Homework 1

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1. List the elements of each of the following sample spaces
   (a) the set of integers between 1 and 50 divisible by 8
   (b) the set $S = \{x|x^2 + 4x - 5 = 0\}$
   (c) the set of outcomes when a coin is tossed until a tail or three heads appear
   (d) the set $S = \{x|x \text{ is a continent }\}$
   (e) the set $S = \{x|2x - 4 \geq 0 \text{ and } x < 1\}$

   Solution:
   $\{8, 16, 24, 32, 40, 48\}$
   $\{-5, 1\}$
   $\{T, HT, HHT, HHH\}$
   Asia, Europe, Africa, North America, South America, Antarctica, Oceania
   $\emptyset$

2. An experiment consists of tossing a die and then flipping a coin once if the number on the die is even. If the number on the die is odd, the coin is flipped twice. Using the notation $4H$, for example, to denote the outcome that the die comes up 4 and then the coin comes up heads, and $3HT$ to denote the outcome that the die comes up 3 followed by a head and then a tail on the coin, construct a tree diagram to show the 18 elements of the sample space S.

   Solution: Depth first: $\{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$

3. Four students are selected at random from a chemistry class and classified as male or female. List the elements of the sample space S_1, using the letter M for male and F for female. Define a second sample space S_2 where the elements represent the number of females selected.

   Solution:
   $$S_1 = \{FFFF, FFFM, FFMM, FMMM, MMMM\}$$
   $$S_2 = \{0, 1, 2, 3, 4\}$$
   Assuming order is irrelevant.

4. For the sample space in 2,
   (a) list the elements corresponding to the event A that a number less than 3 occurs on the die;

(b) list the elements corresponding to the event B that two tails occur;

(c) list the elements corresponding to the event A';

(d) list the element corresponding to the event $A' \cap B$;

(e) list the elements corresponding to the event $A \cup B$

Solution:
$A = \{1HH, 1HT, 1TH, 1TT, 2H, 2T\}$
$B = \{1TT, 3TT, 5TT\}$
$A' = \{3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$
$A' \cap B = \{3TT, 5TT\}$
$A \cup B = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3TT, 5TT\}$

5. If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and $D = \{1, 6, 7\}$, list the elements of the sets corresponding to the following events:

(a) $A \cup C$;

(b) $A \cap B$;

(c) C';

(d) $(C' \cap D) \cup B$;

(e) $(S \cap C)'$;

(f) $A \cap C \cap D'$

Solution:
$\{0, 2, 3, 4, 5, 6, 8\}$
$\emptyset$
$\{0, 1, 6, 7, 8, 9\}$
$\{1, 3, 5, 6, 7, 9\}$
$\{0, 1, 6, 7, 8, 9\}$
$\{2, 4\}$

6. If $S = \{x|0 < x < 12\}$, $M = \{x|1 < x < 9\}$, and $N = \{x|0 < x < 5\}$, find

(a) $M \cup N$;

(b) $M \cap N$;

(c) $M' \cap N'$;

Solution:
$\{x|0 < x < 9\}$
$\{x|1 < x < 5\}$
$\{x|9 \leq x < 12\}$

7. Let A, B, and C be events relative to the sample space S. Using Venn diagrams, shade the areas representing the following events:

(a) $(A \cap B)'$;

(b) $(A \cup B)'$;

(c) $(A \cap C) \cup B$;

Solution: There's no way I'm using tikz for this.

8. A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are availabe to this buyer?

Solution: $4 \times 3 \times 2 \times 2 = 48$

9. (a) In how many ways can 6 people be lined up to get on a bus?
   (b) If 3 specific persons, among 6, insist on following each other, how many ways are possible?
   (c) If 2 specific persons, among 6, refuse to follow each other, how many ways are possible?

Solution:
$6! = 720$
$3! \times 4! = 144$
$720 - 5! \times 2! = 480$

10. (a) How many three-digit numbers can be formed from the digits 0,1,2,3,4,5, and 6 if each digits can be used only once?
    (b) How many of these are odd numbers?
    (c) How many are greater than 330?

Solution:
$6 * 6 * 5 = 180$
$3 * 5 * 5 = 75$
$3 * 6 * 5 + 3 * 5 = 105$