Lecture 10

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1 Electric Field from Electric Potential

The second fundamental postulate for electrostatics is

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

For electrostatics, there is no change with respect to time, meaning the electrostatic field is conservative. That is, it is the gradient of a scalar field, namely V.

$$\vec{E} = -\vec{\nabla} V$$

As $\vec{E}$ only depends on the gradient of V, it can have any reference (setting $V' = V + C$ also works). Conventionally, we define $V = 0$ at $R = \infty$.

Example 1.1. Determine the electric field from a uniformly charged disk of radius, a, using the expression we had for $V(0,0,z)$ from the previous lecture We have found that

$$V(0,0,z) = \frac{\rho_s}{2\varepsilon_0}\left(\sqrt{z^2+a^2} - |z|\right)$$

We know that

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{a}_x - \frac{\partial V}{\partial y}\hat{a}_y - \frac{\partial V}{\partial z}\hat{a}_z$$

However, we do not have enough information to find E_x and E_y! Luckily, in this case, we do not have to, as both vanish due to symmetry. Differentiating gives us

$$\vec{E}(0,0,z) = E_z\hat{a}_z = -\frac{\partial V}{\partial z}\hat{a}_z = \frac{\rho_s}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)$$

for $z > 0$