

Decision Trees

Lecture 4



Decision Trees



- Any problem that can be presented in a decision table can also be graphically represented in a *decision tree*.
- Decision trees are most beneficial when a sequence of decisions must be made.
- All decision trees contain *decision points* or *nodes*, from which one of several alternatives may be chosen.
- All decision trees contain *state-of-nature points* or *nodes*, out of which one state of nature will occur.

Five Steps of Decision Tree Analysis

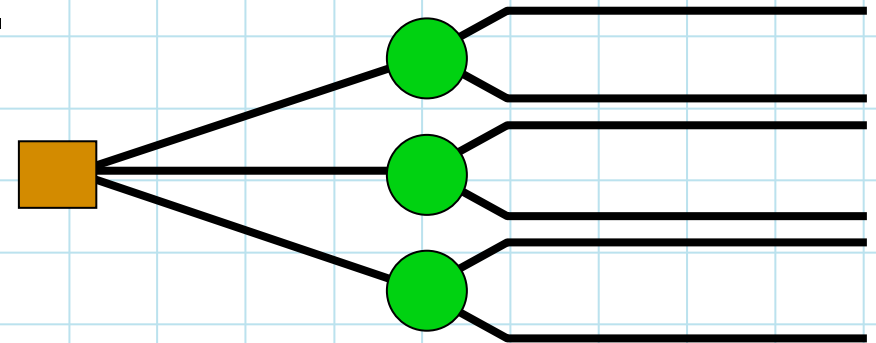


- 1. Define the problem.**
- 2. Structure or draw the decision tree.**
- 3. Assign probabilities to the states of nature.**
- 4. Estimate payoffs for each possible combination of alternatives and states of nature.**
- 5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node.**

Structure of Decision Trees



- Trees start from left to right.
- Trees represent decisions and outcomes in sequential order.
 - Squares represent decision nodes.
 - Circles represent states of nature nodes.
 - Lines or branches connect the decisions nodes and the states of nature.



Thompson's Decision Tree

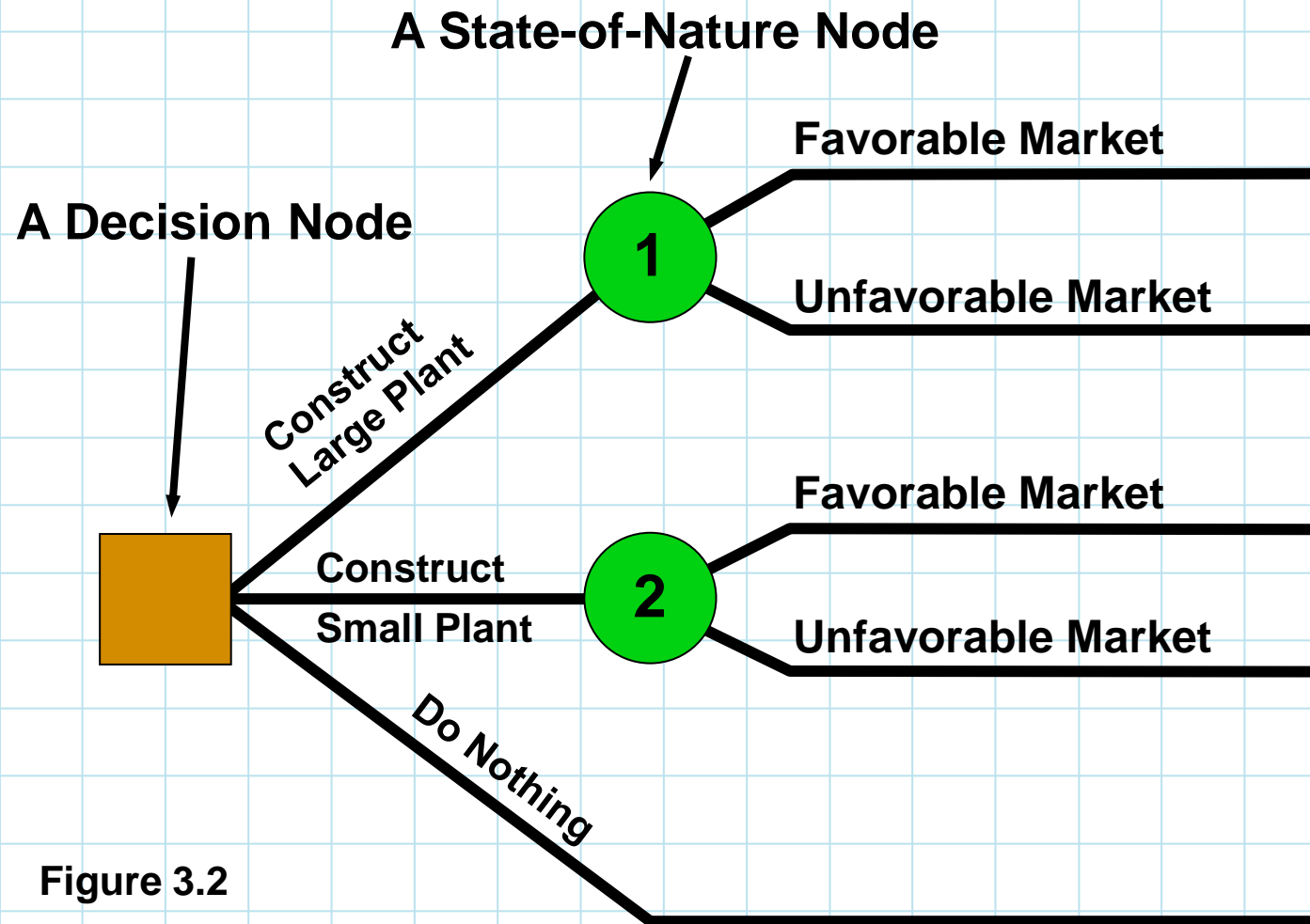


Figure 3.2

Thompson's Decision Tree

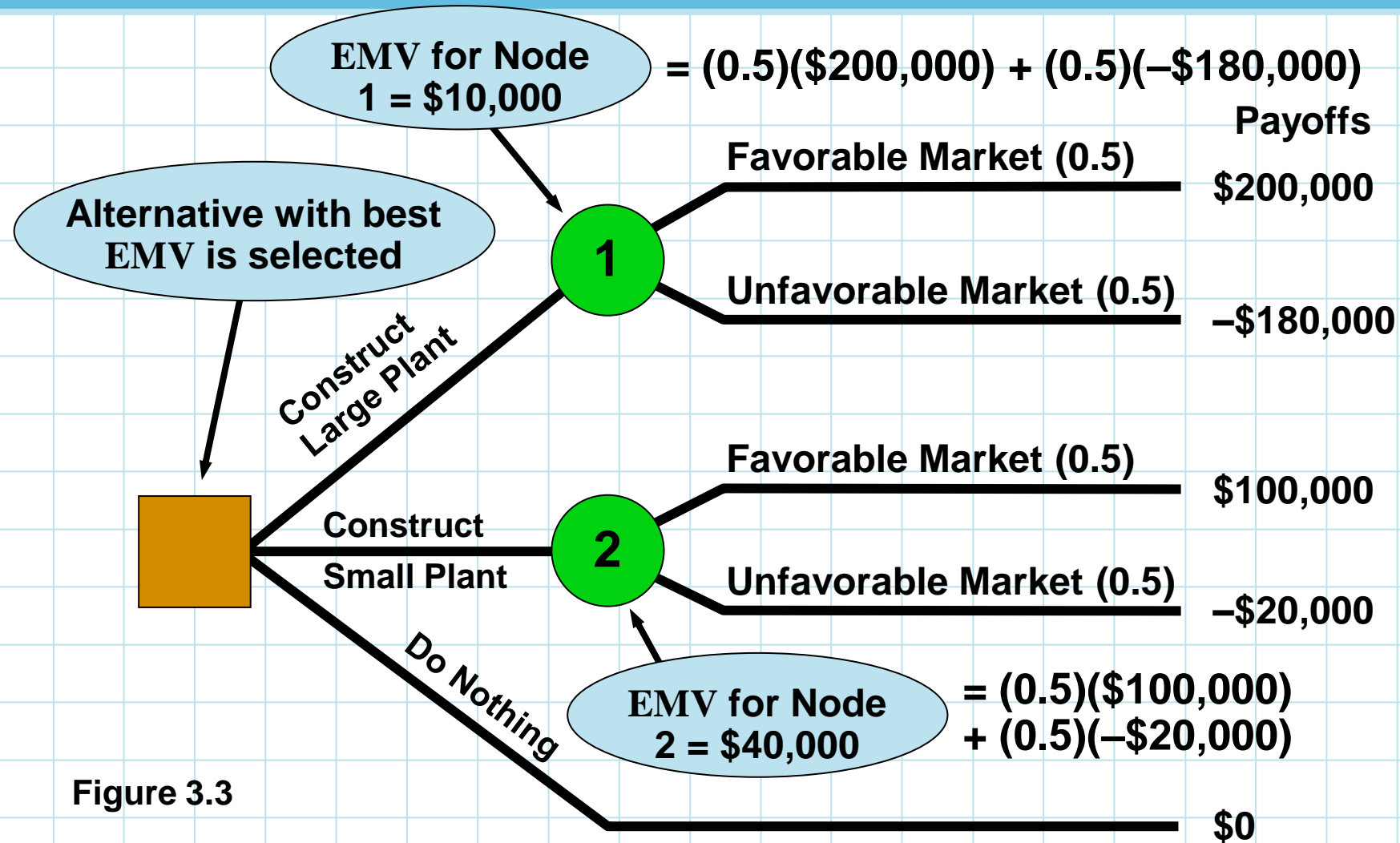


Figure 3.3

Complex problems

- **When sequential decisions need to be made, decision trees are much more powerful tools than decision tables.**
- **For example: when a second decision dependent on the outcome of the first.**

Thompson's Complex Decision Tree

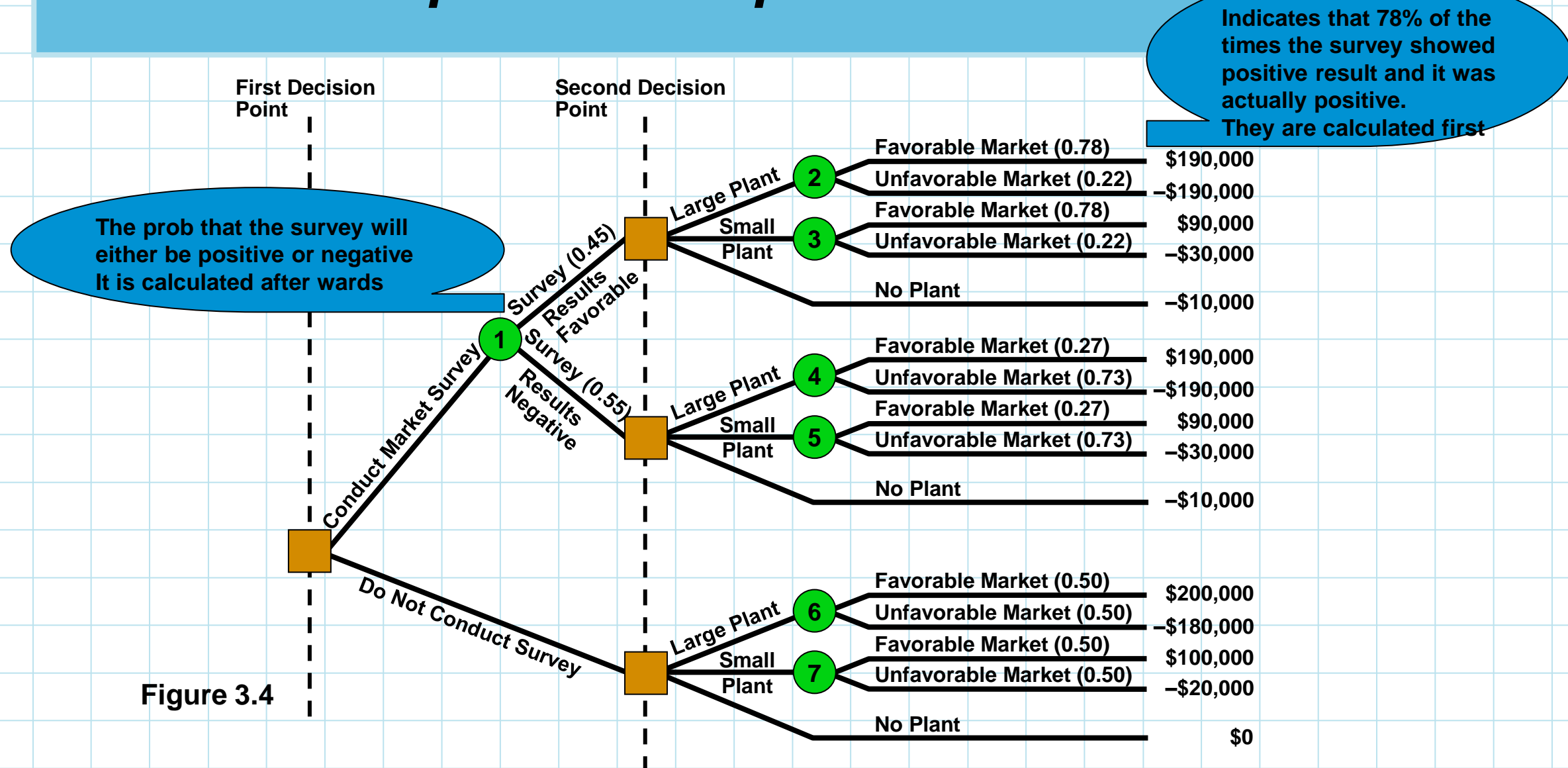


Figure 3.4

Indicates that 78% of the times the survey showed positive result and it was actually positive. They are calculated first

Reading only

How are these probabilities calculated

There are many ways of getting probability data for a problem such as Thompson's. The numbers (such as 0.78, 0.22, 0.27, 0.73 in Figure 3.4) can be assessed by a manager based on experience and intuition.

They can be derived from historical data, or they can be computed from other available data using Bayes' theorem.

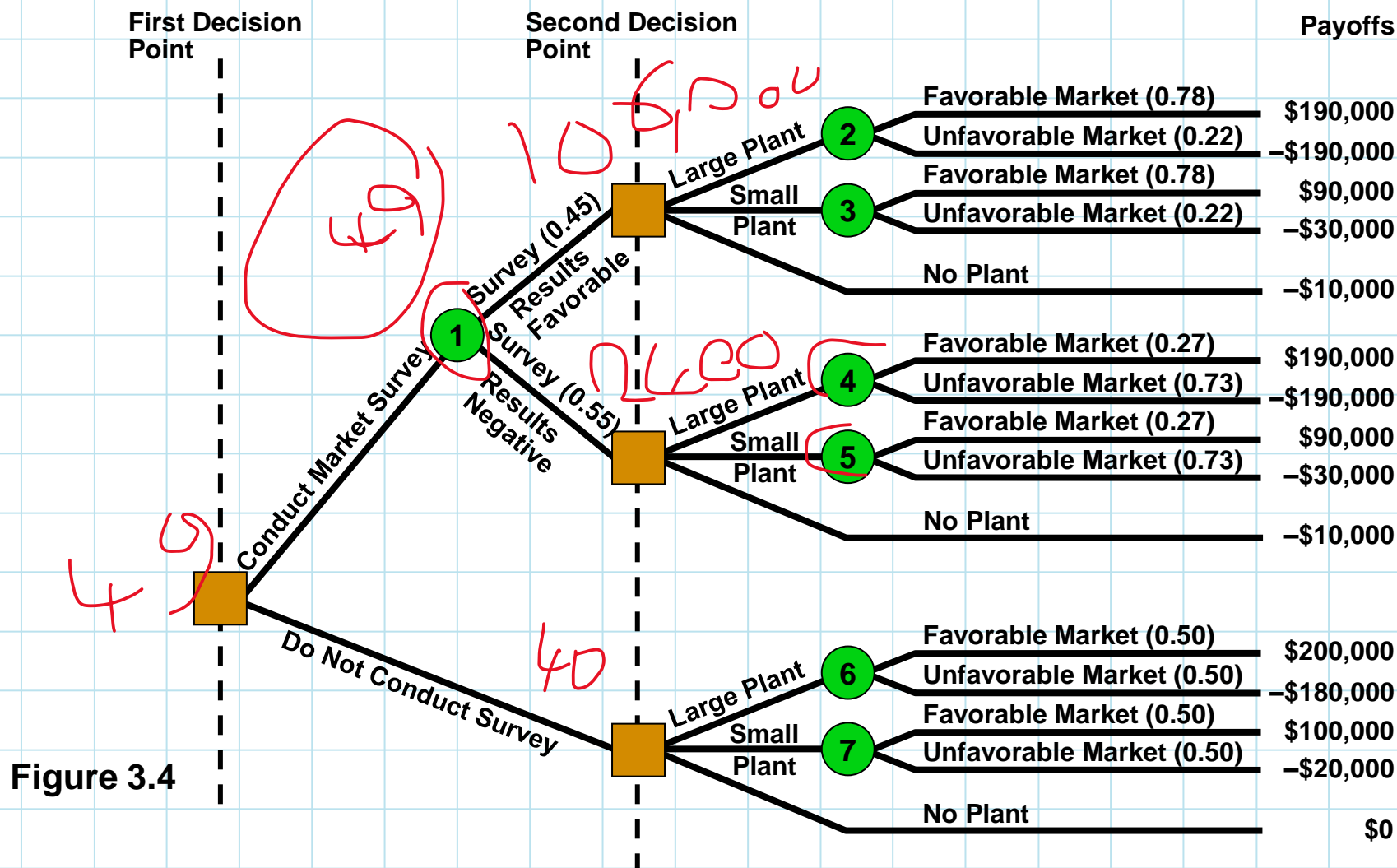
The advantage of Bayes' theorem is that it incorporates both **our initial estimates of the probabilities** as well as information about the **accuracy of the information source** (e.g., market research survey).

Reading Only

- From discussions with market research specialists at the local university, John knows that special surveys such as his can either be **positive** (i.e., predict a favorable market) or be **negative** (i.e., predict an unfavorable market).
- The experts have told John that, **statistically**, of all new products with a favorable market (FM), market surveys were positive and predicted success correctly 70% of the time. Thirty percent of the time the surveys falsely predicted negative results or an unfavorable market (UM).
- On the other hand, when there was actually an unfavorable market for a new product, 80% of the surveys correctly predicted negative results. The surveys incorrectly predicted positive results the remaining 20% of the time.

RESULT OF SURVEY	STATE OF NATURE	
	FAVORABLE MARKET (FM)	UNFAVORABLE MARKET (UM)
Positive (predicts favorable market for product)	$P(\text{survey positive} \mid \text{FM}) = 0.70$	$P(\text{survey positive} \mid \text{UM}) = 0.20$
Negative (predicts unfavorable market for product)	$P(\text{survey negative} \mid \text{FM}) = 0.30$	$P(\text{survey negative} \mid \text{UM}) = 0.80$

Thompson's Complex Decision Tree



Thompson's Complex Decision Tree



1. Given favorable survey results,



$$\begin{aligned}\text{EMV}(\text{node 2}) &= \text{EMV}(\text{large plant} \mid \text{positive survey}) \\ &= (0.78)(\$190,000) + (0.22)(-\$190,000) = \$106,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 3}) &= \text{EMV}(\text{small plant} \mid \text{positive survey}) \\ &= (0.78)(\$90,000) + (0.22)(-\$30,000) = \$63,600\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

2. Given negative survey results,

$$\begin{aligned}\text{EMV}(\text{node 4}) &= \text{EMV}(\text{large plant} \mid \text{negative survey}) \\ &= (0.27)(\$190,000) + (0.73)(-\$190,000) = \underline{-\$87,400}\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 5}) &= \text{EMV}(\text{small plant} \mid \text{negative survey}) \\ &= (0.27)(\$90,000) + (0.73)(-\$30,000) = \underline{\$2,400}\end{aligned}$$

$$\text{EMV for no plant} = \underline{-\$10,000}$$

Thompson's Complex Decision Tree



3. Compute the expected value of the market survey,

$$\begin{aligned}\text{EMV}(\text{node 1}) &= \text{EMV}(\text{conduct survey}) \\ &= (0.45)(\$106,400) + (0.55)(\$2,400) \\ &= \$47,880 + \$1,320 = \underline{\$49,200}\end{aligned}$$

4. If the market survey is not conducted,

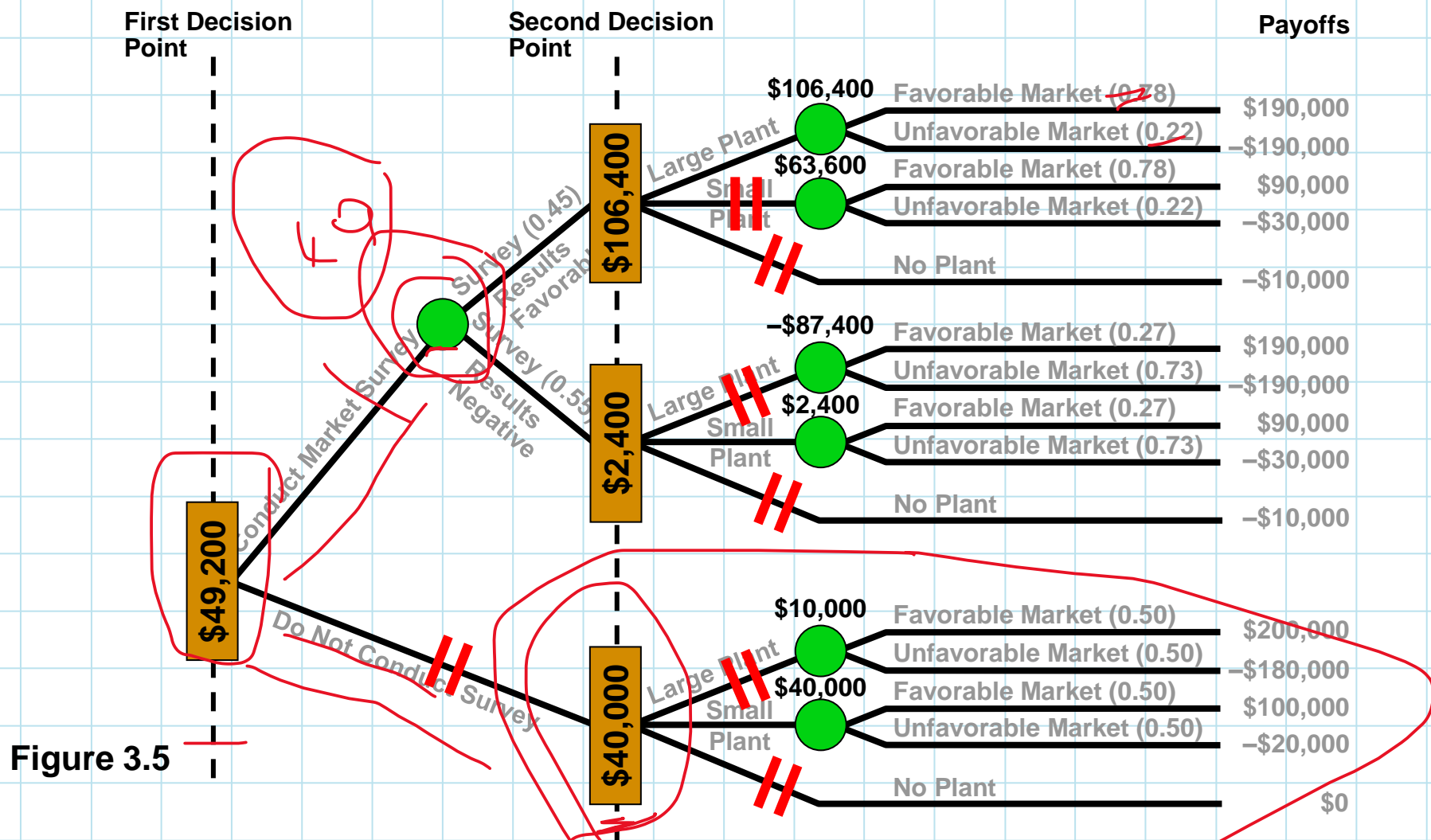
$$\begin{aligned}\text{EMV}(\text{node 6}) &= \text{EMV}(\text{large plant}) \\ &= (0.50)(\$200,000) + (0.50)(-\$180,000) = \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 7}) &= \text{EMV}(\text{small plant}) \\ &= (0.50)(\$100,000) + (0.50)(-\$20,000) = \underline{\$40,000}\end{aligned}$$

$$\text{EMV for no plant} = \$0$$

5. The best choice is to seek marketing information.

Thompson's Complex Decision Tree



Expected Value of Sample Information

$$\text{EVSI} = (\text{EV with SI} + \text{cost}) - (\text{EV without SI})$$

where

EVSI = expected value of sample information

EV with SI = expected value with sample information

EV without SI = expected value without sample information

which is the increase in expected value resulting from the sample information

$$\text{EVSI} = (\$49,200 + \$10,000) - \$40,000 = \$59,200 - \$40,000 = \$19,200$$

Efficiency of Sample Information

There may be many types of sample information available to a decision maker. In developing a new product, information could be obtained from a survey, from a focus group, from other market research techniques, or from actually using a test market to see how sales will be. While none of these sources of information are perfect, they can be evaluated by comparing the EVSI with the EVPI.

If the sample information was perfect, then the efficiency would be 100%. The efficiency of sample information is

efficiency of sample information



$$\text{Efficiency of sample information} = \frac{\text{EVSI}}{\text{EVPI}} 100\%$$

In the Thompson Lumber example,

$$\text{Efficiency of sample information} = \frac{19,200}{60,000} 100\% = \underline{32\%}$$

Thus, the market survey is only 32% as efficient as perfect information.