Homework 5

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1. Given a continuous uniform distribution, show that
   (a) $\mu = \frac{A+B}{2}$
   (b) $\sigma^2 = \frac{(B-A)^2}{12}$

Solution: The mean is

$$\begin{aligned}
\mu &= \int_{-\infty}^{\infty} xf(x)dx \\
&= \int_A^B \frac{x}{B-A}dx \\
&= \frac{B^2-A^2}{2(B-A)} \\
&= \frac{A+B}{2}
\end{aligned}$$

The variance is

$$\begin{aligned}
\sigma^2 &= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx \\
&= \int_A^B \frac{x^2-(A+B)x+\frac{(A+B)^2}{4}}{B-A}dx \\
&= \frac{B^3-A^3}{3(B-A)} - \frac{(A+B)(B^2-A^2)}{2(B-A)} + \frac{(A+B)^2(B-A)}{4(B-A)} \\
&= \frac{A^2+AB+B^2}{3} - \frac{(A+B)^2}{2} + \frac{(A+B)^2}{4} \\
&= \frac{4A^2+4AB+4B^2-6A^2-12AB-6B^2+3A^2+6AB+3B^2}{12} \\
&= \frac{A^2-2AB+B^2}{12} \\
&= \frac{(B-A)^2}{12}
\end{aligned}$$

2. A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.
   (a) What is the probability that the individual waits more than 7 minutes?
   (b) What is the probability that the individual waits between 2 and 7 minutes?

Solution:

$$P[X > 7] = \frac{10-7}{10} = 0.3$$
$$P[2 \leq X \leq 7] = \frac{7-2}{10} = 0.5$$

3. Given a standard normal distribution, find the value of k such that
   (a) $P(Z > k) = 0.2946$
   (b) $P(Z < k) = 0.0427$
   (c) $P(-0.93 < Z < k) = 0.7235$

Solution:

$$0.54, -1.72, 1.28$$

4. The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what percentage of the loaves are
   (a) longer than 31.7 centimeters?
   (b) between 29.3 and 33.5 centimeters in length?
   (c) shorter than 25.5 centimeters?

Solution:

$$P(X > 31.7) = P(Z > 0.85) = 19.8\%$$
$$P(29.3 < X < 33.5) = P(-0.35 < Z < 1.75) = 59.7\%$$
$$P(X < 25.5) = P(Z < -2.25) = 1.22\%$$

5. f a set of observations is normally distributed, what percent of these differ from the mean by
   (a) more than 1.3σ?
   (b) less than 0.52σ?

Solution:

$$P(|Z| > 1.3) = 19.4\%$$
$$P(|Z| < 0.52) = 39.7\%$$