Examples on Queuing models

Example 1:

- A university cafeteria line in the student center is a self-serve facility in which students select the food items they want and then form a single line to pay the cashier. Students arrive at a rate of about four per minute according to a Poisson distribution. The single cashier ringing up sales takes about 12 seconds per customer, following an exponential distribution.
- (a) What is the probability that there are more than two students in the system? More than three students? More than four?
- (b) What is the probability that the system is empty? (c) How long will the average student have to wait before reaching the cashier?
- (d) What is the expected number of students in the queue?
- (e) What is the average number in the system?
- (f) If a second cashier is added (who works at the same pace), how will the operating characteristics computed in parts (b), (c), (d), and (e) change? Assume that customers wait in a single line and go to the first available cashier.

Solution

- $\boldsymbol{\lambda}$= 4 students/min, μ= 5 students/min
- (a) What is the probability that there are more than two students in the system? More than three students? More than four?
- P>2 = $\left(\frac{\lambda}{\mu}\right)^{n+1}$ $=$ (4/5)^3
- P>3= $\left(\frac{\lambda}{\mu}\right)^{n+1}$ $=$ (4/5)^4
- P> 4= $\left(\frac{\lambda}{\mu}\right)^{n+1}$ $=$ (4/5)^5
- (b) What is the probability that the system is empty? (c) How long will the average student have to wait before reaching the cashier?
- P0= 1- $\left(\frac{\lambda}{\mu}\right)$ = 1-(4/5)
- Wq=$\frac{\lambda}{\mu(\mu-\lambda)}$ = 4/5(5-4) =4/5(1)= 0.8 minutes

Cont. Solution

- (d) What is the expected number of students in the queue?
- Lq=$\frac{\lambda^2}{\mu(\mu-\lambda)}$ = (4^2)/5(5-4) = 3.2 students
- (e) What is the average number in the system?
- L= $\frac{\lambda}{(\mu-\lambda)}$= 4 students.
- (f) repeats with M\M\M model equations

Example 2

- Billy's Bank is the only bank in a small town in Arkansas. On a typical Friday, an average of 10 customers per hour arrive at the bank to transact business. There is one single teller at the bank, and the average time required to transact business is 4 minutes. It is assumed that service times can be described by the exponential distribution. Although this is the only bank in town, some people in the town have begun using the bank in a neighboring town about 20 miles away. A single line would be used, and the customer at the front of the line would go to the first available bank teller. If a single teller at Billy's is used, find
- (a) the average time in the line.
- (b) the average number in the line.
- (c) the average time in the system.
- (d) the average number in the system.
- (e) the probability that the bank is empty.

Solution

- λ= 10/hour (what is arrival is 3/ 10 mins)
- μ= 15
- 1 transaction → 4 mins
- ? Transactions → 60 mins

Example 3

- A vending machine dispenses hot chocolate or coffee. Service time is 30 seconds per cup and is constant. Customers arrive at a mean rate of 80 per hour, and this rate is Poisson-distributed. Determine:

a. The average number of customers waiting in line.
b. The average time customers spend in the system.
c. The average number in the system.

Solution

- Model M/D/1
- Given: Mean service time = 30 seconds/customer (constant)

1 cup >> 30 seconds

? Cup >> 60 second ====== 2 cups/ mins

2 cups >> 1 mins

? Cups >> 60 mins ====== 120 cup/ hour

Solution cont.

- Lq= $\frac{\lambda^2}{2\mu(\mu_{-\lambda})} = \frac{\mathbf{80^2}}{2*\mathbf{120(120-80)}}$=0.667
- Wq= $\frac{\lambda}{2\mu(\mu_{-\lambda})}$= $\frac{\mathbf{80}}{2*\mathbf{120(120-80)}}$1/120
- W= Wq+(1/μ)= 0.0166
- L= Lq+(λ /μ)=1.334