

Lecture 5

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1 Adiabatic Process

$$\Delta U = Q - p\Delta V = -p\Delta V$$

We also use $U = \frac{f}{2}NkT$, where $f = 3$ for the single atomic case, and $f = 7$ for the diatomic case. Combining gives

$$-p\Delta V = \frac{f}{2}Nk\Delta T$$

Now from the ideal gas law,

$$\Delta(pV) = Nk\Delta T$$

Using the product rule,

$$\begin{aligned} -p\Delta V &= \frac{f}{2}(p\Delta V + V\Delta p) \\ -\left(\frac{f}{2} + 1\right)p\Delta V &= \frac{f}{2}V\Delta p \\ -\frac{\frac{f}{2} + 1}{\frac{f}{2}}\frac{\Delta V}{V} &= \frac{\Delta p}{p} \\ -\frac{\frac{f}{2} + 1}{\frac{f}{2}}d\ln V &= d\ln p \\ -\left(1 + \frac{2}{f}\right)\ln \frac{V_2}{V_1} &= \ln \frac{p_2}{p_1} \\ \left(\frac{V_1}{V_2}\right)^{1+\frac{2}{f}} &= \frac{p_2}{p_1} \\ V_1^{1+\frac{2}{f}}p_1 &= V_2^{1+\frac{2}{f}}p_2 \end{aligned}$$

Thus $pV^{1+\frac{2}{f}}$ is held constant.
 Similarly, we have isohoric processes, with a constant V , and isobaric processes, with a constant p .

2 Heat Capacity

For constant volume,

$$C_V = \left(\frac{\partial U}{\partial T} \right)_{V,N}$$

The first law of thermodynamics simplifies to

$$\Delta U = Q$$

The general definition is then

$$C_{\text{condition}} = \left(\frac{Q}{\Delta T} \right)_{\text{condition}}$$

For constant pressure,

$$C_p = \left(\frac{Q}{\Delta T} \right)_{p,N} = \left(\frac{\Delta U + p\Delta V}{\Delta T} \right)_{p,N} = \left(\frac{\partial U}{\partial T} \right)_{p,N} + \left(\frac{\partial V}{\partial T} \right)_{p,N}$$

Hence $C_p > C_V$ always holds. For ideal gases,

$$U = \frac{f}{2} NkT$$

Substituting, this gives

$$\left(\frac{\partial U}{\partial T} \right)_{p,N} = \left(\frac{\partial U}{\partial T} \right)_{V,N} = \frac{f}{2} Nk$$

and from the ideal gas law,

$$\left(\frac{\partial V}{\partial T} \right)_{p,N} = \frac{Nk}{p}$$

3 Main Postulate

Consider a closed system with fixed energy. Then all accessible microstates of the system are equally likely.