Lab 1- Answer

Problems

1. **Assume that $f(n) = 5n + 50$ and $g(n) = n$. Is $f(n) = O(g(n))$**

$$f(n) \leq c \cdot g(n) \quad \text{for all } n \geq n_0$$

Given:

- f(n) = 5n + 50
- g(n) = n

We want to find constants c and n_0 such that:

$$5n + 50 \leq c \cdot n$$

Rewriting:

$$50 \leq (c - 5)n \Rightarrow n \geq \frac{50}{c - 5}$$

Choose $c = 6$, then:

$$n \geq \frac{50}{1} = 50$$

So for $c = 6$ and $n_0 = 50$, the inequality holds.

$$f(n) = O(n)$$

2. Find the upper bound for $f(n) = 3n + 8$

We want to express $f(n)$ in Big-O notation.

Step 1: Try $O(n)$

Assume:

$$3n + 8 \leq c \cdot n$$

Then:

$$8 \leq (c - 3)n \Rightarrow n \geq \frac{8}{c - 3}$$

Choose $c = 4$:

$$n \geq \frac{8}{1} = 8$$

Then

$$f(n) = 3n + 8 = O(n)$$

3. Find the upper bound for $f(n) = n^2 + 10$

Try $O(n^2)$.

We want:

$$n^2 + 10 \leq c \cdot n^2 \Rightarrow 10 \leq (c - 1)n^2 \Rightarrow n^2 \geq \frac{10}{c - 1}$$

Let $c = 2$:

$$n^2 \geq 10 \Rightarrow n \geq \sqrt{10} \approx 3.16$$

So inequality holds for n .

$$f(n) = n^2 + 10 = O(n^2)$$

4. Find the lower bound for $f(n) = 10n^2 + 5$

We want to find a Big-Omega bound: $f(n) = (n^2)$

Step 1: Find dominant term ⇒ (n^2)

Assume:

$$10n^2 + 5 \geq c \cdot n^2 \Rightarrow 5 \geq (c - 10)(n^2)$$

This holds for any $c < 10$. Try $c = 9$:

$$10n^2 + 5 \geq 9n^2 \Rightarrow n^2 + 5 \geq 0 \quad \text{Always true}$$

$$f(n) = 10n^2 + 5 = \Omega(n^2)$$

5. Show that $f(n) = n^3 + 3n^2 = \Theta(n^3)$

To prove $f(n) = \Theta(n^3)$, we must show:

1. $f(n) = O(n^3)$
2. $f(n) = \Omega(n^3)$

Step 1: Show $f(n) = O(n^3)$

We want:

$$n^3 + 3n^2 \leq c \cdot n^3 \Rightarrow 1 + \frac{3}{n} \leq c$$

As $n \to \infty, \frac{3}{n} \to 0$. So for large n, this is less than $c = 2.$

Thus, for $c = 2$, inequality holds when $n \geq 3$

Step 2: Show $f(n) = \Omega(n^3)$

We want:

$$n^3 + 3n^2 \geq c \cdot n^3 \Rightarrow 1 + \frac{3}{n} \geq c$$

Choose $c = 1$, then $1 + \frac{3}{n} \geq 1$ is always true.

$$f(n) = n^3 + 3n^2 = \Theta(n^3)$$