Lecture 7

niceguy

March 13, 2023

1 Electronic Paramagnet

Continuing from our last lecture, we see total spin S can be found as $2N_{\text{up}}-N$. Then

$$\Omega(N_{\text{up}}, N) = \binom{N}{N_{\text{up}}} = \frac{N!}{(N-N_{\text{up}})!N_{\text{up}}!}$$

is the multiplicity function.

2 Einstein Solid

This is a collection of N simple harmonic oscillators in 3D (i.e. each atom has 3 degrees of freedom). Each oscillator has energy

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Then each microstate can be written as $\{n_1, \ldots, n_N\}$ where $n_i \in \mathbb{N}$. Then total energy is

$$U = \hbar\omega\left(\frac{N}{2} + \sum_{i=1}^{N} n_i\right) = \hbar\omega\left(\frac{N}{2} + q\right)$$

where q is defined as above, the total number of quanta.

$$\Omega(q, N) = \frac{(N-1+q)!}{(N-1)!q!}$$

Note the -1 term comes from the reduced degree of freedom (stars and banners).

Example 2.1. Consider two isolated states N_A, q_A and N_B, q_B. Then, they are brought together, allowing energy to flow. Then

$$q'_A + q'_B = q_{\text{total}}$$

which is constant due to conservation of energy. Now total microstates is

$$\Omega_{\text{total}} = \Omega_A(q'_A)\Omega_B(q'_B)$$