Lecture 8

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Consider two isolated systems, A and B. Total microscopic states of the entire system $C = A + B$ is

$$\Omega_C(q) = \sum_{q'_A=0}^{q} \Omega_A(q'_A)\Omega_B(q - q'_B)$$

Now consider two systems 1 and 2 with the same energy $\frac{E}{2}$. Then if energy Δ is transferred from 1 to 2,

$$P(\Delta) = \frac{\Omega_1\left(\frac{E}{2} - \Delta\right)\Omega_2\left(\frac{E}{2} + \Delta\right)}{\Omega_{1+2}(E)}$$

The most likely value of Δ is

$$\begin{aligned}
\frac{\partial}{\partial \Delta}\left(\Omega_1\left(\frac{E}{2} - \Delta\right)\Omega_2\left(\frac{E}{2} + \Delta\right)\right) &= 0 \\
-\frac{\partial \Omega_1\left(\frac{E}{2} - \Delta\right)}{\partial\left(\frac{E}{2} - \Delta\right)}\Omega_2\left(\frac{E}{2} + \Delta\right) + \Omega_1\left(\frac{E}{2} - \Delta\right)\frac{\partial \Omega_2\left(\frac{E}{2} + \Delta\right)}{\partial\left(\frac{E}{2} + \Delta\right)} &= 0 \\
\frac{1}{\Omega_1\left(\frac{E}{2} - \Delta\right)}\frac{\partial \Omega_1\left(\frac{E}{2} - \Delta\right)}{\partial\left(\frac{E}{2} - \Delta\right)} &= \frac{1}{\Omega_2\left(\frac{E}{2} + \Delta\right)}\frac{\partial \Omega_2\left(\frac{E}{2} + \Delta\right)}{\partial\left(\frac{E}{2} + \Delta\right)} \\
\frac{\partial}{\partial\left(\frac{E}{2} - \Delta\right)}\left(k \ln \Omega_1\left(\frac{E}{2} - \Delta\right)\right) &= \frac{\partial}{\partial\left(\frac{E}{2} + \Delta\right)}\left(k \ln \Omega_2\left(\frac{E}{2} + \Delta\right)\right) \\
\frac{\partial}{\partial E'_1} k \ln \Omega_1(E'_1) &= \frac{\partial}{\partial E'_2} k \ln \Omega_2(E'_2)
\end{aligned}$$

Where E_1' and E_2' are defined as shown. Now we define tempterature as

$$\frac{1}{T} = \frac{\partial}{\partial E}(k \ln \Omega(E))_{N,V}$$

Definition 1.1. Entropy is defined as

$$S(E, N, V) = k \ln \Omega(E, N, V)$$

Now $S \geq 0$ since $\Omega \geq 1$.

Definition 1.2. Temperature is defined as

$$\frac{1}{T} = \left(\frac{\partial S(E, N, V)}{\partial E}\right)_{N,V}$$

2 Einstein Solids

Recall

$$\Omega(N, q) = \frac{(N - 1 + q)!}{(N - 1)q!} \approx \frac{(N + q)!}{N!q!}$$

Taking the log,

$$\ln \Omega(N, q) = \ln(N + q)! - \ln N! - \ln q!$$

We use Stirling's approximation, with $N >> 1$, $\frac{q}{N} >> 1$.