## Lecture 6

**Boolean Identities:**

$+$ and $\cdot$ are commutative, associative, and distributive

**Any SOP circuit can be implimented as NAND NAND**

- Example: $f = x_1 x_2 + x_2 x_3$ can be rewritten as $f = (x_1 \text{ NAND } x_2) \text{ NAND } (x_2 \text{ NAND } x_3)$

**Any POS circuit can be implemented as NOR NOR**

- Example: $f = (x_1 + x_2)(x_2 + x_3)$ can be rewritten as $f = (x_1 \text{ NOR } x_2) \text{ NOR } (x_2 \text{ NOR } x_3)$

**Design Example**

Gumball factory - $s_2$ normally gives 0, but $s_2 = 1$ if gumball is too large - $s_1$ normally gives 0, but $s_1 = 1$ if gumball is too small - $s_0$ normally gives 0, but $s_0 = 1$ if gumball is too light
Synthesise a logic function $f = 1$ when a gumball is either too large, or both too small and too light - By inspection: $s_2 + s_1 s_0$ - From truth table:

$$
\begin{aligned}
f &= \overline{s_2} s_1 s_0 + s_2 \overline{s_1 s_0} + s_2 \overline{s_1} s_0 + s_2 s_1 \overline{s_0} + s_2 s_1 s_0 \\
&= s_1 s_0 + s_2 \overline{s_1} + s_2 \overline{s_0} \\
&= s_1 s_0 + s_2 (\overline{s_1} + \overline{s_0}) \\
&= s_1 s_0 + s_2 (\overline{s_1 s_0}) \\
&= s_1 s_0 + s_2
\end{aligned}
$$

**Minimal POS Example**

Derive a minimal POS for $f(x_1, x_2, x_3) = \prod M(0, 2, 4)$

$$
\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + \overline{x_2} + x_3)(\overline{x_1} + x_2 + x_3) \\
&= (x_1 + x_3)(x_2 + x_3)
\end{aligned}
$$

Using the minterms of $\overline{f}$

$$
\begin{aligned}
\overline{f} &= \overline{x_1 x_2 x_3} + \overline{x_1} x_2 \overline{x_3} + x_1 \overline{x_2 x_3} \\
&= \overline{x_1 x_3} + \overline{x_2 x_3}
\end{aligned}
$$

So

$$
f = \overline{\overline{f}} = \overline{(\overline{x_1 x_3} + \overline{x_2 x_3})} = \overline{(\overline{x_1 x_3})(\overline{x_2 x_3})} = (\overline{\overline{x_1}} + \overline{\overline{x_3}})(\overline{\overline{x_2}} + \overline{\overline{x_3}}) = (x_1 + x_3)(x_2 + x_3)
$$