Lecture 16

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1 Capacitance

The potential difference between two charged bodies is always proportional to the charge on them.

Definition 1.1. The *capacitance* is

$$C = \frac{Q}{\Delta V} = \frac{Q}{V}$$

with the unit of Farads.

Then

$$C = \frac{Q}{\Delta V} = \frac{\oiint_S \vec{D} \cdot d\vec{S}}{|\int \vec{E} \cdot d\vec{l}|} = \frac{\oiint \varepsilon_0 \varepsilon_r \vec{E} \cdot d\vec{S}}{|\int \vec{E} \cdot d\vec{l}|}$$

which can be solved for just by $\vec{E}$.

Example 1.1. The capacitance of parallel plates is as follows. Since $\vec{E}$ is conservative, ΔV is path independent, i.e.

$$\Delta V = -\int \vec{E} \cdot d\vec{l} = Ed = \frac{\rho_s}{\varepsilon_0 \varepsilon_r} d$$

Q is simply

$$Q = \rho_s S$$

where S is the surface area. Then the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{S \varepsilon_0 \varepsilon_r}{d}$$

This holds assuming $\vec{E}$ is constant throughout, neglecting fringing fields.

Example 1.2. Consider a spherical capacitor with ε_r, between $a \leq R \leq b$. Find the capacitance.
Let Q be the charge on the inner sphere. Gauss' Law gives us

$$E_R = \frac{Q}{4\pi\varepsilon_0\varepsilon_r R^2}$$

Then

$$\Delta V = \left| -\int \vec{E} \cdot d\vec{l} \right| = \frac{Q}{4\pi\varepsilon_0\varepsilon_r R}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Then

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b-a}$$