

Early Classification of Time Series: Cost-based Multi-class Algorithms: Supplementary material

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I. COMPLEXITY ANALYSIS

A. ECONOMY- γ

The computational complexity of each step of learning phase of the ECONOMY- γ algorithm can be estimated as follows:

- Learn a classifier at each time step: $\mathcal{O}(T.Learn)$
- Sort confidence values to make intervals $I_{\tau 1 \leq j \leq K}^j$ at each time step t : $\mathcal{O}(T.|\mathcal{S}|.log(|\mathcal{S}|))$
- Confusion matrices at each time step: $\mathcal{O}(T.|\mathcal{S}|.Predict)$
- Compute prior probabilities for each class in each group: $|\mathcal{Y}|.K.|\mathcal{S}|$
- Estimate transition matrices: $\mathcal{O}(|\mathcal{S}|^2 K^2)$

(where *Learn* and *Predict* are respectively the learning and prediction complexities of the classifiers used). Thus the computational complexity of the ECONOMY- γ algorithm learning part is $\mathcal{O}(T.Learn + T.|\mathcal{S}|.log(|\mathcal{S}|) + T.|\mathcal{S}|.Predict + |\mathcal{Y}|.K.|\mathcal{S}| + |\mathcal{S}|^2 K^2)$.

B. Confidence scores aggregating probabilities

These four approaches differ only in the confidence scores they use. Their complexity only differ from the ECONOMY- γ method in the confidence score function. The four proposed methods use functions whose complexity is $\mathcal{O}(K)$ as they run through a vector of size K , thus giving a total complexity of $\mathcal{O}(T.Learn + T.(K.|\mathcal{S}| + |\mathcal{S}|.log(|\mathcal{S}|)) + T.|\mathcal{S}|.Predict + |\mathcal{Y}|.K.|\mathcal{S}| +$

$|\mathcal{S}|^2 K^2)$ where $T.|\mathcal{S}|$ is the complexity of applying the confidence function to all elements in the set \mathcal{S} .

C. Clustering

Given a clustering complexity *Clustering*, and that calibration requires sorting of size $|\mathcal{S}|$ for each class, we get respectively the following complexities:

$$\mathcal{O}(T.Learn + T.Clustering + T.|\mathcal{S}|.Predict + |\mathcal{Y}|.K.|\mathcal{S}| + |\mathcal{S}|^2 K^2)$$

and

$$\mathcal{O}(T.Learn + T.K.|\mathcal{S}|.log(|\mathcal{S}|) + T.Clustering + T.|\mathcal{S}|.Predict + |\mathcal{Y}|.K.|\mathcal{S}| + |\mathcal{S}|^2 K^2)$$

II. MULTI-CLASS DATASETS

Below are presented the full results obtained for the 33 datasets from [1] with more than 2 classes.

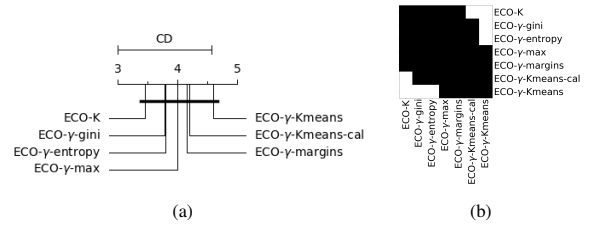


Fig. 1: Comparison of ECONOMY approaches for $\alpha = 0.001$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

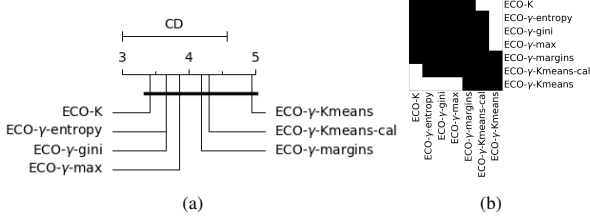


Fig. 2: Comparison of ECONOMY approaches for $\alpha = 0.01$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

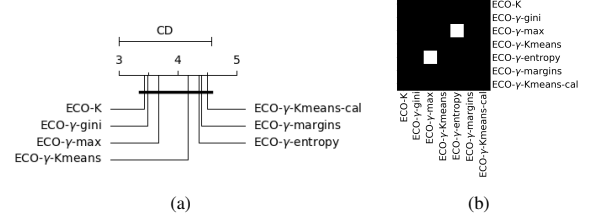


Fig. 7: Comparison of ECONOMY approaches for $\alpha = 0.5$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

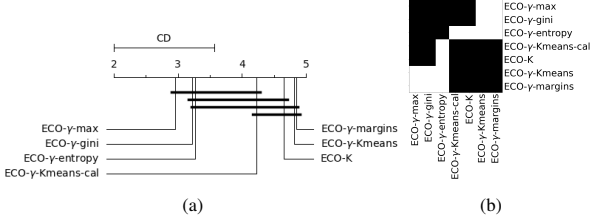


Fig. 3: Comparison of ECONOMY approaches for $\alpha = 0.1$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

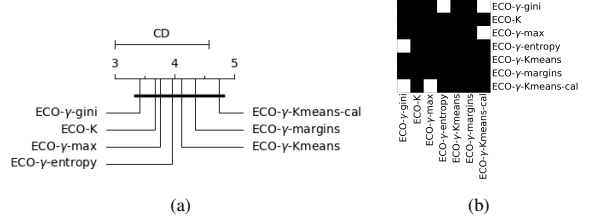


Fig. 8: Comparison of ECONOMY approaches for $\alpha = 0.6$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

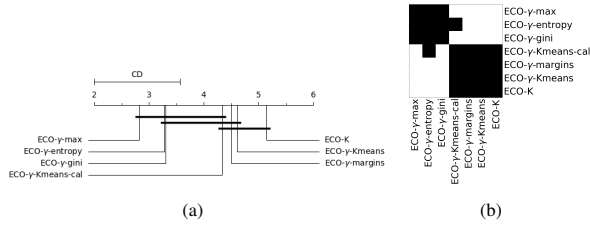


Fig. 4: Comparison of ECONOMY approaches for $\alpha = 0.2$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

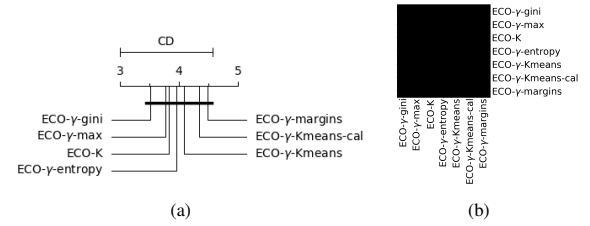


Fig. 9: Comparison of ECONOMY approaches for $\alpha = 0.7$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

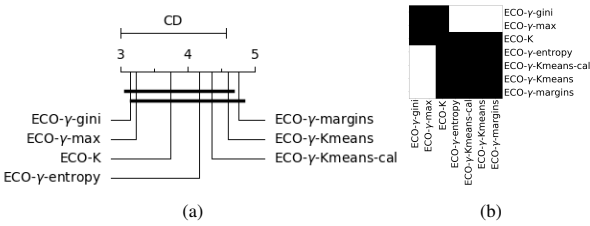


Fig. 5: Comparison of ECONOMY approaches for $\alpha = 0.3$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

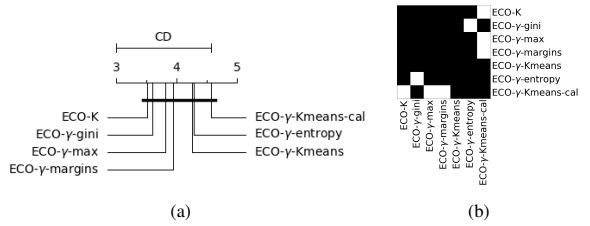


Fig. 10: Comparison of ECONOMY approaches for $\alpha = 0.8$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

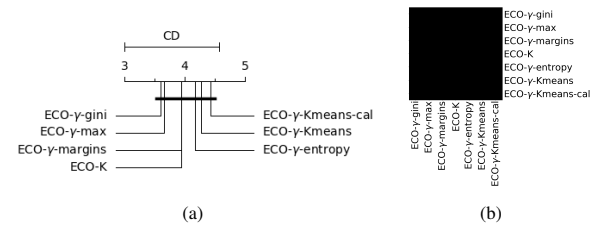


Fig. 6: Comparison of ECONOMY approaches for $\alpha = 0.4$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

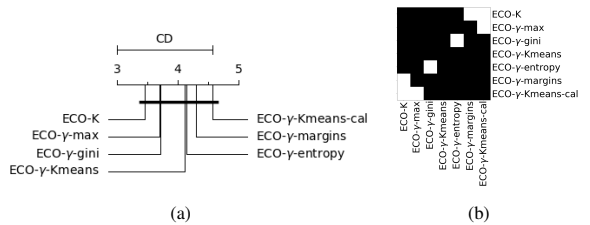


Fig. 11: Comparison of ECONOMY approaches for $\alpha = 0.9$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

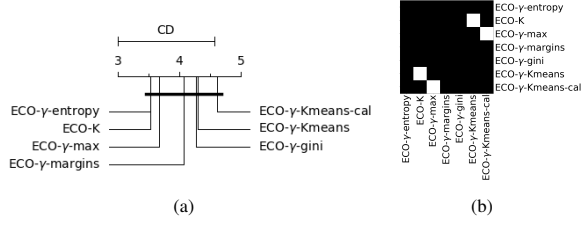


Fig. 12: Comparison of ECONOMY approaches for $\alpha = 1$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

III. FULL BENCHMARK, INCLUDING BINARY CLASSIFICATION

This section presents the same experiments as before, but carried out on the 45 datasets used in [1], including 12 binary classification datasets.

Algorithm	wins	defeats	ties	balance
ECO- γ -gini	22	0	50	+22
ECO- γ -max	15	0	57	+15
ECO- γ -entropy	12	3	57	+9
ECO-K	8	6	58	+2
ECO- γ -margins	2	9	61	-7
ECO- γ -Kmeans-cal	2	18	52	-16
ECO- γ -Kmeans	0	25	47	-25

TABLE I: ECONOMY approaches comparison using Wilcoxon signed-rank test: significant wins / defeats of each approach (against all the other) counted for all α , based on the *AvgCost* criterion.

REFERENCES

- [1] U. Mori, A. Mendiburu, S. Dasgupta, and J. A. Lozano, "Early classification of time series by simultaneously optimizing the

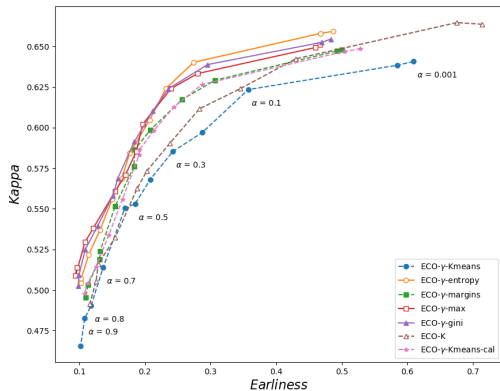


Fig. 13: Average Earliness vs. Average Kappa score obtain over the 45 datasets by varying the slope of the time cost, such as $\alpha \in [10^{-3}, 1]$.

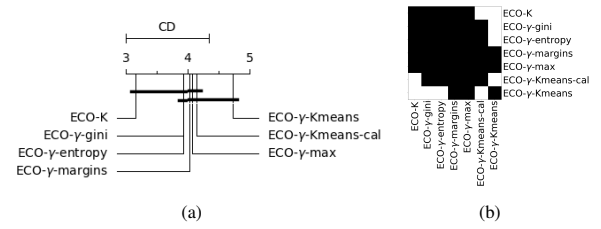


Fig. 14: Comparison of ECONOMY approaches for $\alpha = 0.001$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

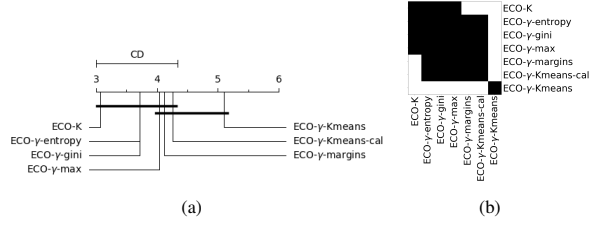


Fig. 15: Comparison of ECONOMY approaches for $\alpha = 0.01$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

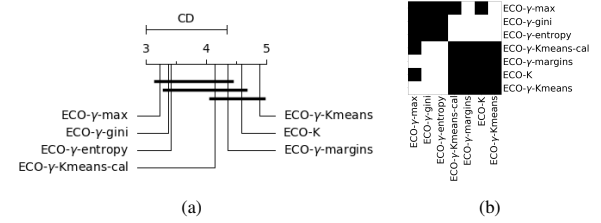


Fig. 16: Comparison of ECONOMY approaches for $\alpha = 0.1$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

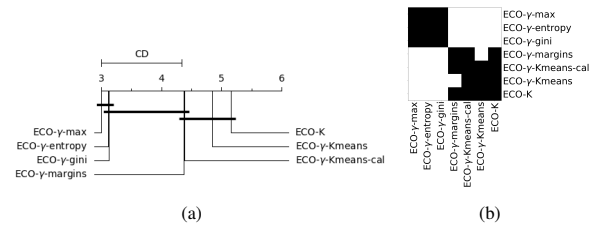


Fig. 17: Comparison of ECONOMY approaches for $\alpha = 0.2$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

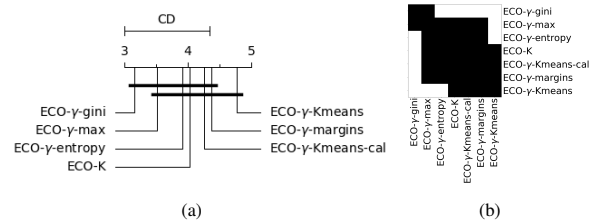


Fig. 18: Comparison of ECONOMY approaches for $\alpha = 0.3$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

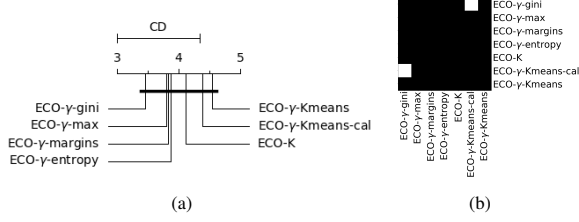


Fig. 19: Comparison of ECONOMY approaches for $\alpha = 0.4$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

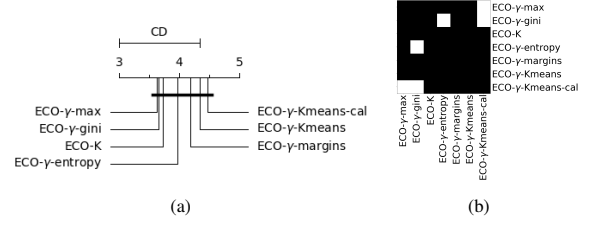


Fig. 24: Comparison of ECONOMY approaches for $\alpha = 0.9$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

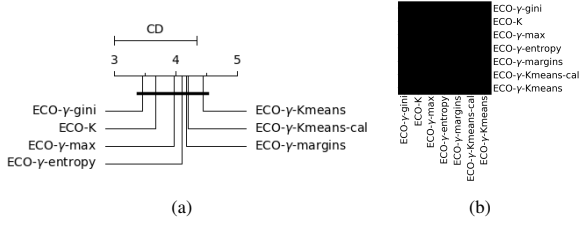


Fig. 20: Comparison of ECONOMY approaches for $\alpha = 0.5$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

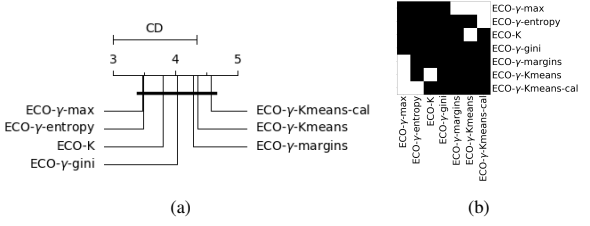


Fig. 25: Comparison of ECONOMY approaches for $\alpha = 1$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

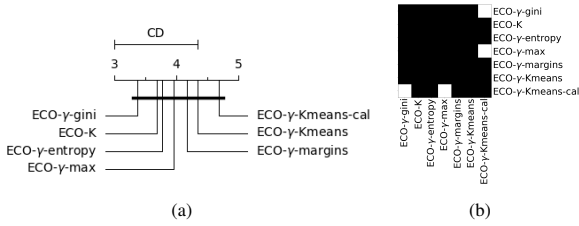


Fig. 21: Comparison of ECONOMY approaches for $\alpha = 0.6$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

accuracy and earliness,” *IEEE transactions on neural networks and learning systems*, vol. 29, no. 10, pp. 4569–4578, 2017.

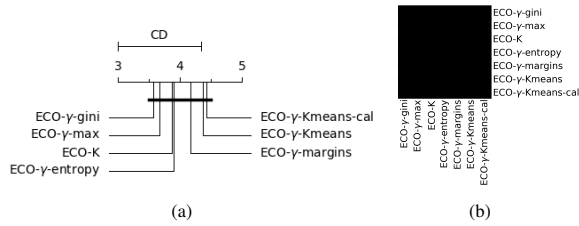


Fig. 22: Comparison of ECONOMY approaches for $\alpha = 0.7$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests

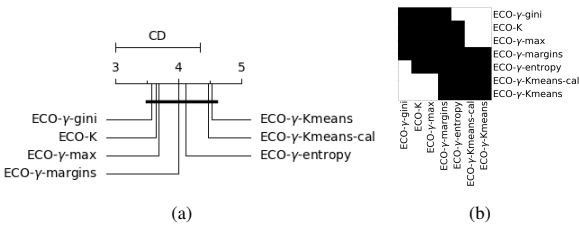


Fig. 23: Comparison of ECONOMY approaches for $\alpha = 0.8$ using (a) Nemenyi and (b) Wilcoxon signed-rank tests