Lecture 18

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1 Hydrostataic Forces acting on Curved Surfaces

Considering the same example

Example 1.1. The surface is parametrised as

$$\begin{aligned} x &= R\sin\theta \\ z &= R - R\cos\theta \\ y &= y \end{aligned}$$

so

$$\vec{r} = R\sin\theta\hat{i} + y\hat{j} + (R - R\cos\theta)\hat{k}$$

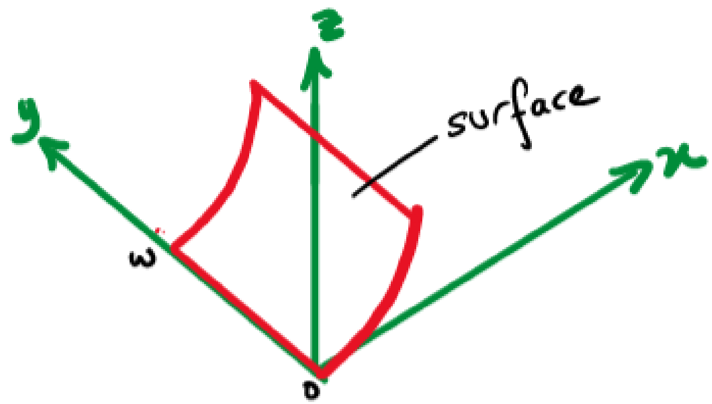
The moment is

$$\begin{aligned} M &= \int d\vec{M} \\ &= \int \vec{r} \times d\vec{F} \\ &= -\int \vec{r} \times pd\vec{S} \end{aligned}$$

However, we only consider the moment about y, as the moments along the other axes are cancelled out by the hinge. Thus

$$dM_{\text{opening}} = ||d\vec{M}_y|| = (\vec{r} \times d\vec{S}) \cdot \hat{j}(-p)$$

Figure 1: Gate



We have

$$\begin{aligned} d\vec{S} &= \vec{r}_\theta \times \vec{r}_y \\ &= (R\cos\theta\hat{i} + R\sin\theta\hat{k}) \times \hat{j} \\ &= -R\sin\theta\hat{i} + R\cos\theta\hat{k} \end{aligned}$$

Then

$$\begin{aligned} \vec{r} \times d\vec{S} &= (R\sin\theta\hat{i} + y\hat{j} + (R - R\cos\theta)\hat{k}) \times (-R\sin\theta\hat{i} + R\cos\theta\hat{k}) \\ &= Ry\cos\theta\hat{i} - R^2\sin\theta\hat{j} + Ry\sin\theta\hat{k} \end{aligned}$$

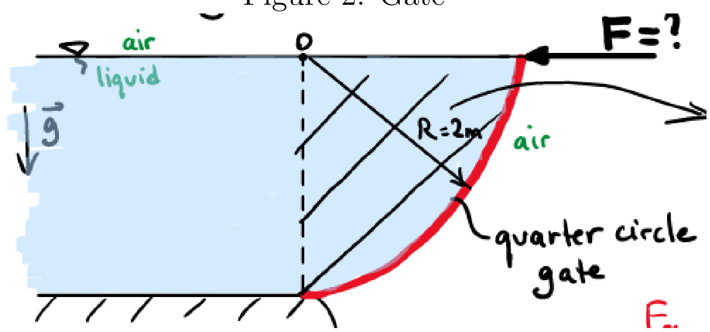
Then the total moment is

$$\begin{aligned} M_y &= \int_S dM_y \\ &= \int_0^{\frac{\pi}{2}} \int_0^w R^2\sin\theta \times \rho g R\cos\theta dy d\theta \\ &= \int_0^{\frac{\pi}{2}} \int_0^w \rho g R^3 \sin\theta\cos\theta d\theta dy \end{aligned}$$

where we again reach to the same integral.

There is also another method, where one considers the free body diagram of the fluid. Consider the body ABC, where AB represents the curve, and AC and BC are horizontal and veertical lines respectively. From equilibrium,

Figure 2: Gate



$$\sum F_x = 0 \Rightarrow F_{x,AB} = F_x$$

$$\sum F_y = 0 \Rightarrow F_{y,AB} = F_y + W$$

where F_x is applied on BC and F_y on AC. Note the the direction of weight is reversed if the surface is above the liquid.

Example 1.2. Find the horizontal force F required to hold the gate closed. Neglect the mass of the gate.

Considering $\sum F_x = 0$, the horizontal component is

$$F_x = \frac{1}{2}\rho g R^2 w = 120000\text{N}$$

Similarly for y,

$$F_y = \rho g \frac{\pi R^2}{4} w = 60000\pi\text{N}$$

Reaction force is

$$\sqrt{F_x^2 + F_y^2} = 223451\text{N}$$

at an angle

$$\alpha = \arctan\left(\frac{F_y}{F_x}\right) = 57.5°$$

The perpendicular distance to the hinge is given by $R\cos\alpha$, so

$$FR = 223451 R\cos\alpha = 120000\text{N}$$