Problem Set 1

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1. Consider a particle moving along x direction according to the function $x(t) = -0.25t^3 + 1.5t^2 + 2t - 1$, where $t \geq 0$ and is measured in seconds and x is measured in meters.

   (a) Determine the initial position, velocity, and acceleration of the particle.

   Solution:
   Initial Position:
   $$x(0) = -1$$
   Initial Velocity:
   $$x'(t) = -0.75t^2 + 3t + 2 \Rightarrow x'(0) = 2$$
   Initial Acceleration:
   $$x''(t) = -1.5t + 3 \Rightarrow x''(0) = 3$$

   (b) Determine the time at which the particle stops

   Solution:
   $$\begin{aligned} x'(t) &= 0 \\ -0.75t^2 + 3t + 2 &= 0 \\ t &= 4.58 \end{aligned}$$
   if we ignore the negative solution.

   (c) Determine the time at which the particle experiences zero acceleration.

   Solution:
   $$\begin{aligned} x''(t) &= 0 \\ -1.5t + 3 &= 0 \\ t &= 2 \end{aligned}$$

2. A uniform bar of length L and mass M is placed along y axis, starting at $y = \frac{L}{9}$ and ending at $y = -\frac{8}{9}L$, as shown in the picture.

   (a) Using parallel axis theorem, show that the moment of inertia of the bar about the origin is $\frac{19}{81}ML^2$.

Solution:

$$I = \frac{1}{12}ML^2 + M\left(\frac{4.5-1}{9}\right)^2 L^2 = \frac{19}{81}ML^2$$

(b) The bar is pivoted at the origin as lifted to the right so that it makes an angle of 30° with the y axis. What is the gravitational torque (moment) exerted on teh bar?

Solution:

$$\frac{8M}{9}g\sin 30^\circ \times \frac{4L}{9} - \frac{M}{9}g\sin 30^\circ \times \frac{L}{18} = \frac{7}{36}MgL$$

in the clockwise direction.

(c) What is the potential energy of the bar in this position. Assume $U(y=0)=0$.

Solution:

$$\frac{7L}{18}\left(1-\cos 30^\circ\right)Mg$$

(d) The bar is released. What is its speed of the bar as it passes through its vertical position if no energy was lost during the motion?

Solution:

$$\begin{aligned}
\frac{1}{2}mv^2 &= \frac{7}{18}\left(1-\cos\left(\frac{\pi}{6}\right)\right)MgL \\
v^2 &= \frac{7}{9}\left(1-\cos\left(\frac{\pi}{6}\right)\right)\frac{g}{L} \\
v &= \sqrt{\frac{7}{9}\left(1-\cos\left(\frac{\pi}{6}\right)\right)\frac{g}{L}}
\end{aligned}$$

3. An oscillator consists of a mass m attached to two springs with spring constants k_1 and k_2, as shown in the picture. At time $t = 0$s the oscillator is displaced distance $x = -x_0$ from equilibrium position and is stationary.

(a) Draw a free body diagram for the mass. Indicate all possible (reasonable) forces.

(b) Write Newton's Second Law equations from the mass in x and y direction assuming there is no friction. Assume x direction to be horizontal.

Solution:

$$m\ddot{x} + (k_1+k_2)x = 0$$

$$mg = N$$

(c) If the period of oscillation is equal to T, what is the first time that the mass is passing the equilibrium?

Solution:

$$\frac{T}{4}$$

(d) What is the direction of the mass' velocity at that instance?

Solution: $+x$

(e) What is the second time that the mass is passing through the equilibrium?

Solution:

$$\frac{3T}{4}$$

4. Consider a function

$$x(\theta) = 2.0 \sin\left(3\theta + \frac{\pi}{3}\right)$$

where θ is measured in radians.

(a) What is the smallest positive value of θ for which $x(\theta)$ is at positive, maximum value?

Solution:

$$\begin{aligned} 3\theta + \frac{\pi}{3} &= \frac{\pi}{2} \\ \theta &= \frac{\pi}{18} \end{aligned}$$

(b) What are the first two, smallest, positive values of θ for which $x(\theta) = 0$?

Solution:
First solution:

$$\begin{aligned} 3\theta + \frac{\pi}{3} &= \pi \\ \theta &= \frac{2\pi}{9} \end{aligned}$$

Second solution:

$$\begin{aligned} 3\theta + \frac{\pi}{3} &= 2\pi \\ \theta &= \frac{5\pi}{9} \end{aligned}$$