

# Lecture 2

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## 1 Ideal Gas

When plotting pressure against temperature for a gas, the experimental data shows that all such lines meet at a pressure of 0 and temperature of  $-273^{\circ}\text{C}$ . This is called the **absolute zero**.

### 1.1 Derivation

We assume that all molecules move with the same typical average velocity, and the velocity in any given physical volume is isotropically distributed. Then by symmetry,

$$\bar{v}_x = \frac{1}{N} \sum_i v_{x_i} = 0$$

And

$$\overline{v_x^2} = \frac{1}{N} \sum_i v_{x_i}^2 = \bar{v}^2 \neq 0$$

Letting  $L$  be the box length, and  $\Delta t$  such that

$$L = \bar{v} \Delta t$$

The number of molecules hitting the wall in time  $\Delta t$  is half of the molecules in the volume, i.e.

$$\frac{1}{2} \frac{N}{V} LA = \frac{1}{2} \frac{N}{V} \bar{v} \Delta t A$$

then the change in momentum per unit area of the wall in the time interval  $\Delta t$  is

$$2m\bar{v} \times \frac{N}{V} \frac{1}{2} \bar{v} \Delta t A$$

Force is but the change in momentum over time, and pressure is force per area, which gives

$$p = m\bar{v}^2 \frac{N}{V}$$

Comparing this with the ideal gas law,

$$kT = m\bar{v}^2 = \frac{1}{3} m \overline{v^2}$$

Rearranging, we get the kinetic energy

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

This means temperature is a measure of average kinetic energy!  
The root mean squared velocity is

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

## 2 Equipartition Theorem

For classical gasses: At thermodynamic equilibrium of a classical gas at source given  $T$  the average energy of

- a translational degree of freedom is  $\frac{kT}{2}$
- a rotational degree of freedom is  $\frac{kT}{2}$
- a vibrational degree of freedom  $kT$

**Example 2.1.** For a system of 2 atoms, there are 3 degrees of freedom for translation, 2 for rotation, and 1 for vibration.

**Example 2.2.** For  $\text{N}_2$ , average energy is

$$3 \times \frac{kT}{2} + 2 \times \frac{kT}{2} + kT = \frac{7kT}{2}$$