Lecture 8

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1 Joint Distribution

Definition 1.1. $f(x)$ is a joint PMF of the discrete random variables X and Y if

- $f(x, y) \geq 0$
- $\sum_x \sum_y f(x, y) = 1$
- $P(X = x, Y = y) = f(x, y)$

Example 1.1. There are 52 cards and 5 in a hand. Let X be the number of queens and Y the number of kings. Then

$$f(x, y) = \begin{cases} \binom{4}{x}\binom{4}{y}\binom{44}{5-x-y}/\binom{52}{5} & x + y \leq 5 \\ 0 & \text{else} \end{cases}$$

Let $A = \{(X, Y)|X + Y = 2\}$. Then

$$P((X, Y) \in A) = \sum_{(X,Y)\in A} f(x, y) = f(2, 0) + f(1, 1) + f(0, 2)$$

Definition 1.2. $F(x, y)$ is a joint PDF of the continuous random variables X and Y if

- $f(x, y) \geq 0$
- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$
- $P((X, Y) \in A) = \int_A f(x, y) dx dy$

Example 1.2. Consider a uniform distribution in $S = \{(x, y) | -1 \le x \le 1, -1 \le y \le 1\}$. Then

$$f(x, y) = \frac{1}{4}$$

in S.

2 Marginal Distribution

Consider a joint distribution $f(x, y)$. Then the marginal distributions are the probabilities of X or Y happening individually. For the discrete case,

$$g(x) = \sum_y f(x, y)$$

$$h(y) = \sum_x f(x, y)$$

For the continuous case,

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$h(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

Example 2.1.

$$f(x, y) = \begin{cases} 1 & |x| + |y| \le 1, y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Then

$$\begin{aligned} g(x) &= \int_{-\infty}^{\infty} f(x, y) dy \\ &= \int_{0}^{1-|x|} dy \\ &= 1 - |x| \end{aligned}$$