Lecture 5

niceguy

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1 Adiabetic Process

$$\Delta U = Q - p\Delta V = -p\Delta V$$

We also use $U = \frac{f}{2}NkT$, where $f = 3$ for the single atomic case, and $f = 7$ for the diatomic case. Combining gives

$$-p\Delta V = \frac{f}{2}Nk\Delta T$$

Now from the ideal gas law,

$$\Delta(pV) = Nk\Delta T$$

Using the product rule,

$$-p\Delta V = \frac{f}{2}(p\Delta V + V\Delta p)$$

$$-\left(\frac{f}{2} + 1\right)p\Delta V = \frac{f}{2}V\Delta p$$

$$-\frac{\frac{f}{2} + 1}{\frac{f}{2}}\frac{\Delta V}{V} = \frac{\Delta p}{p}$$

$$-\frac{\frac{f}{2} + 1}{\frac{f}{2}}d\ln V = d\ln p$$

$$-\left(1 + \frac{2}{f}\right)\ln\frac{V_2}{V_1} = \ln\frac{p_2}{p_1}$$

$$\left(\frac{V_1}{V_2}\right)^{1 + \frac{2}{f}} = \frac{p_2}{p_1}$$

$$V_1^{1 + \frac{2}{f}}p_1 = V_2^{1 + \frac{2}{f}}p_2$$

Thus $pV^{1+\frac{2}{f}}$ is held constant.

Similarly, we have isohoric processes, with a constant V, and isobaric processes, with a constant p.

2 Heat Capacity

For constant volume,
$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N}$$

The first law of thermodynamics simplifies to
$$\Delta U = Q$$

The general definition is then
$$C_{\text{condition}} = \left(\frac{Q}{\Delta T}\right)_{\text{condition}}$$

For constant pressure,
$$C_p = \left(\frac{Q}{\Delta T}\right)_{p,N} = \left(\frac{\Delta U + p\Delta V}{\Delta T}\right)_{p,N} = \left(\frac{\partial U}{\partial T}\right)_{p,N} + p\left(\frac{\partial V}{\partial T}\right)_{p,N}$$

Hence $C_p > C_V$ always holds. For ideal gases,
$$U = \frac{f}{2}NkT$$

Substituting, this gives
$$\left(\frac{\partial U}{\partial T}\right)_{p,N} = \left(\frac{\partial U}{\partial T}\right)_{V,N} = \frac{f}{2}Nk$$

and from the ideal gas law,
$$\left(\frac{\partial V}{\partial T}\right)_{p,N} = \frac{Nk}{p}$$

3 Main Postulate

Consider a closed system with fixed energy. Then all accessible microstates of the system are equally likely.