Lecture 34

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1 Function Approximation

We have n pairs of (x_i, y_i), and we want to approximate $y = f(x)$. It is helpful to define an error, where $e_i = f(x_i) - y_i$.

1.1 Linear Regression

We use the approximate function $y = ax + b$. Then we define the total error, which we wish to minimise.

$$\mathcal{E} = \sum_{i=1}^{n} e_i^2$$

We can minimise this by differentiating the error.

$$\begin{aligned}
\frac{\partial \mathcal{E}}{\partial a} &= 0 \\
\sum_{i=1}^{n} 2(ax_i + b - y_i)x_i &= 0 \\
\sum_{i=1}^{n} ax_i^2 + b\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i y_i &= 0
\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial \mathcal{E}}{\partial b} &= 0 \\ \sum_{i=1}^{n} 2(ax_i + b - y_i) &= 0 \\ a\sum_{i=1}^{n} x_i + bn - \sum_{i=1}^{n} y_i &= 0\end{aligned}$$

Solving the simultaneous equations,

$$\begin{aligned}a &= \frac{\sum_{i=1}^{n}(x_i - \overline{x})y_i}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \\ b &= \overline{y} - a\overline{x}\end{aligned}$$

2 MLE Approximation

We define e_i similarly, but we note that they are realisations of normal random variables with mean 0 and variance σ^2. Then $\max_{a,b} L$ gives the same answer.