Problem Set 11

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1. Consider a monatomic ideal gas that lives at a height z above sea level, so each molecule has potential energy mgz in addition to its kinetic energy.

   (a) Show that the chemical potential is the same as if the gas were at sea level, plus an additional term mgz

   Solution: The new potential energy is given by

   $$U' = U + mgzN$$

   Then

   $$\mu' = \frac{dU'}{dN} = \frac{dU}{dN} + \frac{d}{dN}mgzN = \mu + mgz$$

   (b) Suppose you have two chunks of helium gas, one at sea level and one at height z, each having the same temperature and volume. Assuming that they are in diffusive equilibrium, show that the number of molecules in the higher chunk is

   $$N(z) = N(0)e^{-mgz/kT}$$

   Solution:

   $$-kT\ln[\frac{V}{N_0}\left(\frac{2\pi mkT}{h^2}\right)^3/2] = -kT\ln[\frac{V}{N_B}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}] + mgz$$
   $$-kT\ln\frac{1}{N_0} = -kT\ln\frac{1}{N_B} + mgz$$
   $$\ln\frac{N_0}{N_B} = \frac{mgz}{kT}$$
   $$N(z) = N_0 e^{-\frac{mgz}{kT}}$$