Application of Wavelet Transform in Signal and Image Processing: A Project Report

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***Abstract*—This document serves as my comprehensive final project report, wherein I collaborated with my project partner, An Vu. Despite the joint nature of the project, the report presented here is the result of my individual work. Our research focused on applying wavelet systems to various forms of imagery. The main task was directed toward the analysis of satellite imagery as a mandatory task while the supplementary task explored the application of these systems to medical imaging data.**

***Index Terms*—Wavelet transform, Wavelet analysis, Wavelet-based image enhancement, Noise suppression, Feature augmentation, and Edge sharpening.**

I. Literature Review

A. Paper 1: D.L. Donoho and M. Elad titled 'Optimally-sparse representation in general (non-orthogonal) dictionaries via l1 minimization'

It offers an advanced exploration into the domain of engineering where there is a frequent desire for data representation in the most unambiguous and elementary format. In the realm of signal analysis, there is often an assumption that the signal under scrutiny is sparse within a specific transformation domain, such as the wavelet or Fourier spectrum. However, an understanding has emerged acknowledging that many signals embody a complex mixture of diverse elements, and thus no single method can achieve an optimal characterization. This has led to the consideration of models that formulate sparse combinations of generative elements hailing from multiple disparate transforms. Nonetheless, this approach bears the compromise of forgoing the sparse solutions. This study seeks to address and rectify this limitation. The crux of the problem is to discover the sparse representation alpha, such that the multiplication of the dictionary by alpha will yield the original signal. Potential examples of such dictionaries include wavelet packets and cosine packet dictionaries as proposed by Coifman; wavelet frames such as the directional wavelet frames developed by Ron and Shen; and the integrated ridgelet/wavelet systems introduced by Starck, Candès, and Donoho. Therefore, this paper exemplifies the application of wavelet theory in addressing the sparse representation challenge.

B. Paper 2: "The Geometry of the Extension Principle" by Theodoros Staropoulos

This manuscript explores a significant challenge encountered in the construction of wavelets within the context of digital data processing. Specifically, when the integer translations of a refinable function constitute a Bessel family rather than forming a frame, no lower bound exists. This scenario impedes the desired spatial localization typically achievable in classical Multi-Resolution Analysis (MRA). To counteract this limitation, the authors have formulated the Extension Principles, methodologies specifically designed to facilitate the construction of stable wavelet filters with attributes such as symmetry, antisymmetry, and minimal support. This strategy is viable given the propensity for refinable functions to possess a smooth Fourier transform as desired, in addition to compact support and symmetry within the spatial domain. One fundamental inquiry explored in this paper pertains to a function referred to as the Fundamental Function. This function is defined on the spectrum of the subspace spanned by $T_k(\phi)$, where k is an element of the set of integers. The authors aim to elucidate the geometric significance of the Fundamental Function and to understand its relationship with the conventional refinable function.

C. Paper 3: Atreas. Et.al "On the design of multi Dimensional Compactly supported Paseval Framelets with directional characteristics"

In their paper "On the Design of Multi-Dimensional Compactly Supported Parseval Framelets with Directional Characteristics," Atreas et al. introduce a novel methodology for constructing wavelet-like families of affine frames. These multi-dimensional frames possess distinct directional characteristics, compact spatial support, directional vanishing moments, and axial symmetries or anti-symmetries. The innovative, non-homogeneous family they conceived, termed framelets, originates from readily accessible refinable functions.

The potential applications of these Parseval framelets are wide-ranging: they can discern edges, textures, and surfaces of singularities with high sensitivity in pre-selected orientations. Notably, due to the challenges associated with constructing refinable functions with stable integer shifts, they restricted their attention to refinable functions whose integer shifts form a Bessel family. Their mathematical framework allows for control over the redundancy of the affine family by the number of singular values equal to 1 of certain matrices, which corresponds to the number of high pass filters.

II. Main Task

Initially, I introduced Gaussian noise to the original image, with a variance of $noise_var = 0.08$. The code for this process is contained within the m-file ”$maincodeyoussaf.m$”. Subsequently, I employed wavelet techniques to remove the noise from the image.

A. The efficacy of 5 different types of wavelet systems and their Mean Squared Error (MSE)

Wavelet	Efficacy	MSE
"haar"	44.301	2.4154
"coif3"	44.95	2.0801
"db3"	44.898	2.1049
"sym3"	44.898	2.1049
"sym4"	44.921	2.0942
"db4"	44.916	2.0964

Fig. 1. The efficacy of the 5 Wavelet Systems

Fig. 2. Original Image

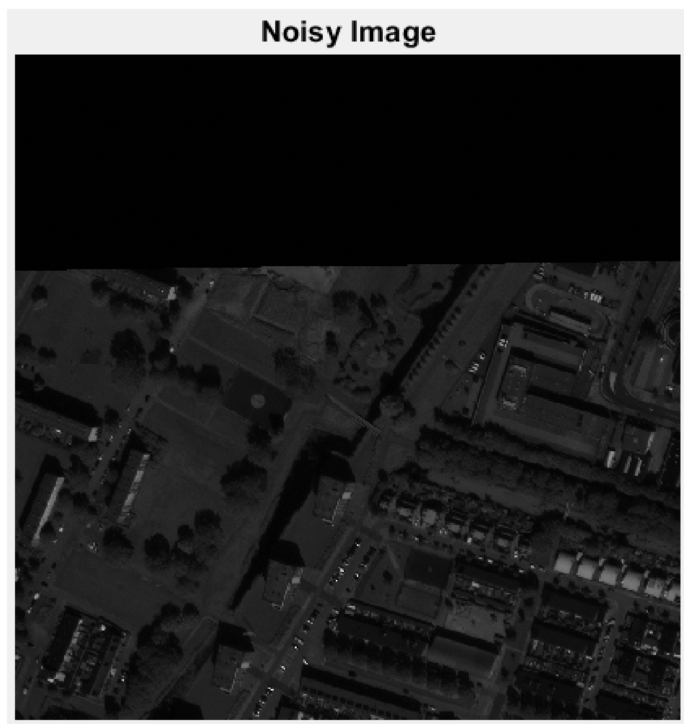

Fig. 3. Noisy Image

Where,

- ”$harr$” stands for Haar wavelet.
- ”$coif3$” denotes Coiflets wavelet.
- ”$db3$” means Daubechies wavelet.
- ”$sym3$” is Symlets wavelet.
- ”$sym4$” stands for Symlets wavelet as well.
- ”$db4$” denotes Daubechies wavelet.

B. Application

Now, I run my code and use it on the provided Matlab image $'PassiveOptical_Image.mat'$. My code finds the variable name inside the $.matfile$. Then, convert to double precision for computations and rescale to range $[0, 255]$ $image = im2double(image)$.
We get

- The best wavelet is $coif3$ with an efficacy of 44.949938
- The MSE of the denoised image using coif3 is 2.080111

Here is the original image we have, This what we get after applying the Guassian noise, Then, we denoisy it, Now, let us try $noise_var = 0.2$ i.e we make the image nosier. Then, we get these values for the efficacy of wavelets, For this noisy image, Then, we denoise it to get this image We observe that

Fig. 4. Denoised Image

Wavelet	Efficacy	MSE
"haar"	43.423	2.9563
"coif3"	44.063	2.5512
"db3"	44.028	2.572
"sym3"	44.028	2.572
"sym4"	44.031	2.5704
"db4"	44.044	2.5625

Fig. 5. Wavelet Efficacy

Fig. 6. Noisy Image

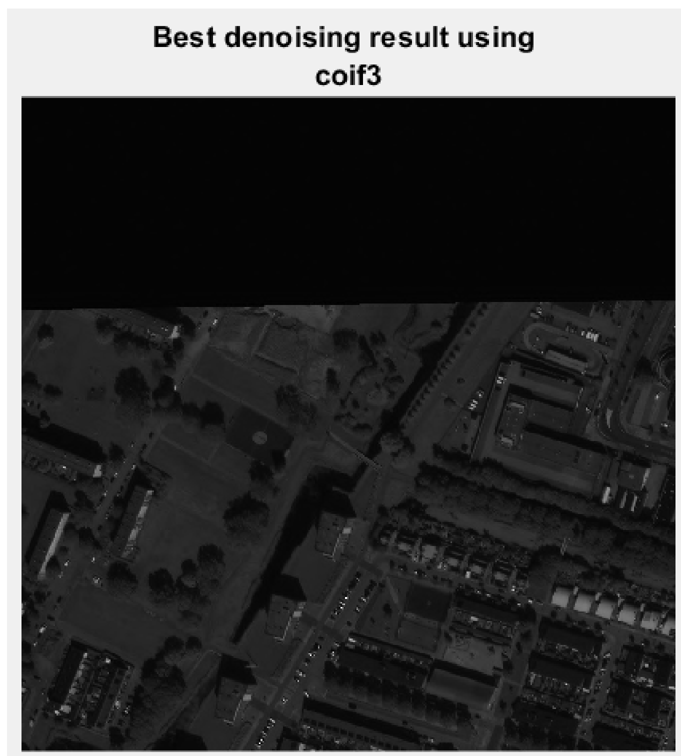

Fig. 7. Denoised Image

- The best wavelet is $coif3$ with an efficacy of 44.063314
- The MSE of the denoised image using $coif3$ is 2.551226

III. The Optional Task I: Biomedical Images

Let us apply the same theory on image $s1channel2.tif$. So, here I modified my code because we changed the type of the input.

We apply the Gaussin noise $noisevar = 0.2$, and this is the noisy image Then, by using Wavelets to denoise the above

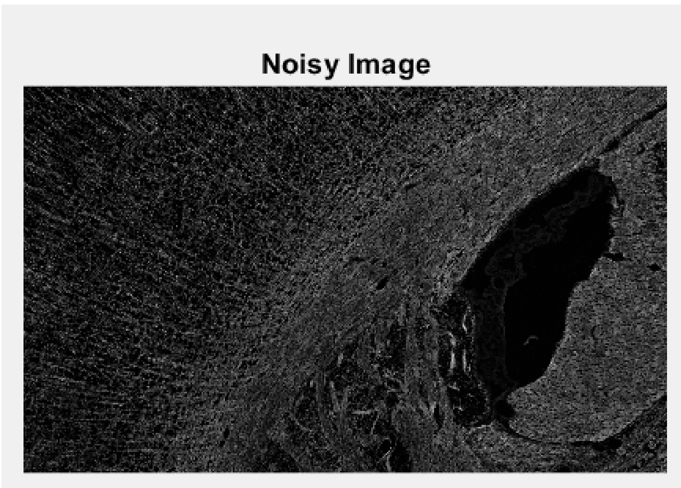

Fig. 8. Noisy Image

image, we get the following wavelet efficacy values And this

Wavelet	Efficacy	MSE
"haar"	24.016	257.9
"coif3"	25.571	180.29
"db3"	25.237	194.72
"sym3"	25.237	194.72
"sym4"	25.422	186.58
"db4"	25.388	188.07

is the denoised image We observe the following remarks:

- The best wavelet is $coif3$ with an efficacy of 25.570979
- The MSE of the denoised image using $coif3$ is 180.294473.

IV. The Optional Task II: Biomedical Images

For the second optional image $s1channel4.tif$, we do the same steps as before and here you are the results, We apply the Gaussin noise $noisevar = 0.01$, and this is the noisy image

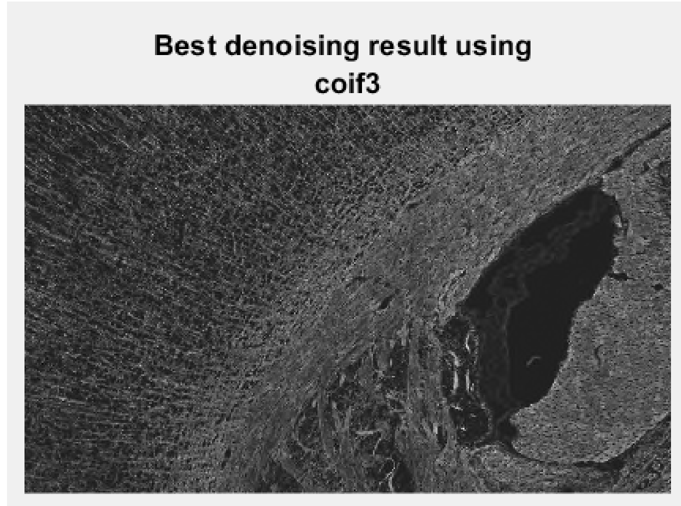

Fig. 9. Denoised Image

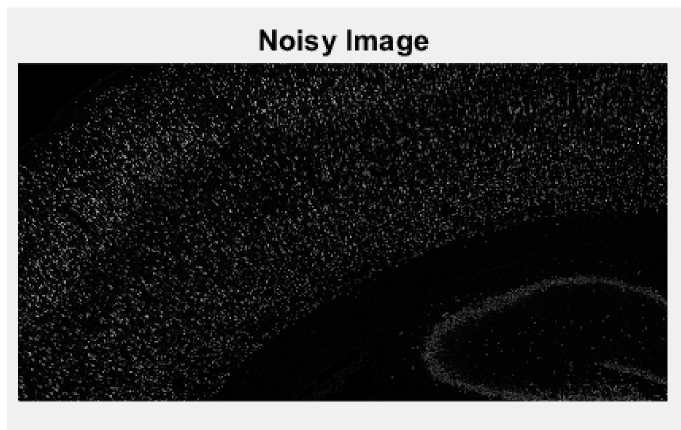

Fig. 10. Noisy Image

Then, by using Wavelets to denoise the above image, we get the following wavelet efficacy values And this is the denoised image We observe the following remarks:

- The best wavelet is $haar$ with an efficacy of 41.864783.
- The MSE of the denoised image using $haar$ is 4.232550.

Acknowledgment

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References

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[2] S. Mallat, "a Wallet tour of signal processing The Sparse way," Third edi. Elsevier 2009 (references)

Wavelet	Efficacy	MSE
"haar"	41.865	4.2326
"coif3"	41.519	4.5832
"db3"	41.364	4.7501
"sym3"	41.364	4.7501
"sym4"	41.512	4.5911
"db4"	41.369	4.744

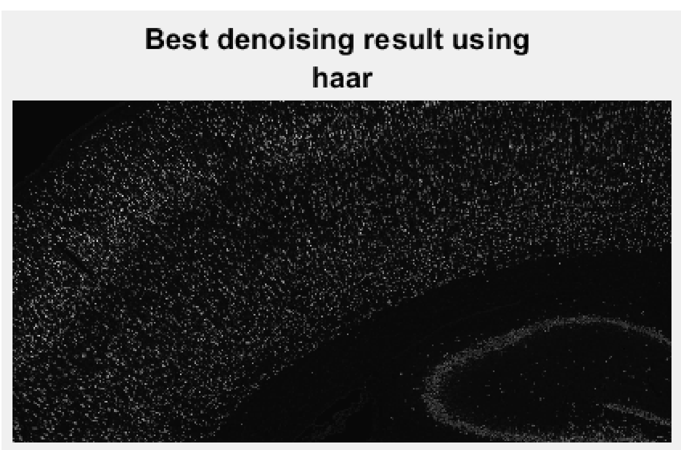

Fig. 11. Denoised Image