

# Database II

## Lecture 5

# Relational Algebra I

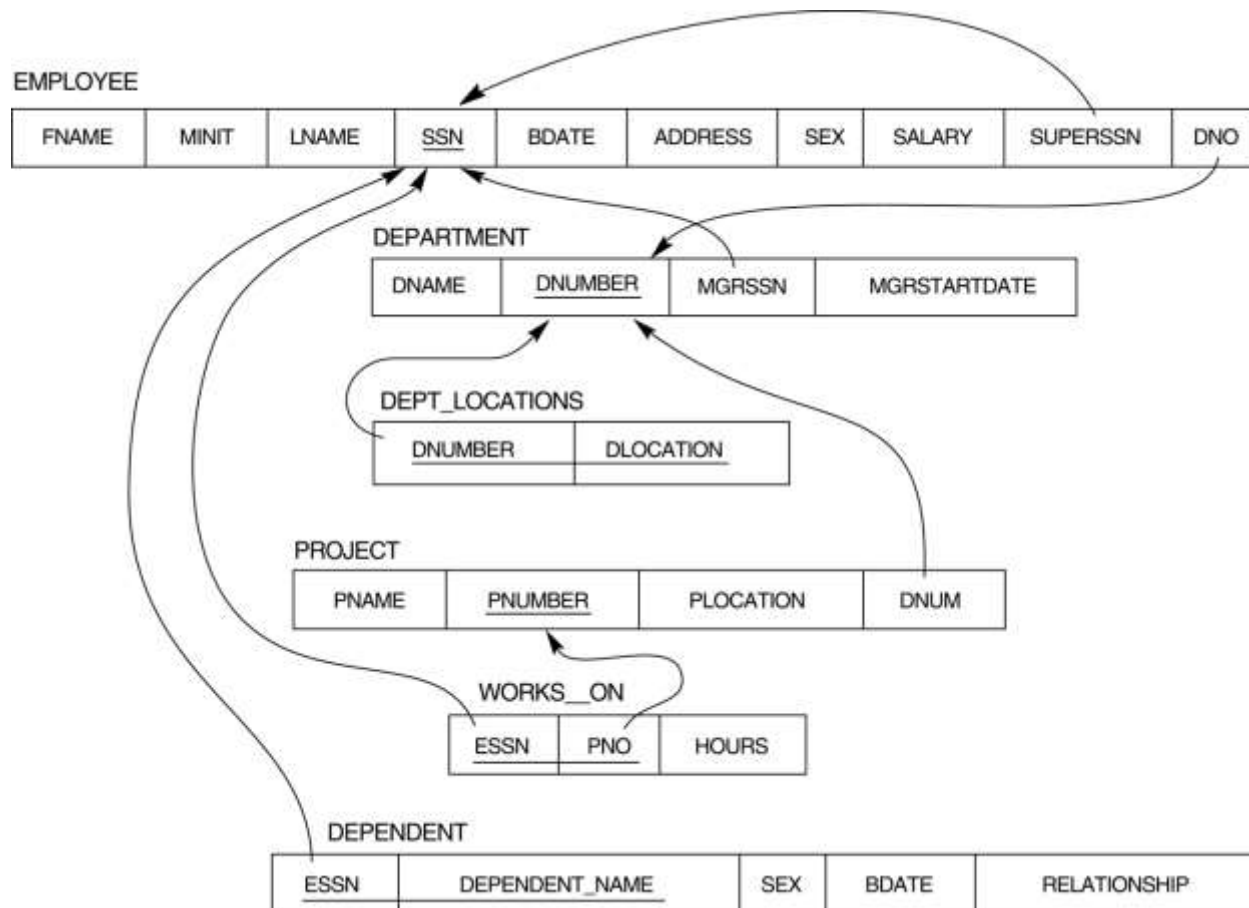
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# Lecture Outline

- Example Database Application (COMPANY)
- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory

# Database State for COMPANY

All examples discussed below refer to the COMPANY database shown here.



**Figure 5.6**

One possible database state for the COMPANY relational database schema.

**EMPLOYEE**

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

**DEPARTMENT**

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

**DEPT\_LOCATIONS**

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

**WORKS\_ON**

<u>Essn</u>	<u>Pno</u>	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

**PROJECT**

Pname	<u>Pnumber</u>	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

**DEPENDENT**

<u>Essn</u>	<u>Dependent_name</u>	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

The query results in this lecture refer to this database state

# Relational Algebra

- The basic set of operations for the relational model is known as the relational algebra. These operations enable a user to specify basic retrieval requests.
- The result of a retrieval is a new relation, which may have been formed from one or more relations. The **algebra operations** thus produce new relations, which can be further manipulated using operations of the same algebra.
- A sequence of relational algebra operations forms a **relational algebra expression**, whose result will also be a relation that represents the result of a database query (or retrieval request).

# Unary Relational Operations

## ● SELECT Operation

SELECT operation is used to select a *subset* of the tuples from a relation that satisfy a **selection condition**. It is a filter that keeps only those tuples that satisfy a qualifying condition – those satisfying the condition are selected while others are discarded.

**Example:** To select the EMPLOYEE tuples whose department number is four or those whose salary is greater than \$30,000 the following notation is used:

$\sigma_{\text{DNO} = 4}(\text{EMPLOYEE})$

$\sigma_{\text{SALARY} > 30,000}(\text{EMPLOYEE})$

In general, the select operation is denoted by  $\sigma_{\langle \text{selection condition} \rangle}(\text{R})$  where the symbol  $\sigma$  (sigma) is used to denote the select operator, and the selection condition is a Boolean expression specified on the attributes of relation R

# Unary Relational Operations

## SELECT Operation Properties

- The SELECT operation  $\sigma_{\langle \text{selection condition} \rangle}(R)$  produces a relation S that has the same schema as R
- The SELECT operation  $\sigma$  is **commutative**; i.e.,  
$$\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(R)) = \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))$$
- A cascaded SELECT operation **may be applied in any order**; i.e.,  
$$\begin{aligned} &\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition3} \rangle}(R))) \\ &= \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition3} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))) \end{aligned}$$
- A cascaded SELECT operation may be replaced by a single selection with a conjunction of all the conditions; i.e.,  
$$\begin{aligned} &\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition3} \rangle}(R))) \\ &= \sigma_{\langle \text{condition1} \rangle \text{ AND } \langle \text{condition2} \rangle \text{ AND } \langle \text{condition3} \rangle}(R) \end{aligned}$$

# Unary Relational Operations (cont.)

## ● PROJECT Operation

This operation selects certain *columns* from the table and discards the other columns. The PROJECT creates a vertical partitioning – one with the needed columns (attributes) containing results of the operation and other containing the discarded Columns.

**Example:** To list each employee's first and last name and salary, the following is used:

$$\pi_{\text{LNAME, FNAME, SALARY}}(\text{EMPLOYEE})$$

The general form of the project operation is  $\pi_{\langle \text{attribute list} \rangle}(\text{R})$  where  $\pi$  (pi) is the symbol used to represent the project operation and  $\langle \text{attribute list} \rangle$  is the desired list of attributes from the attributes of relation R.

The project operation *removes any duplicate tuples*, so the result of the project operation is a set of tuples and hence a valid relation.



# Unary Relational Operations (cont.)

## PROJECT Operation Properties

- The number of tuples in the result of projection  $\pi_{\langle \text{list} \rangle} (R)$  is always less or equal to the number of tuples in R.
- If the list of attributes includes a key of R, then the number of tuples is equal to the number of tuples in R.
- $\pi_{\langle \text{list1} \rangle} (\pi_{\langle \text{list2} \rangle} (R)) = \pi_{\langle \text{list1} \rangle} (R)$  as long as  $\langle \text{list2} \rangle$  contains the attributes in  $\langle \text{list1} \rangle$

# Examples of applying SELECT and PROJECT operations

**Figure 6.1**

Results of SELECT and PROJECT operations. (a)  $\sigma_{(Dno=4 \text{ AND } Salary > 25000) \text{ OR } (Dno=5 \text{ AND } Salary > 30000)}(EMPLOYEE)$ . (b)  $\pi_{Lname, Fname, Salary}(EMPLOYEE)$ . (c)  $\pi_{Sex, Salary}(EMPLOYEE)$ .

(a)

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

# Unary Relational Operations (cont.)

## ● Rename Operation

We may want to apply several relational algebra operations one after the other. Either we can write the operations as a single **relational algebra expression** by nesting the operations, or we can apply one operation at a time and create **intermediate result relations**. In the latter case, we must give names to the relations that hold the intermediate results.

**Example:** To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation. We can write a single relational algebra expression as follows:

$$\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$$

OR We can explicitly show the sequence of operations, giving a name to each intermediate relation:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE})$$

$$\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$$

# Unary Relational Operations (cont.)

## ● Rename Operation (cont.)

The rename operator is  $\rho$

The general Rename operation can be expressed by any of the following forms:

- $\rho_S(B_1, B_2, \dots, B_n)(R)$  is a renamed relation  $S$  based on  $R$  with column names  $B_1, B_1, \dots, B_n$ .
- $\rho_S(R)$  is a renamed relation  $S$  based on  $R$  (which does not specify column names).
- $\rho_{(B_1, B_2, \dots, B_n)}(R)$  is a renamed relation with column names  $B_1, B_1, \dots, B_n$  which does not specify a new relation name.

# Example of applying multiple operations and RENAME

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

**Figure 6.2**

Results of a sequence of operations.

(a)  $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$ .

(b) Using intermediate relations and renaming of attributes.

# Relational Algebra Operations From Set Theory

## ● UNION Operation

The result of this operation, denoted by  $R \cup S$ , is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.

**Example:** To retrieve the social security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5, we can use the union operation as follows:

**DEP5\_EMPS**  $\leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE})$

**RESULT1**  $\leftarrow \pi_{\text{SSN}}(\text{DEP5\_EMPS})$

**RESULT2(SSN)**  $\leftarrow \pi_{\text{SUPERSSN}}(\text{DEP5\_EMPS})$

**RESULT**  $\leftarrow \text{RESULT1} \cup \text{RESULT2}$

The union operation produces the tuples that are in either RESULT1 or RESULT2 or both. The two operands must be “type compatible”.

# Relational Algebra Operations From Set Theory

## ● Type Compatibility

- The operand relations  $R_1(A_1, A_2, \dots, A_n)$  and  $R_2(B_1, B_2, \dots, B_n)$  must have the same number of attributes, and the domains of corresponding attributes must be compatible; that is,  $\text{dom}(A_i) = \text{dom}(B_i)$  for  $i=1, 2, \dots, n$ .
- The resulting relation for  $R_1 \cup R_2$ ,  $R_1 \cap R_2$ , or  $R_1 - R_2$  has the same attribute names as the *first* operand relation  $R_1$  (by convention).

# Relational Algebra Operations From Set Theory

## ● UNION Example

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

INSTRUCTOR	FNAME	LNAME
	John	Smith
	Ricardo	Browne
	Susan	Yao
	Francis	Johnson
	Ramesh	Shah

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

STUDENT  $\cup$  INSTRUCTOR



# Relational Algebra Operations From Set Theory (cont.)

## ● INTERSECTION OPERATION

The result of this operation, denoted by  $R \cap S$ , is a relation that includes all tuples that are in both R and S. The two operands must be "type compatible"

**Example:** The result of the intersection operation (figure below) includes only those who are both students and instructors.

FN	LN
Susan	Yao
Ramesh	Shah

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT  $\cap$  INSTRUCTOR

# Relational Algebra Operations From Set Theory (cont.)

## ● Set Difference (or MINUS) Operation

The result of this operation, denoted by  $R - S$ , is a relation that includes all tuples that are in  $R$  but not in  $S$ . The two operands must be "type compatible".

**Example:** The figure shows the names of students who are not instructors, and the names of instructors who are not students.

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT-INSTRUCTOR

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

INSTRUCTOR-STUDENT

# Relational Algebra Operations From Set Theory (cont.)

- Notice that both union and intersection are *commutative operations*; that is

$$\mathbf{R \cup S = S \cup R, \text{ and } R \cap S = S \cap R}$$

- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative operations*; that is

$$\mathbf{R \cup (S \cup T) = (R \cup S) \cup T, \text{ and } (R \cap S) \cap T = R \cap (S \cap T)}$$

- The minus operation is *not commutative*; that is, in general

$$\mathbf{R - S \neq S - R}$$

# Relational Algebra Operations From Set Theory (cont.)

## ● CARTESIAN (or cross product) Operation

- This operation is used to combine tuples from two relations in a combinatorial fashion. In general, the result of  $R(A_1, A_2, \dots, A_n) \times S(B_1, B_2, \dots, B_m)$  is a relation  $Q$  with degree  $n + m$  attributes  $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order. The resulting relation  $Q$  has one tuple for each combination of tuples—one from  $R$  and one from  $S$ .
- Hence, if  $R$  has  $n_R$  tuples (denoted as  $|R| = n_R$ ), and  $S$  has  $n_S$  tuples, then  $|R \times S|$  will have  $n_R * n_S$  tuples.
- The two operands do NOT have to be "type compatible"

### Example:

**FEMALE\_EMPS**  $\leftarrow \sigma_{SEX='F'}(\text{EMPLOYEE})$

**EMPNAMES**  $\leftarrow \pi_{FNAME, LNAME, SSN}(\text{FEMALE\_EMPS})$

**EMP\_DEPENDENTS**  $\leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$

# Relational Algebra Operations From Set Theory (cont.)

Figure 7.12 An illustration of the CARTESIAN PRODUCT operation.

FEMALE EMPs	FNAME	MINIT	LNAME	SSN	BOATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
	Alice	J	Zelazny	999887777	1969-07-19	3281 Crodle Spring, TX	F	25000	987654321	4
	Jennifer	S	Wallace	987654321	1941-06-20	2191 Bony, Red Lake, TX	F	40000	888995555	4
	Joyce	A	English	453453453	1972-07-14	5631 Rice, Houston, TX	F	25000	333445555	6

EMPNAMEs	FNAME	LNAME	SSN
	Alice	Zelazny	999887777
	Jennifer	Wallace	987654321
	Joyce	English	453453453

EMP_DEPENDENTS	FNAME	LNAME	SSN	ESSN	DEPENDENT_NAME	SEX	BOATE	...
	Alice	Zelazny	999887777	333445555	Alice	F	1969-04-05	...
	Alice	Zelazny	999887777	333445555	Theodora	M	1983-10-25	...
	Alice	Zelazny	999887777	333445555	Joy	F	1959-05-03	...
	Alice	Zelazny	999887777	987654321	Abner	M	1942-02-28	...
	Alice	Zelazny	999887777	123456789	Michael	M	1968-01-04	...
	Alice	Zelazny	999887777	123456789	Alice	F	1969-12-30	...
	Alice	Zelazny	999887777	123456789	Elisabeth	F	1967-05-05	...
	Jennifer	Wallace	987654321	333445555	Alice	F	1969-04-05	...
	Jennifer	Wallace	987654321	333445555	Theodora	M	1983-10-25	...
	Jennifer	Wallace	987654321	333445555	Joy	F	1959-05-03	...
	Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
	Jennifer	Wallace	987654321	123456789	Michael	M	1968-01-04	...
	Jennifer	Wallace	987654321	123456789	Alice	F	1969-12-30	...
	Jennifer	Wallace	987654321	123456789	Elisabeth	F	1967-05-05	...
	Joyce	English	453453453	333445555	Alice	F	1969-04-05	...
	Joyce	English	453453453	333445555	Theodora	M	1983-10-25	...
	Joyce	English	453453453	333445555	Joy	F	1959-05-03	...
	Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
	Joyce	English	453453453	123456789	Michael	M	1968-01-04	...
	Joyce	English	453453453	123456789	Alice	F	1969-12-30	...
	Joyce	English	453453453	123456789	Elisabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS	FNAME	LNAME	SSN	ESSN	DEPENDENT_NAME	SEX	BOATE
	Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28

RESULT	FNAME	LNAME	DEPENDENT_NAME
	Jennifer	Wallace	Abner

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