Lecture 16

niceguy

November 30, 2022

1 Incompressible Fluid at Rest

 ρ is a constant for incompressible fluids. Then from

$$\frac{dp}{dz} = -\rho g$$

we get

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

as derived in the previous lecture.

Definition 1.1. The specific weight is defined as

$$\gamma := \rho g$$

We define specific weight as it appears often in fluid dynamics. It is the weight per unit volume of the fluid. Moreover, since liquids are usually much denser than gases

$$\begin{aligned}
\rho_g &<< \rho_f \\
\rho_g g &<< \rho_f g \\
\left| \left(\frac{dp_g}{dz} \right) \right| &<< \left| \left(\frac{dp_f}{dz} \right) \right|
\end{aligned}$$

Therefore, for small distances $\Delta h \approx 0$, we can take gas pressure to be constant.

Example 1.1. Find the pressure-elevation relationship for isothermal perfect gas.

$$p = \rho RT$$

Then

$$\frac{dp}{dz} = -\rho g$$

$$\frac{dp}{dz} = -\frac{p}{RT}g$$

$$\frac{dp}{p} = -\frac{g}{RT}dz$$

$$\ln \frac{p_2}{p_1} = -\frac{g}{RT}(z_2 - z_1)$$

$$p_2 = p_1 e^{-\frac{g}{RT}(z_2 - z_1)}$$

As a summary, assuming no shear,

$$-\vec{\nabla}p + \rho\vec{q} = \rho\vec{a}$$

where \vec{q} is a body force. If it is gravity, we have

$$-\vec{\nabla}p - \rho q\hat{k} = \rho \vec{a}$$

If the fluid is also at rest, we have

$$\frac{dp}{dz} = -\rho g$$

For incompressible fluids at rest,

$$p_2 = p_1 + \rho g h$$

and for compressible fluids at rest,

$$\int dp = \int \rho g dz$$

which simplifies to a constant p for small ranges Δh .

Example 1.2. For an incompressible fluid at rest, in a gravitational field acting in the -z direction, show that the free surface is horizontal. Given the conditions, we have

$$p = -\rho gz + C$$

We know $p = p_{atm}$ at $z = z_s$, so

$$p_{atm} = -\rho g z_s + C$$

Rearranging yields

$$z_s = \frac{C - p_{atm}}{\rho g}$$

which is an expression of constants. The z component of the surface is a constant, hence the surface is horizontal.

2 Measurements of Pressure

Pressure values are stated with respect to a reference level. The gage pressure is measured with a local atmospheric reference, while absolute pressure is compared with a vacuum reference. Hence

$$p_{abs} = p_{gage} + p_{atm}$$

Several instruments for measuring pressure including

• Mercury Barometer: $P_a = \rho g h$

• Piezometer: $P_a = \rho g h$

• U-tube manometer: $P_a = \gamma_2 h_2 - \gamma_1 h_1$

Example 2.1. A closed tank contains air and water. A piezometer is connected to the tank as shown. The column height are $h_1 = 1 \text{m}, h_2 = 0.5 \text{m}$. If the pressure gage reading of the compressed air shows 10 kPa, determine the height h of the water in the piezometer.

$$\gamma(h-1) = 10000 \Rightarrow h = 2$$