Chapter 6

Inventory Control Models

To accompany
Quantitative Analysis for Management*, *Eleventh Edition*, *Global Edition
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. **Understand the importance of inventory control and ABC analysis.**
2. **Use the economic order quantity (EOQ) to determine how much to order.**
3. **Compute the reorder point (ROP) in determining when to order more inventory.**
4. **Handle inventory problems that allow quantity discounts or non-instantaneous receipt.**

Learning Objectives

After completing this chapter, students will be able to:

5. **Understand the use of safety stock.**
6. **Describe the use of material requirements planning in solving dependent-demand inventory problems.**
7. **Discuss just-in-time inventory concepts to reduce inventory levels and costs.**
8. **Discuss enterprise resource planning systems.**

Sensitivity Analysis with the EOQ Model

- The EOQ model assumes all values are know and fixed over time.
- Generally, however, some values are estimated or may change.
- Determining the effects of these changes is called ***sensitivity analysis.***
- Because of the square root in the formula, changes in the inputs result in relatively small changes in the order quantity.

$$\text{EOQ} = \sqrt{\frac{2DC_o}{C_h}}$$

Sensitivity Analysis with the *EOQ Model*

- **In the Sumco Pump example:**

$$\text{EOQ} = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = 200\ \text{units}$$

- **If the ordering cost were increased four times from \$10 to \$40, the order quantity would only double**

$$\text{EOQ} = \sqrt{\frac{2(1{,}000)(40)}{0.50}} = 400\ \text{units}$$

- **In general, the EOQ changes by the square root of the change to any of the inputs.**

Reorder Point: Determining When To Order

- Once the order quantity is determined, the next decision is ***when to order.***
- The time between placing an order and its receipt is called the ***lead time*** (*L*) or ***delivery time.***
- When to order is generally expressed as a ***reorder point*** (*ROP*).

$$\text{ROP} = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix} \times \begin{pmatrix}\text{Lead time for a}\\ \text{new order in days}\end{pmatrix}$$

$$= d \times L$$

Procomp's Computer Chips

- **Demand for the computer chip is 8,000 per year.**
- **Daily demand is 40 units.**
- **Delivery takes three working days.**

$$\text{ROP} = d \times L = \textbf{40 units per day} \times \textbf{3 days}$$
$$= \textbf{120 units}$$

- **An order based on the EOQ calculation is placed when the inventory reaches 120 units.**
- **The order arrives 3 days later just as the inventory is depleted.**

Reorder Point Graphs

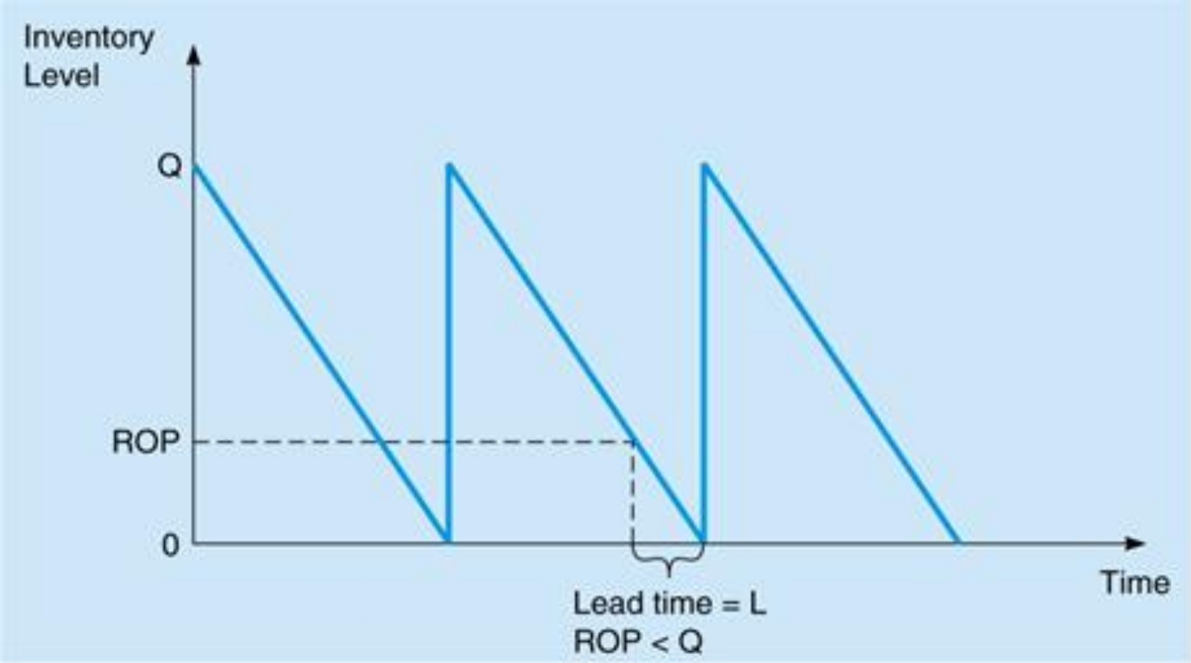

Figure 6.4

EOQ Without The Instantaneous Receipt Assumption

- **When inventory accumulates over time, the *instantaneous receipt* assumption does not apply.**
- **Daily demand rate must be taken into account.**
- **The revised model is often called the *production run model.***

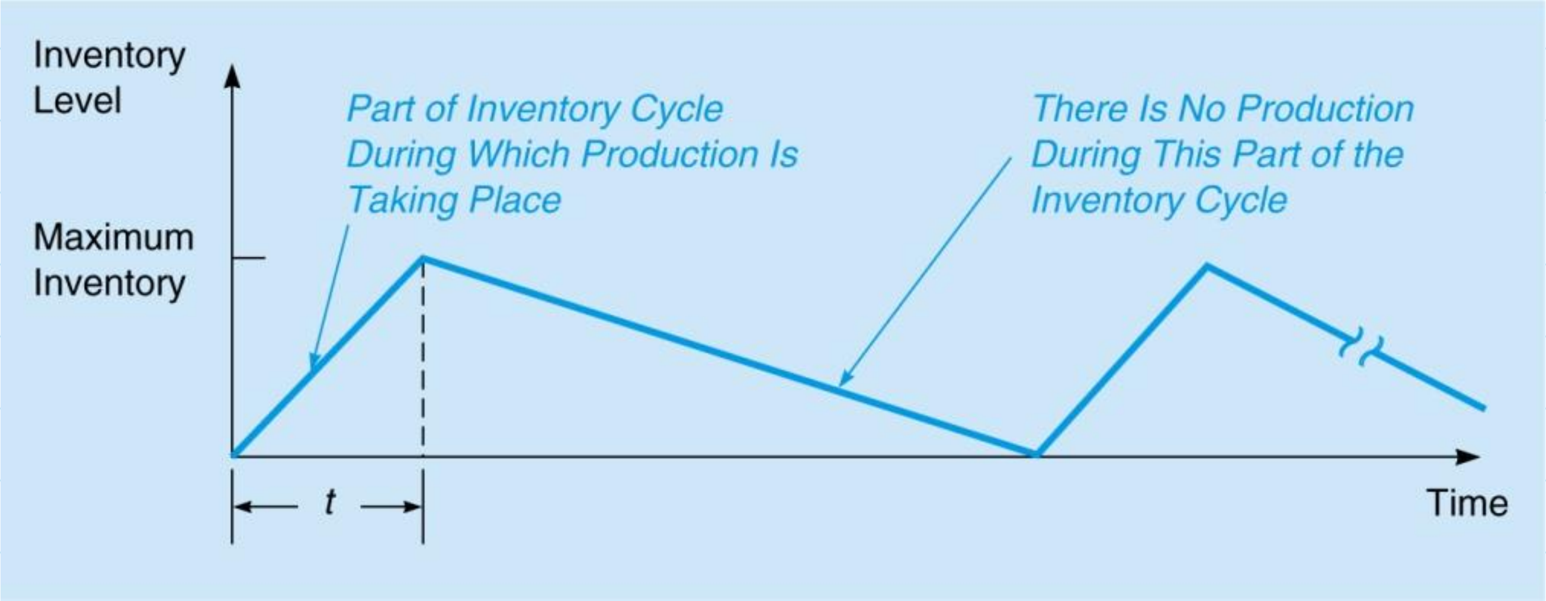

Figure 6.5

Annual Carrying Cost for Production Run Model

- In production runs, ***setup cost*** replaces ordering cost.
- The model uses the following variables:

Q = number of pieces per order, or production run

C_s = setup cost

C_h = holding or carrying cost per unit per year

p = daily production rate

d = daily demand rate

t = length of production run in days

Annual Carrying Cost for Production Run Model

Maximum inventory level

= (Total produced during the production run)
– (Total used during the production run)

= (Daily production rate)(Number of days production)
– (Daily demand)(Number of days production)

$= (pt) - (dt)$

since Total produced $= Q = pt$

we know $t = \dfrac{Q}{p}$

Maximum inventory level $= pt - dt = p\dfrac{Q}{p} - d\dfrac{Q}{p} = Q\left(1 - \dfrac{d}{p}\right)$

Annual Carrying Cost for Production Run Model

Since the average inventory is one-half the maximum:

$$\text{Average inventory} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$

and

$$\text{Annual holding cost} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)C_h$$

Annual Setup Cost for Production Run Model

Setup cost replaces ordering cost when a product is produced over time.

$$\text{Annual setup cost} = \frac{D}{Q}C_s$$

replaces

$$\text{Annual ordering cost} = \frac{D}{Q}C_o$$

Determining the Optimal Production Quantity

By setting setup costs equal to holding costs, we can solve for the optimal order quantity

Annual holding cost = Annual setup cost

$$\frac{Q}{2}\left(1-\frac{d}{p}\right)C_h = \frac{D}{Q}C_s$$

Solving for Q, we get

$$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

Production Run Model

Summary of equations

$$\text{Annual holding cost} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)C_h$$

$$\text{Annual setup cost} = \frac{D}{Q}C_s$$

$$\text{Optimal production quantity } Q^* = \sqrt{\frac{2DC_s}{C_h\left(1 - \frac{d}{p}\right)}}$$

Brown Manufacturing

Brown Manufacturing produces commercial refrigeration units in batches.

$$\text{Annual demand} = D = 10{,}000 \text{ units}$$
$$\text{Setup cost} = C_s = \$100$$
$$\text{Carrying cost} = C_h = \$0.50 \text{ per unit per year}$$
$$\text{Daily production rate} = p = 80 \text{ units daily}$$
$$\text{Daily demand rate} = d = 60 \text{ units daily}$$

1. **How many units should Brown produce in each batch?**
2. **How long should the production part of the cycle last?**

Brown Manufacturing Example

1. $$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

$$Q^* = \sqrt{\frac{2 \times 10{,}000 \times 100}{0.5\left(1-\frac{60}{80}\right)}}$$

$$= \sqrt{\frac{2{,}000{,}000}{0.5\left(\frac{1}{4}\right)}} = \sqrt{16{,}000{,}000}$$

$$= 4{,}000 \text{ units}$$

2. $$\text{Production cycle} = \frac{Q}{p}$$

$$= \frac{4{,}000}{80} = 50 \text{ days}$$

Patterson Electronics supplies microcomputer circuitry to a company that incorporates microprocessors into refrigerators and other home appliances. One of the components has an annual demand of 250 units, and this is constant throughout the year. Carrying cost is estimated to be $1 per unit per year, and the ordering cost is $20 per order.

a. To minimize cost, how many units should be ordered each time an order is placed?
b. How many orders per year are needed with the optimal policy?
c. What is the average inventory if costs are minimized?
d. Suppose the ordering cost is not $20, and Patterson has been ordering 150 units each time an order is placed. For this order policy to be optimal, what would the ordering cost have to be?

a. The EOQ assumptions are met, so the optimal order quantity is

$$\text{EOQ} = Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(250)20}{1}} = 100 \text{ units}$$

b. Number of orders per year $= \dfrac{D}{Q} = \dfrac{250}{100} = 2.5$ orders per year

Note that this would mean in one year the company places 3 orders and in the next year it would only need 2 orders, since some inventory would be carried over from the previous year. It averages 2.5 orders per year.

c. Average Inventory $= \dfrac{Q}{2} = \dfrac{100}{2} = 500$ units

$$Q = \sqrt{\frac{2DC_o}{C_h}}$$

$$C_o = Q^2 \frac{C_h}{2D}$$

$$= \frac{(150)^2(1)}{2(250)}$$

$$= \frac{22{,}500}{500} = \$45$$

Flemming Accessories produces paper slicers used in offices and in art stores. The minislicer has been one of its most popular items: Annual demand is 6,750 units and is constant throughout the year. Kristen Flemming, owner of the firm, produces the minislicers in batches. On average, Kristen can manufacture 125 minislicers per day. Demand for these slicers during the production process is 30 per day. The setup cost for the equipment necessary to produce the minislicers is $150. Carrying costs are $1 per minislicer per year. How many minislicers should Kristen manufacture in each batch?

The data for Flemming Accessories are summarized as follows:

$$D = 6{,}750 \text{ units}$$
$$C_s = \$150$$
$$C_h = \$1$$
$$d = 30 \text{ units}$$
$$p = 125 \text{ units}$$

This is a production run problem that involves a daily production rate and a daily demand rate. The appropriate calculations are shown here:

$$Q^* = \sqrt{\frac{2DC_s}{C_h(1 - d/p)}}$$
$$= \sqrt{\frac{2(6{,}750)(150)}{1(1 - 30/125)}}$$
$$= 1{,}632$$

Dorsey Distributors has an annual demand for a metal detector of 1,400. The cost of a typical detector to Dorsey is $400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is $25 per order. If Dorsey orders in quantities of 300 or more, it can get a 5% discount on the cost of the detectors. Should Dorsey take the quantity discount? Assume the demand is constant.

The solution to any quantity discount model involves determining the total cost of each alternative after quantities have been computed and adjusted for the original problem and every discount. We start the analysis with no discount:

$$\text{EOQ (no discount)} = \sqrt{\frac{2(1{,}400)(25)}{0.2(400)}}$$
$$= 29.6 \text{ units}$$

$$\text{Total cost (no discount)} = \text{Material cost} + \text{Ordering cost} + \text{Carrying cost}$$
$$= \$400(1{,}400) + \frac{1{,}400(\$25)}{29.6} + \frac{29.6(\$400)(0.2)}{2}$$
$$= \$560{,}000 + \$1{,}183 + \$1{,}183 = \$562{,}366$$

The next step is to compute the total cost for the discount:

$$\text{EOQ (with discount)} = \sqrt{\frac{2(1{,}400)(25)}{0.2(\$380)}}$$
$$= 30.3 \text{ units}$$
$$Q(\text{adjusted}) = 300 \text{ units}$$

Because this last economic order quantity is below the discounted price, we must adjust the order quantity to 300 units. The next step is to compute total cost:

$$\text{Total cost (with discount)} = \text{Material cost} + \text{Ordering cost} + \text{Carrying cost}$$
$$= \$380(1{,}400) + \frac{1{,}400(25)}{300} + \frac{300(\$380)(0.2)}{2}$$
$$= \$532{,}000 + \$117 + \$11{,}400 = \$543{,}517$$

The optimal strategy is to order 300 units at a total cost of \$543,517.