Lecture 26

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1 Magnetization

A magnetic dipole is a closed loop of current. Then

$$\vec{m} = IS\hat{a}_n$$

where I is the current and S is the surface area contained in the loop. With N terms,

$$\vec{m} = NIS\hat{a}_n$$

If a magnetic dipole is placed in an applied magnetic field, it will experience a torque equal to

$$\vec{T} = \vec{m} \times \vec{B}$$

Now all materials have small magnetic dipole caused by electrons orbiting the nuclei. If we apply a magnetic field to a magnetic material, the dipoles within the material would align with the applied magnetic field.

Definition 1.1. A material is considered magnetic if all atomic magnetic dipoles can be aligned in the same directions.

Definition 1.2. The magnetization vector is defined as

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_i \vec{m}_i$$

Definition 1.3. This leads to the creation of bound current density

$$\vec{J}_{\mathrm{ms}} = \vec{M} \times \hat{a}_n$$

Definition 1.4. The magnetic field intensity is

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_r \mu_0}$$

Definition 1.5. The magnetic susceptibility is

$$\vec{M} = \chi_m \vec{H}$$

Definition 1.6. The relative permeability is

$$\mu_r = \chi_m + 1$$

Example 1.1. Calculate the magnetic field at $(0, 0, z)$ due to a permanent cylindrical bar magnet. We assume $\vec{M} = M_0 \hat{a}_z$ in the cylinder defined by $-\frac{L}{2} \leq z \leq \frac{L}{2}, 0 \leq r \leq a$.
Along the side wall,

$$\vec{J}_{\text{ms}} = \vec{M} \times \hat{a}_r = M_0 \hat{a}_\phi$$

We use the Biot-Savart Law,

$$\begin{aligned}
\vec{B}(0, 0, Z) &= \iint \frac{\mu_0 \vec{J}_{\text{ms}} \times \vec{R} dS}{4\pi |\vec{R}|^3} \\
&= \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{2\pi} \frac{M_0 \hat{a}_\phi \times (-a\hat{a}_r + (Z - z)hata_z)}{(a^2 + (Z - z)^2)^{1.5}} a d\phi dz \\
&= \frac{\mu_0 M_0}{2} \left[\frac{L}{2\sqrt{a^2 + \left(z - \frac{L}{2}\right)^2}} + \frac{L}{2\sqrt{a^2 + \left(z + \frac{L}{2}\right)^2}} \right]
\end{aligned}$$