Chapter 14

Simulation Modeling

Introduction

- ***Simulation*** **is one of the most widely used quantitative analysis tools.**
- **To *simulate* is to try to duplicate the features, appearance, and characteristics of a real system.**
- **We will build a *mathematical model* that comes as close as possible to representing the reality of the system.**
- ***Physical*** **models can also be built to test systems.**

Definition

Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of either understanding the behavior of the system and/or evaluating various strategies for the operation of the system.

Process of Simulation

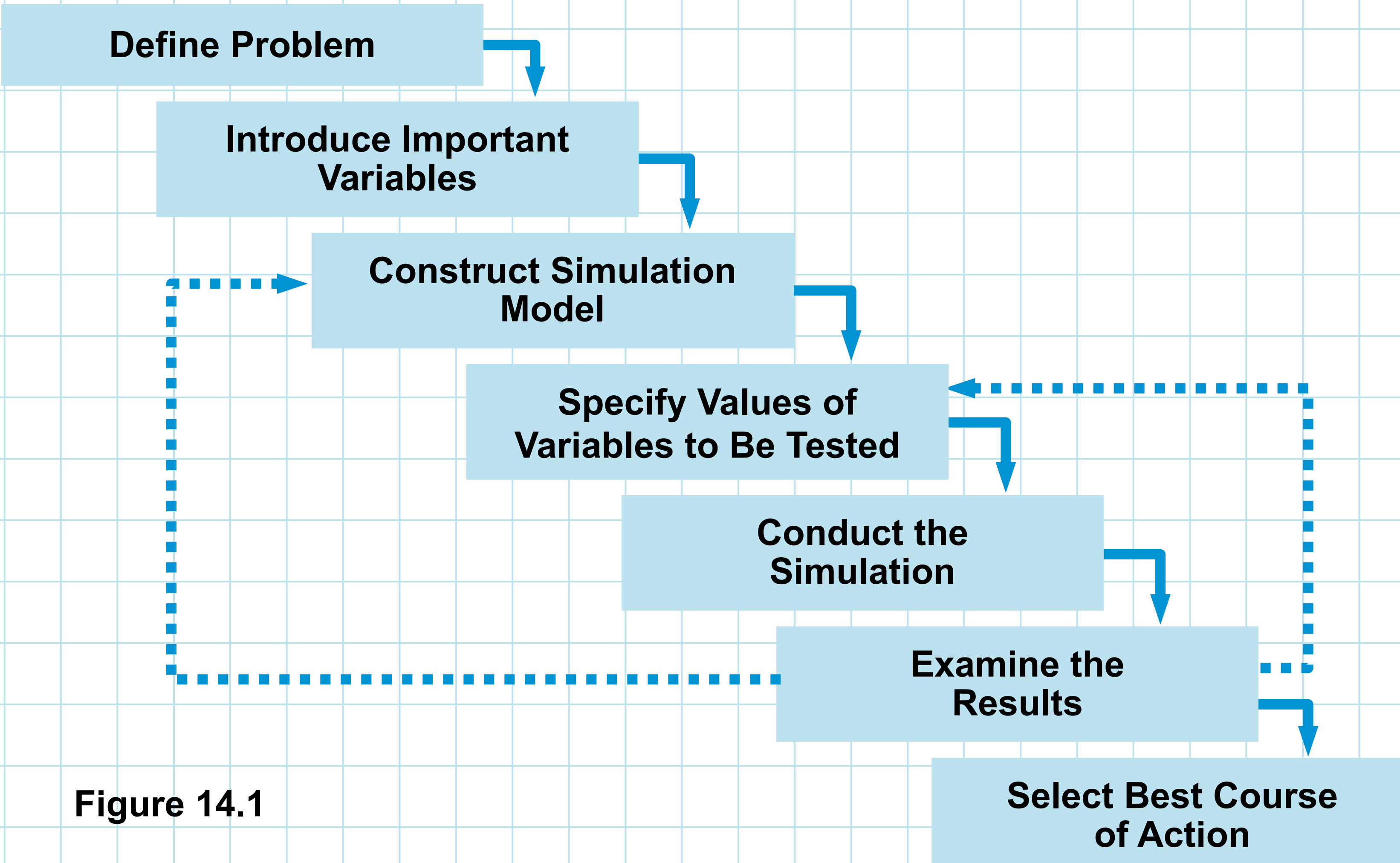

Figure 14.1

Advantages and Disadvantages of Simulation

The main advantages of simulation are:

1. It is relatively straightforward and flexible.
2. Recent advances in computer software make simulation models very easy to develop.
3. Can be used to analyze large and complex real-world situations.
4. Allows “what-if?” type questions.
5. Does not interfere with the real-world system.
6. Enables study of interactions between components.
7. Enables time compression.
8. Enables the inclusion of real-world complications.

Advantages and Disadvantages of Simulation

The main disadvantages of simulation are:

1. It is often expensive as it may require a long, complicated process to develop the model.
2. It does not generate optimal solutions; it is a trial-and-error approach.
3. It requires managers to generate all conditions and constraints of real-world problem.
4. Each model is unique and the solutions and inferences are not usually transferable to other problems.

Applications

- *COMPUTER SYSTEMS:* hardware components, software systems, networks, data base management, information processing, etc..
- *MANUFACTURING:* material handling systems, assembly lines, automated production facilities, inventory control systems, plant layout, etc..
- *BUSINESS:* stock and commodity analysis, pricing policies, marketing strategies, cash flow analysis, forecasting, etc..
- *GOVERNMENT:* military weapons and their use, military tactics, population forecasting, land use, health care delivery, fire protection, criminal justice, traffic control, etc..

Example of sim in Disney world

- *Cruise Line Operation:* Simulate the arrival and check-in process at the dock.
- *Private Island Arrival:* How to transport passengers to the beach area? Drop-off point far from the beach. Used simulation to determine whether to invest in trams, how many trams to purchase, average transport and waiting times, etc..

Monte Carlo Simulation

- **When systems contain elements that exhibit chance in their behavior, the Monte Carlo method of simulation can be applied.**
- **The basic idea in Monte Carlo simulation is to generate values for the variables making up the model being studied.**

Brief history

- World War II

  - “*Monte Carlo*” simulation: originated with the work on the atomic bomb. Used to simulate bombing raids. Given the security code name “Monte-Carlo”.

  - Still widely used today for certain problems which are not analytically solvable (for example: complex multiple integrals…)

Examples of probabilistic vars

- There are a lot of variables in real-world systems that are probabilistic in nature and that we might want to simulate. A few examples of these variables follow:
- Some examples are:
  1. Inventory demand.
  2. Lead time for inventory.
  3. Times between machine breakdowns.
  4. Times between arrivals.
  5. Service times.
  6. Times to complete project activities.
  7. Number of employees absent.

Monte Carlo Simulation

- The basis of the Monte Carlo simulation is experimentation on the probabilistic elements through random sampling.
- It is based on the following five steps:
  1. Establishing a probability distribution for important variables.
  2. Building a cumulative probability distribution for each variable.
  3. Establishing an interval of random numbers for each variable.
  4. Generating random numbers.
  5. Actually simulating a series of trials.

Harry's Auto Tire

- A popular radial tire accounts for a large portion of the sales at Harry's Auto Tire.
- Harry wishes to determine a policy for managing this inventory.
- He wants to simulate the daily demand for a number of days.

Step 1: Establishing probability distributions

- One way to establish a probability distribution for a given variable is to examine historical outcomes.
- Managerial estimates based on judgment and experience can also be used.

Harry's Auto Tire

Historical Daily Demand for Radial Tires at Harry's Auto Tire and Probability Distribution

DEMAND FOR TIRES	FREQUENCY (DAYS)	PROBAILITY OF OCCURRENCE
0	10	10/200 = 0.05
1	20	20/200 = 0.10
2	40	40/200 = 0.20
3	60	60/200 = 0.30
4	40	40/200 = 0.20
5	30	30/200 = 0.15
	200	200/200 = 1.00

Table 14.1

Harry's Auto Tire

Step 2: Building a cumulative probability distribution for each variable

- **Converting from a regular probability to a cumulative distribution is an easy job.**
- **A cumulative probability is the probability that a variable will be less than or equal to a particular value.**
- **A cumulative distribution lists all of the possible values and the probabilities, as shown in Table 14.2.**

Harry's Auto Tire

Cumulative Probabilities for Radial Tires

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY
0	0.05	0.05
1	0.10	0.15
2	0.20	0.35
3	0.30	0.65
4	0.20	0.85
5	0.15	1.00

Table 14.2

Harry's Auto Tire

Step 3: Setting random number intervals

- **Assign a set of numbers to represent each possible value or outcome.**
  - **These are *random number intervals.***
  - **A *random number* is a series of digits that have been selected by a totally random process.**
  - **The range of the random number intervals corresponds *exactly* to the probability of the outcomes as shown in Figure 14.2.**

Harry's Auto Tire

Graphical Representation of the Cumulative Probability Distribution for Radial Tires

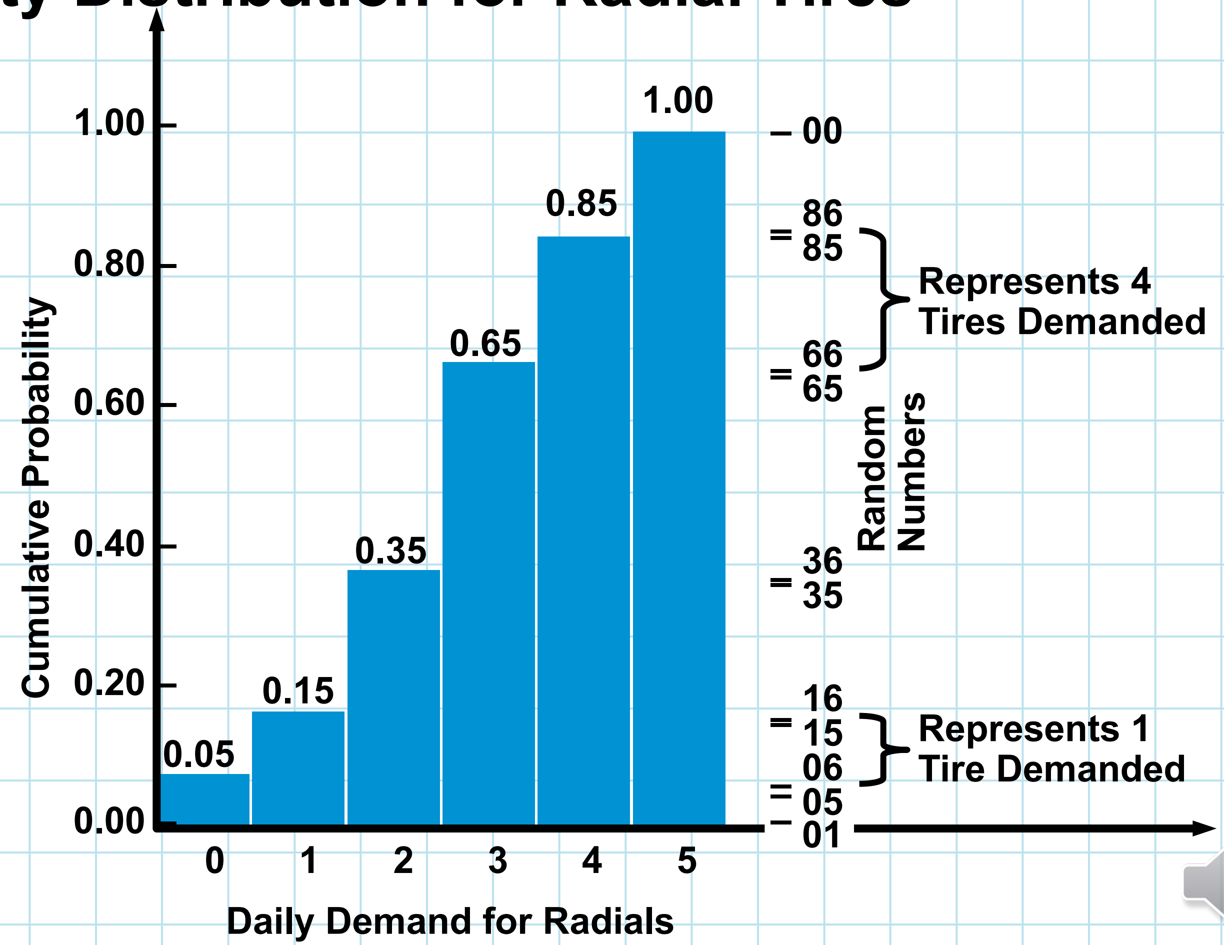

Figure 14.2

Harry's Auto Tire

Assignment of Random Number Intervals for Harry's Auto Tire

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY	INTERVAL OF RANDOM NUMBERS
0	0.05	0.05	01 to 05
1	0.10	0.15	06 to 15
2	0.20	0.35	16 to 35
3	0.30	0.65	36 to 65
4	0.20	0.85	66 to 85
5	0.15	1.00	86 to 00

Table 14.3

Harry's Auto Tire

Step 4: Generating random numbers

- Random numbers can be generated in several ways.
- Large problems will use computer program to generate the needed random numbers.
- For small problems, random processes like roulette wheels or pulling chips from a hat may be used.
- The most common manual method is to use a random number table.
- Because *everything* is random in a random number table, we can select numbers from anywhere in the table to use in the simulation.

Harry's Auto Tire

Table of random numbers (partial)

52	06	50	88	53	30	10	47	99	37
37	63	28	02	74	35	24	03	29	60
82	57	68	28	05	94	03	11	27	79
69	02	36	49	71	99	32	10	75	21
98	94	90	36	06	78	23	67	89	85
96	52	62	87	49	56	59	23	78	71
33	69	27	21	11	60	95	89	68	48
50	33	50	95	13	44	34	62	64	39
88	32	18	50	62	57	34	56	62	31
90	30	36	24	69	82	51	74	30	35

Table 14.4

Harry's Auto Tire

Step 5: Simulating the experiment

- We select random numbers from Table 14.4.
- The number we select will have a corresponding range in Table 14.3.
- We use the daily demand that corresponds to the probability range aligned with the random number.

Harry's Auto Tire

Ten-day Simulation of Demand for Radial Tires

DAY	RANDOM NUMBER	SIMULATED DAILY DEMAND
1	52	3
2	37	3
3	82	4
4	69	4
5	98	5
6	96	5
7	33	2
8	50	3
9	88	5
10	90	5
		39 = total 10-day demand
		3.9 = average daily demand for tires

Table 14.5

Harry's Auto Tire

Note that the average demand from this simulation (3.9 tires) is different from the ***expected*** daily demand.

$$\text{Expected daily demand} = \sum_{i=0}^{5} (\text{Probability of } i \text{ tires})(\text{Demand of } i \text{ tires})$$

$$= (0.05)(0) + (0.10)(1) + (0.20)(2) + (0.30)(3) + (0.20)(4) + (0.15)(5)$$

$$= 2.95 \text{ tires}$$

If this simulation were repeated hundreds or thousands of times it is much more likely the average ***simulated*** demand would be nearly the same as the ***expected*** demand.

The Law of Large Numbers

states something like this:

The results obtained from performing a large number of trials should be close to the expected value. And it will become closer to the true expected value, the more trials you perform.

Higgins Plumbing and Heating maintains a stock of 30-gallon hot water heaters that it sells to homeowners and installs for them. Owner Jerry Higgins likes the idea of having a large supply on hand to meet customer demand, but he also recognizes that it is expensive to do so. He examines hot water heater sales over the past 50 weeks and notes the following:

HOT WATER HEATER SALES PER WEEK	NUMBER OF WEEKS THIS NUMBER WAS SOLD
4	6
5	5
6	9
7	12
8	8
9	7
10	3
	Total 50

a. If Higgins maintains a constant supply of 8 hot water heaters in any given week, how many times will he be out of stock during a 20-week simulation? We use random numbers from the seventh column of Table 14.4, beginning with the random digits 10.
b. What is the average number of sales per week (including stockouts) over the 20-week period?
c. Using an analytic nonsimulation technique, what is the expected number of sales per week? How does this compare with the answer in part (b)?

52	06	50	88	53	30	10	47	99	37	66	91	35	32	00	84	57	07
37	63	28	02	74	35	24	03	29	60	74	85	90	73	59	55	17	60
82	57	68	28	05	94	03	11	27	79	90	87	92	41	09	25	36	77
69	02	36	49	71	99	32	10	75	21	95	90	94	38	97	71	72	49
98	94	90	36	06	78	23	67	89	85	29	21	25	73	69	34	85	76
96	52	62	87	49	56	59	23	78	71	72	90	57	01	98	57	31	95
33	69	27	21	11	60	95	89	68	48	17	89	34	09	93	50	44	51
50	33	50	95	13	44	34	62	64	39	55	29	30	64	49	44	30	16
88	32	18	50	62	57	34	56	62	31	15	40	90	34	51	95	26	14
90	30	36	24	69	82	51	74	30	35	36	85	01	55	92	64	09	85
50	48	61	18	85	23	08	54	17	12	80	69	24	84	92	16	49	59
27	88	21	62	69	64	48	31	12	73	02	68	00	16	16	46	13	85
45	14	46	32	13	49	66	62	74	41	86	98	92	98	84	54	33	40
81	02	01	78	82	74	97	37	45	31	94	99	42	49	27	64	89	42
66	83	14	74	27	76	03	33	11	97	59	81	72	00	64	61	13	52
74	05	81	82	93	09	96	33	52	78	13	06	28	30	94	23	37	39
30	34	87	01	74	11	46	82	59	94	25	34	32	23	17	01	58	73
59	55	72	33	62	13	74	68	22	44	42	09	32	46	71	79	45	89
67	09	80	98	99	25	77	50	03	32	36	63	65	75	94	19	95	88
60	77	46	63	71	69	44	22	03	85	14	48	69	13	30	50	33	24
60	08	19	29	36	72	30	27	50	64	85	72	75	29	87	05	75	01
80	45	86	99	02	34	87	08	86	84	49	76	24	08	01	86	29	11
53	84	49	63	26	65	72	84	85	63	26	02	75	26	92	62	40	67
69	84	12	94	51	36	17	02	15	29	16	52	56	43	26	22	08	62
37	77	13	10	02	18	31	19	32	85	31	94	81	43	31	58	33	51

a.

WEEK	RANDOM NUMBER	SIMULATED SALES	WEEK	RANDOM NUMBER	SIMULATED SALES
1	10	4	11	08	4
2	24	6	12	48	7
3	03	4	13	66	8
4	32	6	14	97	10
5	23	6	15	03	4
6	59	7	16	96	10
7	95	10	17	46	7
8	34	6	18	74	8
9	34	6	19	77	8
10	51	7	20	44	7

With a supply of 8 heaters, Higgins will be out of stock three times during the 20-week period (in weeks 7, 14, and 16).

b. Average sales by simulation $= \dfrac{\text{Total sales}}{\text{20 weeks}} = \dfrac{135}{20} = 6.75$ per week.

c. Using expected value,

$$\begin{aligned} E(\text{sales}) &= 0.12(4 \text{ heaters}) + 0.10(5) + 0.18(6) + 0.24(7) \\ &\quad + 0.16(8) + 0.14(9) + 0.06(10) \\ &= 6.88 \text{ heaters} \end{aligned}$$

With a longer simulation, these two approaches will lead to even closer values.