1 Grassman numbers

- Are tired of integration? Do you enjoy derivatives instead? Boy do I have the number for you!!
- Grassman numbers are named after Hermann Grassman and used in quantum field theory to describe fermions
- The first property of Grassman numbers is anticommutivity under multiplication

$$\eta_1\eta_2 = -\eta_2\eta_1 \tag{1}$$

- This has the interesting consequence that the square, and by extension any higher power, of a Grassman number is 0 since

$$\eta_1\eta_2 = -\eta_2\eta_1 = 0 \tag{2}$$

- Thus we can only express functions of Grassman numbers in the form

$$f(\eta) = a_0 + a_1\eta \quad (a_0, a_1) \in \mathbb{C} \tag{3}$$

- The derivative of a Grassman function works as usual, with the added constraint that the operator must be adjacent to the matching variable by way of commutation

$$\frac{\mathrm{d}}{\mathrm{d}\eta_1}\eta_2\eta_1 = -\frac{\mathrm{d}}{\mathrm{d}\eta_1}\eta_1\eta_2 = -\eta_2 \tag{4}$$