Lecture 22

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1 Resistance and Joule's Law

$$\vec{J} = \sigma \vec{E}$$

or

$$\vec{E} = \rho \vec{J}$$

Conductivity σ and resistivity ρ are microscopic properties, dependent on position, while resistance and conductance are macroscopic properties.

Example 1.1 (Resistance of a Cylinder)**.** Consider a cylinder with a voltage difference V maintained at both ends. To find the resistance, we need to find voltage and current.

$$R = \frac{V}{I} = \left| \frac{\int \vec{E} \cdot d\vec{l}}{\iint_S \sigma \vec{E} \cdot d\vec{S}} \right|$$

In this case,

$$|V| = |-\int \vec{E} \cdot d\vec{l}| = EL$$

and

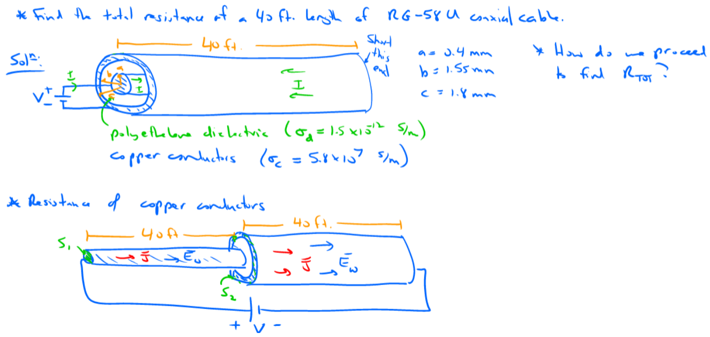
$$I = \iint_S \sigma \vec{E} \cdot d\vec{S} = E\sigma S$$

Dividing,

$$R = \frac{L}{\sigma S}$$

We can also write

$$R = \int \frac{dl}{\sigma S}$$



Example 1.2 (Resistance of a Coaxial Cable)**.** Find the total resistance of a 40 feet length of RG-58U coaxial cable.

$$\begin{aligned}R &= \frac{L_1}{\sigma S_1} + \frac{L_2}{\sigma S_2} \\ &= \frac{40 \times 0.305}{5.8 \times 10^7 \pi a^2} + \frac{40 \times 0.305}{5.8 \times 10^7 \pi (c^2 - b^2)} \\ &= 485 \mathrm{m}\Omega\end{aligned}$$

Using Guass' Law,

$$\vec{E} = \frac{\rho_{sa} a}{\varepsilon_0 \varepsilon_r} \hat{a}_r$$

So

$$R = \frac{\int_a^b \frac{\rho_{sa} a}{\varepsilon_0 \varepsilon_r} dr}{\int_0^{2\pi} \int_0^L \frac{\sigma_d \rho_{sa} a}{\varepsilon_0 \varepsilon_r a} a dz d\phi} = \frac{\ln \frac{b}{a}}{2\pi L \sigma_d} = 1.17 \times 10^{10} \Omega$$

Alternatively,

$$R = \int \frac{dl}{\sigma_d S} = \int_a^b \frac{dr}{\sigma_d \times 2\pi r L} = \frac{\ln \frac{b}{a}}{2\pi L \sigma_d}$$

giving the same result.

2 Joule's Law

Joule's Law addresses power loss, as electric field has to do work to create a current.

$$
\begin{aligned}
\Delta P &= \frac{d\Delta U}{dt} \\
&= \frac{d}{dt}\int \vec{F}_e \cdot d\vec{l} \\
&= \frac{d}{dt}\int Q\vec{E} \cdot d\vec{l} \\
&= \frac{d}{dt}\int \rho_v \Delta V \vec{E} \cdot d\vec{l} \\
&= \int \vec{E} \cdot \left(\rho_v \frac{d\vec{l}}{dt}\right) \Delta V \\
&= \vec{E} \cdot \vec{J} \Delta V
\end{aligned}
$$

Where we substitute the definition of $\vec{J}$ on the last equality.