Dr. M. Yusuf

Solved problems – probability

1. The following table classifies 400 people according to their smoking habits and whether or not they have cancer.

	Smoker (A)	Non-Smoker ($\bar{A}$)
Has cancer (C)	200	50
Does not have cancer ($\bar{C}$)	50	100

If an individual is selected at random from this group, find the probability that he/she is

(a) a smoker and has cancer ,
(b) a smoker or has cancer
(c) a non-smoker or has cancer

Q1 solution . A = Smoker ; $\bar{A}$ = Non-Smoker
C = Has Cancer ; $\bar{C}$ = Does not have Cancer

$n(\Omega) = N = 400$; $n(A) = 250$; $n(\bar{A}) = 150$; $n(C) = 250$; $n(\bar{C}) = 150$

(a) $P(A \cap C) = \dfrac{n(A \cap C)}{n(\Omega)} = \dfrac{200}{400} = 0.5$

(b) $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$$= \frac{n(A)}{n(\Omega)} + \frac{n(C)}{n(\Omega)} - \frac{n(A \cap C)}{n(\Omega)} = \frac{250}{400} + \frac{250}{400} - \frac{200}{400} = \frac{300}{400} = 0.75$$

(c) $P(\bar{A} \cup C) = P(\bar{A}) + P(C) - P(\bar{A} \cap C)$

$$= \frac{n(\bar{A})}{n(\Omega)} + \frac{n(C)}{n(\Omega)} - \frac{n(\bar{A} \cap C)}{n(\Omega)} = \frac{150}{400} + \frac{250}{400} - \frac{50}{400} = \frac{350}{400} = 0.875$$

2 A and B are events defined on the same sample space.

(a) If $P(\bar{A}) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.1$,

Find: (i) $P(A \cup B)$, (ii) $P(A \cap \bar{B})$, (iii) $P(\bar{A} \cap B)$, (iv) $P(\bar{A} \cap \bar{B})$

(b) If $P(A \cap \bar{B}) = 0.3$, $P(A \cap B) = 0.2$ and $P(\bar{A} \cap \bar{B}) = 0.1$,

Find: (i) $P(A)$, (ii) $P(\bar{A} \cap B)$, (iii) $P(A \cup B)$

Q2 solution (a) $P(\bar{A}) = 0.6$; $P(B) = 0.5$; $P(A \cap B) = 0.1$; where $P(\bar{A}) = 1 - P(A)$

$\therefore$ $P(A) = 1 - P(\bar{A}) = 1 - 0.6 = 0.4$; $P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.5$

	B	$\bar{B}$	Total
A	0.1	0.3	0.4
$\bar{A}$	0.4	0.2	0.6
Total	0.5	0.5	1.0

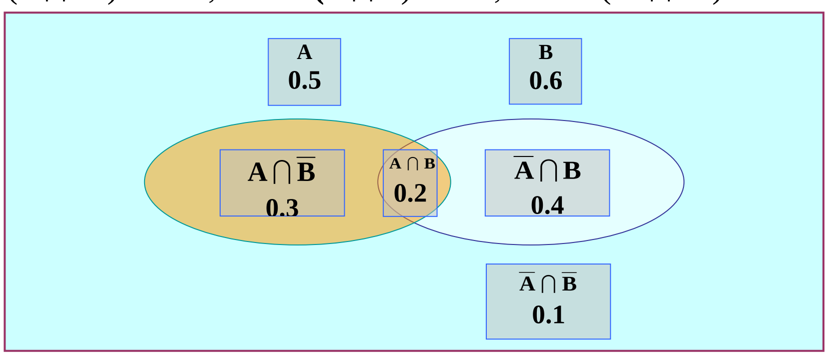
(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - 0.1 = 0.8$

(ii) $P(A \cap \bar{B}) = 0.3$

(iii) $P(\bar{A} \cap B) = 0.4$; (iv) $P(\bar{A} \cap \bar{B}) = 0.2$

(b) $P(A \cap \bar{B}) = 0.3$; $P(A \cap B) = 0.2$; $P(\bar{A} \cap \bar{B}) = 0.1$



(i) $P(A) = P(A \cap \bar{B}) + P(A \cap B) = 0.3 + 0.2 = 0.5$

(ii) $P(\bar{A} \cap B) = 0.4$

(iii) $P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)$

$= 0.3 + 0.2 + 0.4 = 0.9$

3 Refer to the data in question 1,

(a) If an individual is selected at random from the group, find the probability that the person selected:

(i) has cancer given that he/she is a smoker.

(ii) is not a smoker given that he/she does not have cancer.

(b) determine whether smoking and having cancer are independent.

Q3 solution Refer to the data in Q1.

A = Smoker; $\bar{A}$ =Non-Smoker; C =Has Cancer ; $\bar{C}$ =Does not have Cancer

$n(\Omega) = N = 400$; $n(A) = 250$; $n(\bar{A}) = 150$; $n(C) = 250$; $n(\bar{C}) = 150$

(a) (i) $P(C \mid A) = \dfrac{P(C \cap A)}{P(A)} = \dfrac{n(C \cap A)/n(\Omega)}{n(A)/n(\Omega)} = \dfrac{n(C \cap A)}{n(A)} = \dfrac{200}{250} = 0.80$

(ii) $P(\bar{A} \mid \bar{C}) = \dfrac{P(\bar{A} \cap \bar{C})}{P(\bar{C})} = \dfrac{n(\bar{A} \cap \bar{C})/n(\Omega)}{n(\bar{C})/n(\Omega)} = \dfrac{n(\bar{A} \cap \bar{C})}{n(\bar{C})} = \dfrac{100}{150} = 0.67$

(b) By definition:

if events A, C are independent, then: $P(A \cap C) = P(A) \times P(C)$ …….. (1)

$\therefore$ $P(A \cap C) = \dfrac{n(A \cap C)}{n(\Omega)} = \dfrac{200}{400} = 0.50$ **; and**

$P(A) \times P(C) = \dfrac{250}{400} \times \dfrac{250}{400} = 0.625 \times 0.625 = 0.390625 = 0.39$

Since $0.50 \neq 0.39$ (different probabilities on both sides of eq.(1))

$\therefore$ $P(A \cap C) \neq P(A) \times P(C)$

The events A and C are not- independent.

i.e., Smoking and having cancer are not-independent.

Note on Independence

Events A and B are Independent, if

(1) $P(A \cap B) = P(A) \times P(B)$

(2) $P(A \mid B) = P(A)$

(3) $P(B \mid A) = P(B)$