Lecture 10

3-variable K-map

Gray code: only one bit has changed between any 2 consecutive binary representations

$x_3 \backslash x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

To find x_2x_3, we can refer to the K-map, which gives us m_3 and m_7, which can then be simplified as

$$x_2x_3 = \overline{x_1}x_2x_3 + x_1x_2x_3 = x_2x_3(\overline{x_1} + x_1) = x_2x_3$$

If we want $m_2 + m_6$, we can see directly from the map that it corresponds to $x_2\overline{x_3}$. Similarly, $m_2 + m_3 + m_6 + m_7 = x_2$.

Example

z\xy	00	01	11	10
0	1	1	1	1
1	1	0	0	0

f is equal to 1 on the first row and column, so

$$f = \overline{xy} + \overline{z}$$

Terminology

- Implicant: for a function f, an implicant is any product term in f
  - e.g. m_0, m_1, etc
- Prime Implicant: an implicant for which is it not possible to remove any literal and still have a valid implicant
  - e.g. $\overline{z}$ in the above example is a prime implicant, but $\overline{xz}$ is not ($\overline{x}$ can be removed)
- Cover: any set of implicants that include all minterms of a function. Consider $|x_3 \backslash x_1 x_2$ | 00 | 01 | 11 | 10 | | — | — | — | — | — | | 0 | 0 | 0 | 1 | 1 | | 1 | 1 | 0 | 0 | 1 | All prime implicants are $x_1\overline{x_3}, x_1\overline{x_2}, \overline{x_2}x_3$. The **minimum** cost cover, however, is

  $$f = x_1\overline{x_3} + \overline{x_2}x_3$$

- Essential prime implicant: prime implicant that covers at least one minterm not covered by any other prime implicant
  - In the above example, they are $x_1\overline{x_3}$, $\overline{x_2}x_3$

4-variable K-maps

Note that anything beyond is difficult in 2D

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

The first two rows are $\overline{x_3}$.
The four cells in the middle are x_2x_4.

Example

$$f(x_1, x_2, x_3, x_4) = \sum m(2, 4, 5, 8, 10, 11, 12, 13, 15)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

The prime implicants are $x_2\overline{x_3}, x_1\overline{x_3x_4}, x_1x_2x_4, x_1x_3x_4, x_1\overline{x_2}x_3, \overline{x_2}x_3\overline{x_4}, x_1\overline{x_2x_4}$
The essential prime implicants are $x_2\overline{x_3}, \overline{x_2}x_3\overline{x_4}$
The minimum cost cover is

$$f = x_2\overline{x_3} + \overline{x_2}x_3\overline{x_4} + x_1x_3x_4 + x_1\overline{x_2x_4}$$

There can be more than one minimum cost covers (the last term can be replaced by $x_2\overline{x_3x_4}$