Lecture 16

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1 Incompressible Fluid at Rest

ρ is a constant for incompressible fluids. Then from

$$\frac{dp}{dz} = -\rho g$$

we get

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

as derived in the previous lecture.

Definition 1.1. The specific weight is defined as

$$\gamma := \rho g$$

We define specific weight as it appears often in fluid dynamics. It is the weight per unit volume of the fluid. Moreover, since liquids are usually much denser than gases

$$\rho_g << \rho_f$$
$$\rho_g g << \rho_f g$$
$$\left|\left(\frac{dp_g}{dz}\right)\right| << \left|\left(\frac{dp_f}{dz}\right)\right|$$

Therefore, for small distances $\Delta h \approx 0$, we can take gas pressure to be constant.

Example 1.1. Find the pressure-elevation relationship for isothermal perfect gas.

$$p = \rho R T$$

Then

$$\begin{aligned}
\frac{dp}{dz} &= -\rho g \\
\frac{dp}{dz} &= -\frac{p}{RT} g \\
\frac{dp}{p} &= -\frac{g}{RT} dz \\
\ln \frac{p_2}{p_1} &= -\frac{g}{RT}(z_2 - z_1) \\
p_2 &= p_1 e^{-\frac{g}{RT}(z_2 - z_1)}
\end{aligned}$$

As a summary, assuming no shear,

$$-\vec{\nabla} p + \rho \vec{g} = \rho \vec{a}$$

where $\vec{g}$ is a body force. If it is gravity, we have

$$-\vec{\nabla} p - \rho g \hat{k} = \rho \vec{a}$$

If the fluid is also at rest, we have

$$\frac{dp}{dz} = -\rho g$$

For incompressible fluids at rest,

$$p_2 = p_1 + \rho g h$$

and for compressible fluids at rest,

$$\int dp = \int \rho g dz$$

which simplifies to a constant p for small ranges Δh.

Example 1.2. For an incompressible fluid at rest, in a gravitational field acting in the $-z$ direction, show that the free surface is horizontal.
Given the conditions, we have

$$p = -\rho g z + C$$

We know $p = p_{atm}$ at $z = z_s$, so

$$p_{atm} = -\rho g z_s + C$$

Rearranging yields

$$z_s = \frac{C - p_{atm}}{\rho g}$$

which is an expression of constants. The z component of the surface is a constant, hence the surface is horizontal.

2 Measurements of Pressure

Pressure values are stated with respect to a reference level. The gage pressure is measured with a local atmospheric reference, while absolute pressure is compared with a vacuum reference. Hence

$$p_{abs} = p_{gage} + p_{atm}$$

Several instruments for measuring pressure including

- Mercury Barometer: $P_a = \rho g h$
- Piezometer: $P_a = \rho g h$
- U-tube manometer: $P_a = \gamma_2 h_2 - \gamma_1 h_1$

Example 2.1. A closed tank contains air and water. A piezometer is connected to the tank as shown. The column height are $h_1 = 1$m,$h_2 = 0.5$m. If the pressure gage reading of the compressed air shows 10kPa, determine the height h of the water in teh piezometer.

$$\gamma(h - 1) = 10000 \Rightarrow h = 2$$