Lecture 12

niceguy

February 6, 2023

1 Binomial Distribution

For n coin flips with $P(H) = p$, we want the probability of getting k heads in n flips.

$$b(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

The latter part $p^k(1-p)^{n-k}$ gives the probability of k heads out of n flips in a particular sequence. The first part $\binom{n}{k}$ gives the total number of such sequences. Since con flips are independent, we simply multiply both to obtain the desired probability.

2 Hypergeometric Distribution

We assume replacement. We define multinomials similar to binomals, but with more states than 2 (e.g. a deck of 52 cards instead of a coin with 2 sides).

Example 2.1. Given 52 cards, we want the probability of getting 3 red cards in 5 draws.
There are $\binom{52}{5}$ 5 card hands. The number of 5 card hands is $\binom{26}{3} \times \binom{26}{2}$, the number of ways to choose 3 red cards then 2 black cards. The required probability is then

$$\binom{26}{3}\binom{26}{2} \div \binom{52}{5}$$

With N objects, n samples, k successes out of N, then the chances of x successes and $n - x$ failures is

$$h(x;N,n,k) = \frac{\binom{k}{x}\binom{n-k}{n-x}}{\binom{N}{n}}$$

This is valid for $0 \leq x \leq n, x \leq k, x \geq n-(N-k)$. Then

$$\mu = \frac{nk}{N}$$

and

$$\sigma^2 = \frac{kn(N-n)}{N(N-1)}\left(1-\frac{k}{N}\right)$$

3 Negative Binomial

We want to know how many times we have to flip a coin to get k heads.

$$b*(x;k,p) = \binom{x-1}{k-1}p^k(1-p)^{n-k}$$

Similar to the binomial case, $\binom{x-1}{k-1}$ gives the total number of sequences, and $p^k(1-p)^{n-k}$ gives the total number of sequences.

4 Geometric Distribution

It is a negative binomial with $k=1$, i.e. how many trials are needed for the first success. Then

$$g(x;p) = p(1-p)^{k-1}$$

The mean and variance are

$$\mu = \frac{1}{p}$$

$$\sigma^2 = \frac{1-p}{p^2}$$

Example 4.1. We are playing a game against someone better. The probability of lising is 0.9. Then the distribution is

$$g(x) = 0.1(0.9)^{x-1}$$

And the first win is expected on game 10.