

AER4420

Autopilot Project

TEAM3

February 24, 2025



Contents

1 AUTOPILOT LITERATURE REVIEW

- Research
- Flight Mechanics Review
- Numerical solution of ODEs

2 AIRPLANE SIMULATOR PART I (RK4)

- Simulink Model
- Given Data

3 Comparing Results

- Sampling Time Selection in Aircraft Simulation
- Figures
- Error Computation

References40

Research

- Definition of Autopilot

Research

- Definition of Autopilot
- Main Objective

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot
- Stability Augmentation and Control Augmentation Systems

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot
- Stability Augmentation and Control Augmentation Systems
- Onboard Sensors

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot
- Stability Augmentation and Control Augmentation Systems
- Onboard Sensors
- Nonmesurable States

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot
- Stability Augmentation and Control Augmentation Systems
- Onboard Sensors
- Nonmesurable States
- Fly-By-Wire (FBW)

Research

- Definition of Autopilot
- Main Objective
- Early Autopilot Development
- Inputs and Outputs of an Autopilot System
- Human vs Autopilot
- Stability Augmentation and Control Augmentation Systems
- Onboard Sensors
- Nonmesurable States
- Fly-By-Wire (FBW)
- Autopilot Software VS Hardware

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)
- Assumptions in Deriving Airplane Equations of Motion

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)
- Assumptions in Deriving Airplane Equations of Motion
- Mathematical Classification of the Airplane EOM

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)
- Assumptions in Deriving Airplane Equations of Motion
- Mathematical Classification of the Airplane EOM
- Difference Between Body Axes and Earth (Inertial) Axes

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)
- Assumptions in Deriving Airplane Equations of Motion
- Mathematical Classification of the Airplane EOM
- Difference Between Body Axes and Earth (Inertial) Axes
- Difference Between Pitch Angle (θ) vs. Angle of Attack (α) and Sideslip Angle (β) vs. Heading Angle (ψ)

Flight Mechanics Review

- General Rigid Body Dynamics Equations in 3D (12 Equations)
- Classification into Kinetics and Kinematics Kinetic Equations (6 in total)
- Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)
- Assumptions in Deriving Airplane Equations of Motion
- Mathematical Classification of the Airplane EOM
- Difference Between Body Axes and Earth (Inertial) Axes
- Difference Between Pitch Angle (θ) vs. Angle of Attack (α) and Sideslip Angle (β) vs. Heading Angle (ψ)
- Attitude Representations: Advantages and Disadvantages

Numerical solution of ODEs

- Numerical Solving Algorithms

Numerical solution of ODEs

- Numerical Solving Algorithms
- Using Runge-Kutta 4th order Solver (RK4)

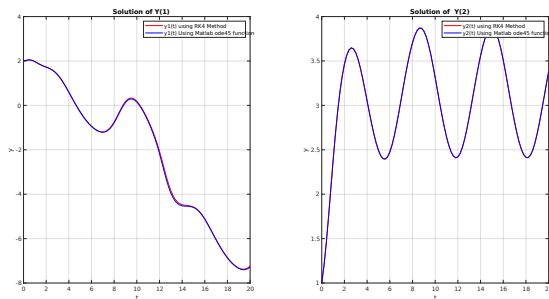


Figure: Our Code VS MATLAB

Simulink Model

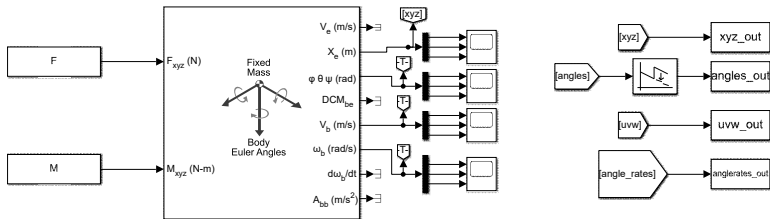


Figure: Simulink Model Used

Given Data

$$m = 11 \text{ kg}$$

$$I = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & -4 \\ -1 & -4 & 0.2 \end{bmatrix} \text{ kg} \cdot \text{m}^2$$

$$[u, v, w, p, q, r, \phi, \theta, \psi, x, y, z]_{t=0} = \left[10, 2, 0, 2 * \frac{\pi}{180}, 1 * \frac{\pi}{180}, 0 * \frac{\pi}{180}, 20 * \frac{\pi}{180}, 15 * \frac{\pi}{180}, 30 * \frac{\pi}{180}, 2, 4, 7 \right]$$

$$F = [2 \quad 8 \quad 3] N$$

$$M = [14 \quad 20 \quad 7] N.m$$

$$t_f = 25 s$$

Sampling Time Selection in Aircraft Simulation

The choice of T_s depends on the fastest dynamics of the system. Based on digital control principles [1, 2], common sampling times for aircraft systems are:

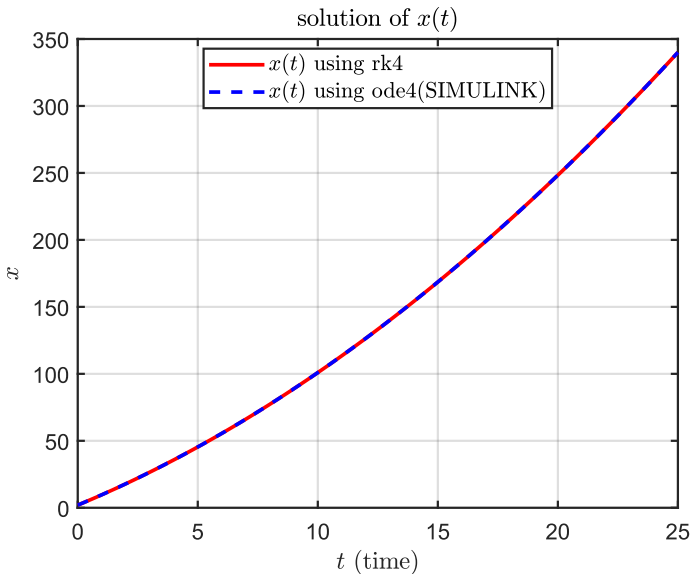
- **Slow response systems (e.g., navigation, autopilot):**
 $T_s = 50 \text{ ms}$ (0.05 s)
- **Flight control (inner loops, stability augmentation):**
 $T_s = 10 \text{ ms}$ (0.01 s)
- **High-bandwidth actuator control:** $T_s = 1 \text{ ms}$ (0.001 s)

Numerical ODE Solver Considerations

When using RK4 for solving the equations of motion, the internal solver step Δt should be smaller than T_s . As recommended in numerical integration literature [3, 4], a good practice is:

$$\Delta t = \frac{T_s}{5} \text{ or } \frac{T_s}{10} \quad (1)$$

States Comparson



States Comparson

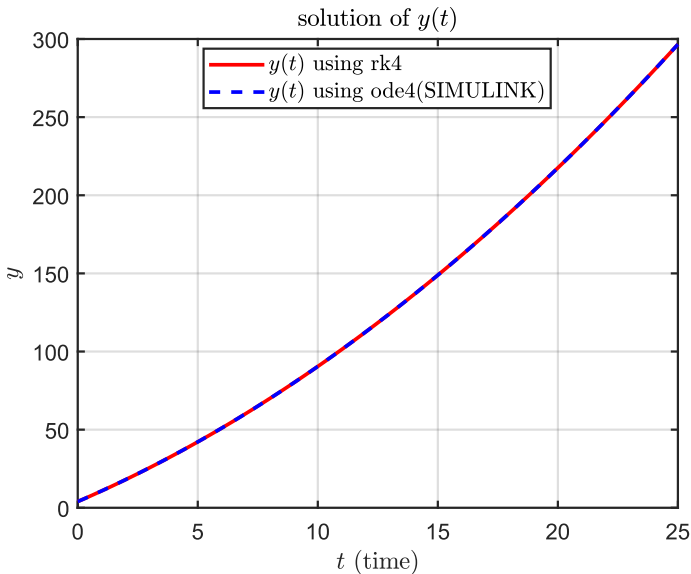
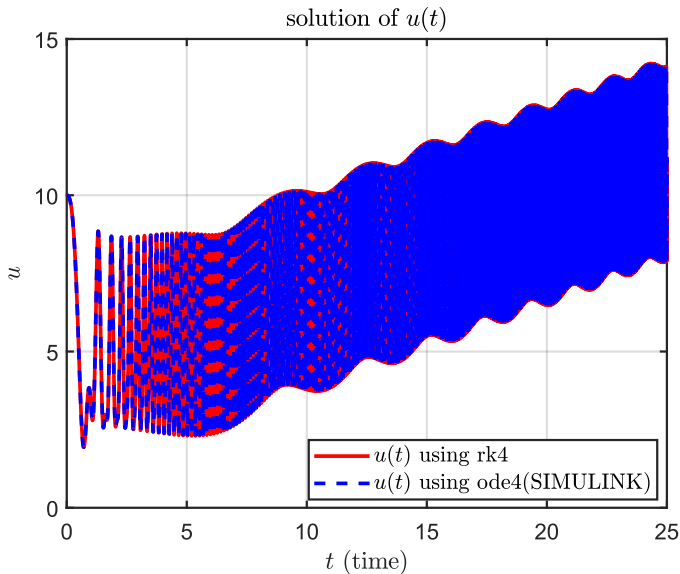
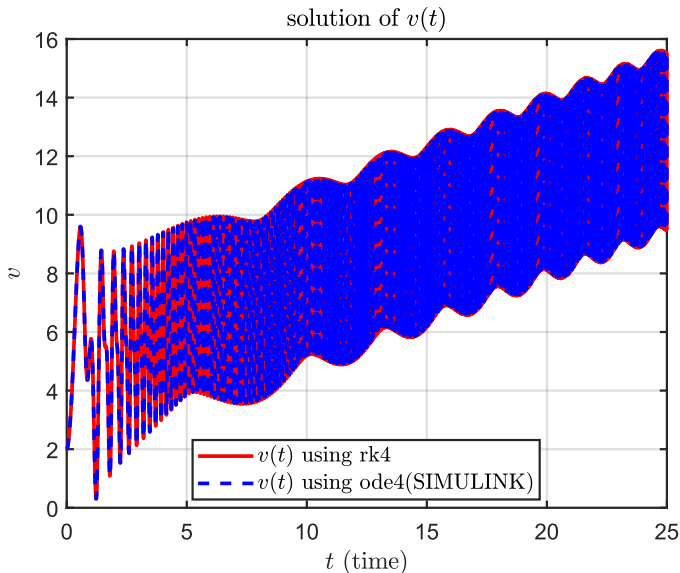


Figure 1 is a line plot titled "solution of $z(t)$ ". The horizontal axis is labeled " t (time)" and ranges from 0 to 25 with major ticks every 5 units. The vertical axis is labeled " z " and ranges from -15 to 10 with major ticks every 5 units. The plot contains two data series: a solid red line representing " $z(t)$ using rk4" and a dashed blue line representing " $z(t)$ using ode4(SIMULINK)". Both curves start at $z \approx 7$ when $t = 0$, decrease to a minimum of $z \approx -11$ at $t \approx 19$, and then slightly increase to $z \approx -10$ at $t = 25$. The two curves are nearly identical, showing excellent agreement between the two numerical methods.

States Comparson



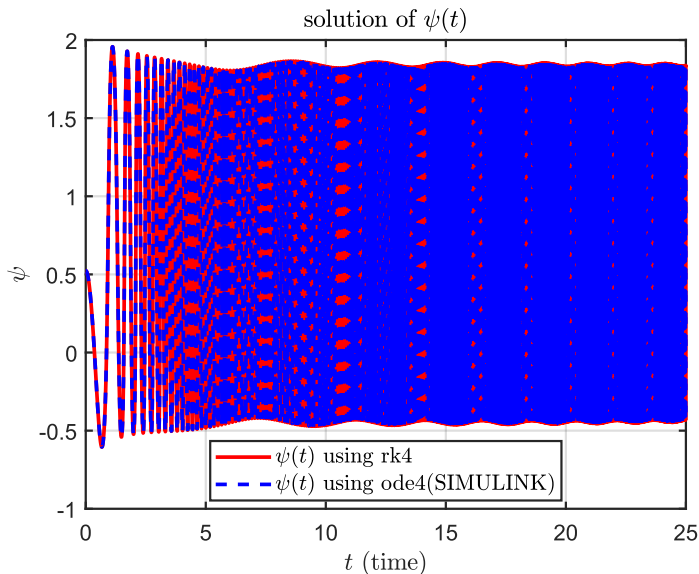
States Comparson



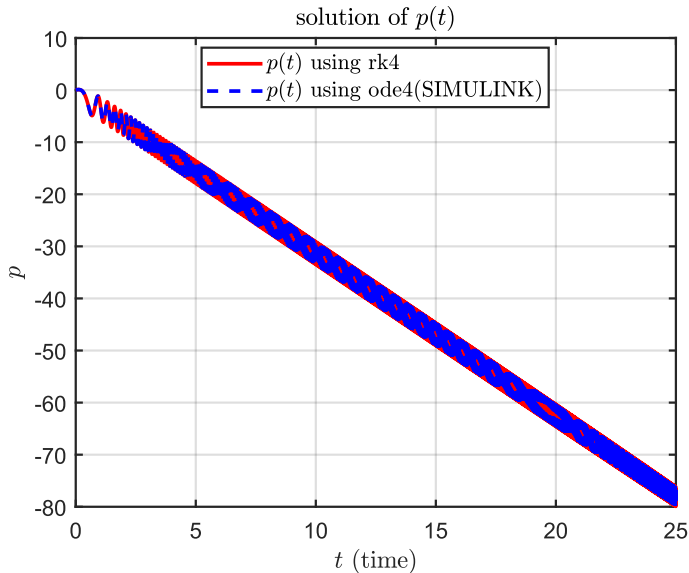
States Comparsion



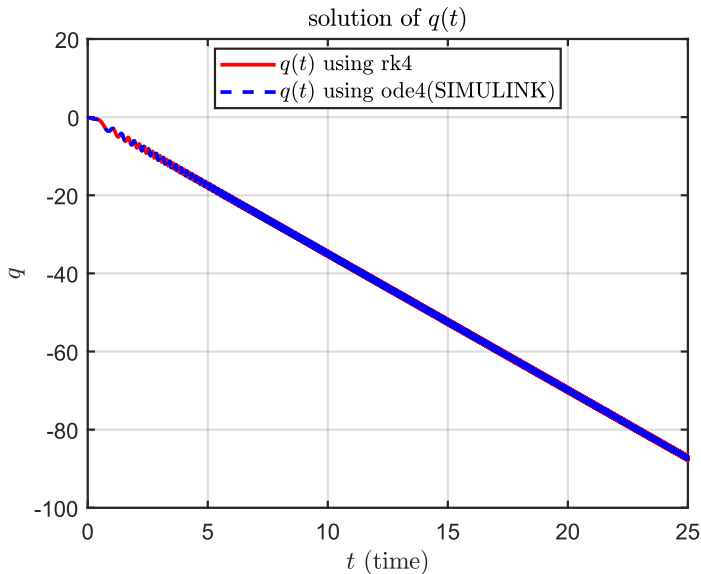
States Comparison



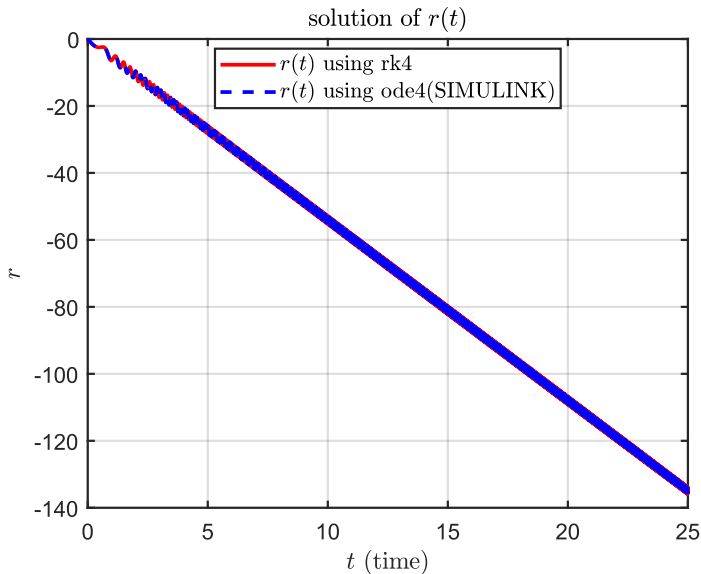
States Comparson



States Comparson



States Comparson



Error Computation

We calculated the deviation between RK4 and Simulink solutions:

- Mean absolute error for each state variable.
- Root Mean Square Error (RMSE) to measure average deviation.
- Maximum error to identify the worst-case difference.

References

Mean Absolute Error (MAE) for each state:

```
1.0e-09 * [0.0037 0.0036 0.0027 0.0050 0.0016 0.0028 0.0031 0.0012 0.0027
0.1684 0.1931 0.0672]
```

Root Mean Square Error (RMSE) for each state:

```
1.0e-09 * [0.0053 0.0052 0.0039 0.0071 0.0022 0.0040 0.0049 0.0019 0.0041
0.2815 0.3240 0.1358]
```

Max Error for each state:

```
1.0e-09 * [0.0132 0.0133 0.0099 0.0156 0.0049 0.0089 0.0193 0.0053 0.0136
0.8145 0.9377 0.4579]
```



G. F. Franklin, J. D. Powell, and M. L. Workman, *Digital Control of Dynamic Systems*, 3rd ed. Addison-Wesley, 2014.



K. Ogata, *Discrete-Time Control Systems*, 2nd ed. Pearson, 2010.



J. C. Butcher, *Numerical Methods for Ordinary Differential Equations*, 3rd ed. Wiley, 2016.



S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*, 7th ed. McGraw-Hill, 2015.



B. Stevens, F. Lewis, and E. Johnson, *Aircraft Control and Simulation*.