Lecture 4

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1 Recall

$$P(A) \in [0,1] \forall A \subseteq S$$

Example 1.1. Consider throwing darts. The probability of landing in region A, given a random throw, is

$$P(A) = \frac{\text{Area of } A}{\text{Area of } S} = \frac{\int_A dx}{\int_S dx}$$

2 Additive Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 2.1. Consider 2 dice. The probability of rolling 7 (A) or at least one two (B) is

$$P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{11}{36} - \frac{1}{18} = \frac{5}{12}$$

Expanding, we have

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B \cup C) - P(A \cap (B \cup C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \end{aligned}$$

3 Conditional Probability

For $A, B \subseteq S$, $P(B|A)$ is the probability of B given A occurred.

Example 3.1. Rolling two dice

- $P(7) = \frac{1}{6}$
- $P(7|\text{first roll is a } 2) = \frac{1}{6}$

Definition 3.1.

$$P(B|A) = \frac{P(A \cup B)}{P(A)}, P(A) > 0$$

Consider being a pro athlete. The probability of that is 10^{-4}. Given the probability of beinga pro who starts at 3 is 9×10^{-5}, and the probability of starting at 3 is 0.01. Then the probability of being a pro given one starts at 3 is then

$$\frac{9 \times 10^{-5}}{0.01} = 0.009$$