Lecture 14

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1 Numerical Methods

$$\frac{dy}{dt} = f(t, y)$$

where f is continuously differentiable. Existance and uniqueness theorems give us a unique solution for every (t_0, y_0) pair.
Unfortunately, most ODEs cannot be solved analytically, so we must use numerical methods.

Example 1.1. Ricatti Equations

$$y'(x) = q_0(x) + q_0(x)y + q_2(x)y^2$$

We consider the specific equation

$$\frac{dy}{dt} = 1 + 2y + ty^2, y(0) = -1$$

This is technically solvable `https://math.stackexchange.com/a/244533`, and you are encouraged to memorise this for coming midterms and finals. However, this can also be done using **Euler's Method**, which makes use of direction fields (they can be sketched without solving the ODE). The smaller the step size, the more accurate the approximation.

1.1 Euler's Method

We form a partition of $[t_0, T]$ by

$$t_0 < t_1 < t_2 < \cdots < t_N = T$$

First, we form an approximation of the solution between $[t_0, t_1]$ using linear interpolation, where

$$y(t_1) = y(t_0) + f(t_0, y_0)(t_1 - t_0)$$

Similarly, $y(t_2), y(t_3), \ldots, y(t_N) = y(T)$ can also be approximated successively, using the previous approximations.
Potential issues

- Length of arrows (Does it depend on the ODE?)
- Accuracy (Are there ODEs where Euler's Method is just bad?)

Euler's method "jumps between" different solution curves, so it is more accurate when solutions converge.