Lecture 3

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1 Continuous Charge Distributions

Consider the field at $P(0,0,z)$ due to a charged plate lying on the xy plane. Electric field is the sum of contributions from all partitions of the plate. Taking the limit,

$$\vec{E}_{\text{TOT}} = \int \frac{(\vec{R}-\vec{R}')dQ'}{4\pi\varepsilon_0|\vec{R}-\vec{R}'|^3} = \iint_S \frac{\rho(\vec{R}-\vec{R}')dS}{4\pi\varepsilon_0|\vec{R}-\vec{R}'|^3}$$

where $\vec{R}'$ refers to the position vector of the plate (element).
Imagine using point charges when we can integrate.

2 Cylindrical Coordinates

- Point $P(r,\phi,z)$
- Unit vectors $\hat{a}_r, \hat{a}_\phi, \hat{a}_z$
- Position vector $r\hat{a}_r + z\hat{a}_z$
- Differential lengths $dr, rd\phi, dz$
- Differential length vector $d\vec{l} = dr\hat{a}_r + rd\phi\hat{a}_\phi + dz\hat{a}_z$
- Differential surface vectors
  - $d\vec{s}_r = rd\phi dz\hat{a}_r$
  - $d\vec{s}_\phi = drdz\hat{a}\phi$

- $d\vec{s}_z = rdrd\phi\hat{a}_z$

- Differential volume $rdrd\phi dz$

3 Spherical Coordinates

- Point $P(R, \theta, \phi)$
- Unit vectors $\hat{a}_R, \hat{a}_\theta, \hat{a}_\phi$
- Position vector $R\hat{a}_R$
- Differential lengths $dR, Rd\theta, R\sin\theta d\phi$
- Differential length vector $dR\hat{a}_R + Rd\theta\hat{a}_\theta + R\sin\theta d\phi\hat{a}_\phi$
- Differential surface vectors
  - $d\vec{s}_R = R^2\sin\theta d\phi d\theta\hat{a}_R$
  - $d\vec{s}_\phi = Rd\theta dR\hat{a}_\phi$
  - $d\vec{s}_\theta = R\sin\theta d\phi dR\hat{a}_\theta$
- Differential volume $dV = R^2\sin\theta dRd\phi d\theta$