# Lecture 18

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## 1 Maxwell Speed Distribution

Pick one particle out of $10^{23}$. The particle is the system, and all the other particles form the reservoir. We know the probability of a microstate is

$$P = \frac{1}{z} e^{-\frac{E}{kT}} = \frac{1}{z} e^{-\frac{mc^2}{2kT}}$$

Then consider a "velocity sphere", with axis $v_x, v_y, v_z$. Then considering the probability of the particle having speed in $[v, v + dv]$,

$$P(v)dv = Ce^{-\frac{mv^2}{2kT}} v^2 dv$$

Since probability is normalised, we can solve for the constant, and

$$P(v)dv = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} dv$$

Then one can also show

$$\left\langle \frac{mv^2}{2} \right\rangle = \int_0^\infty P(v) \frac{mv^2}{2} dv = \frac{3}{2} kT$$

The average energy is (from the last lecture)

$$E = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$

which is approximately $kT$ when $\hbar\omega << kT$ and $\hbar\omega e^{-\frac{\hbar\omega}{kT}}$ when $kT << \hbar\omega$.

$$\frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{\langle E \rangle} = \frac{1}{\sqrt{N}}$$