Lecture 32

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1 Time-Varying Fields

Rationale: we forgot about time in the previous lectures. The Maxwell equations we use only hold when everything is constant with respect to time. Now

$$\begin{aligned}
\vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\vec{\nabla} \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
\vec{\nabla} \cdot \vec{D} &= \rho_v \\
\vec{\nabla} \cdot \vec{B} &= 0
\end{aligned}$$

2 Faraday's Law

A changing magnetic flux causes a current to flow in a closed loop. Then integrating Faraday's Law,

$$V_{\text{emf}} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S} = -\frac{\partial \Phi}{\partial t}$$

3 Lenz' Law

Definition 3.1 (Lenz' Law)**.** The inductive emf will generate a current whose own magnetic field opposes the change in the original magnetic flux which produced the induced emf.

Note that this is a restatement of the conservation of energy. If the current is in the opposite direction, this would cause emf to increase, which causes the current to further increase, creating a positive feedback loop.
Further note that a current is only generated when there is a closed circuit.

Example 3.1. Find the induced emf if $I(t) = I_0 \cos(\omega t)$ for a toriod with a square cross section.
We approximate

$$\vec{B}(t) = \frac{\mu I(t)}{2\pi r}$$

Then

$$\begin{aligned}
\Phi(t) &= \iint_S \vec{B}(t) \cdot d\vec{S} \\
&= \int_0^c \int_a^b \frac{\mu I(t)}{2\pi r} dr dz \\
&= \frac{\mu I(t)}{2\pi} c \ln \frac{b}{a} \\
&= \frac{\mu_0 \mu_r c \ln \frac{b}{a}}{2\pi} I_0 \cos(\omega t)
\end{aligned}$$

Now

$$V_{\text{emf}} = -N \frac{\partial \Phi}{\partial t} = \frac{M \mu_0 \mu_r c \ln \frac{b}{a} I_0 \omega}{2\pi} \sin(\omega t) = V_0 \sin(\omega t) = L \frac{di}{dt}$$