Lecture 18

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1 Capacitors

Example 1.1. Consider an inner and outer cylindrical conductors. $Q = 1\text{nC}, a = 1\text{mm}, b = 3\text{mm}, c = 5.5\text{mm}, \varepsilon_{r1} = 2, \varepsilon_{r2} = 4, L = 5\text{cm}$. Where a, b, c are the radii in ascending order. Find the total stored electric potential energy and the capacitance.
We can use the equation

$$W = \frac{1}{2} \iint_S \rho_s v ds$$

or

$$W = \frac{1}{2} \iiint_V \vec{D} \cdot \vec{E} dv$$

From Gauss' Law,

$$\vec{D} = \begin{cases} \frac{Q}{2\pi r L} \hat{a}_r & a < r < c \\ 0 & \text{else} \end{cases}$$

Then

$$\vec{E} = \begin{cases} \frac{Q}{2\pi \varepsilon_0 \varepsilon_{r1} r L} \hat{a}_r & a < r < b \\ \frac{Q}{2\pi \varepsilon_0 \varepsilon_{r2} r L} \hat{a}_r & b < r < c \\ 0 & \text{else} \end{cases}$$

The second equation then gives

$$\begin{aligned} W &= \frac{1}{2} \int_0^L \int_0^{2\pi} \int_a^b \frac{Q^2}{4\pi^2 \varepsilon_0 \varepsilon_{r1} r^2 L} r dr d\phi dz + \frac{1}{2} \int_0^L \int_0^{2\pi} \int_b^c \frac{Q^2}{4\pi^2 \varepsilon_0 \varepsilon_{r2} r^2 L} r dr d\phi dz \\ &= \frac{Q^2}{4\pi\varepsilon_0} \left(\frac{\ln b - \ln a}{\varepsilon_{r1}} - \frac{\ln c - \ln b}{\varepsilon_{r2}} \right) \\ &= 0.126 \mu \text{J} \end{aligned}$$

And

$$c = \frac{1}{2}\frac{Q^2}{W} = 3.87\text{pf}$$

2 Some Maff

Consider the differential form of Gauss' Law:

$$\vec{\nabla} \cdot (\varepsilon \vec{E}) = \rho_v$$

$$\vec{E} = -\vec{\nabla} V$$

Poisson's equation is then

$$\vec{\nabla} \cdot (\varepsilon \vec{\nabla} V) = -\rho_v$$

If $\rho_v = 0$, we get Laplace's equation

$$\vec{\nabla} \cdot (\varepsilon \vec{\nabla} V) = 0$$

If the material is homogeneous, i.e. σ and ε_r are independent of spatial coordinates, this simplifies to

$$\vec{\nabla}^2 V = -\frac{\rho_v}{\varepsilon}$$

and

$$\vec{\nabla}^2 V = 0$$