Lecture 5

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1 Vector Fields

Vector fields describe physical quantities that only have both a magnitude and direction at every point in space.

1.1 Changing Unit Vectors

Example 1.1. At $(-2, 0, 0)$, we have $\hat{a}_r = -\hat{a}_x$ and $\hat{a}_\phi = -\hat{a}_y$

Note that we can rewrite unit vectors in one system in terms of those of another, e.g.

$$\hat{a}_r = \cos\phi \hat{a}_x + \sin\phi \hat{a}_y$$

in the cylindrical coordinate system.

Example 1.2. At $(1, 1, 0)$

$$\hat{a}_r = \frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y$$

and

$$\hat{a}_\phi = -\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y$$

Example 1.3. Consider a cone. Then

$$dA = r\sin\theta d\phi dr$$

In this case,

$$dQ' = \rho_s r \sin\theta d\phi dr$$

and the integration limits are from 0 to 2π and 0 to R

Example 1.4. Consider a cylinder. Then

$$dA = Rd\phi dz$$

Example 1.5. A wire bent into a semicircular shape lies in teh xy-plane and is defined by $r = a, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$. The wire is charged with a non-uniform charge density defined by $\rho_l = \rho \sin\phi$, where ρ is a constant. Determine an expression for the electric field intensity at any point on the z-axis.
Then $dQ = a\rho \sin\phi d\phi$. By symmetry, it only acts along the y-axis. Letting h be the height,

$$\begin{aligned} E_y &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{a\rho \sin\phi d\phi \times (-a\sin\phi)}{(h^2+a^2)^{3/2}} \\ &= \frac{-a^2\rho}{(h^2+a^2)^{3/2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\phi d\phi \\ &= \frac{-a^2\rho\pi}{2(h^2+a^2)^{3/2}} \end{aligned}$$