Chapter 9

Transportation and Assignment Models

The Assignment Algorithm

- The second special-purpose LP algorithm is the assignment method.
- Each assignment problem has associated with it a table, or matrix.
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things to which we want them assigned.
- The numbers in the table are the costs associated with each particular assignment.
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1.

Assignment Model Approach

- The Fix-It Shop has three rush projects to repair.
- The shop has three repair persons with different talents and abilities.
- The owner has estimates of wage costs for each worker for each project.
- The owner’s objective is to assign the three project to the workers in a way that will result in the lowest cost to the shop.
- Each project will be assigned exclusively to one worker.

Assignment Model Approach

Estimated Project Repair Costs for the Fix-It Shop Assignment Problem

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 9.20

Assignment Model Approach

Summary of Fix-It Shop Assignment Alternatives and Costs

PRODUCT ASSIGNMENT				
1	2	3	LABOR COSTS ($)	TOTAL COSTS ($)
Adams	Brown	Cooper	11 + 10 + 7	28
Adams	Cooper	Brown	11 + 12 + 11	34
Brown	Adams	Cooper	8 + 14 + 7	29
Brown	Cooper	Adams	8 + 12 + 6	26
Cooper	Adams	Brown	9 + 14 + 11	34
Cooper	Brown	Adams	9 + 10 + 6	25

Table 9.21

We need better way

- For example, a problem involving the assignment
- of four workers to four projects requires that we consider (4!) or 24 alternatives.
- A problem with eight workers and eight tasks, which actually is not that large in a realistic situation, yields (8!) or 40,320 possible solutions!
- Since it would clearly be impractical to compare so many alternatives, a more efficient solution method is needed.

The Hungarian Method (Flood's Technique)

- The *Hungarian method* is an efficient method of finding the optimal solution to an assignment problem without having to make direct comparisons of every option.
- It operates on the principle of *matrix reduction.*
- By subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of *opportunity costs.*
- Opportunity costs show the relative penalty associated with assigning any person to a project as opposed to making the *best* assignment.
- We want to make assignment so that the opportunity cost for each assignment is zero.

Three Steps of the Assignment Method

1. *Find the opportunity cost table by*:
   (a) Subtracting the smallest number in each row of the original cost table or matrix from every number in that row.
   (b) Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column.
2. *Test the table resulting from step 1 to see whether an optimal assignment can be made* by drawing the minimum number of vertical and horizontal straight lines necessary to cover all the zeros in the table. If the number of lines is less than the number of rows or columns, proceed to step 3.

Three Steps of the Assignment Method

3. *Revise the opportunity cost table* by *subtracting* the smallest number not covered by a line from all numbers not covered by a straight line. This same number is also added to every number lying at the intersection of any two lines. Return to step 2 and continue the cycle until an optimal assignment is possible.

Steps in the Assignment Method

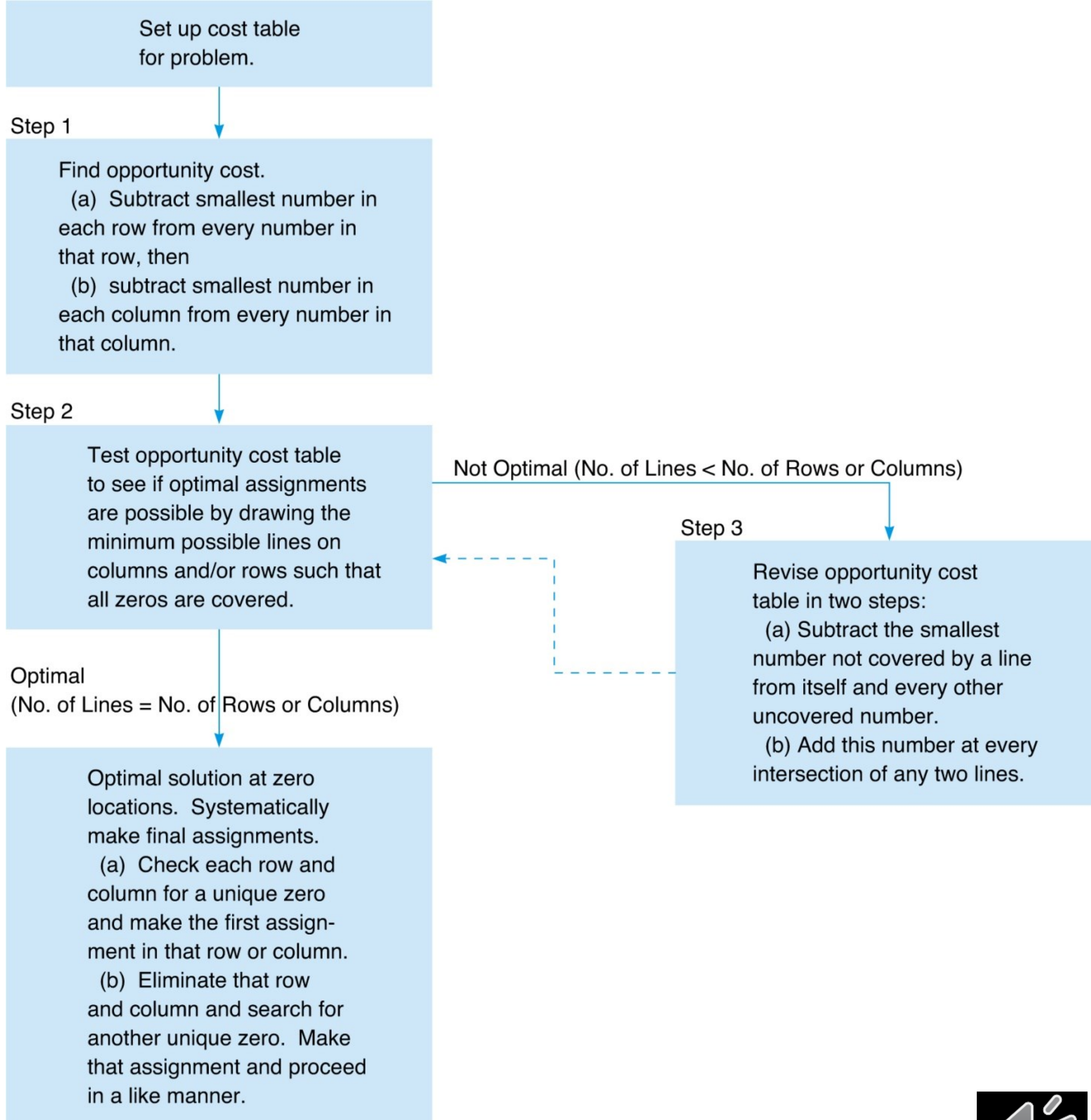

Figure 9.4

The Hungarian Method (Flood's Technique)

- Step 1: Find the opportunity cost table.
  - We can compute *row* opportunity costs and *column* opportunity costs.
  - What we need is the *total* opportunity cost.
  - We derive this by taking the row opportunity costs and subtract the smallest number in that column from each number in that column.

The Hungarian Method (Flood's Technique)

Cost of Each Person-Project Assignment for the Fix-it Shop Problem

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Tables 9.22-9.23

Row Opportunity Cost Table for the Fix-it Shop Step 1, Part (a)

	PROJECT		
PERSON	1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

The opportunity cost of assigning Cooper to project 2 is $12 – $7 = $5.

The Hungarian Method (Flood's Technique)

Derive the total opportunity costs by taking the costs in Table 9.23 and subtract the smallest number in each column from each number in that column.

Row Opportunity Cost Table for the Fix-it Shop Step 1, Part (a)

	PROJECT		
PERSON	1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Total Opportunity Cost Table for the Fix-it Shop Step 1, Part (b)

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 9.24

The Hungarian Method (Flood's Technique)

- Step 2: Test for the optimal assignment.
  - We want to assign workers to projects in such a way that the total labor costs are at a minimum.
  - We would like to have a total assigned opportunity cost of zero.
  - The test to determine if we have reached an optimal solution is simple.
  - We find the *minimum* number of straight lines necessary to cover all the zeros in the table.
  - If the number of lines equals the number of rows or columns, an optimal solution has been reached.

The Hungarian Method (Flood's Technique)

Test for Optimal Solution to Fix-it Shop Problem

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Covering line 1

Covering line 2

Table 9.25

This requires only two lines to cover the zeros so the solution is not optimal.

The Hungarian Method (Flood's Technique)

- Step 3: Revise the opportunity-cost table.
  - We *subtract* the smallest number not covered by a line from all numbers not covered by a straight line.
  - The same number is added to every number lying at the intersection of any two lines.
  - We then return to step 2 to test this new table.

The Hungarian Method (Flood's Technique)

Revised Opportunity Cost Table for the Fix-it Shop Problem

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

The Hungarian Method (Flood's Technique)

Optimality Test on the Revised Fix-it Shop Opportunity Cost Table

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	4
Cooper	0	1	0

Table 9.27

Covering line 1

Covering line 2

Covering line 3

This requires three lines to cover the zeros so the solution is optimal.

Making the Final Assignment

- The optimal assignment is Adams to project 3, Brown to project 2, and Cooper to project 1.
- For larger problems one approach to making the final assignment is to select a row or column that contains only one zero.
  - Make the assignment to that cell and rule out its row and column.
  - Follow this same approach for all the remaining cells.

Making the Final Assignment

Total labor costs of this assignment are:

ASSIGNMENT	COST ($)
Adams to project 3	6
Brown to project 2	10
Cooper to project 1	9
Total cost	25

Making the Final Assignment

Making the Final Fix-it Shop Assignments

(A) FIRST ASSIGNMENT				(B) SECOND ASSIGNMENT				(C) THIRD ASSIGNMENT			
	1	2	3		1	2	3		1	2	3
Adams	3	4	0	Adams	3	4	0	Adams	3	4	0
Brown	0	0	5	Brown	0	0	5	Brown	0	0	5
Cooper	0	1	0	Cooper	0	1	0	Cooper	0	1	0

Table 9.28

Example

You work as a manager for a chip manufacturer, and you currently have 3 people on the road meeting clients. Your salespeople are in Adam, Zak and Brown, and you want them to fly to three other cities: Delhi, Mumbai and Kerala. The table below shows the cost of airline tickets in INR between the cities:

	Delhi	Kerala	Mumbai
Adam	1500	4000	4500
Zak	2000	6000	3500
Brown	2000	4000	2500

Solution

- Row Reduction

Step 1: Subtract minimum of every row.

1500, 2000 and 2000 are subtracted from rows 1, 2 and 3 respectively.

0	2500	3000
0	4000	1500
0	2000	500

Total Reduction

- Col Reduction>> total reduction

```
Step 2: Subtract minimum of every column.
0, 2000 and 500 are subtracted from columns 1, 2 and 3
respectively.

   0      500  2500
   0     2000  1000
   0        0     0
```

Covering all zeros

Step 3: Cover all zeroes with minimum number of horizontal and vertical lines.

0	500	2500
0	2000	1000
0	0	0

Optimal?

Finding the optimal sol

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Step 4:  Since we only need 2 lines to cover all zeroes,
we have NOT found the optimal assignment.

Step 5:  We subtract the smallest uncovered entry
from all uncovered rows. Smallest entry is 500.
 -500    0   2000
 -500  1500   500
    0     0     0
```

Finally

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Then we add the smallest entry to all covered columns, we get
   0      0   2000
   0   1500    500
 500      0      0
```

Now we return to **Step 3:**. Here we cover again using lines. and go to **Step 4:**. Since we need 3 lines to cover, we found the optimal solution.

```
 1500  4000  4500
 2000  6000  3500
 2000  4000  2500
```

```
So the optimal cost is 4000 + 2000 + 2500 = 8500
```