Millikan Oil Drop Experiment and the Charge of the Electron Review

Youssef Waleed Gebreel
201900300

A review presented under the supervision of
Dr. Amr Mohamed

Physics Department
University of Science and Technology in ZewailCity
Egypt
30/5/2020

Youssef Waleed Gebreel

May 2020

Contents

1 Abstract

Overtime, scientists couldn't resist thinking of absolute value of charge, as they couldn't resist imagining the quantization of matter. In this report, the first perfect experiment that proves the quantization of charge into elementary charges and precisely estimates their magnitude will be discussed in detail. It is the oil drop experiment provided by Sir Robert Millikan, which worth the Nobel Prize in physics in 1923 for this fantastic discovery. Robert Millikan determined the magnitude of the elementary charge to be 1.59×10^{-19} coulombs, the value that is very close to our recent value 1.6×10^{-19} coulombs, which required a high modern technology to be obtained. If this proves something, it will be the huge effort exerted by Millikan to obtain this very precise value.

2 Introduction

The charge of the electron is one of the fundamental constants of this world. Determining its value precisely was crucial for the understanding of the world, for all physical and chemical theories and calculations based on it, and for all applications based on it. It is well known today that any charge either it was positive ,or negative is quantized into small particles of this charge, whose quantities must be multiple of a certain number, and can not be fractions of this certain number. This certain number is called "The elementary Charge". the quantity of charge carried by this elementary charge is the same for positive and negative charges, but they are different in the mass of the elementary charge.

Faraday's laws of electrolysis-especially the second one- were the first strong evidence of the quantization of charge. Faraday's second law of electrolysis states that: when equal amounts of charge passes through several electrolytes composed of different pole substances, the mass deposited is directly proportional to the chemical equivalent of the electrolyte substances. Based on this law, Faraday discovered that the amount of charge required to obtain the chemical equivalent mass of any substance is constant and was measured to be 96500C[3].

After his discovery, this constant amount of charge was named after him and was called "Faraday's constant". Faraday's constant provides a great evidence of the quantization of charge. When the same amount of charge causes a certain quantized mass of any substance to be deposited, then, the charge itself must be quantized.

Applying this law to Silver (Ag) and Magnesium (Mg) poles electrolytes will provide an evident present evidence of Faraday's law ability to prove the quantization of charge.

According to Silver, The atomic mass (M_{silver}) equals 107.88 g/mol. To obtain the mass of one atom of Silver ($m_{atom-Ag}$), we should divide the atomic mass by Avogadro's number (N_{avo}) as following.

$$m_{atom-Ag} = \frac{M_{silver}}{N_{avo}} = \frac{107.88}{6.02\times10^23} = 1.79 \times 10^-22g$$

If the amount of charge needed to obtain a chemical equivalent gram of silver which is 107.88 g is 96500 C, then we can calculate the amount of charge needed to obtain the mass of one atom of silver as following

$$96500C <=> 107.88g$$

$$XC <=> 1.79 \times 10^-22g$$

$$X = \frac{96500 \times 1.79 \times 10^-22}{107.88} = 1.6 \times 10^-19C$$

This result indicates that an amount of charge of 1.6×10^-19C is responsible for the deposition of one atom of Silver, which is very consistent with our modern conception of the charge of the elementary charge, and because Silver ions carry one positive charge, it requires one electron to be converted into solid form.
According to Magnesium, the atomic mass ($M_{magnesium}$) equals 24.3 g/mol. To obtain the mass of one atom of Magnesium ($m_{atom-Mg}$), we should divide the atomic mass by Avogadro's number (N_{avo}) as following

$$m_{atom-Mg} = \frac{M_{magnesium}}{N_{avo}} = \frac{24.3}{6.02 \times 10^23} = 4.04 \times 10^-23g$$

If the amount of charge needed to obtain a chemical equivalent gram of Magnesium which is ($\frac{24.3}{2} = 12.15$) is 96500 C, then we can calculate the amount of charge needed to obtain the mass of one atom of silver as following

$$96500C <=> 12.15g$$

$$YC <=> 4.04 \times 10^-23g$$

$$Y = \frac{96500 \times 4.04 \times 10^-23}{12.15} = 3.21 \times 10^-19C$$

This result indicates that the amount of charge of 3.21×10^-19C is responsible for the deposition of one atom of Magnesium, which means that two electrons form one atom of Magnesium, and this is because Magnesium ions carry two positive charges, so two electrons are required for neutralizing one Magnesium ion and convert it into solid form.
Although Faraday's laws of electrolysis provides a strong evidence for the quantiztion of charge as seen, Faraday couldn't go deeper in this idea, and that's because of two reasons. First, at his time, Avogadro's number wasn't determined yet. It was just a concept provided by Amedeo Avogadro that the volume of the gas is directly proportional to the number of atoms or molecules of it, but the numerical value of it wasn't determined yet. Second, at his time, they were approximately in complete ignorance of electricity as he said. There were many imaginations about it, but was neither of them proved. Although his absence of knowledge about the theory of electricity, he predicted that this phenomenon is related to what he called " the Absolute Quantity of Electricity", which we called now the elementary charge, and explained his opinions

about it in his paper in a section named " On the Absolute Quantity of Electricity Associated with the Particles or Atoms of Matter"[3].
When cathode rays-which are radiations originated from the cathode material and travelling to the anode when both cathode and anode are connected to a high voltage source- appeared, J. J .Thomson began to study it. He concluded from his experiments proceeded in 1897 by observing their deflection pattern through electric and magnetic field that these rays consisted of negatively charged particles, and they are the same even when the cathode materials are different[7].
In further investigations, Sir Thomson performed some experiments aimed to obtain the mass-to-charge ratio of these particles forming the cathode rays called after that "Electrons". After proceeding two different complicated types of experiments using different cathode materials, he found out that the mass-to-charge ratio of these particles equals $10^{-}7$, and this ratio was so small compared with this ratio of hydrogen ions which was $10^{-}4$, but he did not know if this huge difference is due to the tiny mass of cathode rays particles compared with those of hydrogen ions, or due to the huge charge of cathode rays particles compared with those of hydrogen ions, or due to both[7].
Until this point of time, it was known that there is two types of charges, which are positive charges like those of hydrogen ions and other positive ions, and Negative charges like those of cathode rays particles and other negative ions, and each type of these charges consists of small indivisible elementary charges, but the magnitude of this elementary charge either that of positive or negative charges was not detected yet.
Meanwhile, C. T. R Wilson succeeded to invent a chamber provided with appropriate conditions for creating clouds. He went further and began to study reasons behind the cloud formation much deeper. It was known that there must be dust particles in the vicinity of water vapor, causing water vapor to condense. These dust particles were called nuclei of condensation. Wilson discovered that the condensation can take place even if the vicinity of water vapor contains no dust particles. This occurs when the expansion ratio reaches a certain value (1.258). Further investigations by Wilson showed that that occurs due to the ionization of air gases, causing ions to act as nuclei of condensation[8]. After that, series of experiments using cloud chamber of Wilson took place to evaluate the value of the absolute elementary charge, employing the new discovery of Rontgen rays as a tool to ionize air. Sir J. J. Thomson started this series of experiments in 1898, and then was followed by other scientists. These experiment were very good in employing the cloud chamber for the sake of obtaining the elementary charge, but it lacked accuracy and were based on some assumptions, that were not correct, however these experiments were ended up with an inspiration to Robert Millikan, who could make use of these experiences and developed them in a way that lacked sources of error. In his oil drop experiment, he could evaluate the elementary charge with a great amount of accuracy.

3 Methodology

Millikan's oil drop experiment is one of the most fantastic experiments in physics. Most scientists were unanimous in the accuracy of this experiment. The troubling problem that was annoying Millikan before thinking of using oil was the quick evaporation of water and alcohol droplets which he used before oil. they were impossible to be observed more than one minute, so Millikan and his assistant Fletcher were very happy when they thought of using oil instead of water or alcohol, because oil is known for its great resistance to evaporation[1].

3.1 The Experimental Apparatus

The apparatus used in this experiment as shown in figure 1 is composed of an atomizer (A), that sprays the droplets of oil in the dust-free chamber (C). Some droplets of oil passes through a pin-hole (p) to enter the space between the two plates M and N forming the horizontal air condenser. Once some droplets pass through the pin-hole (p), it is closed by an electromagnetic cover to prevent air currents from passing through it and affect the droplets motion. M and N are circular plates held 16 mm apart using three small ebonite posts (a) fixed in their place by ebonite screws. A strip of thin sheet ebonite (c) wraps the two plates forming a completely cylindrical enclosed air space.

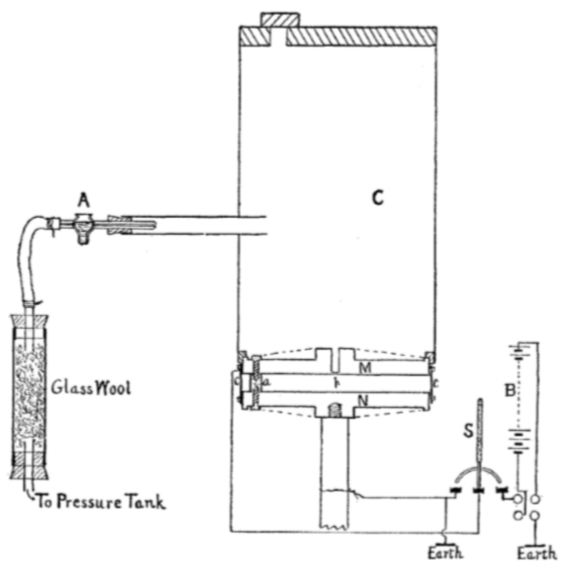

Figure 1: Millikan's 1911 Oil Drop Experiment Apparatus[4]

Three glass windows (1.5 cm square) are fixed in this thin ebonite strip of the angles 0°, 165°and 180°. A thin beam of light from arc lamp enters the con-

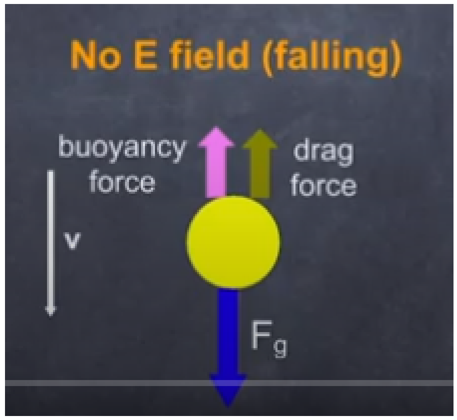

Figure 2: Forces acting on the oil droplet at the first stage of the procedure when there is no electric field[2].

denser through the first window (0°) travelling to the last window (180°). A short focus telescope is fixed in the other window for the sake of observing the oil droplets. There is a standard scale of distance held vertically on the lens of the microscope in the form of cross-hairs for the sake of the measuring velocity by observing the distances taken and their time points. The two plates-which work as a capacitor producing an electric field in the downward connected to a battery (B) by a key (S) as a mediator between the two plates and the battery to control the length of the circuit ,and therefore the strength of the electric field.

3.2 Procedure and Mathematical Approach

When a droplet of oil enters the air condenser space between the two plates when the electric field is turned off, it falls under the effect of gravity towards the lower plate (N), catching number of charges n when passing through the ionized region due to the light coming out from the arc lamp fixed in the thin ebointe strip.
When the downward force due to gravity F_g equals the upward forces of buoyancy F_B and air friction (drug force) F_D, the droplet begins to move with constant terminal velocity V_1 as shown in figure 2.

$$F_g - F_B = F_D \tag{1}$$

The gravity force F_g equals the mass of the oil droplet m_{oil} times the acceleration due to gravity g.

$$F_g = m_{oil} g$$

the mass of the oil droplet can be deduced by substituting its value by the volume of the oil droplet with spherical shape of radius a times the density of oil σ.

$m_{oil} = \frac{4}{3}\pi a^3\sigma$

So, the gravity force equation for the oil droplet will be as following.

$$F_g = \frac{4}{3}\pi a^3\sigma g \tag{2}$$

The buoyancy force F_B equals the mass of the fluid-which in this case is air-covered by the oil droplet m_{air} times the acceleration due to gravity g.

$$F_B = m_{air}g$$

The mass of the air covered by the oil droplet can be deduced by substituting its value by the volume of the air covered by the oil droplet, which equals the volume of the oil droplet itself times the density of air ρ.

$m_{air} = \frac{4}{3}\pi a^3\rho$

So, the buoyancy force equation will be as following.

$$F_B = \frac{4}{3}\pi a^3\rho g \tag{3}$$

The air friction (drug force) F_D of the oil droplet moving at constant terminal velocity V_1 in air of viscosity factor η in our case is determined by Stokes' law as following

$$F_D = 6\pi a\eta V_1 \tag{4}$$

Substituting by equations 2, 3 and 4 in equation 1 will give us the equation for the radius of the oil droplet a as following.

$$a = \sqrt{\frac{9}{2}\frac{\eta V_1}{g(\sigma - \rho})} \tag{5}$$

Before the droplet strikes the lower plate (N), an electric field of strength between 3000 volts and 8000 volts is applied by turning on the switch (S) connecting the plates to the battery (B). The battery positive pole is connected to the upper plate (M), and its negative pole is connected to the lower plate (N), so if the droplet received number of charges n, it will experience an upward electric force F_E.
When the downward forces due to gravity F_g and air friction (drug force) F_D equal the upward forces due to electric field F_E and buoyancy F_B, the droplet begins to move with constant terminal velocity V_2 as shown in figure 3.

$$F_E = F_g + F_D + F_B \tag{6}$$

The air friction (drug force) F_D of oil drop moving at constant terminal velocity V_2 in air of viscosity factor η is determined by Stokes's law as following.

$$F_D = 6\pi a\eta V_2 \tag{7}$$

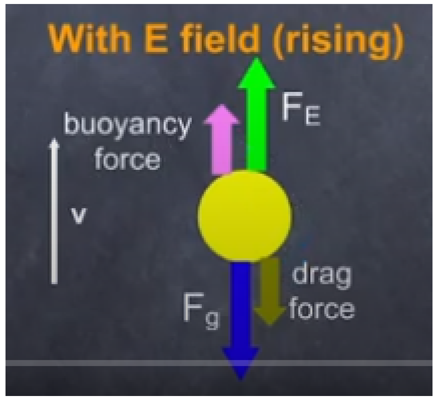

Figure 3: Forces acting on the oil drop at the second stage of the procedure when an electric field is applied[2]

The electric force F_E acting on an oil droplet carrying charge e_n moving in an electric field of strength E is determined as following.

$$F_E = e_n E \tag{8}$$

Substituting by equations 2, 7, 3,5 and 8 in equation 6 and rearranging the variables in the equation will give us the value of the charge e_n carried by an oil droplet as following.

$$e_n = \frac{4}{3}\pi(\frac{9\eta}{2})^{\frac{3}{2}}(\frac{1}{g(\sigma-\rho)})^{\frac{1}{2}}\frac{(V_1+V_2)V_1^{\frac{1}{2}}}{E}[4] \tag{9}$$

3.3 Observations and Results

For the sake of applying principles explained above to obtain the value of the elementary charge, an experiment was performed using single oil drop, that was held under observation for four hours and half, allowing it to fall under the action of gravity, and then, ascends under the effect of the electric field repeatedly.
Figure shows the observations of this experiment. The column (G sec) indicates the time taken by the drop to fall under the action of gravity a cross-hair distance. The column (F sec) indicates the time taken by the drop to ascend under the effect of electric field a cross-hair distance. The column (e_n) indicates the value of charge carried by the drop evaluated from equation 9. The column (n) indicates the number by which e_n values divided to obtain the number of the last column, so this column (n) can be guessed to express the number of charges carried by the drop, since numbers at the last column are very close in their values. These observations are divided into groups, where each group has its mean G and F values, and the value of the voltage applied (V).

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	22.8	29.0	7	34.47	4.923
	22.0	21.8	8	39.45	4.931
	22.3	17.2			
$G=$ 22.28	22.4	—	9	44.42	4.936
$V=7950$	22.0	17.3			
	22.0	17.3			
	22.0	14.2	10	49.41	4.941
	22.7	21.5	8	39.45	
	22.9	11.0	12	59.12	4.927
	22.4	17.4	9	44.42	
	22.8	14.3	10	49.41	
$V=7920$	22.8	12.2	11	53.92	4.902
$G=$ 22.80	22.8	12.3			
	23.0	—			
	22.8	14.2			
$F=$ 14.17	—	—	10	49.41	4.941
	22.8	14.0			
	22.8	17.0			
$F=$ 17.13	—	17.2	9	44.42	4.936
	22.9	17.2			
	22.8	10.9			
$F=$ 10.73	22.8	10.9	12	59.12	4.927
	22.8	10.6			
	22.8	12.2	11	53.92	4.902
$V=7900$	22.8	8.7	14	68.65	4.904
$G=$ 22.82	22.7	6.8	17	83.22	4.894
$F=$ 6.7	22.9	6.6			
	22.8	7.2			
	—	7.2			
	—	7.3			
$F=$ 7.25	—	7.2	16	78.34	4.897
	23.0	7.4			
	—	7.3			
	—	7.2			
$F=$ 8.65	22.8	8.6	14	68.65	4.904
	23.1	8.7			
	23.2	9.8	13	63.68	4.900
	—	9.8			
$F=$ 10.63	23.5	10.7	12	59.12	4.927
	23.4	10.6			

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.2	9.6			
	23.0	9.6			
	23.0	9.6			
	23.2	9.5			
$V=7820$	23.0	9.6	13	63.68	4.900
$G=$ 23.14	—	9.4			
$F=$ 9.57	22.9	9.6			
	—	9.6			
	22.9	9.6			
	—	10.6	12	59.12	4.927
$F=$ 8.65	—	8.7	14	68.65	4.904
	23.4	8.6			
	23.0	12.3			
	23.3	12.2			
$F=$ 12.25	—	12.1	11	53.92	4.902
	23.2	12.4			
	Change forced with radium.				
	23.4	72.4			
	22.9	72.4			
$F=$ 72.10	23.2	72.2	5	24.60	4.920
	23.5	71.8			
	23.0	71.7			
	23.0	39.2	6		
$V=7800$	23.2	39.2			
$G=$ 23.22	—	27.4	7	34.47	
	—	20.7	8	39.38	4.922
	—	26.9	7	34.47	4.923
	—	27.2			
	23.3	39.5			
	23.3	39.2	6	29.62	4.937
$F=$ 39.20	23.4	39.0			
	23.3	39.1			
	23.2	71.8	5	24.60	4.920
	23.4	382.5	4		
	23.2	374.0			
	23.4	71.0	5	24.60	4.920
	23.8	70.6			
$V=7760$	23.4	38.5	6		
$G=$ 23.43	23.1	39.2			
	23.5	70.3			
	23.4	70.5			
	23.6	71.2	5	24.60	4.920
	23.4	71.4			
	23.6	71.0			
	23.4	71.4			

	G sec.	F sec.	n	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	23.5	380.6			
	23.4	384.6			
	23.2	380.0			
$F=$ 379.6	23.4	375.4	4	19.66	4.915
	23.6	380.4			
	23.3	374.0			
	23.4	383.6			
	—	39.2			
$F=$ 39.18	23.5	39.2	6	29.62	4.937
$V=7730$	23.5	39.0			
$G=$ 23.46	23.4	39.6			
	—	70.8			
$F=$ 70.65	—	70.4	5	24.60	4.920
	—	70.6			
	23.6	378.0	4	19.66	
	Saw it. here, at end of 305. sec., pick up two negatives.				
	23.6	39.4	6	29.62	4.937
	23.6	70.8	5	24.60	4.920
				Mean of all e_1s $=4.917$	

Differences.

$24.60 - 19.66 = 4.94$
$29.62 - 24.60 = 5.02$
$34.47 - 29.62 = 4.85$
$39.38 - 34.47 = 4.91$
Mean dif. $= 4.93$

Figure 4: Preliminary Observation of The Oil Drop Experiment[4]

It is evident from the very slight increase in G times of the drop during the whole 4.5 hours that the evaporation rate of oil drop is very slow. It can be observed from the second half of the table- which Millikan described to be more accurate- that there are sudden jumps and falls in the values of F. For instance, in the group, whose average G is 23.43, Values of F starts with 71.8 seconds, and then, jumps to about 380, and then falls again to 71, and so on. It can be concluded that when F value suddenly increases, the oil drop catches positive ions, which rises the downward force, and thus reducing the drop's upward velocity, and when F value suddenly falls, the oil drop catches electrons, which rises the upward force, and hence, rises the upward velocity. When observing values of n opposite to these values of F, we see that when F value is 71, n value is 5, and when F value suddenly increases to 380, n value becomes 4, and when F value falls to its initial value again (71), n value returns to its initial value (5). We can conclude from these observations that n represents the net negative charge carried by the drop, and that positive ion charge equals approximately the electron negative charge, as the value of F jumps and falls with the same value.
It also can be concluded that if we obtain the difference between values of e_n, when their n opposing values have difference of one, we can obtain the value of the elementary charge. Applying this to the second half of the table, shows that the mean of the differences give the value of the elementary charge to be (4.93×10^{-10}) **esu**, as shown at the end of the table, which equals (1.644×10^{-19}) **coulomb**.

3.4 Improvements

Millikan observed irregularities in some measurements when providing drops of same conditions, and these irregularities were unpredictable, so he suspected that some assumptions- especially, the assumption that there is no slip at the bounding surface between the medium and the drop- included in Stokes' law must be false. Millikan made some experiments to investigate this issue, and his concerns were in order. In order to simulate the new assumptions with equations, he concluded that there must be a term that is proportional to the ratio between the mean free path ι and the radius of the drop, which he called the correction term A, and the corrected Stokes' law became as following.

$$x = 6\pi\eta a V(1 + \frac{A\iota}{a})^{-1} \tag{10}$$

Consequently, when substituting in equation 9, the new elementary charge e relative to the old one e_1 will be as following.

$$e^{\frac{2}{3}}(1 + \frac{A\iota}{a}) = {e_1}^{\frac{2}{3}} \tag{11}$$

Let : ${e_1}^{\frac{2}{3}} = Y$, and $\frac{\iota}{a} = X$. Then, $Y = e^{\frac{2}{3}}(1 + AX)$

$$\frac{dY}{dX} = Ae^{\frac{2}{3}} \tag{12}$$

The graph in figure represents ${e_1}^{\frac{2}{3}}$ on Y-axis, and $\frac{\iota}{a}$ on X-axis. Equation 9 was used to obtain values of ${e_1}^{\frac{2}{3}}$. The mean free path ι was deduced from the formula $\eta = \frac{1}{3}\rho u \iota$, where u is the square root of the mean square velocity of the drop. The radius of the drop was estimated by the microscope cross-hairs.

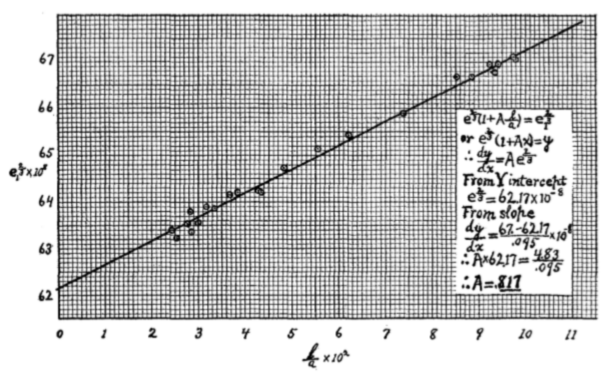

Figure 5: A graph displays the relation between ${e_1}^{\frac{2}{3}}$ and $\frac{\iota}{a}$ [4]

The slope of the graph, which is $\frac{67-62.17}{0.095}$ represents $\frac{dY}{dX}$ and the Y intercept represents $e^{\frac{2}{3}}$. By substituting in equation 12, we get the required value of correction Term A, which is 0.817. Millikan repeated the experiment as shown in figure and got the value of the elementary charge to be 4.891×10^{-10} esu, which equals 1.63×10^{-19} coulombs.[4]

No.	Tem. °C.	Dew Point °C.	$l \times 10^8$ cm.	Velocity cm./sec.	a (=radius) cm.	l/a	$e_1 \times 10^{10}$	Max. Obs'l Error. %	$e \times 10^{10}$	Dif. from Mean. %
1	24.0	5.3	945	.001315	.0000313	.3020	7.384	6.		
2	26.0	10.8	954	.001673	358	.2172	6.864	4.		
3	23.8	9.3	944	.001927	386	.1993	6.142	2.5		
4	19.9	1.8	929	.006813	755	.1230	5.605	1.5		
5	24.6	3.7	948	.01085	967	.0980	5.490	.5	4.892	.20
6	26.4	6.0	955	.01107	979	.0975	5.496	.7	4.889	.26
7	24.0	0.0	945	.01164	.0001004	.0941	5.483	.4	4.903	.03
8	20.0	1.8	929	.01176	1006	.0923	5.482	.4	4.916	.28
9	24.8	0.0	949	.01193	1016	.0934	5.458	.8	4.891	.22
10	26.3	6.0	955	.01339	1084	.0883	5.448	.5	4.908	.10
11	23.6	3.7	943	.01415	1109	.0850	5.448	.4	4.921	.42
12	24.3	11.0	947	.01868	1281	.0739	5.349	.5	4.900	.03
13	24.0	0.0	945	.02613	1521	.0621	5.293	.5	4.910	.17
14	27.0	6.0	959	.03337	1730	.0554	5.257	.5	4.918	.34
15	23.2	−1.2	942	.04265	1954	.0483	5.208	.5	4.913	.21
16	27.6	12.2	959	.05360	2205	.0435	5.143	.4	4.884	.36
17	26.8	6.0	958	.05534	2234	.0429	5.145	.5	4.885	.34
18	25.2	4.0	951	.06800	2481	.0384	5.143	.7	4.912	.21
19	23.8	5.0	944	.07270	2562	.0369	5.139	.5	4.913	.01
20	23.2	13.5	942	.08843	2815	.0325	5.102	.3	4.901	.01
21	24.6	1.7	948	.09822	2985	.0318	5.107	.4	4.915	.27
22	25.0	9.2	950	.1102	3166	.0300	5.065	.4	4.884	.36
23	27.7	15.0	959	.1219	3344	.0287	5.042	.5	4.882	.40
24	22.6	1.6	939	.1224	3329	.0282	5.096	.5	4.923	.44
25	24.0	3.7	944	.1267	3393	.0278	5.061	.5	4.894	.15
26	23.8	5.0	944	.15145	3712	.0254	5.027	.5	4.880	.44
27	25.2	0.3	948	.1644	3876	.0245	5.050	.3	4.903	.03
28	22.3	−0.7	938	.2027	4297	.0218	4.989	.7	4.858	.85
29	21.8	−0.1	936	.2175	4447	.0211	5.046	.4	4.918	.36
30	22.3	4.2	938	.3089	5315	.0177	4.980	1.		
31	24.4	1.0	947	.3969	6047	.0157	5.060	1.		
32	22.8	1.0	940	.4074	6104	.0154	5.033	1.		
33	25.2	2.7	951	.4735	6581	.0144	4.911	1.5		

Figure 6: Observations of The Oil Drop Experiment After Stokes' Law Correction [4]

In 1913, after two years from publishing this paper[4], Millikan repeated the oil drop experiment with taking many advanced considerations such as reevaluating some constants, making some other corrections and developing his experimental apparatus to be more accurate, leading finally to a value of elementary charge of 4.774×10^{-10} e.s.u, which equals 1.59×10^{-19} coulombs[5].

4 Discussion

Millikan oil drop Experiment had a great deal of accuracy, not just in measurements, but the most important is in the assumptions, and the mathematical equations resulted from these assumptions. The experiments performed before Millikan's experiment began with Sir J. J. Thomson Experiment. He used the cloud chamber of Mr C. T .R Wilson and Rontgen rays to ionize air as explained in the introduction section. His method relied in this equation.

$$\frac{I}{A} = neV_d \tag{13}$$

Where I is the current is the current passing through ionized air, n is the number of ions, which he suppose to equals the number of drops per unit volume, e is the magnitude of elementary charge, A is the section area current passes through and V_d is the drift velocity. The purpose of the whole experiment is to calculate n value, with which he could deduce the value of e, since

he measure all other variables (I, A), and V_d was estimated by Mr Rutherford in a previous paper. This methodology has a great deal of errors, but the most remarkable mistake was assuming that each ion carries up only one drop, which was experimentally proved to be wrong[6].
Harold. A. Wilson came after Sir Thomson, and performed an experiment, which agreed in its preliminary principles with Millikan's oil drop experiment. Wilson used the cloud chamber also in his experiment, but this time, he gave no care to the number of ions carried by each drop, or the number of drops. He used two capacitor plates and made a voltage between them like what Millikan did later, and based his mathematical model in the following equation

$$\frac{mg}{mg + Ee} = \frac{V_1}{V_2} \tag{14}$$

Where m is the mass of the drop, g is the acceleration due to gravity, E is the strength of electric field, V_1 is the velocity of the falling drop under the action of gravity, and V_2 is the velocity of the falling drop under the action of gravity and electrostatic force. This equation based on the proportionality between V_1 and the gravitational force mg, and the proportionality of V_2 and both the gravitational force mg and the electrostatic force Ee[9]. That was a new and a good way of thinking a bout a method for evaluating the elementary charge, but it ignores the presence of other forces acting on the drops like buoyancy force and drug force. It should be mentioned that Millikan's first trials to evaluate the elementary charge in 1908 was just a development of this experiment of Wilson performed five years earlier in 1903, so it is evident that Millikan based his experiments on what Wilson reached, and then, began to develop it[1].
After this, there were contributions and evolution in the mathematical models of by Ehrenhaft and Broglie, but still there were sources of error, which Millikan mentioned in his paper to be: the absence of perfect stagnancy in air, the electric field produced was not completely uniform, the evaporation of water or ethanol drop over time, making the mission of observing them for more than one minute very difficult, and finally, the assumption of the validity of Stokes's law, the matter that he worked on later and developed it.

5 Conclusion

After a long deal of developing theories and assumptions, making use of new inventions like cloud chamber and Rontgen rays and conducting very complicated experiments. Sir Robert Millikan was the one who possessed the credit of discovering and estimating the elementary charge, which was 1.59×10^{-19} coulombs. This constant value was the base on which many of other constants were estimated, and many application became available.

References

[1] Michael. F .Perry. “Remembering the oil-drop experiment. ” In: *Physics Today* 60.5 (2007), pp. 56–60. DOI: http://dx.doi.org/10.1063/1.2743125.

[2] High School Physics Explained. *Millikan's oil drop experiment explained.* URL: https://www.youtube.com/watch?v=-tLRfIu3h54&t=688s.

[3] Michael Faraday. “VI. Experimental researches in electricity-Seventh Series. ” In: *Philosophical Transactions of the Royal Society of London* 124 (1834), pp. 77–122. DOI: https://doi.org/10.1098/rstl.1834.0008.

[4] R. A. MILLIKAN. “THE ISOLATION OF AN ION, A PRECISION MEASUREMENT OF ITS CHARGE, AND THE CORRECTION OF STOKES'S LAW.” In: *Physical Review (Series I)* 32.4 (1911), pp. 349–397. DOI: https://doi.org/10.1103/PhysRevSeriesI.32.349.

[5] R. A. Millikan. “On the Elementary Electrical Charge and the Avogadro Constant. ” In: *Physical Review* 2.2 (1913), pp. 109–143. DOI: https://doi.org/10.1103/PhysRev.2.109.

[6] J. J. Thomson. “LVII. On the charge of electricity carried by the ions produced by Röntgen rays.” In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 46.283 (1898), pp. 528–545. DOI: https://doi.org/10.1080/14786449808621229.

[7] J. J. Thomson. “XL. Cathode Rays. ” In: *Philosophical Magazine* 44.269 (1897), pp. 293–316. DOI: https://doi.org/10.1080/14786449708621070.

[8] C. T. R Wilson. “Condensation of water vapour in the presence of dust-free air and other gases.” In: *Proceedings of the Royal Society of London* 61.369-377 (1897), pp. 240–242. DOI: https://doi.org/10.1098/rspl.1897.0030.

[9] Harold A. Wilson. “A determination of the charge on the ions produced in air by Röntgen rays.” In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 5.28 (1903), pp. 429–441. DOI: https://doi.org/10.1080/14786440309462941.