Lecture 6

Boolean Identities: $+$ and $\cdot$ are commutative, associative, and distributive

Any SOP circuit can be implimented as NAND NAND

- Example: $f = x_1x_2 + x_2x_3$ can be rewritten as $f = (x_1 \text{ NAND } x_2) \text{ NAND } (x_2 \text{ NAND } x_3)$

Any POS circuit can be implemented as NOR NOR

- Example: $f = (x_1 + x_2)(x_2 + x_3)$ can be rewritten as $f = (x_1 \text{ NOR } x_2) \text{ NOR } (x_2 \text{ NOR } x_3)$

Design Example Gumball factory - s_2 normally gives 0, but $s_2 = 1$ if gumball is too large - s_1 normally gives 0, but $s_1 = 1$ if gumball is too small - s_0 normally gives 0, but $s_0 = 1$ if gumball is too light
Synthesise a logic function $f = 1$ when a gumball is either too large, or both too small and too light - By inspection: $s_2 + s_1s_0$ - From truth table:

$$\begin{aligned} f &= \overline{s_2}s_1s_0 + s_2\overline{s_1s_0} + s_2\overline{s_1}s_0 + s_2s_1\overline{s_0} + s_2s_1s_0 \\ &= s_1s_0 + s_2\overline{s_1} + s_2\overline{s_0} \\ &= s_1s_0 + s_2(\overline{s_1} + \overline{s_0}) \\ &= s_1s_0 + s_2(\overline{s_1s_0}) \\ &= s_1s_0 + s_2 \end{aligned}$$

Minimal POS Example Derive a minimal POS for $f(x_1, x_2, x_3) = \prod M(0, 2, 4)$

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x_2} + x_3)(\overline{x_1} + x_2 + x_3) \\ &= (x_1 + x_3)(x_2 + x_3) \end{aligned}$$

Using the minterms of $\overline{f}$

$$\begin{aligned} \overline{f} &= \overline{x_1x_2x_3} + \overline{x_1}x_2\overline{x_3} + x_1\overline{x_2x_3} \\ &= \overline{x_1x_3} + \overline{x_2x_3} \end{aligned}$$

So

$$f = \overline{\overline{f}} = \overline{(\overline{x_1x_3} + \overline{x_2x_3})} = \overline{(\overline{x_1x_3})}\,\overline{(\overline{x_2x_3})} = (\overline{\overline{x_1}} + \overline{\overline{x_3}})(\overline{\overline{x_2}} + \overline{\overline{x_3}}) = (x_1 + x_3)(x_2 + x_3)$$