Lecture 4

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January 16, 2023

1 Electric Fields

Example 1.1. Electric Field Above a Charged Disk

$$\vec{E} = \iint_S \frac{dQ'(\vec{R} - \vec{R}')}{4\pi\varepsilon_0 |\vec{R} - \vec{R}'|^3}$$

By definition,

$$dQ' = \rho_S ds'$$

and we can use cylindrical coordinates, so

$$ds' = r dr d\phi$$

The position vectors are given by

$$\vec{R} = h\hat{a}_z$$
$$\vec{R}' = r\hat{a}_r$$

Note that r is constant. Then

$$\vec{R} - \vec{R}' = -r\hat{a}_r + h\hat{a}_z = -r\cos\phi\hat{a}_x - r\sin\phi\hat{a}y + h\hat{a}_z$$

And

$$d\vec{E} = \frac{\rho_S r dr d\phi}{4\pi\varepsilon_0 (r^2 + h^2)^{3/2}}$$

By symmetry, $\vec{E}$ must be a multiple of $\hat{a}_z$. Integrating gives

$$\frac{\rho_s}{2\varepsilon_0}\left(\frac{h}{|h|} - \frac{h}{\sqrt{a^2 + h^2}}\right)$$

Then as $h \to 0$ or $a \to \infty$, this simplifies to $\frac{\rho_S}{2\varepsilon_0}$. Then a capacitor can be build by placing two sheets of oppositely charged plates next to each other in parallel.