Lecture 15

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1 Normal Distribution

Example 1.1. X is normal with $n(x; 5, 2)$. Find $P(-1 \leq X \leq 4)$.
Using the cumulative distribution function

$$\Phi(x) = \int_{-\infty}^{x} n(t; 0, 1) dt$$

Set $z = \frac{X-5}{2}$. Then Z has $n(z; 0, 1)$, so

$$P(-1 \leq X \leq 4) = P\left(-3 \leq Z \leq -\frac{1}{2}\right) = \Phi\left(-\frac{1}{2}\right) - \Phi(-3) = 0.3072$$

We know that the binomial distribution converges to the Poisson distribution when $n \to \infty, p \to 0, np = \lambda$.
If we have the mean $\mu = np$ and variance $\sigma^2 = np(1-p)$. Letting

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

Then as $n \to \infty$, then the distribution of Z approaches $n(z; 0, 1)$.

2 Gamma Distribution

Definition 2.1. The Gamma function is defined as

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx, \alpha \geq 0$$

Fun facts:

- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
- $\Gamma(n) = (n-1)!$ for $n \in \mathbb{Z}^+$

The Gamma Distribution is

$$F(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The mean is $\mu = \alpha\beta$ and the variance is $\sigma^2 = \alpha\beta^2$.

2.1 Chi-Squared Distribution

With a parameter $v \in \mathbb{Z}^+$,

$$f(x;v) = \begin{cases} \frac{1}{2^{\frac{v}{2}} \Gamma\left(\frac{v}{2}\right)} x^{\frac{v}{2}-1} e^{-\frac{x}{2}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

2.2 Exponential Distribution

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & x \leq 0 \\ 0 & x < 0 \end{cases}$$

Which is the Gamma distribution with $\alpha = 1$, so its mean is $\mu = \beta$ and its variance is $\sigma^2 = \beta^2$.