

Lecture 14

niceguy

March 30, 2023

At equilibrium, a system of two gases would tend to the same pressure and temperature, where entropy is maximised. This means

$$\frac{\partial S_1}{\partial N_1} = \frac{\partial S_2}{\partial N_2}$$

Now we define this as

$$-\frac{\mu}{T} = \frac{\partial S}{\partial N}$$

where μ is the chemical potential, with units of energy. Using our favourite equation

$$S = kN \left[\ln \left(\frac{V}{N} \left(\frac{U}{3N} \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right]$$

Then differentiating and skipping the steps

$$\begin{aligned} \frac{\mu}{T} &= -k \left[\ln \left(\frac{V}{N} \left(\frac{U}{3N} \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right] + kN \frac{5}{2N} \\ \mu &= -kT \ln \left[\frac{V}{N} \left(\frac{1}{2}kT \right)^{\frac{3}{2}} \left(\frac{4\pi m}{h^2} \right)^{\frac{3}{2}} \right] \\ &= -kT \left[\ln \left(\frac{1}{n} \frac{1}{\lambda_{th}^3} \right) + C \right] \end{aligned}$$

where C is a constant and n is the number density. With the negative sign, μ increases with n , which makes sense.

1 Saha Equation

$$\frac{n_p n_e}{n_h} \frac{1}{\left(\frac{4\pi}{h^2} m_e \frac{kT}{2}\right)^{\frac{3}{2}}} = e^{-\frac{I}{kT}}$$

Where $I = 13.6\text{eV}$. And $n_p = \frac{N_p}{V}$, the density of protons. We similarly define the density of electrons and neutrons.