Proof of the transcendence of the Dottie number

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1 Abstract

Dottie number, a unique real solution to the following equation:cos(d)=d. This constant d which value is approximately equal to: 0.7390851332151607 appears naturally in the Iterative cosine function. It has drawn attention to the mathematical community due to its interesting properties. One question arises that is whether this constant is algebraic or transcendental. In this paper, we prove that it is transcendental by applying the Lindemman-Weierstrass theorem through a proof of contradiction.

2 introduction

In this paper,we prove the transcendence of the Dottie number using a classical method:a proof by contradiction based on the Lindemman-Weierstrass theorem and the exponential form of the cosine function.

3 Main Proof

We begin by stating the assumptions needed for this proof:Let $d \in \mathbb{R}$ be a unique real number satisfying the equation $\cos(d) = d$, suppose for the sake of contradiction that d is algebraic. Recall that the cosine function can be written using Euler's formula as $\cos(d) = \frac{e^{id}+e^{-id}}{2}$ substituing $\cos(d) = d$ and multiplying both sides by 2 we get the following $2d = e^{id} + e^{-id}$. Let us define $z = e^{id}$ where $e^{-id} = \frac{1}{z}$ then the equation becomes:$2d = z + \frac{1}{z}$ multiplying both sides by z and rearranging we get the quadratic equation:$z^2 - 2dz + 1 = 0$ since d is algebraic by assumption,the coefficients of this quadratic are algebraic numbers therefore z is algebraic as a root of a polynomial with algebraic coefficients. Now by the Lindemman-Weierstrass theorem for distinct algebraic numbers 0 and id in our case, where $id \neq 0$ so $e^0 = 1$ and e^{id} are algebraically independant over the field of algebraic numbers. however from the polynomial equation we got with algebraic coefficients, we see that $z = e^{id}$ satisfies a non trivial algebraic relation with 1 which contradicts the Lindemman-Weierstrass theorem Hence d cannot be algebraic and must be transcendential.

4 conclusion

In this paper we have proven the transcendence of the Dottie number by a proof by contradiction and applying the Lindemman-Weierstrass theorem and using Euler's Formula for the cosine function. This establishes that the unique real solution to the equation $\cos(d) = d$ is not algebraic. This result contributes to the study of transcendental numbers defined as solutions to transcendental equations.

5 References

References

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