Lecture 15

niceguy

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1 Thermodynamic Identity for Entropy

We call entropy S the thermodynamic potential (but entropy "wants" to increase while potential "wants" to decrease). If we take the total differential,

$$dS = \frac{\partial S}{\partial U}dU + \frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial N}dN = \frac{1}{T}dU + \frac{p}{T}dV - \frac{\mu}{T}dN \tag{1}$$

This holds under quasistatic infinitesimal changes of U, V, N. This needs to be quasistatic or else we do not have good definitions for T, p, etc, without equilibrium.

2 Thermodynamic Identity for Energy

Rearranging $S = k \ln \Omega$ to put U as the subject, we obtain

$$e^{\frac{S}{kN}} = U^{\frac{3}{2}} f(N, V)$$

where f is a constant function of N and V. Multiplying Equation 1 by T,

$$dU = TdS - pdV + \mu dN$$

Note also U is a function of S, V, N, so

$$dU = \frac{\partial U}{\partial S}dS + \frac{\partial U}{\partial V}dV + \frac{\partial U}{\partial N}dN$$

Then comparing like terms,

$$T = \frac{\partial U}{\partial S}, p = -\frac{\partial U}{\partial V}, \mu = \frac{\partial U}{\partial N}$$