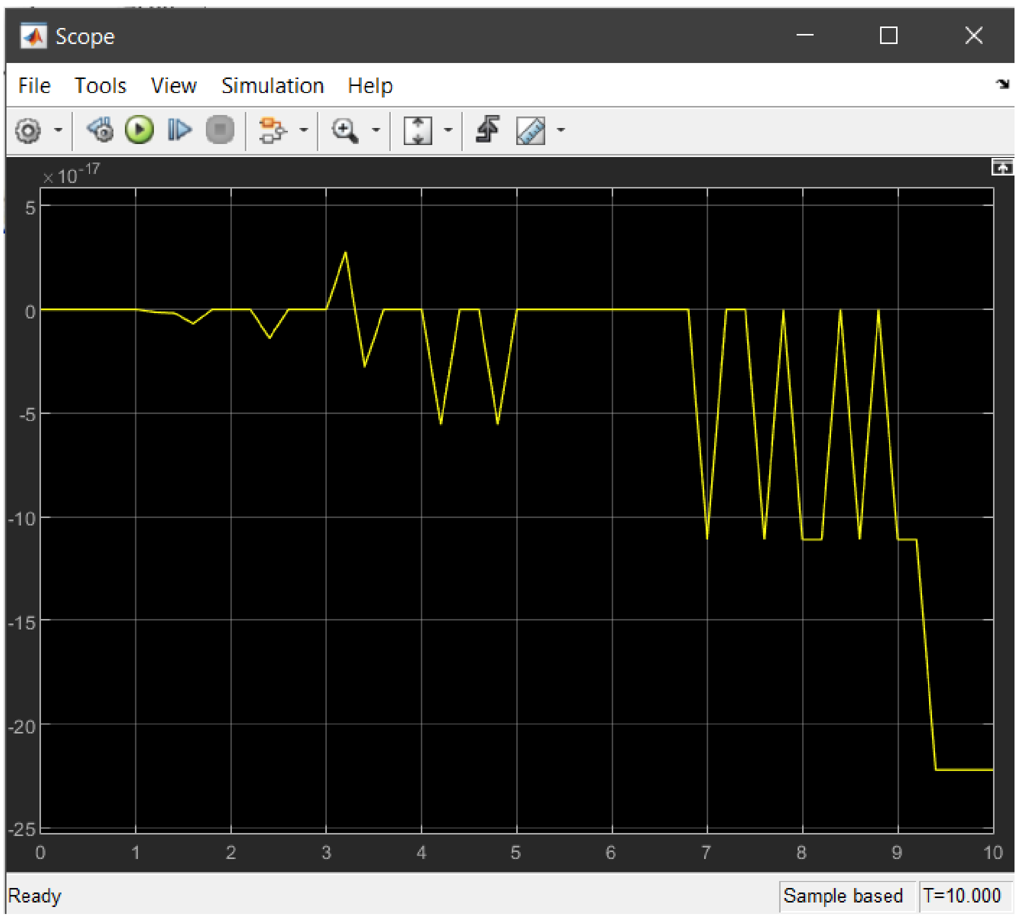

Figure 3.2: Matlab plots to show the difference between the step response of the block diagram and the transfer function. It is shown that the difference is approximately 1.0 x 10^-17 when A is 10 and this confirms that the transfer function is correct.

3.2.1 Table for experimental cases

Using the script file with the equations written in it for damping ratio, ω_n, and ω_d with adjusting different values for A we can generate the table below labeled Table 1.

Table 1: Data Obtained from the Characteristics of Servo System

Case	A	ζ	ω_n	ω_d	Behavior
1	4	2.07	0.2472	0 + 0.4480j	Over Damped
2	17	1.0041	0.5096	0 + 0.0461j	Critically Damped
3	35	0.6998	0.7312	0.5223	Under Damped
4	300	0.2390	2.1407	2.0786	Under Damped

3.2.2 simulation of the step response of the servo with position feedback

$$\text{f.v.} = \lim_{t\to\infty} \phi_2(t) = \lim_{s\to 0} s\Phi_2(s) = \lim_{s\to 0} s\left[H(s)\Phi_1(s)\right].$$

Using the above equation, the final value expected was computed as seen in Table 2 below.

Table 2: Servo's System Final Value without Rate Feedback Loop

Case	A	Final Value
1	4	1
2	17	1
3	35	1
4	300	1

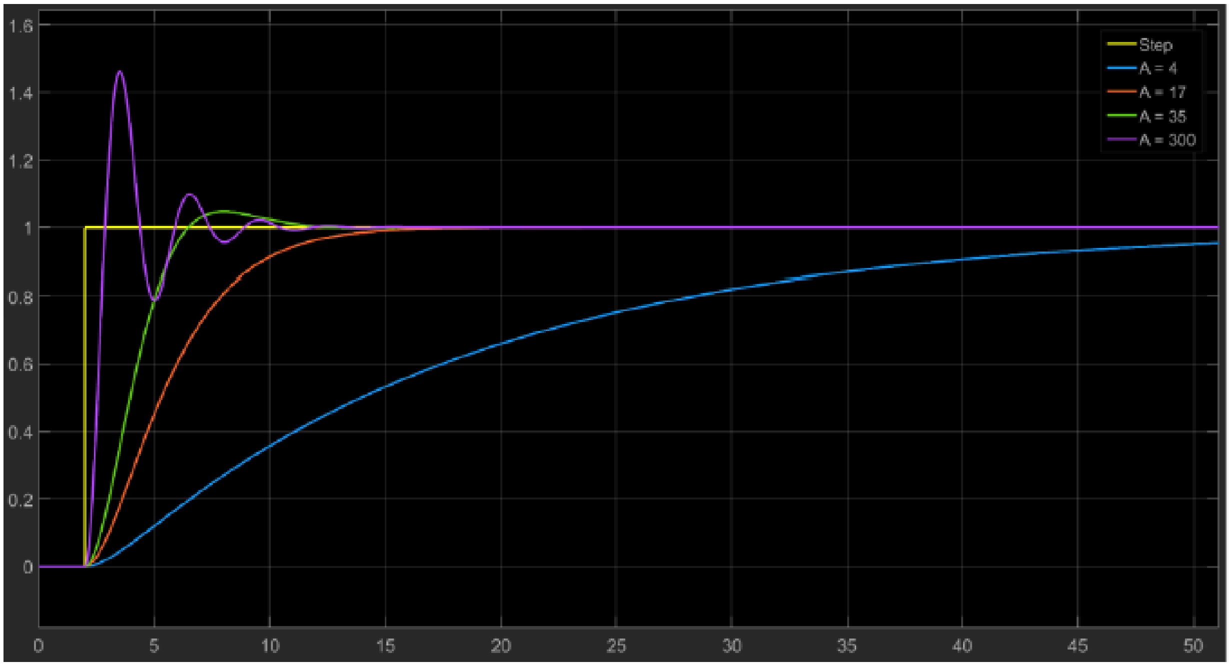

Figure 3.3: Output Step Response for Multiple A Values

As shown in Figure 3.3 we can see that for A= 4 one can see that the final value of the function is 1 which was the same as the expected value calculated in Table 2 above. We can also see that the figure for A=4 shows that the system is overdamped with no overshoot and no oscillation with a slow step response. These characteristics agree with what was calculated in Table 1. For A= 17, we can see that we also obtained the same final value which is 1 and it matches the expected calculated value in Table 2. We can see that the system is critically damped because it has a faster step response and no overshoot or oscillation. This correspond to the calculated values in Table 1 as well. For A = 35, the final value is 1 which is the same as the expected value in Table 2. The system is under damped because the plot has an overshoot and a faster step response. This under damped response is causing oscillations before it reaches the final value. These oscillations happen at Wd = 0.5223 which matches what was calculated in Table 1. For the last A value which is A= 300, one can see that we got the same final value which is 1 as predicted in Table 2. The system is under damped because it has oscillation and a faster step response. The under damped behavior is causing the oscillations before reaching the final value, the oscillations are happening at ω_d = 2.0786 which is the same as the value in Table 1.

3.2.3 Verification of the transfer function with the block diagram implementation

$$H_{rate}(s) = \frac{k_1 AB/N}{s^2 + (C + ABk_r)s + k_2 AB/N}$$

The function above is the transfer function for a servo with rate feedback where k_r can now be used to change and adjust the damping ratio without effecting ω_n.

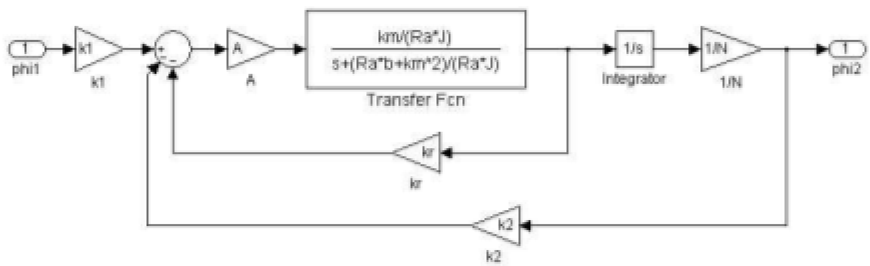

Figure 3.4: Simulink diagram used to simulate the subsystem that realizes the servo with rate feedback.

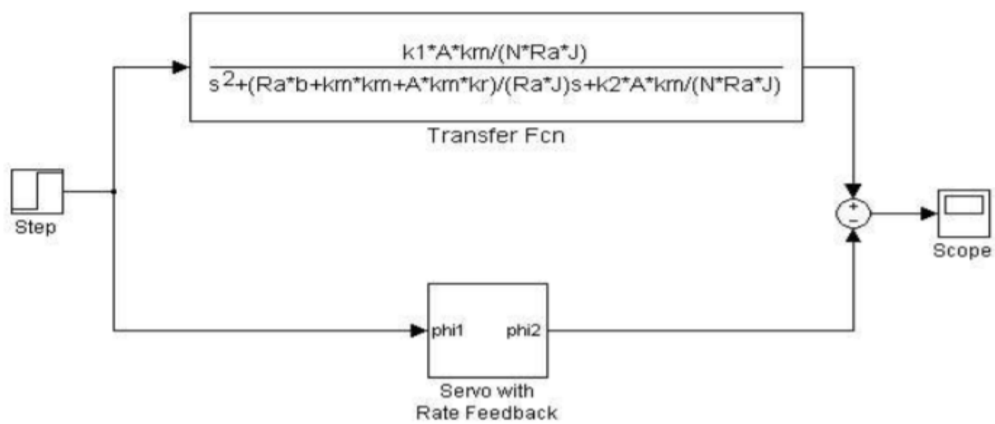

Figure 3.5: Simulink diagram used to compare the step response of the block diagram for the servo with rate feedback and its transfer function

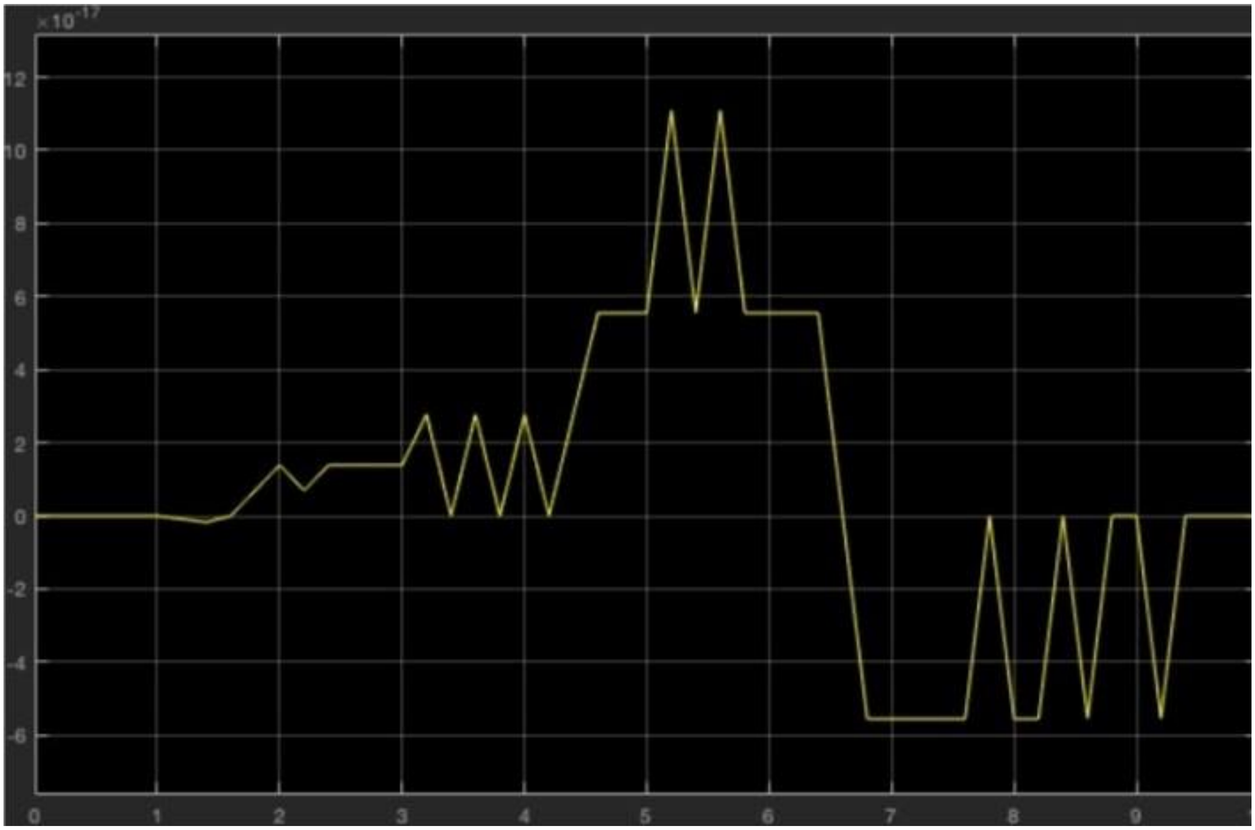

Figure 3.6: Matlab plots to show the difference between the step response of the block diagram for the servo with rate feedback and the transfer function. It is shown that the difference is approximately 1.0 x 10^-17 when A is 10 and this confirms that the transfer function is correct.

3.2.4 Step Responses of Servos with and without Rate Feedback

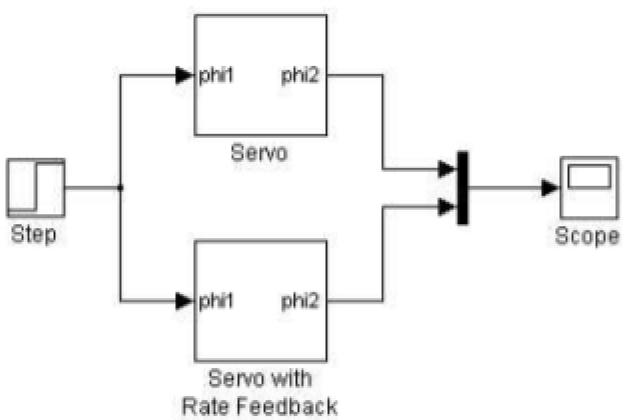

Figure 3.7: Simulink simulation that compares the step response of the block diagram for the servo with rate feedback and its transfer function.

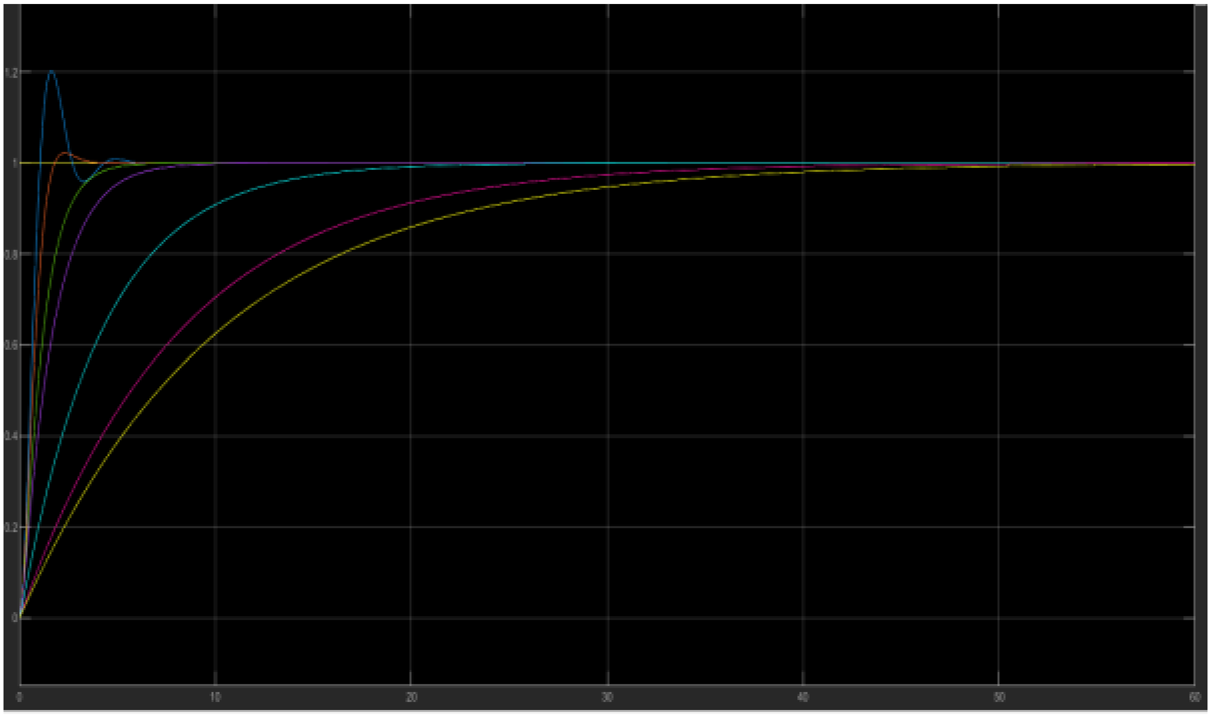

Figure 3.8: Matlab generated plot for output Step Response for A=300 for k_r=0.2(blue), k_r=0.4(orange), k_r=1(green), k_r=1.6 (pink), k_r=0.2(blue), k_r=3.9(purple), k_r=7.8(cyan), k_r=10(yellow)

As shown in Figure 3.8 above, the k_r value is increased one can see that the system will change from under damped to over damped because the increased k_r will cause he system to have slower step response and less oscillations. System is underdamped if the $k_r$value is increased while still being lower that the critical value and this results in less overshoot. The system is overdamped when k_r value is increases over the critical value and this results in the system taking longer time to stabilize

3.2.5 Selecting Damping Ratio for the Servo with Rate Feedback

The equation for k_r will be:

$$k_r = \frac{2\omega_n\zeta - C}{A * B}$$

To have the step response of the servo with rate feedback to be critically damped we must have the damping factor to be 1.

The calculated $k_r$value is 0.79 which is used in Figure 3.9 below along with another k_r value that is reduced to k_r = 0.68

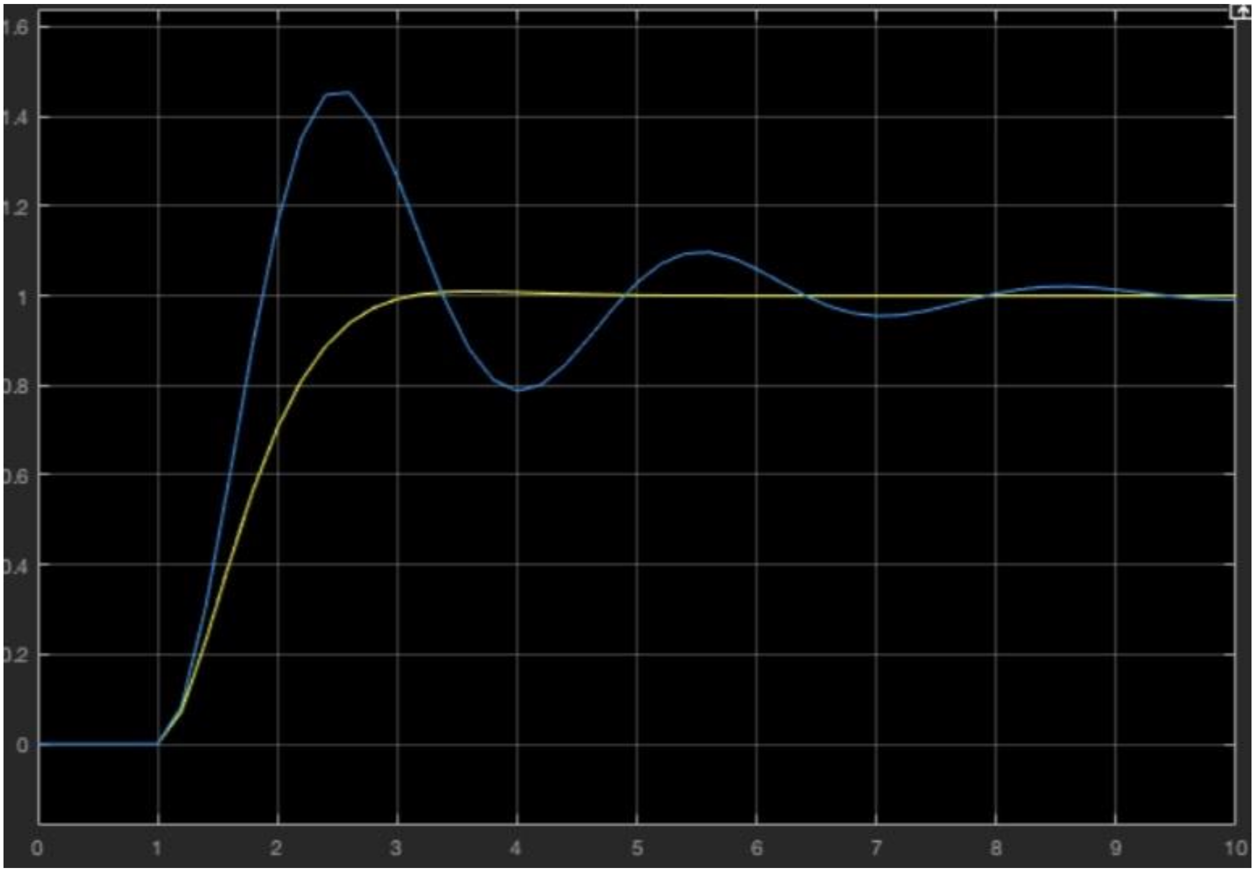

Figure 3.9: Output Step Response with k_r = 0.68, the blue curve is the servo without rate feedback and the yellow curve is the servo with rate feedback. When k_r is decreases the system will go from critically damped to under damped. When k_r is decreased it will cause a small overshoot in the plot.

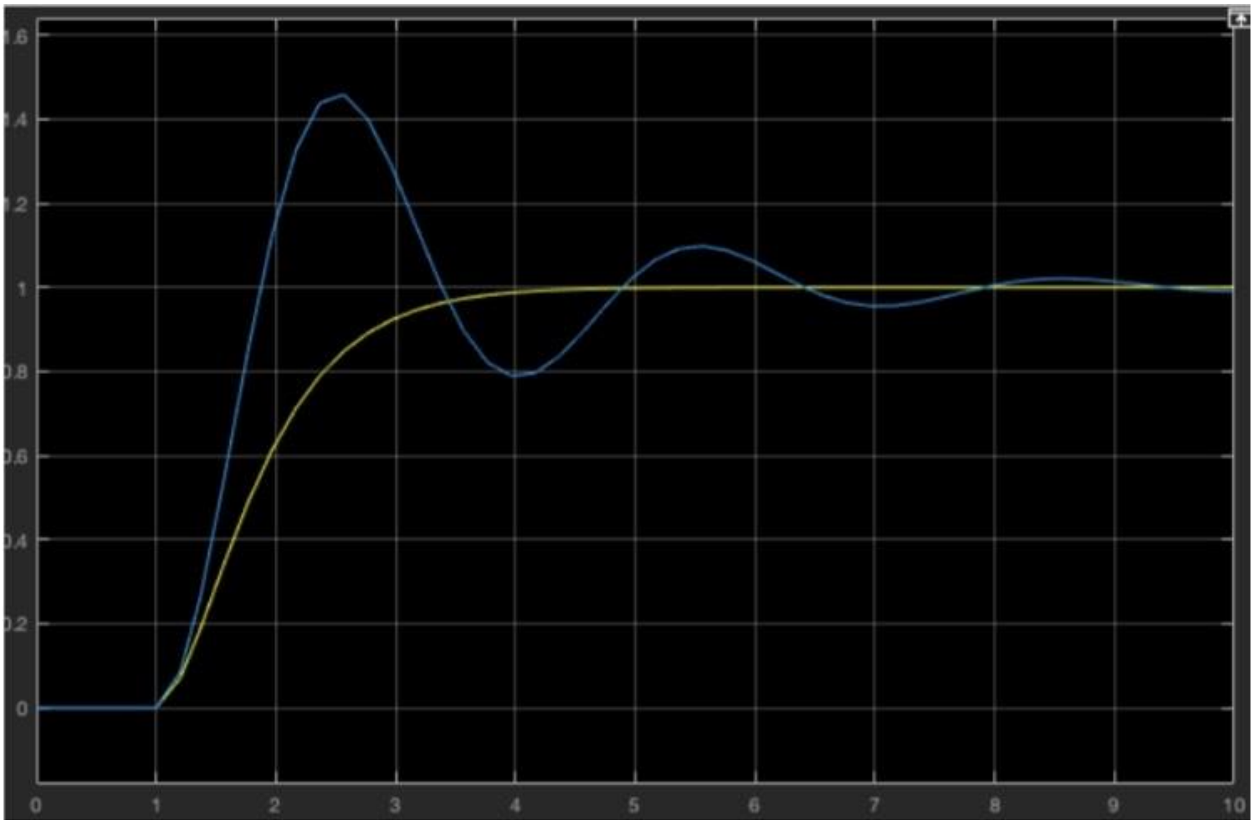

Figure 3.10: Output Step Response with k_r = 0.79, the blue curve is the servo without rate feedback and the yellow curve is the servo with rate feedback. The damping factor is 1 which means that the system is critically damped

4.0 DISCUSSION

In conclusion, the purpose of this lab was to examines the dynamics of a position controller (servo system) for a motor and gain further experience in digital simulation using Simulink. The results of the first part of the lab which included using the positional feedback got effected by the oscillations, adjusting the feedback gains will give a proper representation of the controller's position in the system. In the second part of the lab, using the rate feedback showed noticeable improvement to the system. Also, the damping ratio plays a big role in the behavior of the step response.