Lecture 36

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1 LTI systems

Consider the current state x_k, and the next state $x_{k+1} = Ax_k$. Of course, this isn't always linear, but it would be cool if it were.

2 Markov Chains

There are n states, and a probability $p_i(k)$ of being in state i at time k. Then

$$\sum_i p_i(k) = 1$$

We define P_{ij} to be the probability of moving from state i to j (it is time independent)

$$\sum_j P_{ij} = 1$$

2.1 Dynamics

$$p_i(k+1) = \sum_j P_{ji} p_j(k)$$

Now define $p(k)$ to be a column matrix with $p(k)_i = p_i(k)$, and M to be a matrix with $M_{ij} = P_{ji}$. Then we have

$$p(k+1) = Mp(k)$$

By induction,

$$p(k+s) = M^s p(k)$$

Note that

$$\mathbf{1}^T M = \mathbf{1}^T, M^T \mathbf{1} = \mathbf{1}$$

where **1** denotes the column vector of 1s. Now consider the eigenvectors of M with eigenvalue 1, i.e.

$$q = Mq$$

Given a steady state, we will have

$$q = \lim_{k \to \infty} M^k p$$