Lecture 8

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November 16, 2022

1 Twitch vs Mixer

Let $x(t)$ and $y(t)$ be the users on Twitch and Mixer respectively. We assume

- For every existing user on Twitch, Twitch gains 2 more users
- For every existing user on Mixer, Mixer gains 4 more users
- Twitch loses 1 viewer for every viewer on Mixer
- Mixer loses 3 viewers for every viewer on Twitch
- Twitch naturally gains 200 new viewers
- Mixer naturally gains 50 new viewers

What will happen to both platforms?

$$\frac{dx}{dt} = 2x - y + 200$$

$$\frac{dy}{dt} = 4y - 3x + 50$$

This can be rewritten as

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 200 \\ 50 \end{pmatrix}$$

Based on the phase potrait (Fig 1), it is possible for both Twitch and Mixer to succeed. However, in reality, Mixer failed.

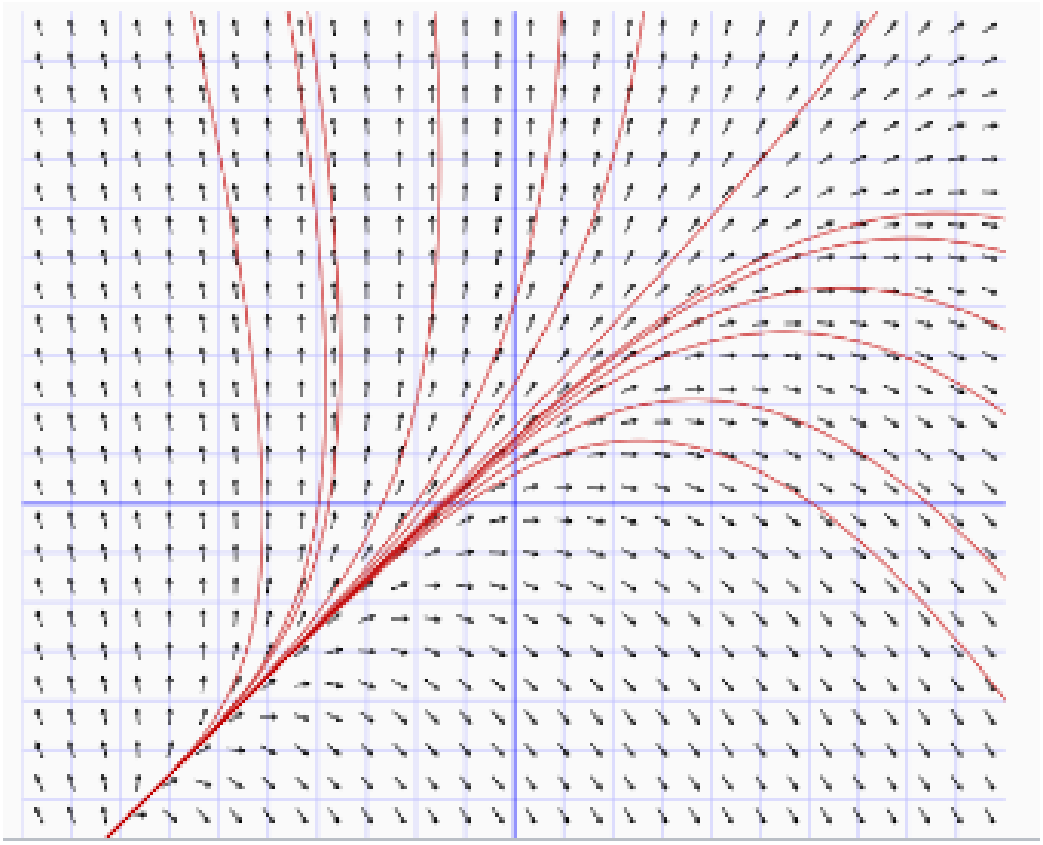

Figure 1: Phase Potrait (stolen from Beamer Slides)

Definition 1.1. A first order linear system of ODEs of dimension two is given by

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix}$$

If $g_1(t) = g_2(t) = 0$, this system is called homogeneous.

Theorem 1.1. *Suppose we have a first order linear system of ODEs as defined above with the initial conditions*

$$\begin{pmatrix} x(t_0) \\ y(t_0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Let $I = (\alpha, \beta)$ be the largest interval satisfying

- $t_0 \in I$
- $p_{11}(t), p_{12}(t), p_{21}(t), p_{22}(t)$ *are all continuous on* I
- $g_1(t)$ *and* $g_2(t)$ *are continuous on* I

Then the initial value problem has a unique solution defined on all of I*.*

Therefore, for autonomous systems, where all p and g are independent of t (i.e. constants), we are guaranteed a unique solution for $t \in \mathbb{R}$. (The proof is trivial and is left to the reader as an exercise)

2 Twitch Partnerships

Twitch also forms partnerships. Say Twitch partners with Apex Legends. We assume

- For every Apex player, Twitch gains one new subscriber
- For every two Twitch users, Apex gains one player

Let's consider a setting where both are losing consumers due to school

- For every user on Twitch, Twitch loses 2 more users
- For every player on Apex, Apex loses 2 more players

Despite the losses, they still gain a fixed number of users/players

- Twitch still gains a consistent 200 viewers
- Apex still gains a consistent 20 players

Then the system becomes

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 0.5 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 200 \\ 20 \end{pmatrix}$$

By observation (Fig 2), the solutions converge to a point. This gives us

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 0.5 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 200 \\ 20 \end{pmatrix}$$

Solving this, we have

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 120 \\ 40 \end{pmatrix}$$

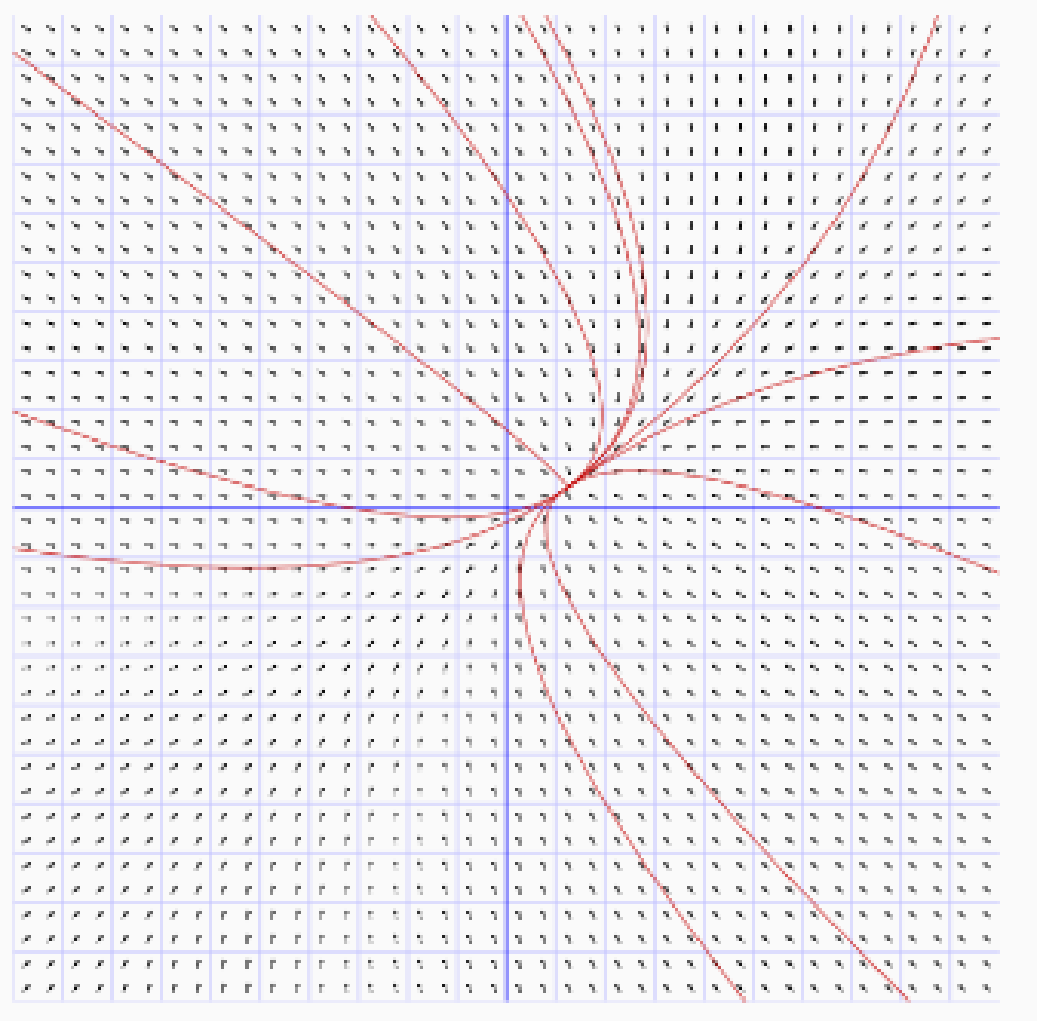

Figure 2: Phase Potrait (also stolen from Beamer)