

1 Introduction

- From the german for number ring, Zahlring, a **ring** is a set, S , with two binary operations $*$ and $+$
- All rings must satisfy the 5 following properties
 1. Additive associativity
 2. Additive commutativity
 3. Additive identity
 4. Additive inverse
 5. Left and right distributivity

Rings may also satisfy the following conditions (these are commonly satisfied, though not always though as necessary. Note property 6 is usually required)

 6. Multiplicative associativity (associative ring)
 7. Multiplicative commutativity (commutative ring)
 8. Multiplicative identity (unit ring or ring with identity)
 9. Multiplicative inverse
- A ring that satisfies all the above properties is called a **field**
- Recall the integers, \mathbb{Z} is a ring without a multiplicative inverse
- The even integers are a ring that also does not have a multiplicative identity