Lecture 2

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1 Counting

Example 1.1.
- Coin flip: $\{H\}$ has 1 element
- Die toss: even has 3 elements
- 2 dice: $6 \times 6 = 36$

Example 1.2. We can multiply options. For 3 appetizers, 4 mains, and 2 desserts, we have $3 \times 4 \times 2 = 24$ choices. If not ordering is also a choice, we have $4 \times 5 \times 3 = 60$ choices.

Example 1.3. To choose a president and vice president out of n people, we have $n(n-1)$ choices.

Example 1.4. How many 4 digit numbers can we make from $\{0, 1, 2, 5, 6, 9\}$?

$$5 \times 4 \times 3 + 2 \times 4 \times 4 \times 3 = 156$$

2 Permutations

Where order matters.

Example 2.1. Given n items, there are $n!$ permutations.

We may also pick r out of n items.

Example 2.2. 2 letters out of a,b,c,d,e,f gives $6 \times 5 = 30$ permutations.

The general formula for the number of permutations is

$$\frac{n!}{(n-r)!}$$

Example 2.3. How many 5 card hands are there from a normal deck?

$$\frac{52!}{47!} = 311875200$$

Example 2.4. Permutations with identical items: a,b,b
There are $\frac{3!}{2!} = 3$ permutations.

3 General Formula

For n items, m types each with n_m entries, the total number of permutations is

$$\frac{n!}{\prod_{i=1}^{m} n_i!}$$

Example 3.1. How many distinct reorderings of ATLANTIC there are?

$$\frac{8!}{2!2!} = 10080$$

Example 3.2. Flip 10 coins in a row. How many sequences have 4 heads?

$$\frac{10!}{4!6!} = 210$$

The probability is then

$$\frac{210}{2^{10}} \approx \frac{1}{5}$$