Lecture 8

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1 Recap

To find the electric field from sources, we can

1. Use Coulomb's Law and superposition
2. Apply Gauss' Law
3. Use potential theory
4. Numerical techniques

2 Electric Scalar Potential

If we move a point charge Q_B in the presence of another point charge Q_A, work done by external agent is

$$W_{\text{ext}} = \int \vec{F} \cdot d\vec{l} = -\int \vec{F}_{AB} \cdot d\vec{l} = -\int_{P_1}^{P_2} Q_B \vec{E}_A \cdot d\vec{l} = \Delta U$$

where U is potential energy. Then we define the voltage as

$$\Delta V = V_2 - V_1 = V_{21} = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$$

and ΔV is the electric scalar potential difference.

Example 2.1. ΔV between two points given a point charge at the origin.

$$\begin{aligned}
\Delta V &= -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} \\
&= -\int_{P_1}^{P_2} \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R \cdot d\vec{l} \\
&= -\frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dR}{R^2} \\
&= \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)
\end{aligned}$$

Where we switch the limits from position to radius as it is the only coordinate that matters. In addition, $\vec{E}$ is conservative, so the path integral can be evaluated directly at the limits.

From the definition of ΔV, if we let R_1 go to infinity, we obtain

$$\Delta V = V_2 = \frac{Q}{4\pi\varepsilon_0 R^2}$$

3 Equipotential Surfaces

A surface which has the same value of V over the entire surface is called an equipotential surface. It can be a real or imaginary surface. Note that all perfect conductors are equipotential surfaces. The electric field is hence always perpendicular to such equipotential surfaces.