Lecture 5

niceguy

January 18, 2023

1 Conditional Probability

$$P(B|A) = \frac{P(A \cup B)}{P(A)}, P(A) > 0$$
$$P(A \cap B) = P(B|A)P(A)$$

Definition 1.1. Events A and B are independent if

$$P(A|B) = P(A)$$

or

$$P(B|A) = P(B)$$

It is trivial to see both statements are equivalent given $P(A), P(B) \neq 0$. Note that independence is not mutual exclusivity.

Example 1.1. For a coin flip, H and T are mutually exclusive, but $P(H) = 0.5 \neq 0 = P(H|T)$

Example 1.2. For a die roll, 2 and 3 are mutually exclusive. However, even and 2 are not mutually exclusive, as $P(2) = \frac{1}{6} \neq \frac{1}{3} = P(2|\text{even})$

2 Bayes' Rule

$$P(A|B)P(B) = P(B|A)P(A)$$

sectionTotal Probability

Recall partitions. $B_1, \ldots, B_k$ is a partition if

$$B_i \cap B_j = \emptyset \forall i \neq j$$

and

$$\bigcup_{i=1}^{k} B_i = S$$

Then the **Law of Total Probability** is

$$\begin{aligned} P(A) &= \sum_{i=1}^{k} P(A \cap B_i) \\ &= \sum_{i=1}^{k} P(A|B_i)P(B_i) \end{aligned}$$

This can be easily proved by noting

$$P\left(\bigcup_{i=1}^{k} A_i\right) = \sum_{i=1}^{k} P(A_i)$$

given $A_i \cap A_j = \emptyset \forall i \neq j$. This then leads to the first equality. The proof that

$$A = \bigcup_{i=1}^{k} A \cap B_i$$

is trivial and is left to the reader as an exercise.

Example 2.1. Consider machines 1, 2, 3 producing the same product. B_i refers to the product being made by machine i. We are given that $P(B_1) = 0.3, P(B_2) = 0.45, P(B_3) = 0.25$. The probability that there are defects is $2\%, 3\%, 2\%$ for machines 1, 2, 3 respectively. The probability that a product is defective is

$$0.3 \times 2\% + 0.45 \times 3\% + 0.25 \times 2\% = 2.45\%$$