Chapter 3

Decision Analysis

To accompany
Quantitative Analysis for Management*, *Eleventh Edition*, *Global Edition
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. **List the steps of the decision-making process.**
2. **Describe the types of decision-making environments.**
3. **Make decisions under uncertainty.**
4. **Use probability values to make decisions under risk.**

Introduction

- What is involved in making a good decision?
- Decision theory is an analytic and systematic approach to the study of decision making.
- A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here.

The Six Steps in Decision Making

1. Clearly define the problem at hand.
2. List the possible alternatives.
3. Identify the possible outcomes or states of nature.
4. List the payoff (typically profit) of each combination of alternatives and outcomes.
5. Select one of the mathematical decision theory models.
6. Apply the model and make your decision.

Thompson Lumber Company

Step 1 – Define the problem.

- The company is considering expanding by manufacturing and marketing a new product – backyard storage sheds.

Step 2 – List alternatives.

- Construct a large new plant.
- Construct a small new plant.
- Do not develop the new product line at all.

Step 3 – Identify possible outcomes.

- The market could be favorable or unfavorable.

Thompson Lumber Company

Step 4 – List the payoffs.

- Identify ***conditional values*** for the profits for large plant, small plant, and no development for the two possible market conditions.

Step 5 – Select the decision model.

- This depends on the environment and amount of risk and uncertainty.

Step 6 – Apply the model to the data.

- Solution and analysis are then used to aid in decision-making.

Thompson Lumber Company

Decision Table with Conditional Values for Thompson Lumber

ALTERNATIVE	STATE OF NATURE FAVORABLE MARKET ($)	 UNFAVORABLE MARKET ($)
Construct a large plant	200,000	–180,000
Construct a small plant	100,000	–20,000
Do nothing	0	0

Table 3.1

Types of Decision-Making Environments

Type 1: Decision making under certainty

- The decision maker ***knows with certainty*** the consequences of every alternative or decision choice.

Type 2: Decision making under uncertainty

- The decision maker ***does not know*** the probabilities of the various outcomes.

Type 3: Decision making under risk

- The decision maker ***knows the probabilities*** of the various outcomes.

Decision Making Under Uncertainty

There are several criteria for making decisions under uncertainty:

1. **Maximax (optimistic)**
2. **Maximin (pessimistic)**
3. **Criterion of realism (Hurwicz)**
4. **Equally likely (Laplace)**
5. **Minimax regret**

Maximax

Used to find the alternative that maximizes the maximum payoff.

- Locate the maximum payoff for each alternative.
- Select the alternative with the maximum number.

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	200,000	–180,000	200,000 Maximax
Construct a small plant	100,000	–20,000	100,000
Do nothing	0	0	0

Table 3.2

Maximin

Used to find the alternative that maximizes the minimum payoff.

- **Locate the minimum payoff for each alternative.**
- **Select the alternative with the maximum number.**

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MINIMUM IN A ROW ($)
Construct a large plant	200,000	–180,000	–180,000
Construct a small plant	100,000	–20,000	–20,000
Do nothing	0	0	0

Table 3.3

Maximin

Criterion of Realism (Hurwicz)

This is a *weighted average* compromise between optimism and pessimism.

- **Select a coefficient of realism α, with $0 \leq \alpha \leq 1$.**
- **A value of 1 is perfectly optimistic, while a value of 0 is perfectly pessimistic.**
- **Compute the weighted averages for each alternative.**
- **Select the alternative with the highest value.**

$$\text{Weighted average} = \alpha(\text{maximum in row}) + (1 - \alpha)(\text{minimum in row})$$

Criterion of Realism (Hurwicz)

- **For the large plant alternative using $\alpha = 0.8$:**
  $(0.8)(200{,}000) + (1 – 0.8)(–180{,}000) = 124{,}000$
- **For the small plant alternative using $\alpha = 0.8$:**
  $(0.8)(100{,}000) + (1 – 0.8)(–20{,}000) = 76{,}000$

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	CRITERION OF REALISM (α = 0.8) $
Construct a large plant	200,000	–180,000	124,000 Realism
Construct a small plant	100,000	–20,000	76,000
Do nothing	0	0	0

Table 3.4

Equally Likely (Laplace)

Considers all the payoffs for each alternative

- Find the average payoff for each alternative.
- Select the alternative with the highest average.

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	ROW AVERAGE ($)
Construct a large plant	200,000	–180,000	10,000
Construct a small plant	100,000	–20,000	40,000
Do nothing	0	0	0

Equally likely

Table 3.5

Minimax Regret

Based on ***opportunity loss*** or ***regret***, this is the difference between the optimal profit and actual payoff for a decision.

- Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative.
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column.
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number.

Minimax Regret

- **It’s the price we pay for not knowing the right state of nature, so we avoid the worst scenario**

Minimax Regret

Determining Opportunity Losses for Thompson Lumber

STATE OF NATURE	
FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
200,000 – 200,000	0 – (–180,000)
200,000 – 100,000	0 – (–20,000)
200,000 – 0	0 – 0

Table 3.6

Minimax Regret

Opportunity Loss Table for Thompson Lumber

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Table 3.7

Minimax Regret

Thompson's Minimax Decision Using Opportunity Loss

ALTERNATIVE	STATE OF NATURE		
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
Do nothing	200,000	0	200,000

Minimax

Table 3.8

Decision Making Under Risk

- This is decision making when there are several possible states of nature, and the probabilities associated with each possible state are known.
- The most popular method is to choose the alternative with the highest ***expected monetary value (EMV).***

EMV (alternative *i*) = (payoff of first state of nature)
x (probability of first state of nature)
+ (payoff of second state of nature)
x (probability of second state of nature)
+ … + (payoff of last state of nature)
x (probability of last state of nature)

EMV for Thompson Lumber

- Suppose each market outcome has a probability of occurrence of 0.50.
- Which alternative would give the highest EMV?
- The calculations are:

$$\begin{aligned}
\text{EMV (large plant)} &= (\$200{,}000)(0.5) + (-\$180{,}000)(0.5) \\
&= \$10{,}000 \\
\text{EMV (small plant)} &= (\$100{,}000)(0.5) + (-\$20{,}000)(0.5) \\
&= \$40{,}000 \\
\text{EMV (do nothing)} &= (\$0)(0.5) + (\$0)(0.5) \\
&= \$0
\end{aligned}$$

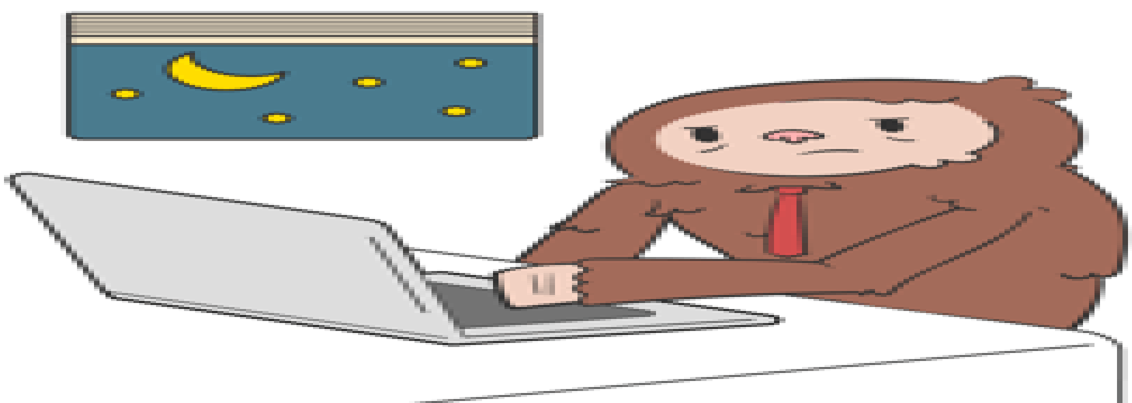
Example

Maria Rojas is considering the possibility of opening a small dress shop on Fairbanks Avenue, a few blocks from the university. She has located a good mall that attracts students. Her options are to open a small shop, a medium-sized shop, or no shop at all. The market for a dress shop can be good, average, or bad. The probabilities for these three possibilities are 0.2 for a good market, 0.5 for an average market, and 0.3 for a bad market. The net profit or loss for the medium-sized and small shops for the various market conditions are given in the following table. Building no shop at all yields no loss and no gain.

a. What do you recommend?

[redacted]

c. Develop the opportunity loss table for this situation. What decisions would be made using the minimax regret criterion and the minimum EOL criterion?



Example

Maria Rojas is considering the possibility of opening a small dress shop on Fairbanks Avenue, a few blocks from the university. She has located a good mall that attracts students. Her options are to open a small shop, a medium-sized shop, or no shop at all. The market for a dress shop can be good, average, or bad. The probabilities for these three possibilities are 0.2 for a good market, 0.5 for an average market, and 0.3 for a bad market. The net profit or loss for the medium-sized and small shops for the various market conditions are given in the following table. Building no shop at all yields no loss and no gain.

a. What do you recommend?
b. Calculate the EVPI.
c. Develop the opportunity loss table for this situation. What decisions would be made using the minimax regret criterion and the minimum EOL criterion?

ALTERNATIVE	GOOD MARKET ($)	AVERAGE MARKET ($)	BAD MARKET ($)
Small shop	75,000	25,000	–40,000
Medium-sized shop	100,000	35,000	–60,000
No shop	0	0	0

Solution

a. Since the decision-making environment is risk (probabilities are known), it is appropriate to use the EMV criterion. The problem can be solved by developing a payoff table that contains all alternatives, states of nature, and probability values. The EMV for each alternative is also computed, as in the following table:

ALTERNATIVE	STATE OF NATURE GOOD MARKET ($)	AVERAGE MARKET ($)	BAD MARKET ($)	EMV ($)
Small shop	75,000	25,000	–40,000	15,500
Medium-sized shop	100,000	35,000	–60,000	19,500
No shop	0	0	0	0
Probabilities	0.20	0.50	0.30	

$$\text{EMV(small shop)} = (0.2)(\$75{,}000) + (0.5)(\$25{,}000) + (0.3)(-\$40{,}000) = \$15{,}500$$
$$\text{EMV(medium shop)} = (0.2)(\$100{,}000) + (0.5)(\$35{,}000) + (0.3)(-\$60{,}000) = \$19{,}500$$
$$\text{EMV(no shop)} = (0.2)(\$0) + (0.5)(\$0) + (0.3)(\$0) = \$0$$

ALTERNATIVE	STATE OF NATURE GOOD MARKET ($)	AVERAGE MARKET ($)	BAD MARKET ($)	MAXIMUM ($)	EOL ($)
Small shop	25,000	10,000	40,000	40,000	22,000
Medium-sized shop	0	0	60,000	60,000	18,000
No shop	100,000	35,000	0	100,000	37,500
Probabilities	0.20	0.50	0.30		

The best payoff in a good market is 100,000, so the opportunity losses in the first column indicate how much worse each payoff is than 100,000. The best payoff in an average market is 35,000, so the opportunity losses in the second column indicate how much worse each payoff is than 35,000. The best payoff in a bad market is 0, so the opportunity losses in the third column indicate how much worse each payoff is than 0.

The minimax regret criterion considers the maximum regret for each decision, and the decision corresponding to the minimum of these is selected. The decision would be to build a small shop since the maximum regret for this is 40,000, while the maximum regret for each of the other two alternatives is higher as shown in the opportunity loss table.

The decision based on the EOL criterion would be to build the medium shop. Note that the minimum EOL ($18,000) is the same as the EVPI computed in part *b*. The calculations are

$$\text{EOL(small)} = (0.2)25{,}000 + (0.5)10{,}000 + (0.3)40{,}000 = 22{,}000$$

$$\text{EOL(medium)} = (0.2)0 + (0.5)0 + (0.3)60{,}000 = 18{,}000$$

$$\text{EOL(no shop)} = (0.2)100{,}000 + (0.5)35{,}000 + (0.3)0 = 37{,}500$$