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Question 1:

a) $Ax + b$

b) $\nabla g(h(x)) = g'(h(x))\,\nabla h(x)$

c) $f(x) = 1/2\, x^T A x + b^T x \therefore \nabla^2 f(x) = A$, ($\nabla f(x)^T = Ax + b$) from a)

d) $\nabla f(x) = \nabla g(a^T x) \therefore g'(a^T x) a \therefore \nabla^2 f(x) = \nabla^2 g(a^T x) = g''(a^T x) a a^T$

Question 2:

a) Symmetric proof: $A^T = (zz^T)^T = zz^T = A$
value proof: $x^T A x = x^T z z^T x = x^T z (x^T z)^T = (x^T z)^2 \geq 0$

b) $N(A) = \{x \in R^n ; x^T z = 0\}$
$R(A) = R(zz^T) = 1$

c) Symmetric proof: $(BAB^T)^T = BA^T B^T = BAB^T$
value proof: $x^T BAB^T x = (x^T B) A (x^T B)^T \geq 0$

Question 3:

a) $A = TAT^{-1} = AT = TA$ $\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ \vdots & & \ddots \\ 0 & 0 & \lambda_n \end{bmatrix} \therefore [At^{(1)}\ At^{(2)} \ldots At^{(n)}] = [\lambda_1 t^{(1)}\ \lambda_2 t^{(2)} \ldots \lambda_{(n)} t^{(n)}]$

$\boxed{\therefore At^{(i)} = \lambda_i t^{(i)}}$

b) $Au = uAu^T u = uA \therefore A[u^{(1)}\ u^{(2)} \ldots u^{(n)}] = [u^{(1)}\ u^{(2)} \ldots u^{(n)}]$
$[Au^{(1)}\ Au^{(2)} \ldots Au^{(n)}] = [\lambda_1 u^{(1)}\ \lambda_2 u^{(2)} \ldots \lambda_n u^{(n)}]$
$\boxed{Au^{(i)} = \lambda u^{(i)}}$

c) $At^{(i)} = \lambda_i t^{(i)} \therefore (t^{(i)})^T A t^{(i)} = \lambda_i \|t^{(i)}\|_2 = \lambda_i \geq 0$