Lecture 6

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January 20, 2023

1 Review

We can calculate electric field due to static charge for discrete charges

$$\vec{E}_{\text{total}} = \sum_i \frac{Q_i}{4\pi\varepsilon_0|\vec{R}-\vec{R}'|^3}(\vec{R}-\vec{R}')$$

and continuous charges

$$\vec{E}_{texttotal} = \int \frac{dQ'}{4\pi\varepsilon_0|\vec{R}-\vec{R}'|^3}(\vec{R}-\vec{R}')$$

2 Fundamental Postulates of Electrostatics

In differential form, they are

$$\vec{\nabla} \times \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

or in integral form

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$
$$\oiint_S \vec{D} \cdot d\vec{S} = Q_{\text{enc}}$$

where the first equation comes from the fact that

$$\vec{E} = \vec{\nabla} V$$

and the curl of a gradient is zero.

ε is the electrical permittivity of the material, with

$$\varepsilon = \varepsilon_r \varepsilon_0$$

The value of ε_r in vacuum is 1, and it is 1.0006 in air, which is often approximated as 1. This relates $\vec{D}$ and $\vec{E}$

$$\vec{D} = \varepsilon \vec{E}$$

It is possible for ε to vary over a region, e.g. for an anisotropic crystal.

Example 2.1. For the electric flux density given by $\vec{D} = \frac{5}{4}\left(R^2 - \frac{1}{R^2}\right)\hat{a}_R$ for $R < 2\text{m}$. Use Gauss's law to determine the volume charge density in this region.
The gradient in spherical coordinates is given by

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{R^2}\frac{\partial(R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Only the first term is nonzero, as $\vec{D}$ is a function of R only. Substituting, we have

$$\begin{aligned}\rho_v &= \frac{1}{R^2}\frac{\partial}{\partial R}\left(\frac{5}{4}(R^4 - 1)\right)\\ &= \frac{1}{R^2}(5R^3)\\ &= 5R\end{aligned}$$

Note that from Coulomb's law, we have

$$\vec{D} = \varepsilon\vec{E} = \frac{Q}{4\pi R^2}\hat{a}_R$$

Then integrating over the surface of a sphere,

$$\oiint_S \vec{D}\cdot d\vec{S} = \oiint_S \frac{Q}{4\pi R^2} dS = Q$$