

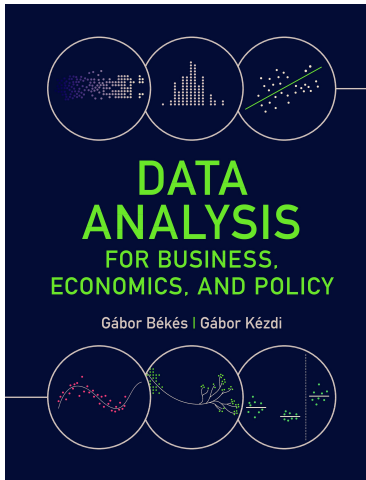
# 12. Time series regression

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Data Analysis 2: Regression analysis

2019

# Slideshow for the Békés-Kézdi Data Analysis textbook



- ▶ Cambridge University Press, 2021 January
- ▶ Available in paperback, hardcover and e-book
- ▶ Slideshow be used and modified for educational purposes only
- ▶ **[gabors-data-analysis.com](https://gabors-data-analysis.com)**
  - ▶ Download all data and code
  - ▶ Additional material, links to references

# Motivation 1

- ▶ Investing in a company stock, and you want to know how risky that investment is.
- ▶ In finance, a relevant measure of risk relates returns on a company stock to market returns: a company stock is considered risky if it moves in the direction of the market, and the more it moves in that direction the riskier it is.
- ▶ Data on daily stock prices for 21 years.
- ▶ How to define returns?
- ▶ How to assess whether and to what extent returns on the company stock move together with market returns?

## Motivation 2

- ▶ People heat and cool in most places
- ▶ Heating and cooling are potentially important uses of electricity.
- ▶ How does weather conditions affect electricity consumption?
- ▶ Monthly data on temperature and residential electricity consumption in a hot region.
- ▶ What model to estimate, how best define variables?
- ▶ How best take into account seasonal patterns?

# Time series and time series regressions

- ▶ Time series data is somewhat special
- ▶ Data preparation is a bit hard, need to make decisions
- ▶ Linear regression with time series data.
- ▶ Special features of time series regression
- ▶ Time series data presents additional opportunities as well as additional challenges to compare variables.
- ▶ Three key issues to deal with: trends, seasonality and serial correlation.

## Time series regressions: data preparation

- ▶ Frequency of time series = time elapsed between observations of a variable
- ▶ Frequency may be yearly, monthly, weekly, daily, hourly, etc
- ▶ Practical problems with frequency
- ▶ Frequency may be irregular with gaps in-between.
- ▶ Often: ignore them, think as day1, day2, ...
- ▶ Sometimes it matters: weekends in financial markets may bring on news. Can add a dummy.
- ▶ Extreme values, spikes

## Time series regressions: data preparation

- ▶ Regressions: to condition  $y$  on values of  $x$  in time series data the two variables need to be on the same frequency.
- ▶ When the frequency of  $y$  and  $x$  is different we need to adjust one of them. Most often - aggregating the variable at higher frequency (e.g., from weekly to monthly).
- ▶ Flow variables, such as sales, aggregation means adding up;
- ▶ Other kinds of variables, such as prices, it is often taking an average or picking one value
  - ▶ Stock price varies by transaction (e.g. second). Daily stock price is closing price on a given day.

## Time series comparisons - S&P 500 case study

- Daily price of Microsoft stock and value of S&P 500 stock market index
- The data covers 21 years starting with December 31 1997 and ending with December 31 2018.
- Many decisions to make
- Look at data first



## Case study: Stock price and stock market index value



Microsoft, daily close price



SP 500 index value, daily close

## Time series comparisons - S&P 500 case study

- ▶ Daily price of Microsoft stock and value of S&P 500 stock market index
- ▶ The data covers 21 years starting with December 31 1997 and ending with December 31 2018.
- ▶ Key decisions:
  - ▶ Daily price = closing price
  - ▶ Gaps will be overlooked
    - ▶ Friday-Monday gap ignored
    - ▶ Holidays (Christmas, 4 of July (when would be a weekday))
- ▶ All values kept, extreme values part of process

## Time series comparisons - S&P 500 case study

- ▶ In finance, portfolio managers often focus on monthly returns - this is the time horizon for which performance are measured and communicated to clients.
- ▶ Hence, we choose monthly returns to analyze.
- ▶ Take the last day of each month

## Case study: Stock price and stock market index value



Microsoft, monthly price



S&P 500 index value, monthly close

# What is special in time series

- ▶ Time series regressions is special for several reasons.
- ▶ Many aspects of regression analysis remains.
  - ▶ Generalization, confidence intervals
  - ▶ Time series regression uncover patterns rather than evidence of causality
  - ▶ Practical data issues, missing observations, extreme values etc, remain
  - ▶ Coefficient interpretation is based on conditional comparison

## What is special in time series

- ▶ Ordering matters – key difference to cross section
- ▶ Complications...
- ▶ Trend - variables for later time periods will tend to be higher (lower)
- ▶ Seasonality - cyclical component, such 4 seasons, months, - every e.g. December value is expected to be different.
- ▶ Time series values are often not independent

## What is special in time series: Trend

Define change (or first difference):  $\Delta x_t = x_t - x_{t-1}$

$$\text{Positive trend: } E[\Delta x_t] > 0 \quad (1)$$

$$\text{Negative trend: } E[\Delta x_t] < 0 \quad (2)$$

- ▶ A time series variable follows a *positive trend* if its change is positive on average. It follows a *negative trend* if its change is negative on average
- ▶ Trend is *linear* if the change is the same on average.
- ▶ Trend is *exponential* if the change in the log of the variable is the same on average.

$$\text{Linear trend: } E[\Delta x_t] = \text{constant} \quad (3)$$

$$\text{Exponential trend: } E[\Delta \ln(x_t)] = \text{constant} \quad (4)$$

## What is special in time series: Seasonality

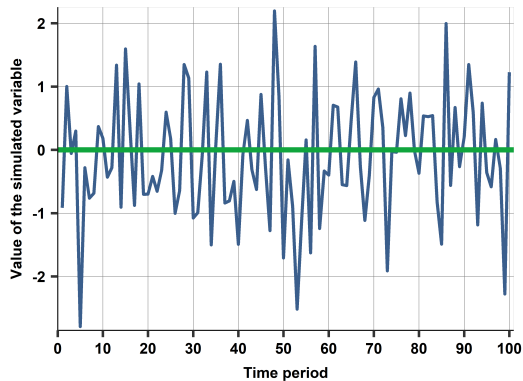
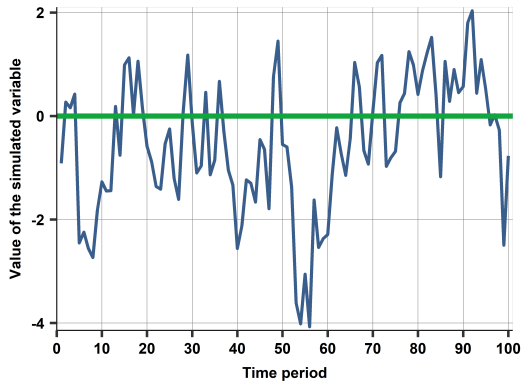
- ▶ There is seasonal variation, or simply *seasonality*, in a time series variable if its expected value changes periodically.
- ▶ Follows the seasons of the year, days of the week, hours of the day.
- ▶ Seasonality may be linear, when the seasonal differences are constant; it may be exponential, if relative differences (that may be approximated by log differences) are constant.
- ▶ Important real life phenomenon - many economic activities follow seasonal variation over the year, through the week or day.



## What is special in time series: Serial correlation

- ▶ Serial correlation means correlation of a variable with its previous values
- ▶ The 1st order serial correlation coefficient is defined as  $\rho_1 = \text{Corr}[x_t, x_{t-1}]$ 
  - ▶ the 2nd order serial correlation coefficient is defined as  $\rho_2 = \text{Corr}[x_t, x_{t-2}]$  ;
- ▶ For a *positively serially correlated* variable, if its value was above average last time, it is more likely that it is above average this time, too.
- ▶  $\rho_1 = 0$  - no serial correlation. "White Noise"
  - ▶ Like cross-section, order does not matter.
  - ▶ Example?

Two simulated series:  $\rho=0.8$  (left),  $\rho=0$  (right)



## What is special in time series: Stationarity

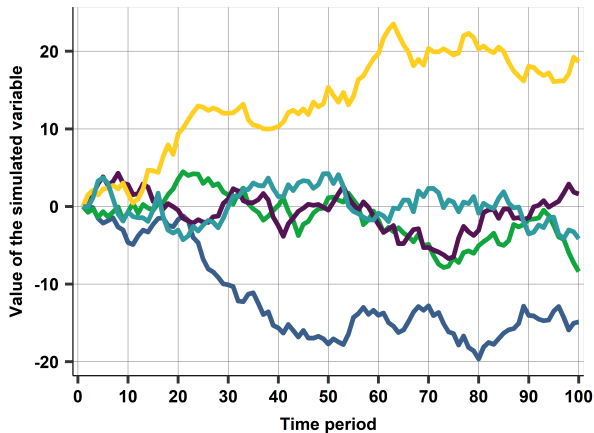
- ▶ Stationarity = a feature of the series itself. Key new concept.
- ▶ **Stationary time series** have the same expected value and same distribution, at all times.
- ▶ Stationarity means stability (in expectations).
- ▶ **Non-stationary** time series are those that are not stable for some reason.
- ▶ Trends and seasonality violate stationarity because the expected value is different at different times.
- ▶ Unstable patterns also lead to non-stationary series

## What is special in time series: Stationarity

- ▶ Another example of nonstationary time series is the *random walk*.
- ▶ Random walk when  $\rho = 1$  – also called a unit root.
- ▶ Time series variables that follow random walk change in completely random ways.
- ▶ Whatever the previous change was the next one may be anything. Wherever it starts, a random walk variable may end up anywhere after a long time.

## What is special in time series: Random walk

- 5 simulated random walk series
- Each random walk series wanders around randomly.
- Further and further away as time passes



# What is special in time series: Random walk

- ▶ Random walks are impossible to predict
- ▶ after a change, they don't revert back to some value or trend line but continue their journey from that point.
- ▶ Spread rising from one interval to another
- ▶ For stationary series, we need stability of patterns
- ▶ Avoid series with random walk when running regressions

## What is special in time series: Unit root

- ▶ Testing is complicated. FYI
- ▶ Phillips-Perron test is based on this model:

$$x_t = \alpha + \rho x_{t-1} + e_t \quad (5)$$

- ▶ This model represents a random walk if  $\rho = 1$  ( with drift if  $\alpha \neq 0$ )
- ▶ The Phillips-Perron test has hypothesis  $H_0 : \rho = 1$  against the alternative  $H_A : \rho < 1$ .
- ▶ Statistical software calculate the p-value for this test.
- ▶ When the p-value is large (e.g., larger than 0.05), we don't reject the null, concluding that the time series variable follows a random walk (perhaps with drift).

## What is special in time series: Trends and seasonality

- ▶ Stationary series are those where the expected value does not change, variance does not change over time: two observations at different points in time have the same mean and variance.
- ▶ A series is stationary if all time intervals are similar in this sense.
- ▶ We have seen three examples of non-stationarity:
  - ▶ Trend - Expected value is different in later time periods than in earlier time periods
  - ▶ Seasonality - Expected value is different in periodically recurring time periods
  - ▶ Random walk and similar series – Variance keeps increasing over time
- ▶ We care about this because regression with time series data variables that are not stationary are likely to give misleading results.



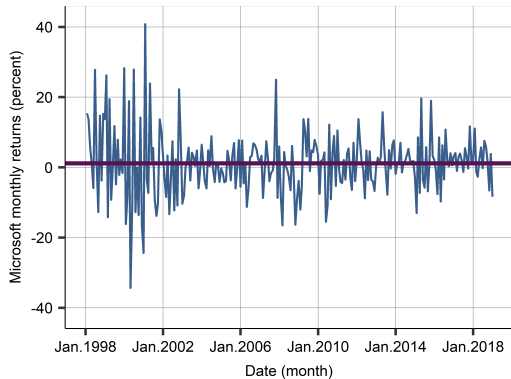
## Returns on a company stock and market returns

- ▶ Started with looking at prices
- ▶ Prices series are random walk
- ▶ They have a unit root – using the Phillips-Perron test, we find a very high p-value (and go for random walk if  $p > 0.05$ ), we are very certain that process is random walk → need action
- ▶ Need to use difference (= return)
- ▶ A: First difference of log price
- ▶ B: Percent change – choose this as more used in Finance

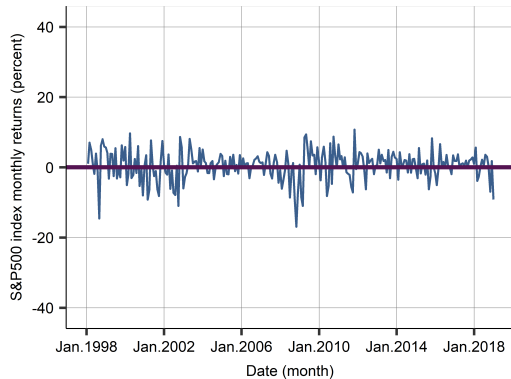
## Returns on a company stock and market returns

- ▶ Take percent return
- ▶ Correlation in time series show visually
- ▶ We can estimate the regression formally

## Case study: Stock price and stock market index return (pct)

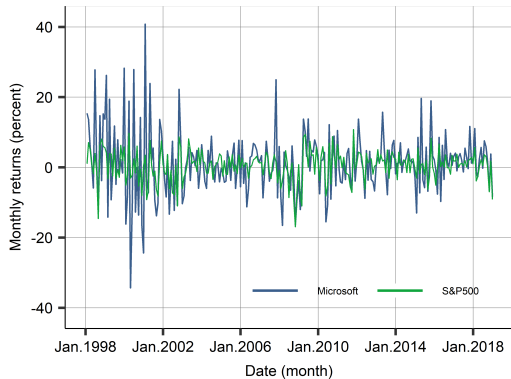


Microsoft, monthly return (pct)

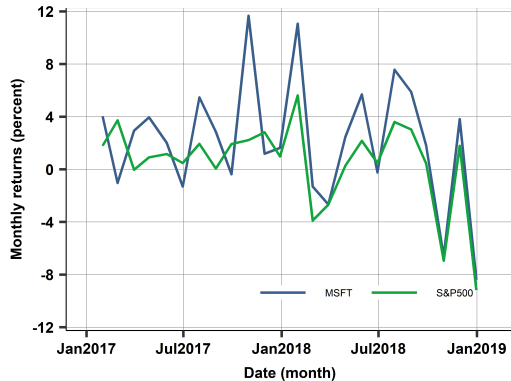


S&P 500 index value, monthly return (pct)

## Case study: Stock and market returns over time (pct)



The entire time series, 1998-2018



2017-18

## Returns on a company stock and market returns

- ▶ Correlation in time series: the price of the Microsoft stock tends to increase when market prices increase, and it tends to decrease when market prices decrease.
- ▶ Market changes are smaller
- ▶ Focus on two years, we can see it better
- ▶ We can estimate the regression formally
  - ▶ Monthly
  - ▶ Percent return

## Returns on a company stock and market returns

$$pctchange(MSFT_t) = \alpha + \beta pctchange(SP500_t) \quad (6)$$

►  $\alpha = 0.54; \beta = 1.26$

## Returns on a company stock and market returns

$$pctchange(MSFT_t) = \alpha + \beta pctchange(SP500_t) \quad (6)$$

- ▶  $\alpha = 0.54$ ;  $\beta = 1.26$
- ▶ Intercept: returns on the Microsoft stock tend to be 0.54 percent when the S%P500 index doesn't change.
- ▶ Slope: returns on the Microsoft stock tend to be 1.26% higher when the returns on the S&P500 index are 1% higher.
- ▶ The 95% confidence interval is [1.06, 1.46].
- ▶ R-squared: 0.36
- ▶ First difference of log prices. Estimate is 1.24
- ▶ Daily returns (percent), beta is 1.10

## Returns on a company stock and market returns

- ▶ Slope is actually the well-known "beta" in finance
- ▶ Beta - measure of the riskiness of the company stock.
  - ▶ Close to one?
  - ▶ Greater than one?
  - ▶ Positive, less than one?
  - ▶ Negative?



## Returns on a company stock and market returns

- ▶ We have seen challenges that make time series regression more complicated
- ▶ Now let's review what we do
- ▶ It will turn out to be simple...

## Time series regressions

- Regression in time series data is defined and estimated the same way as in other data.

$$y_t^E = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \dots \quad (7)$$

- Interpretations similar to cross-section
- $\beta_0$ : We expect  $y$  to be  $\beta_0$  when all explanatory variables are zero.
- $\beta_1$ : Comparing time periods with different  $x_1$  but the same in terms of all other explanatory variables, we expect  $y$  to be higher by  $\beta_1$  when  $x_1$  is higher by one unit.

## Time series regressions - list of issues

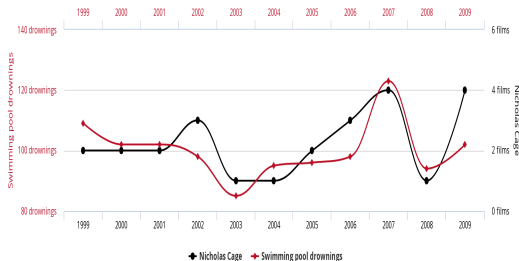
- ▶ Handling trend and seasonality
- ▶ Checking and dealing with unit roots
- ▶ Transforming the series, such as taking first differences
- ▶ Dealing with serial correlation (in  $y_t$ ) – specifying the proper standard errors
- ▶ Considering lags

## Time series regressions: Trends and seasonality

- ▶ Trends, seasonality, and random walks can present serious threats to uncovering meaningful patterns in time series data.
- ▶ Example: time series regression in levels  $y_t^E = \alpha + \beta x_t$ .
- ▶ If both  $y$  and  $x$  have a positive trend, the slope coefficient  $\beta$  will be positive whether the two variables are related or not.
- ▶ That is simply because in later time periods both tend to have higher values than in earlier time periods.
- ▶ Associations between variables only because of the effect of trends are said to be **spurious correlation**

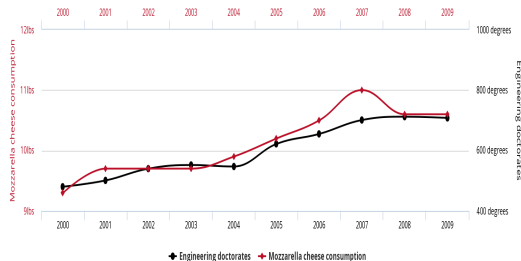
# Correlated time series. But....

Number of people who drowned by falling into a pool  
correlates with  
Films Nicolas Cage appeared in



tylervigen.com

Per capita consumption of mozzarella cheese  
correlates with  
Civil engineering doctorates awarded



tylervigen.com

These and similar graphs from <http://tylervigen.com/spurious-correlations>

## Time series regressions: Trends and seasonality

- ▶ Spurious - could be very far fetched reason, randomness
  - ▶ especially with small samples!
- ▶ One frequent reason: trend and seasonality as confounders.
- ▶ Trend/seasonality is a confounder if both  $y_t$  and  $x_t$  have trend / seasonal variation.
- ▶ If not included while they should be - omitted variables
- ▶ A trend may capture omitted global tendencies in population growth, economic activity, fashion, technology.
- ▶ A seasonality may capture variation in weather, holidays and leisure time, sleeping and eating habits, open and close time of shops, etc.

## Time series regressions: Trends and seasonality

- Example, a regression of the price of college education in the U.S. on the GDP of Germany over the past few decades would result in a positive slope coefficient even though that two may not be related in any fundamental way.

## Time series regressions: Trends and seasonality

- ▶ A good solution to trends is replacing variables in the regression with their first differences
- ▶ Variables in differences do not have trends and are therefore more likely to be stationary.
  - ▶ Could be log difference for exponential trends
- ▶ A good solution to seasonality is including *binary season variables* in regressions.
  - ▶ Look at pattern, figure out if quarters, months, weeks, days of week, etc.
- ▶ Another good solution to handle seasonality is working with year-on-year changes instead of first differences.



## Time series regressions – first difference

We use the  $\Delta$  notation to denote a first difference:

$$\Delta y_t = y_t - y_{t-1} \quad (8)$$

A linear regression in differences is the following

$$\Delta y_t^E = \alpha + \beta \Delta x_t \quad (9)$$

- ▶ Coefficients same interpretation as before, but use "when"
- ▶  $\alpha$  is the average left-hand-side variable when all right-hand-side variables are zero,
- ▶  $\beta$  shows the difference in the average left-hand-side variable for observations with different  $\Delta x_t$ .

## Time series regressions – first difference

$$\Delta y_t^E = \alpha + \beta \Delta x_t$$

- ▶ Because variables denote changes...
- ▶  $\alpha$  is the average change in  $y$  when  $x$  doesn't change.
- ▶ The slope coefficient on  $\Delta x_t$  shows how much more  $y$  is expected to change when  $x$  changes by one more unit.
- ▶ "more" – needed as we expect  $y$  to change anyway, by  $\alpha$ , when  $x$  doesn't change.
  - ▶ The slope shows how  $y$  is expected to change when  $x$  changes, in addition to  $\alpha$ .

## Practice of time series regressions

- ▶ If you think there is a simple stable trend, having levels and a simple trend variable can be a solution. Rarely the case
- ▶ For most applications, time series regression involving using differences or log differences.
- ▶ Take differences unless you have a good reason not to.
  - ▶ One such case is when your variable is already a difference, GDP growth = difference of levels of GDP in percentage

## Practice of time series regressions

- ▶ Capturing seasonality also important
- ▶ Higher frequency – the more important
  - ▶ People behave differently on different hours and days
  - ▶ Weather varies over months
  - ▶ Holidays, ect
- ▶ Have seasonal dummies if seasonality is stable. Often good enough
- ▶ Pattern may vary over time. If it does, solutions must capture exact pattern – difficult
  - ▶ Example?

## Time series regressions: Standard errors

- Serial correlation makes the usual standard error estimates wrong.

## Time series regressions: Standard errors

- ▶ Serial correlation makes the usual standard error estimates wrong.
- ▶ When the dependent variable is serially correlated - heteroskedasticity robust SE is wrong - sometimes very wrong, with a large bias.
  - ▶ More precisely it is serial correlation in residuals, but think about it as serial correlation in  $y_t$  is okay
- ▶ Use new SE - the **Newey-West** SE
  - ▶ procedure incorporates the structure of serial correlation of the regression residuals
  - ▶ Fine if heteroskedasticity as well
  - ▶ Need to specify lags. If enough data, frequency and seasonality should help, Months - 12 should be good
  - ▶ An alternative solution is to have lagged dependent variable in the regression

## Electricity consumption and temperature

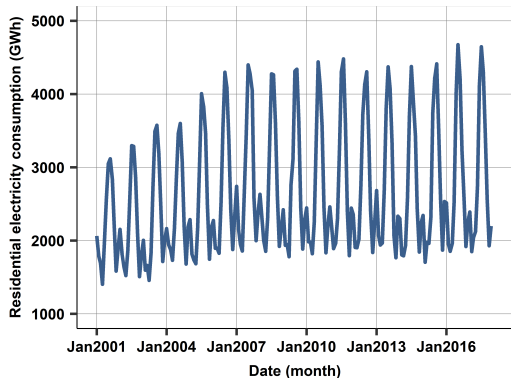
- ▶ Monthly weather and electricity data for Phoenix, Arizona
- ▶ January 2001 and ends in Dec 2017– 204 month
- ▶ The weather data includes “cooling degree days” and “heating degree days” per month.
- ▶ Cooling degree days and heating degree days are daily temperatures transformed in a simple way and then added up within a month.

## Electricity consumption and temperature

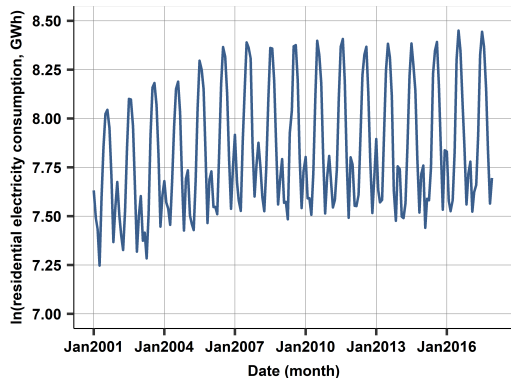
- ▶ The cooling degree days measure takes the average temperature within each day, subtracts a reference temperature (65F, or 18C), and adds up these daily values.
- ▶ If the average temperature in a day is, say, 75F (24C), the cooling degree is 10F (6C). This would be the value for one day.
- ▶ Then we would calculate the corresponding values for each of the days in the month and add them up.
  - ▶ Days when the average temperature is below 65F have zero values.
- ▶ For heating degree days it's the opposite: zero for days with 65F or warmer, and the difference between the daily average temperature and 65F when lower.
  - ▶ For example, with 45F (7C), the heating degree is 20F (11C).



# Electricity consumption and temperature

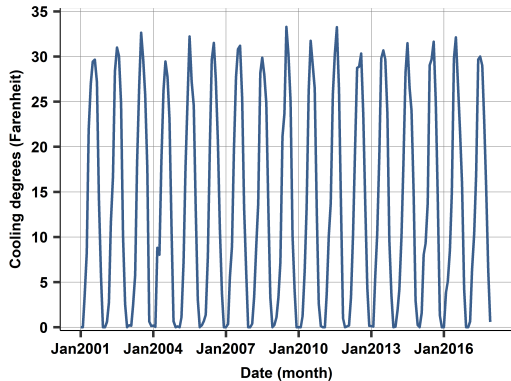


Electricity consumption

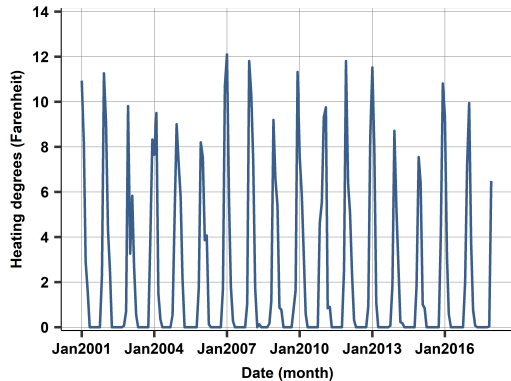


Log electricity consumption

# Electricity consumption and temperature



Average cooling degrees



Average heating degrees

## Electricity consumption and temperature

- ▶ No unit root.
- ▶ There is a trend in electricity for sure, exponential  $\rightarrow$  log difference
- ▶ For easier interpretation, take FD of cooling days and heating days.
- ▶ Natural question: How much does electricity consumption change when temperature changes?
- ▶ In this example, taking first difference does not make a huge difference, would not be a mistake to keep in levels
  - ▶ Another option could be to take 12-month difference
- ▶ Add monthly dummies, January (December to January) is reference
- ▶ Newey-West standard errors in parentheses; \*\*  $p < 0.01$ , \*  $p < 0.05$

# Electricity consumption and temperature

VARIABLES	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$
$\Delta CD$	0.031** (0.001)	0.017** (0.002)
$\Delta HD$	0.037** (0.003)	0.014** (0.003)
month = 2, February		-0.274**
month = 3, March		-0.122**
....		
month = 7, July		0.058**
month = 8, August		-0.085**
month = 9, September		-0.176**
....		
month = 12, December		0.067**
Constant	0.001 (0.002)	0.092** (0.013)
Observations	203	203

## Electricity consumption and temperature

- ▶ In months when cooling degrees increase by one degree and heating degrees do not change, electricity consumption increases by 3.1 percent, on average.
  - ▶ When heating degrees increase by one degree and cooling degrees do not change, electricity consumption increases by 3.7 percent, on average.
- ▶ Monthly dummies matter, reduce slope coefficient estimates
- ▶ How to think about monthly dummies?
- ▶ Monthly dummies may be interpreted. Not easy.

## Electricity consumption and temperature

- ▶ The reference month is January;
- ▶ constant (when cooling and heating degrees stay the same), electricity consumption increases by about 9% from December to January.
- ▶ The other season coefficients compare to this change.
- ▶ February – the January to February change is 28 percentage points lower than in the reference month, December to January.
- ▶ That was +9%, so electricity consumption decreases by about 19% on average to February from January when cooling and heating degrees stay the same.

## Time series regressions: changes and lags

- Useful tool, potential causal scenario where changes take an impact in several periods later

$$\Delta y_t^E = \alpha + \beta_0 \Delta x_t + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} \quad (10)$$

- Coefficients – how  $y$  is expected to change after a one-time change in  $x$ , i.e., when  $x$  changes in one time period *but not afterwards*.
- $\beta_0$  shows the contemporaneous association: what to expect in the same time period.
- $\beta_1$  shows the once-lagged association: what to expect in the next time period.

## Time series regressions: Lags

$$\Delta y_t^E = \alpha + \beta_0 \Delta x_t + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2}$$

- ▶  $\beta_0$  = how many units more  $y$  is expected to change within the same time period when  $x$  changes by one more unit (and it didn't change in the previous two time periods).
- ▶  $\beta_1$  = how much more  $y$  is expected to change *in the next time period* after  $x$  changed by one more unit – provided that it didn't change at other times.
- ▶ Cumulative effect?



## Time series regressions: Lags

$$\Delta y_t^E = \alpha + \beta_0 \Delta x_t + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2}$$

- ▶  $\beta_0$  = how many units more  $y$  is expected to change within the same time period when  $x$  changes by one more unit (and it didn't change in the previous two time periods).
- ▶  $\beta_1$  = how much more  $y$  is expected to change *in the next time period* after  $x$  changed by one more unit – provided that it didn't change at other times.
- ▶ Cumulative effect?

$$\beta_{cumul} = \beta_0 + \beta_1 + \beta_2 \tag{11}$$

## Time series regressions: Lags

- To get a SE on the cumulative effect, do a trick and transformation, and estimate a different model

$$\Delta y_t^E = \alpha + \beta_{cumul} \Delta x_{t-2} + \delta_0 \Delta(\Delta x_t) + \delta_1 \Delta(\Delta x_{t-1}) \quad (12)$$

- the  $\beta_{cumul}$  in this regression is exactly the same as  $\beta_0 + \beta_1 + \beta_2$  in the previous regression.
  - Other two right-hand-side variables strange and we do not care
- Typically estimate both. One with lags to see patterns. One with cumulative second to test the cumulative value.

## Time series regressions: choosing lags

- ▶ Lag selection is a practical question
- ▶ Think about theory, domain knowledge. This may drive your call.
- ▶ Try out a few lags. Few depends on the size of your dataset.
  - ▶ Few dozen observations - need to be picky
  - ▶ 10-20 years of monthly data, can try all months
- ▶ watch for seasonality. Often need lags to capture 12 months, 4 quarters, etc.
- ▶ Try a few versions. Choose based on coefficient significance.

## Electricity consumption and temperature

- ▶ Go back to model
- ▶ Add 2 lags - for both cooling and heating days
- ▶ And keep monthly dummies

## Electricity consumption and temperature

VARIABLES	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$
$\Delta CD$	0.020** (0.002)	
$\Delta CD$ 1st lag	0.006** (0.002)	
$\Delta CD$ 2nd lag	0.001 (0.002)	
$\Delta HD$	0.019** (0.003)	
$\Delta HD$ 1st lag	0.011** (0.003)	
$\Delta HD$ 2nd lag	0.000 (0.003)	
$\Delta CD$ cumulative coeff		0.027** (0.005)
$\Delta HD$ cumulative coeff		0.030** (0.007)

## Electricity consumption and temperature

- ▶ Interestingly evidence of lagged effect
- ▶ Cumulative effect is now slightly larger.
- ▶ Not straightforward answer why
  - ▶ People take time to react to weather change
  - ▶ Or captures some correlated other variable
- ▶ Overall: Temperature is strongly associated with residential electricity consumption in Arizona.
- ▶ Even when seasonality is captured

# Weather and electricity

How is residential electricity consumption related to weather in Arizona?

Files:

- ▶ ch12\_arizona\_electricity (.do / .R)

# Weather and electricity

Data:

- ▶ Residential electricity consumption in Arizona, US (US Energy Information Administration (EIA))
  - ▶ monthly data
  - ▶ state-level
  - ▶ 2001-2018
- ▶ Temperature data (National Oceanic and Atmospheric Administration (NOAA))
  - ▶ monthly data
  - ▶ weather station level (100 stations in Arizona, picked one: Phoenix Airport, which is close to most population)
  - ▶ 1989-2018 in total, but coverage varies a lot by station



## Data preparation

Cross-sectional unit:

- ▶ Discrepancy between level of aggregation in two datasets:
  - ▶ electricity: Arizona state
  - ▶ temperature: Phoenix Airport
- ▶ 60% of state population lives in Phoenix metropolitan area
- ▶ 2nd and 3rd largest cities are also located close to Phoenix

Frequency:

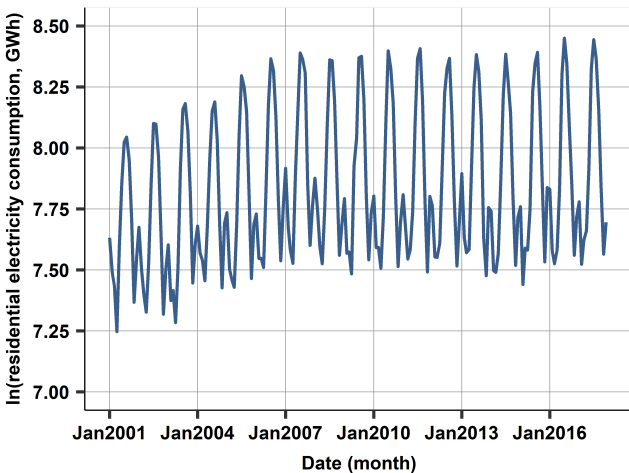
- ▶ Everything is at monthly level
- ▶ Combined data covers January 2001 - December 2017 (204 months)

## Measure of heating / cooling degree days

We need a measure of how hot or cold days are and how likely it is that people use electricity for heating / cooling.

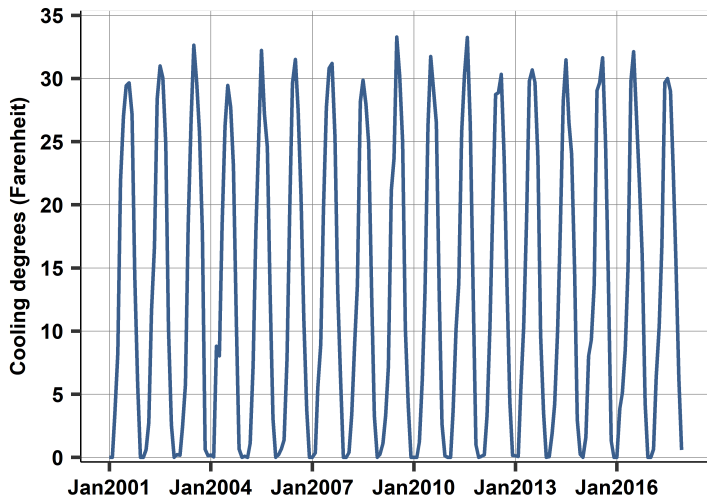
- ▶ reference temperature: 18C (65F)
- ▶ cooling degree days:
  - ▶ take average temperature each day,
  - ▶ subtract reference temperature,
  - ▶ calculate average of all these within a month (count below-18C as 0)
  - ▶ e.g. avg. temp. in a day is 24C (75F), then the cooling degree is 6C (10F)
- ▶ heating degree days:
  - ▶ take average temperature each day,
  - ▶ subtract FROM reference temperature,
  - ▶ calculate average of all these within a month (count above-18C as 0)
  - ▶ e.g. avg. temp. in a day is 7C (45F), then the heating degree is 11C (20F)

## Residential electricity consumption in Arizona

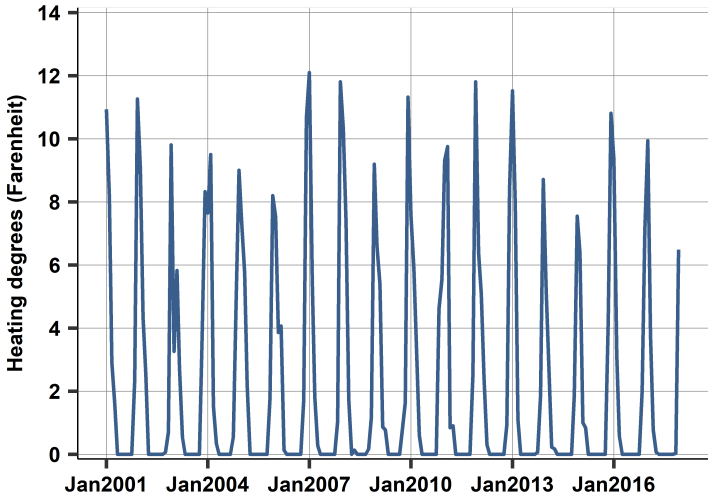


- We use logs
  - for easier interpretation
  - for statistical reasons: in level terms, the variance was increasing over time
- Patterns:
  - upward trend until 2008
  - strong seasonality (highest consumption is in summertime, there is a smaller peak during winter)

## Cooling degree days (F)



Heating degree days (F)



# Time-series Regression Models

$$\ln Q_t^E = \beta_0 + \beta_1 CD_t + \beta_2 HD_t \quad (13)$$

$$\Delta \ln Q_t^E = \gamma_0 + \gamma_1 \Delta CD_t + \gamma_2 \Delta HD_t \quad (14)$$

## Results of time-series regressions

Results look similar in the two models, but interpretation is different.

Model in levels:

- ▶ Average residential electricity consumption in Arizona is **3.2 percent higher in months with one higher** cooling degree in Phoenix (in Fahrenheit), comparing months with the same heating degrees.
- ▶ Average electricity consumption is **4.5 percent higher in months with one higher** heating degree, comparing months with the same cooling degrees.

Model in differences:

- ▶ In months, when cooling degrees **increase by one degree** and heating degrees do not change, electricity consumption **increases by 3.1 percent more**, on average.
- ▶ When heating degrees **increase by one degree** and cooling degrees do not change, electricity consumption **increases by 3.7 percent more**, on average.

## Handling trend and seasonality

If both LHS and RHS variables have trend and/or seasonality we might see a spurious relationship (e.g. age of US spending on science and technology and number of divorces).

→ We need to get rid of them.

- ▶ Trend appears only in LHS variable, so it does not matter,
- ▶ but seasonality is true for all variables in the model, it might drive our results

Solution: we include binary variables for every months



## Regression results

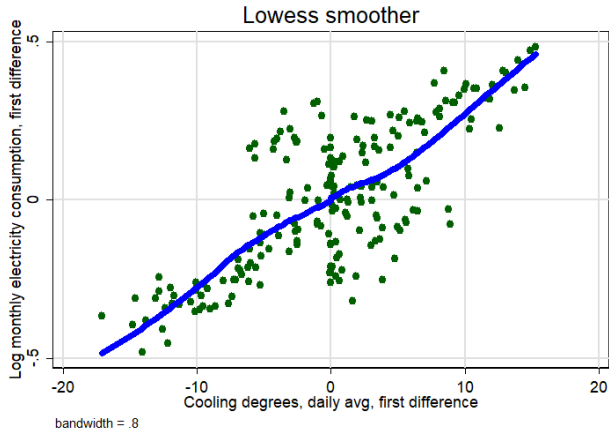
- ▶ Comparing the same months across years in Arizona, electricity consumption is **1.8 percent higher** when cooling degrees are **one degree Fahrenheit higher** (and heating degrees are the same).
- ▶ Comparing the same months across years in Arizona, electricity consumption is **1.5 percent higher** when heating degrees are **one degree Fahrenheit higher** (and cooling degrees are the same).
- ▶ Model in differences has very similar results, so it seems potential trends don't matter here
- ▶ Coefficient estimates are substantially lower than in the original model:
  - ▶ Part of the association is attributable to months as opposed to temperature itself.
  - ▶ The part that months take away from the temperature coefficients is larger in winter months.

## Can we talk about causality here?

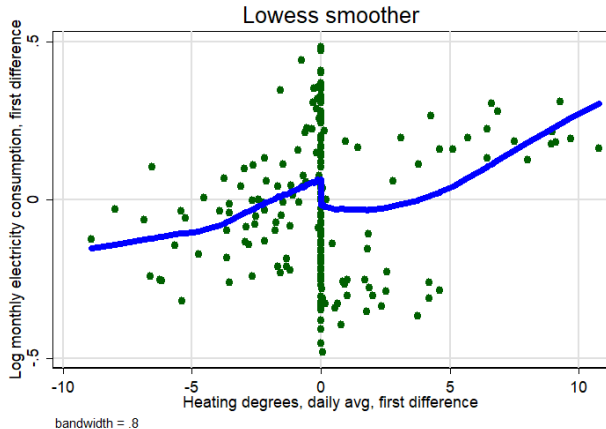
- ▶ Does electricity use affect temperature?
  - ▶ Not really plausible in the short run
- ▶ Is there a third variable that explains both?
  - ▶ E.g. daylight, different activities by season (more time at home during holidays)
  - ▶ mostly captured by months dummies

We can be kind of convinced that comparing electricity usage in the same months across years will capture the causal effect of temperature.

# Potential nonlinearities



# Potential nonlinearities



## Correct Standard Errors

Serial correlation makes the usual standard error estimates wrong.

Two strategies to get correct standard error estimates:

- ▶ Newey-West standard errors (include a full period of seasonal variation)
- ▶ include lags of dependent variable

We do both in our example.

## Estimation results with corrected S.E.

- ▶ The Newey-West standard error estimates are slightly larger for the regression in levels than the simple standard error estimates were. For the regression in differences they are practically the same.
  - ▶ The reason is that in the level-regression there is serial correlation, whereas in the diff-regression we don't see serial correlation.
- ▶ In level model there is a big difference in S.E. in Newey West and in lag model, whereas in the diff model, they are the same.
  - ▶ This is also due to the presence of serial correlation in the level model.

## Estimate cumulative effects

$$\Delta y_t^E = \alpha + \beta_0 \Delta x_t + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} \quad (15)$$

$$\Delta y_t^E = \alpha + \beta_{cumul} \Delta x_{t-2} + \delta_0 \Delta(\Delta x_t) + \delta_1 \Delta(\Delta x_{t-1}) \quad (16)$$

## Main lessons learnt

- ▶ Temperature explains a large part of electricity consumption, i.e. hotter than average summers and cooler than average winters lead to substantially higher electricity consumption.
  - ▶ Months matter on their own right as well.
- ▶ We had to deal with the strong seasonality in both electricity consumption and temperature.
  - ▶ We included month binary variables, and the estimated coefficients became smaller (about half the original for cooling degree days, and about one third the original value for heating degree days)
- ▶ If there is serial correlation in the dependent variable, we need to adjust standard error estimation.
  - ▶ Most general solution is to use Newey-West standard errors.
  - ▶ We saw it does matter, when we have serial correlation.



## Time series regressions: Summary of the process

- ▶ Decide on frequency; deal with gaps if necessary.
- ▶ Plot the series. Identify features and issues.
- ▶ Handle trends by transforming variables (Often: first difference).
- ▶ Specify regression that handles seasonality, usually by including season dummies.
- ▶ Include or don't include lags of the right-hand-side variable(s).
- ▶ Handle serial correlation.
- ▶ Interpret coefficients in a way that pays attention to potential trend and seasonality.
- ▶ Time series econometrics very complicated beyond this
- ▶ But: These steps often good enough