Lecture 25

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1 Ampère's Law

Example 1.1. An infinitely long metallic strip of negligible thickness and width $2a$ carries a total current of I. The strip lies in the xy-plane, is centered about the z-axis, and is infinitely long in the x-direction. The current is uniformly distributed over the width of the strip and flows in the $+x$-direction. Dtermine the magnetic field due to this current strip at a point $P(0, 0, z)$.
Using Biot-Savart law,

$$d\vec{B} = \frac{\mu_0 \vec{J} \times (\vec{R} - \vec{R}')dS}{4\pi|\vec{R} - \vec{R}'|^3}$$

Plugging this into an integral,

$$\begin{aligned}
\vec{B} &= \int_{-\infty}^{\infty} \int_{-a}^{a} \frac{\mu(\frac{I}{2a})\hat{a}_x \times (-x\hat{a}_x - y\hat{a}_y + z\hat{a}_z)}{(x^2+y^2+z^2)^{1.5}} dydx \\
&= \frac{\mu_0 I}{8\pi a} \int_{-\infty}^{\infty} \int_{-a}^{a} \frac{-z\hat{a}_y - y\hat{a}_z}{(x^2+y^2+z^2)^{1.5}} dydx \\
&= \frac{\mu_0 I}{8\pi a} \int_{-\infty}^{\infty} \int_{-a}^{a} \frac{-z\hat{a}_y}{(x^2+y^2+z^2)^{1.5}} dydx \\
&= -\frac{\mu_0 I}{2\pi a} \tan^{-1}\left(\frac{a}{z}\right) \hat{a}_y
\end{aligned}$$

Note that the $\hat{a}_z$ term is ignored. By the right hand rule, we know the resulting field is parallel to $\hat{a}_y$. Moreover, the $\hat{a}_z$ term is odd, so the integral yields a 0.

The forms of **Ampère's Law** are its differential form

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

and its integral form

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{S} = I$$

This means at any point in space, the magnetic field has a non zero curl if and only if a current density $\vec{J}$ is present.

1.1 Integral Form

In the integral form, we will use an **Ampèrian Loop**. We choose a loop such that

- $\vec{H}$ is either tangential or normal to the loop, so $\vec{H} \cdot d\vec{l}$ is Hdl or 0
- $\vec{H}$ has a constant value when $\vec{H}$ is tangential, so $\int Hdl = H \int dl = HL$

Example 1.2. Find the magnetic fields within each region of a coaxial cable. First observe that $\vec{H} = H_\phi \hat{a}_\phi$. Now for $0 < r < a$, the inner cabe,

$$\oint_C \vec{H} \cdot d\vec{l} = H_\phi(2\pi r) = I_{\text{enc}} = J\pi r^2 = \left(\frac{I_0}{\pi} a^2\right) \pi r^2 \Rightarrow \vec{H} = \frac{I_0 a^2 r}{2\pi}$$