# Functional Programming


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# Week1 :

## call by value vs call by name :

**Call-by-value :** evaluate the parameters *then* execute the function on these values.

e.g :

```
sumOfSquares(3, 2+2)
sumOfSquares(3, 4) // evaluated 2+2 BEFORE executing sumOfSquares
square(3) + square(4)
3 * 3 + square(4)
9 + square(4)
9 + 4 * 4
9 + 16
25
```

Call-by-value has the advantage that it evaluates every function argument only once.

**Call-by-name :** Apply the function to unreduced arguments.

e.g :

```
sumOfSquares(3, 2+2)
square(3) + square(2+2) // executing sumeOfSquare before evaluating 2+2
3 * 3 + square(2+2)
9 + square(2+2)
9 + (2+2) * (2+2)
9 + 4 * (2+2)
9 + 4 * 4
```

Call-by-name has the advantage that a function argument is not evaluated if the corresponding parameter is unused in the evaluation of the function body.

e.g :

```
def loop : Int = loop
def f(a:Int , b : Int ) : Int = a // Call-by-value
f(1,loop) // does not terminate
def f(a:Int, b : => Int) : Int = a // Call-by-Name ( => )
f(1,loop) // terminates
```

Both strategies reduce to the same final values as long as
- the reduced expression consists of pure functions, and
- both evaluations terminate.

## Conditionals :

```
def abs(x: Int) = if x ≥ 0 then x else -x
```

`if-then-else` in Scala is an expression (has a value) not a statement .

## Blocks , Scopes and first functional program :

Sqrt with Newton's method :

```
def sqrt(x: Double) = {
    def sqrtIter(guess: Double, x: Double): Double =
        if isGoodEnough(guess, x) then guess
        else sqrtIter(improve(guess, x), x)
    def improve(guess: Double, x: Double) =(guess + x / guess) /
    def isGoodEnough(guess: Double, x: Double) =abs(square(guess) - x) < 0.001

    sqrtIter(1.0, x)
}
```

`sqrtIter` , `improve` and `isGoodEnough` should not be outside sqrt because :

- they pollute the global scope.
- they should not be accessible to the user of the sqrt code given they are specific to sqrt .

the {} could be removed for more clarity.

## Tail recursion :

```
def gcd(a: Int, b: Int): Int =if b == 0 then a else gcd(b, a % b)
```

```
gcd(14, 21)
→ if 21 == 0 then 14 else gcd(21, 14 % 21)
→ if false then 14 else gcd(21, 14 % 21)
→ gcd(21, 14 % 21)
→ gcd(21, 14)
→ if 14 == 0 then 21 else gcd(14, 21 % 14)
↠ gcd(14, 7)
```

`gcd(14,21) = gcd(21,14) = gcd(14,7) = gcd(.., ...) = ... = 7` : flat structure, `gcd` only calls `gcd` => **tail recursion.**

```
def factorial(n: Int): Int = if n == 0 then 1 else n * factorial(n - 1)
```

```
factorial(4)
→ if 4 == 0 then 1 else 4 * factorial(4 - 1) 3-> ↠ 4 * factorial(3)
↠ 4 * (3 * factorial(2))
↠ 4 * (3 * (2 * factorial(1)))
↠ 4 * (3 * (2 * (1 * factorial(0)))
↠ 4 * (3 * (2 * (1 * 1)))
```

`factorial` calls factorial *and multiplies* , nested structure `⇒` **not tail recursion**

**To tell the compiler than a function is tail recursive:**

```
import scala.annotation.tailrec
@tailrec
def gcd(a: Int, b: Int): Int = ...
```

# Week 2 :

### Higher order functions :

*Def :* Functions that take other functions as parameters and return functions.

example :

```
def sum( f : Int ⇒ Int )( a : Int , b : Int ) : Int
```

- takes as argument : the function `f : Int ⇒ Int` .
- returns : a function that has 2 arguments `a:Int,b:Int` and returns `Int`

**Function types :** `A⇒B` is the type of a function that takes an argument of type A and returns a result of type B. e.g : `Int⇒Int` maps integers to integers.

### Anonymous Functions :

```
(x: Int, y: Int) ⇒ x + y
```

Use case :

```
def sumCubes(a: Int, b: Int) = sum(x ⇒ x * x * x, a, b) // type of function is inferred by the compiler
```

## Currying :

Suppose we have this header for `sum` : `def sum(f : Int=>Int , a : Int , b : Int)`

```
def sumInts(a: Int, b: Int) = sum(x => x, a, b)
def sumCubes(a: Int, b: Int) = sum(x => x * x * x, a, b)
def sumFactorials(a: Int, b: Int) = sum(fact, a, b)
```

Notice that a and b get passed unchanged from `sumInts` to `sum` .

*Q :* Can we do it shorter ?

```
def sum(f: Int => Int): (Int, Int) => Int =
    def sumF(a: Int, b: Int): Int =
        if a > b then 0
        else f(a) + sumF(a + 1, b)
sumF

def sumInts = sum(x => x)
def sumCubes = sum(x => x * x * x)
def sumFactorials = sum(fact)
```

`sum` is now a function that takes 1 argument ( `f` ) and returns a function that takes 2 arguments `a` and `b` .

We can still do shorter :

```
def sum(f: Int => Int)(a: Int, b: Int): Int =
    if a > b then 0 else f(a) + sum(f)(a + 1, b)
```

*Q :* What is the type of `sum` ?

*A :* `(Int=>Int) => ((Int,Int)=>Int)` equivalent to

`(Int=>Int) => (Int,Int)=>Int`

Note that function types **associate to the right** so it is equivalent to :

`Int => Int => Int` is equivalent to `Int => (Int => Int)`

## Classes :

we can define classes in Scala :

```
class Rational(x: Int, y: Int):
    def numer = x // numer and denom here are functions (recalculated each call)
    def denom = y

    // adding operations
    def addRational(r: Rational, s: Rational):
        Rational = Rational(r.numer * s.denom + s.numer * r.denom,r.denom *
```

```
    // overriding toString
    override def toString = s"$numer/$denom" // s means formatted string
    // what is after a $ is evaluated

val x = Rational(1, 2) // x: Rational = Rational@2abe0e27
x.numer // 1
x.denom // 2
val y = Rational(5, 7)
val z = Rational(3, 2)
x.add(y).add(z)
```

Classes are very similar to Java.

Suppose that `numer` and `denom` need more calculations , like the following :

```
class Rational(x: Int, y: Int):
    private def gcd(a: Int, b: Int): Int =
        if b == 0 then a else gcd(b, a % b)
    def numer = x / gcd(x, y) // these are recalcaled at each call
    def denom = y / gcd(x, y) // BAD IDEA

    // BETTER IDEA : VARIABLES :
    val numer = x / gcd(x, y)
    val denom = y / gcd(x, y)
```

It is possible to use `this.(..)` like is Java.

### Substitution and Extensions :

#### 1. Substitution :

Suppose that we have a class definition :

`class C(x1, ..., xm){ ... def f(y1, ..., yn) = b ... }`

*Q :* How is the following expression evaluated ? `C(v1, ..., vm).f(w1, ..., wn)`

*A :* Three substitutions happen written :

`[w1/y1, ..., wn/yn][v1/x1, ..., vm/xm][C(v1, ..., vm)/this] b`

- Substitution of `y1, ..., yn` of f by the arguments `w1, ..., wn` .
- Substitution of the `x1, ..., xm` of the class C by the class arguments `v1, ..., vm` .
- Substitution of the self reference `this` by the value of the object `C(v1, ..., vn)` .

#### 2. Extension methods (1) :

Having to define all methods that belong to a class inside the class itself can lead to very large classes, and is not very modular.

Methods that do not need to access the internals of a class can alternatively be defined as extension methods

For instance, we can add min and abs methods to class Rational like this:

```
extension (r: Rational):
    def min(s: Rational): Boolean = if s.less(r) then s else r
    def abs: Rational = Rational(r.numer.abs, r.denom)
```

- Extension methods **CANNOT** access internals ( `private` ) or `this.(..)`.

**3. Operators :**

▶ We write `x + y`, if x and y are integers, but

▶ We write `r.add(s)` if r and s are rational numbers.

In Scala, we can eliminate this difference with **operators**.

```
extension (x: Rational):
    def + (y: Rational): Rational = x.add(y)
```

**Precedence rules :**

```
(all letters)
|
^
&
< >
= !
:
+ -
* / %
(all other special characters)
```

*(all other special characters)* having the **highest priority**.

# Week3 :

## Abstract classes :

```
abstract class IntSet:
    def incl(x: Int): IntSet  // ABSTRACT MEMBERS : no implementation provided
    def contains(x: Int): Boolean
// incl returns the union of {x} and this set
```

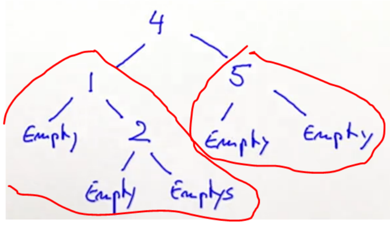
Like in Java, abstract classes cannot be instantiated.

We will try to implement a set as a binary tree.

A set can either be :

1. A tree for the **empty set**.

2. A tree consisting of **one** integer with two subtrees.



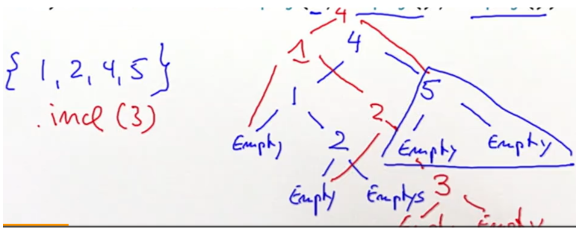

```
class Empty() extends IntSet:
    def contains(x: Int): Boolean = false
    def incl(x: Int): IntSet = NonEmpty(x, Empty(), Empty())

class NonEmpty(elem: Int, left: IntSet, right: IntSet) extends IntSet:
        def contains(x: Int): Boolean =
            if x < elem then left.contains(x)
            else if x > elem then right.contains(x) else true

        def incl(x: Int): IntSet =
        if x < elem then NonEmpty(elem, left.incl(x), right)
        else if x > elem then NonEmpty(elem, left, right.incl(x))
    else this
end NonEmpty
```

1. **Persistence :**

   Note that we the sets are immutable, we always return a new tree while **reusing some subtrees.**(blue one in the following e.g)




A data structure that is creating by maintaining the old one is called **persistent**.

2. **Dynamic binding :**

```
class Empty() extends IntSet:
     ...
    def union(other : Inset): Intset = other
class NonEmpty((elem: Int, left: IntSet, right: IntSet)) extends Inset :
     ...
    def union(other : Inset ): Intset =
lelf.union(right).union(that).incl(elem)

// Why does this terminate ? Union is called with strictly smaller sets each
time so union( "an empty set " ) will be called at some point.
```

Note that a call to `union` doesn't execute the same function if s is `Empty` or `NonEmpty`. That is called Dynamic **binding**. ( Polymorphism )

### Object definition :

In the `IntSet` example, one could argue that there is really **only a single** empty `IntSet`.

This defines a **singleton** object named Empty. No other Empty instance can be (or needs to be) created.

```
 object Empty extends IntSet:
    def contains(x: Int): Boolean = false
    def incl(x: Int): IntSet = NonEmpty(x, Empty, Empty) end Empty
```

#### Companion object :

If a class and object with the same name are given in the same sourcefile, we call them companions. Example:

```

class IntSet ...
object IntSet:
    def singleton(x: Int) = NonEmpty(x, Empty, Empty)
```

Similar to Java, static nested class.

How is naming a class and an object the same name possible ? Scala has **two** namespaces , one for objects and the other for classes.

### Programs :

Like Java , scale source files can have a main methods

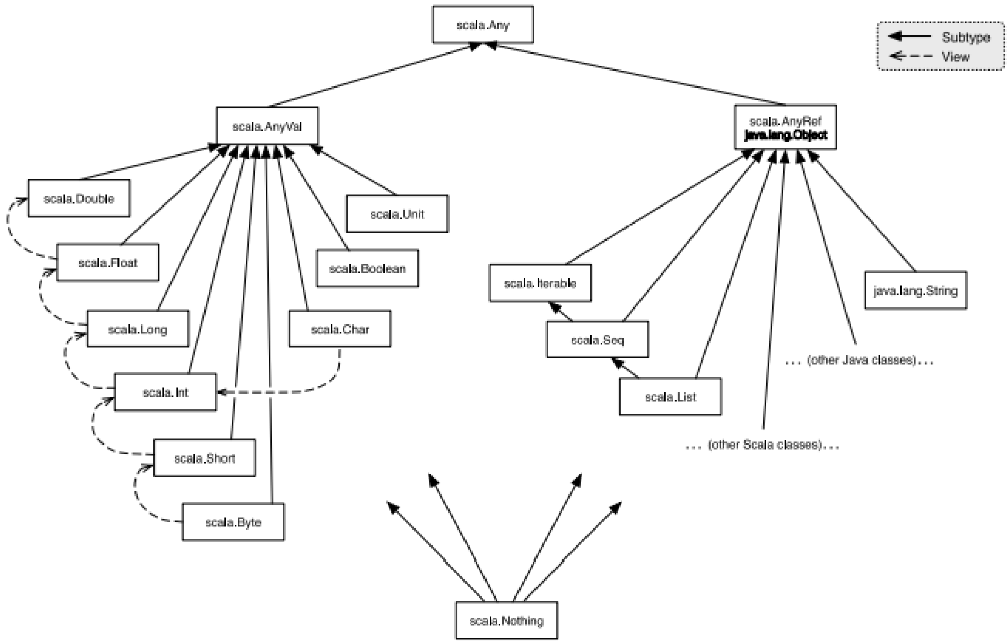
```
object Hello:
    def main(args: Array[String]): Unit = println(”hello world!”)
// written Shorter as the following :
@main def birthday(name: String, age: Int) =
println(s”Happy birthday, $name! $age years old already!”)
```

to run : `scala Hello`

### Traits :

Java `Interface` but stronger :they can have parameters and can contain fields and concrete methods.

```
trait Planar:
    def height: Int
    def width: Int
    def surface = height * width
class Square extends Shape, Planar, Movable ...
```




`- - ->` : can be converted (viewed) as

## Cons List :

A fundamental data structure in many functional languages is the **immutable linked list.**

It is constructed from two building blocks:

1. `Nil` :the empty list
2. `Cons` :a cell containing an element and the remainder of the list

```
trait IntList ...
class Cons(val head: Int, val tail: IntList) extends IntList ...
// NOTE THE USE OF val
// this defines at the same time a parameter and a field of a class
class Nil() extends IntList ...
```

A `IntList` is either a `Nil()` or a `Cons(x,xs)` .

### A complete definition with Generics :

```
trait List[T]:
    def isEmpty: Boolean
    def head: T
    def tail: List[T]
class Cons[T](val head: T, val tail: List[T]) extends List[T]:
    def isEmpty = false
class Nil[T] extends List[T]:
    def isEmpty = true
    def head = throw new NoSuchElementException(”Nil.head”)
    def tail = throw new NoSuchElementException(”Nil.tail”)
```

### Generic functions :

```scala
def singleton[T](elem: T) = Cons[T](elem, Nil[T])
// We can then write:
singleton[Int](1)
singleton[Boolean](true)
```

## Pure Object Orientation :

*Def :* A pure object-oriented language is one in which every value is an object.

*Q :* Is Scala a pure OO language ?

At first primitive types and functions seem like exceptions.

**Primitive types** are in fact not implemented as Class. (E.g `scala.Int` : 32bits , `scala.Boolean` as Java Boolean ) , for reasons of efficiency.

But they are not *conceptually* treated differently. In fact one can implement them using classes :

```scala
package idealized.scala
abstract class Boolean extends AnyVal:
    def ifThenElse[T](t: => T, e: => T): T
    def && (x: => Boolean): Boolean = ifThenElse(x, false)
    def || (x: => Boolean): Boolean = ifThenElse(true, x)
    def unary_!: Boolean = ifThenElse(false, true)
    def == (x: Boolean): Boolean = ifThenElse(x, x.unary_!)
    def != (x: Boolean): Boolean = ifThenElse(x.unary_!, x)
...
end Boolean
object true extends Boolean:
def ifThenElse[T](t: => T, e: => T) = t
object false extends Boolean:
def ifThenElse[T](t: => T, e: => T) = e
```

Here is how to implement `Scala.Int`.

**Functions :** In scala functions are `objects` with `apply` methods

The function type `A => B` is just an abbreviation for the class `scala.Function1[A, B]` , which is defined as follows.

```scala
package scala
trait Function1[A, B]:
    def apply(x: A): B
```

Example :

```scala
f = (x:Int) => x * x
// is expended to
f = new Function1[Int, Int]:
    def apply(x: Int) = x * x
```

This anonymous class can itself expend to

```
{ class $anonfun() extends Function1[Int, Int]:
def apply(x: Int) = x * x
$anonfun()
}
f(7) // expends to
f.apply(7)
```

# Week 4 :

## Decomposition :

`Expr` Suppose you want to write a small interpreter for arithmetic

/ \ expressions.

`Number` `Sum` `Sum(Number(1) , Number(2)).eval` = 3

```
trait Expr :
    def eval(e: Expr): Int =
        if e.isNumber then e.numValue
        else if e.isSum then eval(e.leftOp) + eval(e.rightOp)
        else throw Error("Unknown expression " + e)
```

1. `isNumber` , `isSum` gets quickly gets tedious when adding other datatypes .
2. There's no static guarantee you use the right accessor functions.
   You might hit an Error case if you are not careful.

### Non solution :Type Tests and Type casts

use `def isInstanceOf[T]: Boolean` to type *test* and `def asInstanceOf[T]: T` to cast.

Their use in Scala is discouraged, because there are better alternatives

### Solution 1 : Object Oriented Decomposition

```
trait Expr:
def eval: Int
class Number(n: Int) extends Expr:
    def eval: Int = n
class Sum(e1: Expr, e2: Expr) extends Expr:
    def eval: Int = e1.eval + e2.eval
```

▶ OO decomposition mixes data with operations on the data.

▶Good for encapsulation and data abstraction.

▶Dependencies between classes => Increases **Complexity**.

▶ It makes it easy to add new kinds of data but **hard** to add new kinds of operations

OO decomposition only works well if operations are on a single object.
NOT for expressions of type `a * b + a * c -> a * (b + c)`

**Solution : Pattern Matching :**

```
def eval(e: Expr): Int = e match
    case Number(n) => n // pattern => expression
    case Sum(e1, e2) => eval(e1) + eval(e2)
```

A `MatchError` exception is thrown if no pattern matches the value of the selector.

To be able to do pattern matching `Number` and `Sum` should be **case classes**.

```
trait Expr
case class Number(n: Int) extends Expr
case class Sum(e1: Expr, e2: Expr) extends Expr
```

## Lists and more pattern matching :

▶ the empty list `Nil`, and
▶ the construction operation `::` (pronounced cons):
`x :: xs` gives a new list with the first element x, followed by the elements of `xs`

```
val fruit: List[String] = List("apples", "oranges", "pears")
fruit = "apples" :: ("oranges" :: ("pears" :: Nil)) // similar
```

**Right associativity convention** : Operators ending in ":" associate to the right.
`A :: B :: C` is interpreted as `A :: (B :: C).`

All operations on lists can be expressed in terms of the following three:

- `head` the first element of the list
- `tail` the list composed of all the elements except the first.
- `isEmpty` true if the list is empty, false otherwise .

example of using lists and pattern matching :

```
def isort(xs: List[Int]): List[Int] = xs match
    case List() => List()
    case y :: ys => insert(y, isort(ys))

def insert(x: Int, xs: List[Int]): List[Int] = xs match
    case List() => List(x)
    case y :: ys => if x > y then y :: insert(x,ys) else x :: xs
```

### Enum :

```
Here's our case class hierarchy for expressions again:
trait Expr
object Expr:
    case class Var(s: String) extends Expr
    case class Number(n: Int) extends Expr
    case class Sum(e1: Expr, e2: Expr) extends Expr
```

This is so common is scala that there's a shorthand for that.

```
enum Expr:
    case Var(s: String)
    case Number(n: Int)
    case Sum(e1: Expr, e2: Expr)
    case Prod(e1: Expr, e2: Expr)
```

There's more to Enumeration , they can take parameters and can define methods.

```
enum Direction(val dx: Int, val dy: Int):
    case Right extends Direction( 1, 0)
    case Up extends Direction( 0, 1)
    case Left extends Direction(-1, 0)
    case Down extends Direction( 0, -1)
def leftTurn = Direction.values((ordinal + 1) % 4)
end Direction

val r = Direction.Right
val u = x.leftTurn // u = Up
val v = (u.dx, u.dy) // v = (1, 0)
```

### Type Bounds :

`assertPos` should return the set itself if all elements are positive and throw `exception` otherwise.

```
def assertAllPos(s: IntSet): IntSet
```

The above definition doesn't show that `assertAllPos` of an `Empty` returns an `Empty` and `assertAllPos` of `NonEmpty` returns an `NonEmpty` . There's better :

```
def assertAllPos[S <: IntSet](r: S): S = ...
```

Here, `<: IntSet` is an upper bound of the type parameter S.

- `S <: T` means: S is a **subtype** of T, and
- `S >: T` means: S is a **supertype** of T, or T is a subtype of S.

We can *mix* them : `[S >: NonEmpty <: IntSet]`

> **The Liskov Substitution Principle**
>
> If A <: B, then everything one can to do with a value of type B
> one should also be able to do with a value of type A.

## Variance :

Given: `NonEmpty <: IntSet` is `List[NonEmpty] <: List[IntSet]` ? yes.

when `A <: B => C[A] <: C[B]` we call `C[T]` **covariant**.

Does this work for all types ? let's consider `Arrays` (**mutable**)

```
val a: Array[NonEmpty] = Array(NonEmpty(1, Empty(), Empty()))
val b: Array[IntSet] = a // TYPE ERROR HEERE
b(0) = Empty() // CAN DO WITH ARRAY[INTSET] BUT NOT WITH ARRAY[NONEMPTY]
val s: NonEmpty = a(0)
```

Type Error Line 2 : `Array[NonEmpty]` not a subtype of `Array[Inset]` .

Why ? Because otherwise it would contradict *Liskov Principle* **not** all you can do with `Array[Inset]` , you can do with `Array[NonEmpty]` . (Line 3,4)

Say `C[T]` is a parameterized type and A, B are types such that `A <: B` .

- `C[A] <: C[B]` C is **covariant** (e.g `List` )
- `C[A] >: C[B]` C is **contravariant**
- neither `C[A]` nor `C[B]` is a subtype of the other C is **nonvariant** (e.g `Array` )

```
class C[+A] { ... } C is covariant
class C[-A] { ... } C is contravariant
class C[A] { ... } C is nonvariant
```

> **Function types :**
>
> `A1 => B1 <: A2 => B2` because `A2 <: A1 and B1 <: B2 .`
>
> **Covariant** in return type. **Variant** in input type

```
trait Function1[-T, +U]:
    def apply(x: T): U
```

# Week 5 :

## List methods :

`xs.length` , `xs(n) <=> xs.apply(n)` , `xs.reverse` , `xs.updated(n, x)` , `xs.indexOf(x)` (returns -1 if `x` not in `xs` ), `xs.contains(x)` .

| Creating new Lists | All of the following are LINEAR |
|---|---|
| `xs.last` | The list's last element, exception if `xs` is empty. |
| `xs.take(n)` | A list consisting of the first n elements of `xs` , or `xs` itself if it is shorter than n. |
| `xs.drop(n)` | The rest of the collection after taking n elements. |
| `xs ++ ys` | Concatenation. `def ++ (ys: List[T]): List[T] = xs match case Nil => ys case x :: xs1 => x :: (xs1 ++ ys)` |
| `xs(n)` | (or, written out, `xs.apply(n)` ). The element of `xs` at index n.` |
| `splitAt(n)` | pair of : ( `xs[1 to n]` , `xs[n to xs.length]` ) |

### Simple merge sort implementation :

```
def msort[T](xs: List[T])( (T, T) => Boolean ): List[T] =
    val n = xs.length / 2
    if n == 0 then xs
    else
        def merge[T](xs: List[T], ys: List[T]) = (xs,ys) match
            case (Nil,ys) => ys
            case (xs,Nil) => xs
            case (x :: xs1 , y :: ys1 ) =>
                if lt(x,y) then x :: merge(xs1,ys)
                else y :: merge(xs,ys1)

        val (fst, snd) = xs.splitAt(n)
        merge(msort(fst), msort(snd))
```

`lt : (T, T) => Boolean` is a comparison function.

### Pairs :

```
val label = pair._1
val value = pair._2

val (label, value) = pair
```

### Filtering and mapping :

| methods | all LINEAR |
|---|---|
| `xs.filter(p)` | Elements of `xs` verifying `p.` |
| `xs.filterNot` | Same as `xs.filter(x => !p(x))` ; |
| `xs.partition(p)` | Same as `(xs.filter(p), xs.filterNot(p))` , but computed in a single traversal of the list `xs` |
| `xs.takeWhile(p)` | The longest prefix of list `xs` consisting of elements that all satisfy the predicate `p` . |
| `xs.dropWhile(p)` | The remainder of the list `xs` after any leading elements satisfying `p` have been removed. |
| `xs.span(p)` | Same as `(xs.takeWhile(p), xs.dropWhile(p))` but computed in a single traversal of the list `xs` . |
| `xs.map` | create a new list with `f` applied to all elements of `xs` . |

### Reductions :

Combining elements of lists with a given operator.

```
def sum(xs: List[Int]) = (0 :: xs).reduceLeft((x, y) => x + y)

([0,xs1,xs2, ... ,xsn])
=> [ 0+xs1 , xs2 , ... , xsn ]
=> [ 0+xs1+xs2 , ... , xsn ]
=> 0+xs1+xs2+ ... +xsn
```

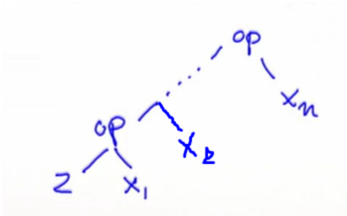
In more generality: `List(x1, ..., xn).reduceLeft(op) = x1.op(x2). ... .op(xn)`

In the same way there's `reduceRight` :

`List(x1, ..., x{n-1}, xn).reduceRight(op) = x1.op(x2.op( ... (x{n-1}.op(xn)) )`

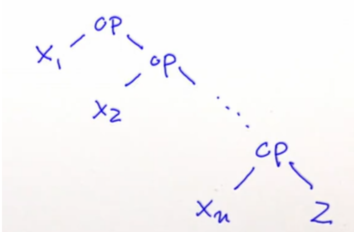
### FoldLeft :

```
def reduceLeft(op: (T, T) => T): T = this match
    case Nil => throw IllegalOperationException("Nil.reduceLeft")
    case x :: xs => xs.foldLeft(x)(op)
// takes an accumulator
def foldLeft[U](z: U)(op: (U, T) => U): U = this match
    case Nil => z
    case x :: xs => xs.foldLeft(op(z, x))(op)
```




### FoldRight:

```
def reduceRight(op: (T, T) => T): T = this match
    case Nil => throw UnsupportedOperationException("Nil.reduceRight")
    case x :: Nil => x
    case x :: xs => op(x, xs.reduceRight(op))
// takes an accumulator
def foldRight[U](z: U)(op: (T, U) => U): U = this match
    case Nil => z
    case x :: xs => op(x, xs.foldRight(z)(op))
```




`FoldLeft` and `FoldRight` are equivalent when `op` is associative and commutative.

`FoldLeft` is `tailrec` so more efficient .

### FoldLeft/FoldRight are very useful :

```
def concat[T](xs: List[T], ys: List[T]): List[T] =
    xs.foldRight( ys )(_::_)
```

```
def reverse[a](xs: List[T]): List[T] =
    xs.foldLeft( List[T]() ) ( ( x , zs ) => x :: zs )

// List[T]()  is neccesary for type inference
```

```
def mapFun[T, U](xs: List[T], f: T => U): List[U] =
    xs.foldLeft( List[T]()  )( (x,z) => f(x)::z )
def lengthFun[T](xs: List[T]): Int =
    xs.foldRight(0)((y, n) => n + 1)
```

## Structural Induction :

We would like to verify that concatenation is associative, and that it admits the empty list Nil as neutral element to the left and to the right:

```
(xs ++ ys) ++ zs = xs ++ (ys ++ zs)
```

We will use *Structural Induction* : To prove a property `P(xs)` for all lists `xs` ,

- show that `P(Nil)` holds (base case),
- for a list `xs` and some element x, show the induction step: if `P(xs)` holds, then `P(x :: xs)` also holds

recall the implementation of `++` :

```
extension [T](xs: List[T]
def ++ (ys: List[T]) = xs match
    case Nil => ys
    case x :: xs1 => x :: (xs1 ++ ys)
```

two facts :

1. `Nil ++ ys` = `ys`
2. `(x :: xs1) ++ ys` = `x :: (xs1 ++ ys)`

*Proof : Induction on* `xs`

- **base case** : Nil

  (Nil ++ ys) ++ zs = ys ++ zs by 1st clause

  Nil ++ ( ys ++ zs ) = ys ++ zs also by 1st clause
- **Inductive step :** suppose (xs ++ ys) ++ zs = xs ++ (ys ++ zs) holds , prove it on x :: xs

  on LHS :

    ((x :: xs) ++ ys) ++ zs

    = (x :: (xs ++ ys)) ++ zs // by 2nd clause of ++

    = x :: ((xs ++ ys) ++ zs) // by 2nd clause of ++

    = x :: (xs ++ (ys ++ zs)) // by induction hypothesis

    on RHS :

    (x :: xs) ++ (ys ++ zs)

    = x :: (xs ++ (ys ++ zs)) // by 2nd clause of ++

# Week 6 :

## Collections :

Collection Hierarchy :

```
                Iterable
              /    |    \
             /     |     \
 ...........Seq   Set    Map
 .         /  \
 .        /    \
Array  List   Vector
```

### Arrays and String :

- Arrays and Strings support the same operations as `Seq` ( `filter` , `map` ... )
- They cannot be subclasses of `Seq` because they come from Java.

### Sequences :

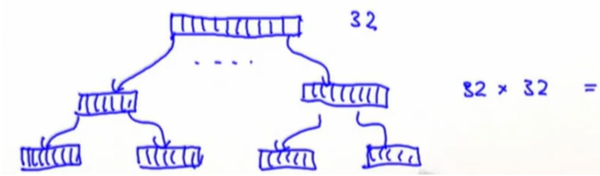
### Range :

```
val r: Range = 1 until 5 // 5 EXCLUDED
val s: Range = 1 to 5 // 5 INCLUDED
1 to 10 by 3 // 3 IS THE STEP
6 to 1 by -2
```

### Vector :

`List` has *linear* time access : Access to the middle element is slower than first...

On the contrary, `Vector` has similar access time for all its elements.




It is implement (as shown above) in the following way : (n=number of elements)

- if n <= 32 then it is an array of 32 elements
- if n <= 32*32 then it is an array of 32 elements each containing an array of 32 elements .
- if n <= 32 *32 *32 : array of array of array

| **Sequence operations :** | |
|---|---|
| `xs.exists(p)` | true if there is an element x of xs such that p(x) holds, false otherwise. |

| **Sequence operations :** | |
|---|---|
| `xs.zip(ys)` | A sequence of pairs drawn from corresponding elements of sequences xs and ys. |
| `xs.unzip` | Splits a sequence of pairs `xs` into two sequences consisting of the first, respectively second halves of all pairs. |
| `xs.flatMap(f)` | Applies collection-valued function f to all elements of `xs` and concatenates the results. `xs.flatMap(f) = xs.map(f).flatten` |
| `xs.sum` , `xs.product` , `xs.max` , `xs.min` | the sum,product,min and max |

Examples showing use of `seq` operations for conciseness :

```
def scalarProduct(xs: Vector[Double], ys: Vector[Double]): Double =
    xs.zip(ys).map(_ * _).sum

def isPrime(n: Int): Boolean =
(2 to n - 1).forall(d ⇒ n % d == 0)
```

## Combinatorial Search and For-Expressions :

### Generate all pairs ( i , j ) 1 <= j < i <= n :

```
(1 until n).flatMap(i ⇒
    (1 until i).map(j ⇒ (i, j)))
```

`For` helps for more conciseness and clarity :

```
for
i ← 1 until n
j ← 1 until i
if isPrime(i + j)
yield (i, j)

// THIS RETURNS A LIST OF THE (i,j)
```

> **IMPORTANT**
>
> The for-expression maybe seem similar to loops in imperative languages, except that it **builds a list** of the results of all iterations.

### For and If :

```
for p ← persons if p.age > 20 yield p.name
// EQUIVALENT TO
persons
.filter(p ⇒ p.age > 20)
.map(p ⇒ p.name)
```

> **IMPORTANT 2 :** for
>
> `i <- 1 until n`
>
> `j <- 1 until n`
>
> is equivalent to **nested** loops.

```
def scalarProduct(xs: List[Double], ys: List[Double]) : Double =
(for (x, y) ← xs.zip(ys) yield x * y).sum

NOT EQUIVALENT TO

(for x ← xs; y ← ys yield x * y).sum // HERE ALL PAIRS (X,Y) are considered
```

**Maps :**

Class `Map[Key, Value]` extends the collection type `Iterable[(Key, Value)]` .

```
val romanNumerals = Map("I" → 1, "V" → 5, "X" → 10)
val capitalOfCountry = Map("US" → "Washington", "Switzerland" → "Bern")
```

The syntax `key -> value` is just an alternative way to write the pair `(key, value)` .

`toList` on a `Map` produces a `List` of pairs `(key,value)` .

**Query on map :**

```
capitalOfCountry("Andorra")
// java.util.NoSuchElementException: key not found: Andorra

capitalOfCountry.get("US") // Some("Washington")
capitalOfCountry.get("Andorra") // None
```

- The `Option` type :

```
trait Option[+A]
    case class Some[+A](value: A) extends Option[A]
    object None extends Option[Nothing]
```

we can decompose the `Option` type with pattern matching :

```scala
def showCapital(country: String) =
capitalOfCountry.get(country) match
    case Some(capital) => capital
    case None => "missing data"


showCapital("US") // "Washington"
showCapital("Andorra") // "missing data"
```

- Updates :
  - `m + (k -> v)` : The map that takes key `k` ‘ to value `v` and is otherwise equal to `m` .
  - `m ++ kvs` The map `m` updated via `+` with all key/value pairs in `kvs` .

    ```scala
    val m1 = Map("red" -> 1, "blue" -> 2) // > m1 = Map(red -> 1, blue -> 2)
    val m2 = m1 + ("blue" -> 3) // > m2 = Map(red -> 1, blue -> 3)
    val m3 = m1 ++  ("blue" -> 0 ,  "yellow" -> 4) // m2 = Map(red -> 1, blue -
    > 0 , yellow -> 4 )
    ```

  - `m - k` : The map `m` without key `k` .
- Default values : `capitalOfCountry.withDefaultValue( v )` producues a new map that has `v` as defaut value.

Example use of maps on `Polynoms` ( `+` operation ) :

```scala
class Polynom(nonZeroTerms: Map[Int, Double]):

    def terms = nonZeroTerms.withDefaultValue(0.0)
    // ===== Solution 1 : More efficient
    def + (other: Polynom) =
        Polynom(other.terms.foldLeft(terms)(addTerm))
    def addTerm(terms: Map[Int, Double], term: (Int, Double)) =
        val (exp, coeff) = term
        terms + (exp, coeff + terms(exp))
    // ===== Solution 2 : More concise
    def + (other: Polynom) =
        Polynom(terms ++ other.terms.map((exp, coeff)
                                    => (exp, terms(exp) + coeff)))
```