Lecture 18

niceguy

February 27, 2023

1 Renewables and Electricity Markets

Renewables have *zero marginal cost*. Let λ be the price of electricity, μ^- be the fee for underproducing, and μ^+ be the fee for overproducing. Let p be the power production, and $\hat{p}$ be the forecast. The revenue is then

$$\tilde{J} = \lambda p - \mu^-(\hat{p} - p)^+ - \mu^+(p - \hat{p})^+$$

where

$$(x)^+ = \max(x, 0)$$

Then given a probability density function f and a cumulative density function F for p, we define

$$J = \mathbb{E}_p[\tilde{J}]$$

Substituting,

$$\begin{aligned}
J &= \lambda \int_{-\infty}^{\infty} pf(p)dp - \mu^- \int_{-\infty}^{\infty} (\hat{p} - p)^+ f(p)dp - \mu^+ \int_{-\infty}^{\infty} (p - \hat{p})^+ f(p)dp \\
&= \lambda \int_{-\infty}^{\infty} pf(p)dp - \mu^- \int_{-\infty}^{\hat{p}} (\hat{p} - p)^+ f(p)dp - \mu^+ \int_{\hat{p}}^{\infty} (p - \hat{p})^+ f(p)dp \\
\frac{dJ}{d\hat{p}} &= -\mu^- \int_{-\infty}^{\hat{p}} f(p)dp + \mu^+ \int_{\hat{p}}^{\infty} f(p)dp \\
&= -\mu^- F(\hat{p}) + \mu^+(1 - F(\hat{p}))
\end{aligned}$$

Then the $\hat{p}$ that satisfies this equation is given by

$$F(\hat{p}) = \frac{\mu^+}{\mu^- + \mu^+}$$

or

$$\hat{p} = F^{-1}\left(\frac{\mu^+}{\mu^- + \mu^+}\right)$$

The limiting behaviour shows that if $\mu^- >> \mu^+$, $F(\hat{p}) \to 0$, so $\hat{p} \to 0$. If $\mu^+ >> \mu^-$, then $F(\hat{p}) \to 1$, so $\hat{p} \to \infty$. If $\mu^+ = \mu^-$, then $F(\hat{p}) = 0.5$, so $\hat{p}$ is the median.

This is also the **Newsvendor problem** (self explanatory), where one wishes to predict how much of a perishable good to obtain.