Chapter 10

Transportation and Assignment Models

To accompany
Quantitative Analysis for Management, *Tenth Edition*,
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Power Point slides created by Jeff Heyl

Unbalanced Transportation Problems

- In real-life problems, total demand is frequently not equal to total supply
- These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations*
- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply

Unbalanced Transportation Problems

- In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped
- Any units assigned to a dummy destination represent excess capacity
- Any units assigned to a dummy source represent unmet demand

Demand Less Than Supply

- Suppose that the Cairo factory increases its rate of production from 100 to 250 desks
- The firm is now able to supply a total of 850 desks each period
- Warehouse requirements remain the same (700) so the row and column totals do not balance
- We add a dummy column that will represent a fake warehouse requiring 150 desks
- This is somewhat analogous to adding a slack variable
- We use the northwest corner rule and either stepping-stone or MODI to find the optimal solution

Demand Less Than Supply

- Initial solution to an unbalanced problem where demand is less than supply

FROM \ TO	A	B	C	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	250 / $5	$4	$3	0	250
E	50 / $8	200 / $4	50 / $3	0	300
F	$9	$7	150 / $5	150 / 0	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Table 10.16

New Cairo capacity

Demand Greater than Supply

- **The second type of unbalanced condition occurs when total demand is greater than total supply**
- **In this case we need to add a dummy row representing a fake factory**
- **The new factory will have a supply exactly equal to the difference between total demand and total real supply**
- **The shipping costs from the dummy factory to each destination will be zero**

Demand Greater than Supply

- **Unbalanced transportation table for Happy Sound Stereo Company**

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	\$6	\$4	\$9	200
PLANT *X*	\$10	\$5	\$8	175
PLANT *Y*	\$12	\$7	\$6	75
WAREHOUSE DEMAND	250	100	150	500 \ 450

Totals do not balance

Table 10.17

Demand Greater than Supply

- **Initial solution to an unbalanced problem in which demand is greater than supply**

TO / FROM	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 | $6	| $4	| $9	200
PLANT *X*	50 | $10	100 | $5	25 | $8	175
PLANT *Y*	| $12	| $7	75 | $6	75
PLANT *Y*	| 0	| 0	50 | 0	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

Table 10.18

Degeneracy in Transportation Problems

- *Degeneracy* occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1
- Such a situation may arise in the initial solution or in any subsequent solution
- Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method or to calculate the *R* and *K* values needed for the MODI technique

Degeneracy in Transportation Problems

- To handle degenerate problems, create an artificially occupied cell
- That is, place a zero (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied
- The square chosen must be in such a position as to allow all stepping-stone paths to be closed
- There is usually a good deal of flexibility in selecting the unused square that will receive the zero

Degeneracy in an Initial Solution

- The Martin Shipping Company example illustrates degeneracy in an initial solution
- They have three warehouses which supply three major retail customers
- Applying the northwest corner rule the initial solution has only four occupied squares
- This is less than the amount required to use either the stepping-stone or MODI method to improve the solution (3 rows + 3 columns – 1 = 5)
- To correct this problem, place a zero in an unused square, typically one adjacent to the last filled cell

Degeneracy in an Initial Solution

- Initial solution of a degenerate problem

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	100 ($8)	($2)	($6)	100
WAREHOUSE 2	($10)	100 ($9)	20 ($9)	120
WAREHOUSE 3	($7)	($10)	80 ($7)	80
CUSTOMER DEMAND	100	100	100	300

Table 10.19

Possible choices of cells to address the degenerate solution

Degeneracy in an Initial Solution

- Initial solution of a degenerate problem

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	100 ($8)	0 ($2)	($6)	100
WAREHOUSE 2	0 ($10)	100 ($9)	20 ($9)	120
WAREHOUSE 3	($7)	($10)	80 ($7)	80
CUSTOMER DEMAND	100	100	100	300

Table 10.19

Possible choices of cells to address the degenerate solution

Degeneracy During Later Solution Stages

- A transportation problem can become degenerate after the initial solution stage if the filling of an empty square results in two or more cells becoming empty simultaneously
- This problem can occur when two or more cells with minus signs tie for the lowest quantity
- To correct this problem, place a zero in one of the previously filled cells so that only one cell becomes empty

Degeneracy During Later Solution Stages

- Bagwell Paint transportation table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	$8	$5	$16	70
FACTORY *B*	$15	$10	$7	130
FACTORY *C*	$3	$9	$10	80
WAREHOUSE REQUIREMENT	150	80	50	280

Table 10.20

Degeneracy During Later Solution Stages

- Bagwell Paint transportation table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	70 ($8)	($5)	($16)	70
FACTORY *B*	50 ($15)	80 ($10)	($7)	130
FACTORY *C*	30 ($3)	($9)	50 ($10)	80
WAREHOUSE REQUIREMENT	150	80	50	280

Table 10.20

Degeneracy During Later Solution Stages

- Bagwell Paint Example
  - After one iteration, the cost analysis at Bagwell Paint produced a transportation table that was not degenerate but was not optimal
  - The improvement indices are

factory *A* – warehouse 2 index = +2
factory *A* – warehouse 3 index = +1
factory *B* – warehouse 3 index = –15
factory *C* – warehouse 2 index = +11

Only route with a negative index

Degeneracy During Later Solution Stages

- Tracing a closed path for the factory B – warehouse 3 route

FROM \ TO	WAREHOUSE 1	WAREHOUSE 3
FACTORY *B*	50 ($15)	($7)
FACTORY *C*	30 ($3)	50 ($10)

Table 10.21

- **This would cause two cells to drop to zero**
- **We need to place an artificial zero in one of these cells to avoid degeneracy**

More Than One Optimal Solution

- It is possible for a transportation problem to have multiple optimal solutions
- This happens when one or more of the improvement indices zero in the optimal solution
- This means that it is possible to design alternative shipping routes with the same total shipping cost
- The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources

Example:

TO / FROM	PROJECT *A*	PROJECT *B*	PROJECT *C*	PLANT CAPACITIES
PLANT 1	\$10	\$4	\$11	70
PLANT 2	\$12	\$5	\$8	50
PLANT 3	\$9	\$7	\$6	30
PROJECT REQUIREMENTS	40	50	60	150

Initial solution

TO / FROM	PROJECT A	PROJECT B	PROJECT C	PLANT CAPACITIES
PLANT 1	\$10 40	\$4 30	\$11	70
PLANT 2	\$12	\$5 20	\$8 30	50
PLANT 3	\$9	\$7	\$6 30	30
PROJECT REQUIREMENTS	40	50	60	150