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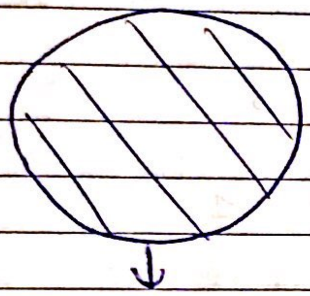
HW # 4 | Stochactics and Biology

Pb # 01,

We would like to share 1,000,000 USD between two players, sons.

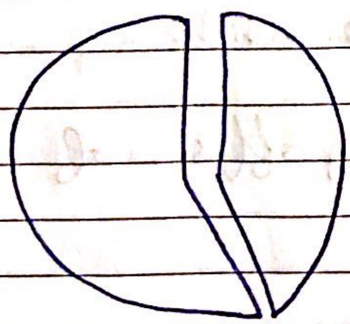
Where s_1 denotes the sum of money for son 1

And s_2 denotes the sum of money for son 2.



1,000,000 USD

The idea of cooperation and every is satisfy is that one player divides the amount like



then the other one chooses. In a way that everyone is satisfy.

Hence the best way is 50:50

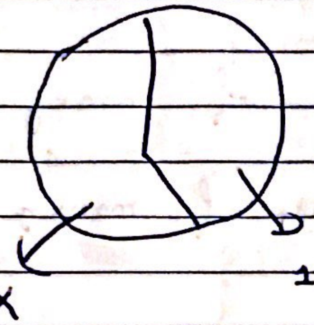
So $\boxed{S_1 = S_2 = 500,000 \text{ USD}}$

Therefore Nash equilibrium is

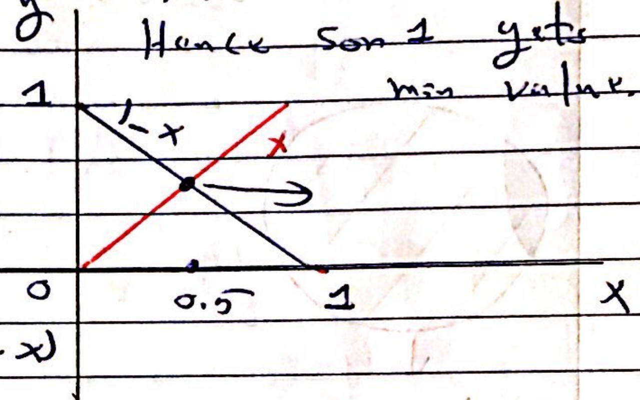
(500,000 ; 500,000)

Mathematically

Son 1 cuts the amount to



Then the 2^{nd} player chooses the max between $(x, 1-x)$. Hence Son 1 gets the min value.



So for every to be satisfy

$$\min(x, 1-x) = \max(x, 1-x)$$

$$\Rightarrow x = 1 - x \Rightarrow \boxed{x = \frac{1}{2} = 0.5 = 50\%}$$

Hence both son get 50% of the money.

Therefore, Nash equilibrium is $\left(\frac{1}{2}, \frac{1}{2}\right)$ with ~~rewards~~ payoffs of (500,000; 500,000)

Pb#02.

The payoff matrix is given by R on a single ground: mutual cooperation.

P: payoff matrix for mutual defection.

T: in case only one player cheats and

S to the cooperating player.

Assumption, $T > R > P > S$

$\boxed{a}$ We consider two strategies GRIM and ALLD such that,

GRIM, ~~except~~ cooperates on the 1^{st} move and then cooperates as long as the opponent does not defect. However, if the opponent defects, GRIM will always defect afterward.

ALLD: always defect.

We suppose that two opponents play the game repeatedly for m rounds

For the 1^{st} round,

	Player 02
Player 01	

	Player 02	
Player 01	GRIM	ALLD
GRIM	(R, R)	(S, T)
ALLD	(T, S)	(P, P)

The payoff matrix,

$$A = \begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

Suppose a small fraction, p, of the population uses strategy GRIM.

The expected payoff to ALLD is ; $Tx + (1-x)P$

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix}\begin{pmatrix} x \\ 1-x \end{pmatrix} = \begin{pmatrix} Rx + S(1-x) \\ Tx + P(1-x) \end{pmatrix}$$

If player 01 uses strategy ALLD all the time

i.e $x = 1$. Then the expected payoff to ALLD is : T

Now, as player 02 knew that the first player cheated, GRIM becomes ALLD on the next '$m-1$' rounds.

i.e

		Player 02	
		GRIM	ALLD
Player 01	GRIM	(R, R)	(P, P)
	ALLD	(P, P)	(P, P)

$$\begin{pmatrix} R & P \\ P & P \end{pmatrix} \begin{pmatrix} x \\ 1-x \end{pmatrix} = \begin{pmatrix} Rx + (1-x)P \\ Px + (1-x)P \end{pmatrix}$$

Hence, the expected payoff to ALLD for "$m-1$"

i.e $x=1$ is $\boxed{(m-1)xP + (m-1)(1-x)P = (m-1)P}$

Therefore, the expected payoff to ALLD

for m steps is $\boxed{T + (m-1)P}$.

(b) Strategy GRIM is stable against invasion of ALLD if the expected payoff to GRIM > The expected payoff to ALLD

i.e $mR > T + (m-1)P$

$\Rightarrow mR > T + mP - P$

$\Rightarrow m(R-P) > T - P$

$\Rightarrow \boxed{m > \dfrac{T-P}{R-P}}$

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$\boxed{c}$

We introduce a new strategy $GRIM^*$, which is the same as GRIM, but always defects on the last round.

For the first $m-1$ rounds, we get

	Player 2: $GRIM^*$	Player 2: GRIM
Player 1: $GRIM^*$	(R, R)	(R, R)
Player 1: GRIM	(R, R)	(R, R)

$$\boxed{A_{m-1} = \begin{pmatrix} R & R \\ R & R \end{pmatrix}}$$

The payoff matrix

Hence, the payoff expected to $GRIM^*$ is $(m-1)R$

For the last round, we get

	Player 2: $GRIM^*$	Player 2: GRIM
Player 1: $GRIM^*$	(P, P)	(T, S)
Player 1: GRIM	(S, T)	(R, R)

$$\boxed{A_1 = \begin{pmatrix} P & T \\ S & R \end{pmatrix}}$$

The payoff matrix

The expected payoff to $GRIM^*$ is T.

Therefore the expected payoff to $GRIM^*$ for m rounds is $\boxed{T + (m-1)R}$

The expected payoff to GRIM for the 1^{st} $(m-1)$ rounds is $(m-1)R$.

The expected payoff to GRIM on the last round is S.

Then the expected payoff to GRIM for m rounds is $\boxed{S+(m-1)R}$.

Since $T>S$, then

$$T+(m-1)R > S+(m-1)R$$

Therefore, $GRIM^*$ dominates GRIM.

$\boxed{d}$ Following the same argument as in $\boxed{c}$, we define,

$GRIM^{**}$, which is GRIM for the first "$m-2$" rounds, but it becomes ALLD on the last two rounds.

Hence ~~the~~ for the first "$m-2$" round, we get

Player 02

		GRIM**	GRIM*
Player 01	GRIM**	(R, R)	(R, R)
	GRIM*	(R, R)	(R, R)

The payoff matrix is $A_{m-2} = \begin{pmatrix} R & R \\ R & R \end{pmatrix}$

The expected payoff to GRIM** is $(m-2)R$.

Similarly, the payoff to GRIM* is $(m-2)R$.

Now, the "m-1" round, we get

Player 02

		GRIM**	GRIM+
Player 01	GRIM**	(P, P)	(T, S)
	GRIM*	(S, T)	(R, R)

$$A_2 = \begin{pmatrix} P & T \\ S & R \end{pmatrix}$$

Hence, the expected payoff to GRIM** is

T.

And, the expected payoff to GRIM* is

S.

Finally, the last round gives

		Player 02	
		GRIM**	GRIM*
Player 01	GRIM**	(P,P)	(P,P)
	GRIM*	(P,P)	(P,P)

$$A_1 = \begin{pmatrix} P & P \\ P & P \end{pmatrix}$$

Hence, the expected payoff to GRIM** = the expected payoff to GRIM* = P

Since $T > S$, then

The expected payoff to GRIM** for "m" rounds $= (m-2)R + P + T$ $>$ The expected payoff to GRIM* for "m" rounds $= (m-2)R + P + S$

Therefore, the new strategy, GRIM** dominates against GRIM*.

$\boxed{e}$ If we continue this argument, we will get ALLD strategy, i.e. defecting all the time.

$\boxed{\text{Pb \# 03}}$

$\boxed{a}$ Let δ be the probability of the game continuing after each round. Hence $1-\delta$ is the probability of the game to stop.

m is the number of games that occur.

If m $\longrightarrow$ geometric distribution, with

Probability of success $= 1-\delta =$ "when the game stop".

Probability of failure $= \delta =$ " Keep going".

Then the expected number of rounds of this geometric r.v $= \langle m \rangle = \dfrac{1}{1-\delta}$

$$= \frac{1-\delta+\delta}{1-\delta}$$

$$= 1 + \frac{\delta}{1-\delta}$$

$\boxed{b}$ We consider a matchup between GRIM and ALLD. Then the payoff matrix is

$$
\begin{array}{ccc}
 & \text{GRIM} & \text{ALLD} \\
\text{GRIM} & & \\
\text{ALLD} & &
\end{array}
\quad
A = \begin{pmatrix} R\sum_{k=0}^{m-1} \delta^k , & \delta S + \sum_{k=1}^{m-1} \delta^k P \\ T + \sum_{k=1}^{m-1} \delta^k P , & P\sum_{k=0}^{m-1} \delta^k \end{pmatrix}
$$

For $m \to \infty$

$$A_\infty = \begin{pmatrix} \frac{R}{1-\delta} , & \delta S + P\frac{\delta}{1-\delta} \\ T + \frac{\delta P}{1-\delta} , & \frac{P}{1-\delta} \end{pmatrix}$$

$$\lim_{m\to\infty} \sum_{k=1}^{m-1} \delta^k = \lim_{m\to\infty} \sum_{k=0}^{m-1} \delta^k - 1 = \frac{1}{1-\delta} - 1 = \boxed{\frac{1-1+\delta}{1-\delta} = \frac{\delta}{1-\delta}}$$

The expected payoff for GRIM vs GRIM $= \frac{R}{1-\delta}$

The expected payoff for GRIM vs ALLD $= \delta S + \frac{\delta P}{1-\delta}$

The expected payoff for ALLD vs GRIM $= T + \frac{\delta P}{1-\delta}$

The expected payoff for ALLD vs ALLD $= \frac{P}{1-\delta}$

For $\boxed{\delta < 1}$

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C

GRIM is stable against ALLD if

The expected payoff of GRIM vs GRIM > The expected payoff to ALLD vs GRIM

i.e $\frac{R}{1-\delta} > T + \frac{\delta P}{1-\delta}$

$\Rightarrow R > (1-\delta)T + \delta P = T + \delta(P-T)$

$\Rightarrow \delta(T-P) > T-R$

$\Rightarrow \boxed{\delta > \frac{T-R}{T-P}}$

Hence GRIM is stable against ALLD

if $\boxed{\delta > \frac{T-R}{T-P}}$.

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Pb #⑭ | Remark: For this pb, I work side to side with ALan

Now we consider an infinitely repeated game with execution errors for each player/round with proba ε.

(i) strategy ①: Tit-for-Tat (TFT) which has strategy $P_T = \{1-\varepsilon, \varepsilon, 1-\varepsilon, \varepsilon\}$

(ii) Strategy ②: GRIM which has strategy $P_G = \{1-\varepsilon, \varepsilon, \varepsilon, \varepsilon\}$

(iii) Always cooperate ALLC which has strategy $P_C = \{1-\varepsilon, 1-\varepsilon, 1-\varepsilon, 1-\varepsilon\}$

[a] Writting the 6 transition matrices for the 6 pairwise: (TFT, GRIM), (TFT, TFT), (TFT, ALL C), (GRIM, ALL C), (GRIM, GRIM) (ALLC, ALLC).

We use the definition in the notes, then,

$$\Pi_{TFT-GRIM} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \\ \varepsilon(1-\varepsilon), & (1-\varepsilon)^2, & \varepsilon^2, & \varepsilon(1-\varepsilon) \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \end{pmatrix}$$

$$\Pi_{TFT-TFT} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \\ \varepsilon(1-\varepsilon), & (1-\varepsilon)^2, & \varepsilon^2, & \varepsilon(1-\varepsilon) \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \end{pmatrix}$$

$$\Pi_{TFT-ALLC} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \\ (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \end{pmatrix}$$

$$\Pi_{GRI-ALLC} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \\ \varepsilon(1-\varepsilon), & \varepsilon^2, & (1-\varepsilon)^2, & \varepsilon(1-\varepsilon) \end{pmatrix}$$

$$M_{GRIM-GRIM} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \\ \varepsilon^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & (1-\varepsilon)^2 \end{pmatrix}$$

$$M_{ALLC-ALLC} = \begin{pmatrix} (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \\ (1-\varepsilon)^2, & \varepsilon(1-\varepsilon), & \varepsilon(1-\varepsilon), & \varepsilon^2 \end{pmatrix}$$

$\boxed{b}$ Finding the stationary distribution for each of the 6 chains in terms of ε.

Here, we develop a technique.

Let $M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}$

Then

$$\pi = (\pi_1, \pi_2, \pi_3, \pi_4) \; M = (\pi_1, \pi_2, \pi_3, \pi_4)$$

Where $\boxed{\sum_{j=1}^{4} \Pi_j = 1}$

Then,

$$\Pi_j = \sum_{i=1}^{4} m_{ij} \Pi_i \quad ; \quad j = 1, \ldots, 4$$

We substitute Π_1 by $\Pi_1 = 1 - \Pi_2 - \Pi_3 - \Pi_4$ for each Π_j $j = 2, 3, 4$. Therefore,

$$\Pi_2 = m_{12} - m_{12}\Pi_2 \boxed{- m_{12}\Pi_3} - m_{12}\Pi_4 + m_{22}\Pi_2 + \boxed{+ m_{32}\Pi_3} + m_{42}\Pi_4$$

$$\Rightarrow \boxed{(1 + m_{12} - m_{22})\Pi_2 + (m_{12} - m_{32})\Pi_3 + (m_{12} - m_{42})\Pi_4 = m_{12}}$$

$$\Pi_3 = m_{13}(1 - \Pi_2 \boxed{-\Pi_3} - \Pi_4) + m_{23}\Pi_2 \boxed{+ m_{33}\Pi_3} + m_{43}\Pi_4$$

$$\Rightarrow \boxed{(m_{13} - m_{23})\Pi_2 + (1 + m_{13} - m_{33})\Pi_3 + (m_{13} - m_{43})\Pi_4 = m_{13}}$$

$$\Pi_4 = m_{14}(1 - \Pi_2 \boxed{-\Pi_3} - \Pi_4) + m_{24}\Pi_2 + \boxed{m_{34}\Pi_3} + m_{44}\Pi_4$$

$$\boxed{(m_{14} - m_{24})\Pi_2 + (m_{14} - m_{34})\Pi_3 + (1 + m_{14} - m_{44})\Pi_4 = m_{14}}$$

Therefore we get

$$\underbrace{\begin{pmatrix} 1+m_{12}-m_{22}, & m_{12}-m_{32}, & m_{12}-m_{42} \\ m_{13}-m_{23}, & 1+m_{13}-m_{33}, & m_{13}-m_{43} \\ m_{14}-m_{24}, & m_{14}-m_{34}, & 1+m_{14}-m_{44} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \pi_2 \\ \pi_3 \\ \pi_4 \end{pmatrix}}_{u} = \underbrace{\begin{pmatrix} m_{12} \\ m_{13} \\ m_{14} \end{pmatrix}}_{b}$$

Matlab $\longrightarrow$ $\begin{cases} u = A\backslash b \\ u(4) = 1 - \text{sum}(u) \end{cases} \Longrightarrow \boxed{\pi = u}$

Algorithm

→ Input the matrix M for each of 6 transition matrices in (a).

→ Solve $Au = b \Longrightarrow u = A\backslash b$
$u(4) = 1 - \text{sum}(u)$

→ $\boxed{\pi = u}$

Note: Please check the attached m.file for these computations.

By using the algorithm we got,

$$\Pi_1 = \left[\frac{1}{2(2\varepsilon-3)} + \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{-1}{2(2\varepsilon-3)} \right]$$

$\varepsilon \to 0$ $\quad \Pi_1 = \left[\frac{1}{2} - \frac{1}{6}, \frac{1}{4}, \frac{1}{4}, +\frac{1}{6} \right]$

TFT_TFT

$$\boxed{\Pi_1 = \left[\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6} \right]}$$

$$\Pi_2 = \left[\frac{2\varepsilon(\varepsilon-1)}{2\varepsilon+1} - 2\frac{2\varepsilon^3-2\varepsilon^2+\varepsilon}{2\varepsilon+1} - \frac{\varepsilon}{2\varepsilon+1} + 1, \right.$$

$$\left. -\frac{2\varepsilon(\varepsilon-1)}{2\varepsilon+1}, \frac{\varepsilon}{2\varepsilon+1}, \frac{2(2\varepsilon^3-2\varepsilon^2+\varepsilon)}{2\varepsilon+1} \right]$$

$\varepsilon \to 0$

$$\boxed{\Pi_2 = [1, 0, 0, 0]}$$

TFT-GRIM

$$\Pi_3 = \left[\frac{\varepsilon(-2\varepsilon^3+\varepsilon^2+\varepsilon-1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1} + \frac{2\varepsilon^2(\varepsilon-1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1} + \right.$$

$$+ \frac{2\varepsilon(\varepsilon-1)(2\varepsilon^3-3\varepsilon^2+1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1} + 1, \frac{-\varepsilon(-2\varepsilon^3+\varepsilon^2+\varepsilon-1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1},$$

$$\left. \frac{-2\varepsilon(\varepsilon-1)(2\varepsilon^3-3\varepsilon^2+1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1}, \frac{-2\varepsilon^2(\varepsilon-1)}{4\varepsilon^4-4\varepsilon^3-\varepsilon^2+\varepsilon+1} \right]$$

TFT- ALLC

$\varepsilon \longrightarrow 0$

$\pi_3 = [1, 0, 0, 0]$

$$\pi_4 = \left[\frac{2\varepsilon(\varepsilon-1)}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3} - \frac{4\varepsilon^4 - 6\varepsilon^3 + 2\varepsilon^2 + \varepsilon}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3} + \right.$$

GRIM - ALLC

$$+ \frac{2(2\varepsilon+1)(\varepsilon-1)(\varepsilon^2 - 2\varepsilon + 1)}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3} + 1, \frac{4\varepsilon^4 - 6\varepsilon^3 + 2\varepsilon^2 + \varepsilon}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3},$$

$$\left. - \frac{2(2\varepsilon+1)(\varepsilon-1)(\varepsilon^2 - 2\varepsilon + 1)}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3}, \frac{-2\varepsilon(\varepsilon-1)}{8\varepsilon^3 - 12\varepsilon^2 + 2\varepsilon + 3} \right]$$

$\varepsilon \longrightarrow 0$

$$\pi_4 = \left[1 - \frac{2}{3}, 0, \frac{2}{3}, 0 \right] = \left[\frac{1}{3}, 0, \frac{2}{3}, 0 \right]$$

$$\pi_5 = \left[2\varepsilon(\varepsilon-1) - 2\varepsilon + 5\varepsilon^2 - 4\varepsilon^3 + 1, -\varepsilon(\varepsilon-1), -\varepsilon(\varepsilon-1), 4\varepsilon^3 - 5\varepsilon^2 + 2\varepsilon \right]$$: GRIM_GRIM

$\varepsilon \longrightarrow 0$ $\quad \pi_5 = [1, 0, 0, 0]$

$$\pi_6 = \left[2\varepsilon(\varepsilon-1) - \varepsilon^2 + 1, -\varepsilon(\varepsilon-1), -\varepsilon(\varepsilon-1), \varepsilon^2 \right]$$

$\varepsilon \longrightarrow 0$

$\pi_6 = [1, 0, 0, 0]$: ALLC_ALLC

π_j is the stationary distribution for the indicated pair for $j = 1, 2, 3, 4, 5, 6$.

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(c) Using the stationary distribution of (b) to calculate the payoff for each pair

$$\Pi_1 = \Pi_{TFT,TFT} = \begin{array}{c} \begin{array}{cccc} cc & cd & dc & dd \end{array} \\ \left[\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6}\right] \end{array}$$

		Player 2	
		C	D
Player 1	C	R	S
	D	T	P

$$\boxed{\text{The payoff} = \frac{1}{3}R + \frac{1}{4}S + \frac{1}{4}T + \frac{1}{6}P}$$

Similarly, $\Pi_2 = \Pi_{TFT, GRIM} = \begin{array}{c} \begin{array}{cccc} cc & cd & dc & dd \end{array} \\ [1, 0, 0, 0] \end{array}$

$$\boxed{\text{The payoff} = 1.R = R}$$

$\Pi_3 = \Pi_{TFT\_ALLC} = \begin{array}{c} \begin{array}{cccc} cc & cd & dc & dd \end{array} \\ [1, 0, 0, 0] \end{array}$

$$\boxed{\text{The payoff} = 1.R = R} = R$$

$\Pi_4 = \Pi_{GRIM\_ALLC} = \left[\frac{1}{3}, 0, \frac{2}{3}, 0\right]$

$$\boxed{\text{The payoff} = \frac{1}{3}R + \frac{2}{3}T}$$

$\Pi_5 = \Pi_{GRIM\_GRIM} = [1, 0, 0, 0]$

$$\boxed{\text{The payoff} = 1.R = R}$$

$\Pi_6 = \Pi_{ALLC-ALLC} = [1, 0, 0, 0]$

$$\Rightarrow \boxed{\text{The payoff} = 1.R = R}$$

(d) Nash Equilibrium

Player 01 \ Player 02	TFT	GRIM
TFT	$\frac{1}{3}R + \frac{1}{4}S + \frac{1}{4}T + \frac{1}{6}P$	R
GRIM	R	R

$\Rightarrow$ we have 3 Nash equilibrium

(GRIM, TFT), (GRIM, GRIM), (TFT, GRIM)

Player 01 \ Player 02	TFT	ALLC
TFT	$\frac{1}{3}R + \frac{1}{4}S + \frac{1}{4}T + \frac{1}{6}P$	R
ALLC	R	R

$\Rightarrow$ The same Nash equilibrium points.

Player 01 \ Player 02	ALLC	GRIM
ALLC	R	$\frac{1}{3}R + \frac{2}{3}T$
GRIM	$\frac{1}{3}R + \frac{2}{3}T$	R

$\Rightarrow$ Two Nash equilibrium points (GRIM, ALLC) and (ALLC, GRIM).