

# Lecture 33

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## 1 1 versus 2 Tailed Tests

In our example,

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu \neq \mu_0$$

With  $\mu_0 = 10$ . We define the critical region to be  $\bar{x} < 8, \bar{x} > 12$ , where  $H_0$  is rejected.

In a one-sided case,

$$H_0 : \mu = 0$$

$$H_1 : \mu > 0$$

with the critical region  $\bar{x} > 2$ .

## 2 Hypothesis Testing vs Confidence Interval

For a Type I error (false positive), we specify a critical region  $R$ , then find  $\alpha$ . Recall

$$P(\bar{X} \in R) = 1 - \alpha$$

Now we can also specify  $\alpha$  and find a critical region.

## 3 P Value

**Definition 3.1.** The  $p$  value is defined as

$$p = P(|Z| > |z|)$$

Then  $p = 2P(Z \geq |z|)$ . If  $p$  is close to 1, then the values of  $z$  are very close to zero, suggesting  $H_0$  is likely true. If  $p$  is close to 0, then conversely,  $H_0$  is likely false.

**Example 3.1.**  $H_0 : \mu = 5, H_1 : \mu \neq 5$  with sample  $n = 40, \bar{x} = 5.5, \sigma = 1$ . Then

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{5.5 - 5}{1/\sqrt{40}} = 3.16$$

Then the  $p$  value is

$$p = 2P(Z > 3.16) = 2(1 - \Phi(3.16)) = 0.0016$$

So we reject  $H_0$  and take  $H_1$ .