Lecture 9

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1 Electric Scalar Potential

For a point charge which is not at the origin, we can generalise as

$$V = \frac{Q}{4\pi\varepsilon_0|\vec{R} - \vec{R}'|}$$

For a collection of point charges, we have

$$V = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0|\vec{R} - \vec{R}_i'|}$$

For a continuous charge distribution,

$$V = \int \frac{dQ'}{4\pi\varepsilon_0|\vec{R} - \vec{R}'|}$$

Example 1.1. Determine the electric potential at any point on the axis of a uniformly charged disk of radius a.

$$\begin{aligned}
V &= \int_0^{2\pi} \int_0^a \frac{\rho_s r dr d\phi}{4\pi\varepsilon_0\sqrt{r^2 + z^2}} \\
&= \frac{\rho_s}{2\varepsilon_0} \int_0^a \frac{r}{\sqrt{r^2 + z^2}} dr \\
&= \frac{\rho_s}{2\varepsilon_0} \left(\sqrt{a^2 + z^2} - |z|\right)
\end{aligned}$$

Example 1.2. It is known that for a specific charge distribution it has an electric field given by

$$\vec{E} = \begin{cases} 1 \times 10^{-5} r \hat{a}_r & r \leq 1\text{cm} \\ \frac{1 \times 10^{-9}}{r} \hat{a}_r & r > 1\text{cm} \end{cases}$$

Determine the volume charge density that creates this field.
For the first case,

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \frac{d}{dr}(rE_r) &= \frac{\rho r}{\varepsilon_0} \\ 2 \times 10^{-5} r &= \frac{\rho r}{\varepsilon_0} \\ \rho &= 2 \times 10^{-5} \varepsilon_0 \end{aligned}$$

For the second case,

$$\begin{aligned} \frac{d}{dr}(rE_r) &= \frac{\rho r}{\varepsilon_0} \\ 0 &= \frac{\rho r}{\varepsilon_0} \\ \rho &= 0 \end{aligned}$$