Lecture 14

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At equilibrium, a system of two gases would tend to the same pressure and temperature, where entropy is maximised. This means

$$\frac{\partial S_1}{\partial N_1} = \frac{\partial S_2}{\partial N_2}$$

Now we define this as

$$-\frac{\mu}{T} = \frac{\partial S}{\partial N}$$

where μ is the chemical potential, with units of energy. Using our favourite equation

$$S = kN \left[\ln \left(\frac{V}{N} \left(\frac{U}{3N} \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right]$$

Then differentiating and skipping the steps

$$\frac{\mu}{T} = -k \left[\ln \left(\frac{V}{N} \left(\frac{U}{3N} \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right] + kN \frac{5}{2N}$$

$$\mu = -kT \ln \left[\frac{V}{N} \left(\frac{1}{2} kT \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right]$$

$$= -kT \left[\ln \left(\frac{1}{n} \frac{1}{\lambda_{th}^3} \right) + C \right]$$

where C is a constant and n is the number density. With the negative sign, μ increases with n, which makes sense.