CS341 Assignment Solution

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Assignment 1: Brute-Force Polynomial Evaluation

Problem: Evaluate the polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

at a given point $x = x_0$ using a brute-force algorithm, and calculate the time efficiency.

Brute-force Algorithm

The brute-force approach calculates each term $a_i x^i$ independently.

Algorithm 1 BruteForcePolynomialEvaluation

```
Input: coefficients a[0...n], value x
Output: value of polynomial p(x)
result ← 0
for i = 0 to n do
    power ← 1
    for j = 1 to i do
        power ← power × x
    end for
    result ← result + a[i] × power
end for
return result
```

Time Complexity

- The inner loop runs i times for each i from 0 to n. - Total operations: $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ - Therefore, the time complexity is $\mathcal{O}(n^2)$.

Assignment 2: The Assignment Problem

Problem: Given a cost matrix $C[i][j]$ of assigning person i to job j, find the assignment with the minimum total cost using exhaustive search.

Cost Matrix

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Exhaustive Search Algorithm

Try all $n!$ possible assignments and pick the one with the lowest total cost.

Algorithm 2 ExhaustiveAssignment

Input: Cost matrix $C[0 \ldots n-1][0 \ldots n-1]$
Output: Minimum total cost
$minCost \leftarrow \infty$
for each permutation P of jobs $[0 \ldots n-1]$ **do**
    $cost \leftarrow 0$
    for $i = 0$ to $n-1$ **do**
        $cost \leftarrow cost + C[i][P[i]]$
    end for
    if $cost < minCost$ **then**
        $minCost \leftarrow cost$
        $bestAssignment \leftarrow P$
    end if
end for
return $minCost$, $bestAssignment$

Result

All permutations checked (24 total for $n = 4$). The minimum cost assignment is:

$$\text{Person } 0 \to \text{Job } 1, \quad \text{Person } 1 \to \text{Job } 2, \quad \text{Person } 2 \to \text{Job } 0, \quad \text{Person } 3 \to \text{Job } 3$$

$$\text{Total cost} = 2 + 3 + 5 + 4 = \boxed{14}$$

Time Complexity

- Number of permutations: $n! = 4! = 24$ - For each permutation, we do n additions $\to$ Total: $O(n \cdot n!)$ - Therefore, time complexity: $\mathcal{O}(n \cdot n!)$