

# Lecture 16

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## 1 Incompressible Fluid at Rest

$\rho$  is a constant for incompressible fluids. Then from

$$\frac{dp}{dz} = -\rho g$$

we get

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

as derived in the previous lecture.

**Definition 1.1.** The specific weight is defined as

$$\gamma := \rho g$$

We define specific weight as it appears often in fluid dynamics. It is the weight per unit volume of the fluid. Moreover, since liquids are usually much denser than gases

$$\begin{aligned}\rho_g &<< \rho_f \\ \rho_g g &<< \rho_f g \\ \left| \left( \frac{dp_g}{dz} \right) \right| &<< \left| \left( \frac{dp_f}{dz} \right) \right|\end{aligned}$$

Therefore, for small distances  $\Delta h \approx 0$ , we can take gas pressure to be constant.

**Example 1.1.** Find the pressure-elevation relationship for isothermal perfect gas.

$$p = \rho RT$$

Then

$$\begin{aligned}\frac{dp}{dz} &= -\rho g \\ \frac{dp}{dz} &= -\frac{p}{RT}g \\ \frac{dp}{p} &= -\frac{g}{RT}dz \\ \ln \frac{p_2}{p_1} &= -\frac{g}{RT}(z_2 - z_1) \\ p_2 &= p_1 e^{-\frac{g}{RT}(z_2 - z_1)}\end{aligned}$$

As a summary, assuming no shear,

$$-\vec{\nabla}p + \rho\vec{g} = \rho\vec{a}$$

where  $\vec{g}$  is a body force. If it is gravity, we have

$$-\vec{\nabla}p - \rho g\hat{k} = \rho\vec{a}$$

If the fluid is also at rest, we have

$$\frac{dp}{dz} = -\rho g$$

For incompressible fluids at rest,

$$p_2 = p_1 + \rho gh$$

and for compressible fluids at rest,

$$\int dp = \int \rho g dz$$

which simplifies to a constant  $p$  for small ranges  $\Delta h$ .

**Example 1.2.** For an incompressible fluid at rest, in a gravitational field acting in the  $-z$  direction, show that the free surface is horizontal. Given the conditions, we have

$$p = -\rho gz + C$$

We know  $p = p_{atm}$  at  $z = z_s$ , so

$$p_{atm} = -\rho gz_s + C$$

Rearranging yields

$$z_s = \frac{C - p_{atm}}{\rho g}$$

which is an expression of constants. The  $z$  component of the surface is a constant, hence the surface is horizontal.

## 2 Measurements of Pressure

Pressure values are stated with respect to a reference level. The gage pressure is measured with a local atmospheric reference, while absolute pressure is compared with a vacuum reference. Hence

$$p_{abs} = p_{gage} + p_{atm}$$

Several instruments for measuring pressure including

- Mercury Barometer:  $P_a = \rho gh$
- Piezometer:  $P_a = \rho gh$
- U-tube manometer:  $P_a = \gamma_2 h_2 - \gamma_1 h_1$

**Example 2.1.** A closed tank contains air and water. A piezometer is connected to the tank as shown. The column height are  $h_1 = 1m, h_2 = 0.5m$ . If the pressure gage reading of the compressed air shows 10kPa, determine the height  $h$  of the water in the piezometer.

$$\gamma(h - 1) = 10000 \Rightarrow h = 2$$