Lecture 17

niceguy

December 9, 2022

1 Manometers

Example 1.1. Determine an equation for $p_A - p_B$ along the pipe in terms of the specific weight of the folwing fluid γ_1 in the pipe, the specific weight of the gage fuild γ_2, and the various heights indicated (Fig 1.1).

$$p_A - h_1\gamma_1 - h_2\gamma_2 + (h_1 + h_2)\gamma_1 = p_B$$

Rearranging,

$$p_1 - p_B = h_2(\gamma_2 - \gamma_1)$$

Example 1.2. Inclined tube manometer (Fig 1.2):

$$p_A + h_1\gamma_1 - l_2\gamma_2 \sin\theta - h_3\gamma_3 = p_B$$

Rearranging,

$$p_A - p_B = l_2\gamma_2 \sin\theta + h_3\gamma_3 - h_1\gamma_1$$

If A and B are gases,

$$p_A - p_B = l_2\gamma_2 \sin\theta$$

2 Hydrostatic Forces on Submerged Surfaces

We assume

- $\vec{F}_p \perp$ surface
- No shear stress
- For incompressible fluids at rest, pressure varies linearly with depth

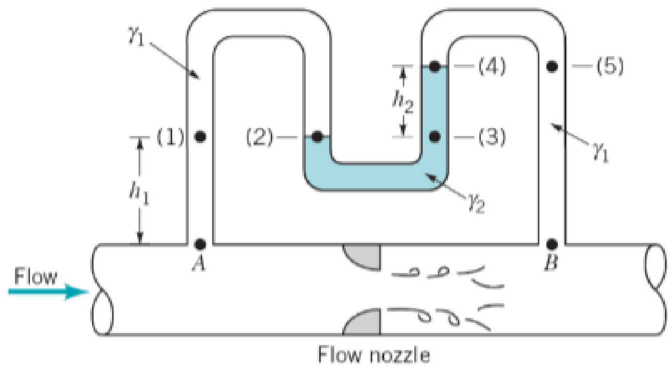

Figure 1: Pipe

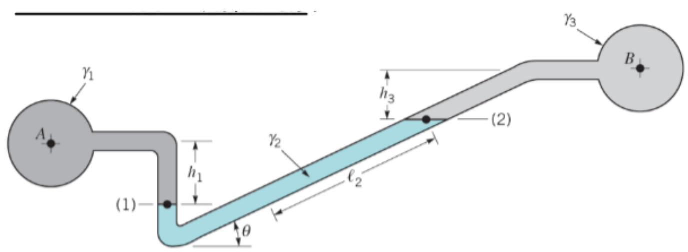

Figure 2: Inclined tube manometer

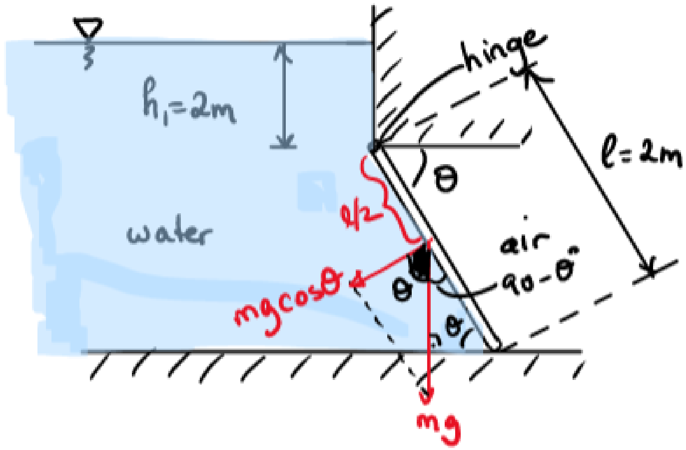

Figure 3: Gate

3 Hydrostatic Forces acting on Planar Surfaces

This can be done by integration.

Example 3.1. Find the mass of the gate such that it will open when the water level exceeds 2m (Fig 3.1). ($\cos\theta = 0.6$, width is 1.8m)

Closing moment:

$$M = \frac{L}{2} \times mg\cos\theta = 0.6mg$$

Opening moment:
Defining x as the distance along the gate,

$$\begin{aligned} M &= \iint_A xp dA \\ &= \int_0^l x\rho ghw dx \\ &= \int_0^l x\rho g(h_1 + x\sin\theta)w dx \\ &= \rho gw \times \frac{18.4}{3} \end{aligned}$$

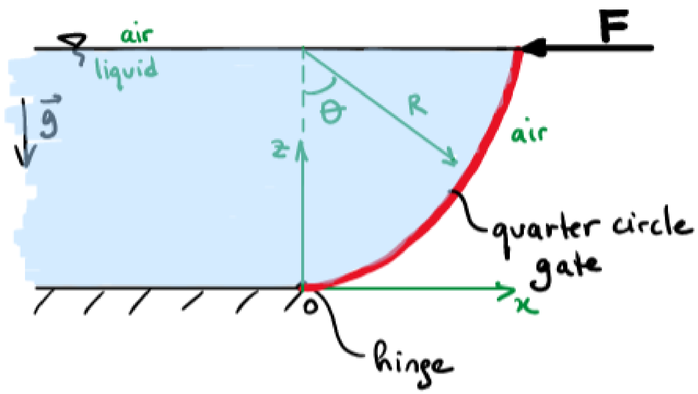

Figure 4: Gate

Equating both yields $m = 18400$kg.

The second method is by using a pressure prism. The average pressure is given by

$$p_{av} = \frac{\rho g h}{2}$$

And the force is then

$$F = \frac{1}{2}\rho g h^2 b$$

In the case if pressure is not 0 at the lowest point, the formulae for pressure and force are trivial, and are left to the reader as an exercise. Note that atmospheric pressure can (usually) be ignored as it is cancelled out.

4 Hydrostatic Force acting on Curved Surfaces

This can be done through integration.

Example 4.1. Find the horizontal force required to hold the gate (Fig 4) closed. Neglect the mass of the gate. (width = 6m, radius $R = 2$m)

We first paramatrise the curve

$$x = R\sin\theta$$
$$z = R - R\cos\theta$$

Moment is then

$$\begin{aligned}
M &= \int x|dF_z| + z|dF_x| \\
&= \int_{\theta=0}^{\theta=\pi/2} R\sin\theta pw\cos\theta R d\theta + (R - R\cos\theta) pw \sin\theta R d\theta \\
&= \int_0^{\pi/2} Rpw\sin\theta R d\theta \\
&= \int_0^{\pi/2} R^3\rho g w \sin\theta\cos\theta d\theta \\
&= R^3\rho g w \int_0^{\pi/2} \sin\theta\cos\theta d\theta \\
&= R^3\rho g w \int_0^{\pi/2} \frac{\sin^2\theta}{2} d\theta \\
&= \frac{R^3\rho g w}{4}(-\cos 2\theta)\Big|_{\theta=0}^{\theta=\pi/2} \\
&= \frac{R^3\rho g w}{4}(\cos 0 - \cos\pi) \\
&= \frac{R^3\rho g w}{2}
\end{aligned}$$

Then the force is

$$\frac{R^2\rho g w}{2} = 120000\text{N}$$

The second method is to parametrise the surface.

Example 4.2.

$$\begin{aligned}
x &= R\sin\theta \\
z &= R - R\cos\theta \\
y &= y
\end{aligned}$$

Then

$$\vec{r} = R\sin\theta\hat{i} + y\hat{j} + (R - R\cos\theta)\hat{i}$$

giving

$$\vec{r}_\theta = R\cos\theta\hat{i} + R\sin\theta\hat{k}$$

and

$$\vec{r}_y = \hat{j}$$

so

$$\vec{r}_\theta \times \vec{r}_y = -R\sin\theta\hat{i} + R\cos\theta\hat{k}$$

whose magnitude is R. The moment is then

$$\begin{aligned} M_y &= \int_S x|dF_z| + z|dF_x| \\ &= \iint R\sin\theta(pR\cos\theta)dyd\theta + \iint (R - R\cos\theta)pR\sin\theta dyd\theta \end{aligned}$$

which gives the same integral as above.