الجامعة اللبنانية الأميركية
Lebanese American University

Turbomachinery

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Semester project: twisted blades

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Abstract

Twisted blades have been used for a while now in turbines, they are very efficient, and although not discussed in the paper, have a good resistance reduction when rotating, as the turbine rotates the closer to the tip one is the faster the blade is spinning, which when taking into consideration the resistance of the fluid around results in an immense stress on the blade and limits the speed at which it can rotate, with turbine blades this issue is tackled. Now coming back to the paper, twisted blades are a revolutionary innovation when it comes to turbomachinery, their outlet relative angle and reaction change along the blade and from my design it results in a decreased stress on the blades, higher efficiency, and decreased total loss coefficient.

Contents

1. Introduction

Twisted blades have been used for around 10 years now, they are becoming majorly popular in the turbomachinery field, this paper aims to depict and understand how and why twisted blades are more effective than non-twisted blades. The design is simple, it is a blade that has a twist of 20 degrees from root to tip, and therefore all properties will be dependent on where one is position his analysis on the blade, the design was primarily done using continuous equations to find the velocities, angles, reaction, power output, mass flowrate. Then the blade was broken down into 3 points (2 segments) the first point being at the root (or the hub), the second point being at the middle point of the blade, and the third point is at the tip of the blade (or casing).

2. Calculation/modeling

2.1 Assumptions, design parameters, and parameters to be taken into consideration

2.1.1 Simplifications

1. Isentropic nozzle, lossless at the nozzle
2. This is an axial turbine and therefore the rotational velocity of the turbine U is constant at points 2 and 3
3. The distance from the shaft center to the blade tip is a sum to be determined through iterations in MATLAB (L) (diameter of the turbine is 2L)

Notes: the velocity changes with x, $\frac{2\pi \times rpm}{60}(x + L_0)$, therefore it must be taken into consideration

2.1.2 Calculations and research

At point 2, at the inlet the blade is not bent, as this angle is meant to correctly introduce the air in the turbine blade, while the outlet can be designed to either increase or decrease the deflection of the fluid by increase or decreasing β_3, the angle as most commonly done is decrease by 20 degree, to an angle of 50 degrees, a base angle will be 70 degree and will subsequently be increase to 50 degrees, an angle higher than 70 degree would lead to complications,

$$\beta_3 = -1 \times (-\frac{20}{L_1}x + \beta_2)$$

By taking an infinitely small element and applying the work output on it, and then integrating over the length of the blade, we end up with the following equation:

$$w = \frac{2\pi \times rpm \times W_{at}}{60 \times L_1} \int_0^{L_1} (x + L_0)(\tan(\beta_2) - \tan(- 1 \times (-\frac{20}{L_1} x + \beta_2)) \, dx$$

Since this equation is very complex to solve by hand due to the existence of absolute functions in it, it was solved using MATLAB by numeric integration.

Next for the reaction of the turbine, we followed the most common design being, an impulse design at the root of the blade, and that will be a 100% impulse blade design at the root (R=0%).

A good and helpful assumption would be that at 50% reaction the exit velocity is V_1 the rest will be different, this exit velocity and inlet velocity could be assumed to be axial which in its part will result in knowing α_1 and α_3 at the tip (0 degrees); and assuming at the impulse (x=0 and R=0%), C_R = 1 and W_2= W_3, these assumptions will be helpful.

No losses in the turbine, axial velocity is constant, W_{a2}=W_{a3}=V_{a2}=V_{a3} ,from this assumption many equations may be derived, the following are the derived equations,

$$W_{a2} = W_{a3} = V_{a2} = V_{a3} = W_{at}$$

Or

$$W_2 \cos(\beta_2) = W_3 \cos(-1 \times \left(-\frac{20}{L_1} x + \beta_2\right)) = V_2 \cos(\alpha_2) = V_3 \cos(\alpha_3)$$

Next is the relationship with V_u, W_u, and U, and we know, $V_u = W_u + U$, in our case we have 2 equations, they are the following,

$$W_2 \sin(\beta_2) + \frac{2\pi \times rpm}{60}(x + L_0) = V_2 \sin(\alpha_2)$$

$$W_3 \sin(-1 \times \left(-\frac{20}{L_1} x + \beta_2\right)) + \frac{2\pi \times rpm}{60}(x + L_0) = V_3 \sin(\alpha_3)$$

Currently with many unknowns some must be assumed, for instance what would the RPM be? The turbine is located in America, it has an rpm of 3600 for a grid of 60hz (Niemkiewicz, 2004) but what must be known next is the blade tip distance to the center of the turbine, the farther it is the higher the velocity and the forces exerted on it, and therefore this speed must be taken into consideration.

There are 3 forces to pay attention to, the drag, the lift, and the centrifugal forces, the longer the blade and the farther the blade tip the higher the forces exerted on the blade.

What is the efficiency of the turbine? It is around 0.6 (office of fossile energy, n.d.)

$$\eta_{ts} = \frac{w}{w_s} = \frac{w(already\ shown\ previously)}{h_{02} - h_{03s}} = \frac{w(already\ shown\ previously)}{h_1 + \frac{{V_1}^2}{2} - h_{3s} - \frac{{V_3}^2}{2}}$$

It is a normal stage, V_1=V_3(L_1) we get:

$$\eta_{ts} = \frac{w(already\ shown\ previously)}{h_1 + \frac{{W_a}^2}{2} - h_{3s} - \frac{W_a \times {L_1}^2}{2\int_0^{L_1} \cos^2\left(-1 \times \left(-\frac{20}{L_1}x + \beta_2\right)\right) dx}}$$

The turbine between 1 and 3s is isentropic, we just need to find its pressure drop,

Now we must estimate the inlet temperature and pressure, currently, with the advancement in technology, Mitsubishi has achieved an inlet temperature of 1600 K into the turbine, this value will be taken (Mitsubishi, 2011)

My final assumption is that the turbine produce 2000 kW

2.2 Calculating the parameters of the flow (angles and velocities)

2.2.1 Calculate lengths, efficiency, axial velocity, and work per kilogram

And since $\alpha_3(0) = 0$ then, and $W_3 = \frac{W_a}{\cos(\beta_3)}$

$$W_a \tan(-1 \times (\beta_2)) + \frac{2\pi \times rpm}{60}(L_0) = 0$$

Given the rpm is 3600, or 60hz, we can deduce W_a as a function of $L_1 + L_0$ or the total length, which is set to 1.5m

From here we get $W_a = 205.8\ m/s$

And $L_1 + L_0 = Lsum$ thus, $L_0 = Lsum - L_1$

The efficiency of the turbine is 0.90, or 90%,

$$T_{03s} = 1400\ K$$

I took 1400 K since at this outlet stagnation temp my design has an efficiency of 90%

The enthalpy of air is only dependent on temperature, and therefore we can deduce h_2 and h_{3s} (from table A-17 thermodynamics)

$$h_{02} = 1757.57\ \mathrm{kJ/kg}$$

$$h_{03s} = 1515.42\ kJ/kg$$

Using MATLAB and iterating for L_1 until the efficiency is near 0.9, to find L_0,

$$\eta_{ts} = \frac{\frac{2\pi \times rpm \times W_{at}}{60 \times L_1} \int_0^{L_1} (x + 3 - L_1)(\tan(\beta_2) - \tan(-1 \times (-\frac{20}{L_1} x + \beta_2))\, dx}{h_1 + \frac{W_a{}^2}{2} - h_{3s} - \frac{(W_a \times L_1)^2}{2(\int_0^{L_1} \cos\left(-1 \times \left(-\frac{20}{L_1} x + \beta_2\right)\right) dx)^2}}$$

$$w = 218489\ J/kg$$

For $L_1 + L_0 = 1.5m$ is 0.5m and the base length is 1m

2.2.2 Calculate W_2 and W_{u2}

$$W_{u2} = W_{a2} \tan \beta_2 = \tan 70 = 565.5$$

$$W_2 = \sqrt{W_{u2}{}^2 + W_{a2}{}^2} = 601.8\ m/s$$

2.2.3 Calculate α_2

$$W_2 \sin(\beta_2) + \frac{2\pi \times rpm}{60}(x + L_0) = V_2 \sin(\alpha_2)$$

$$565.5 + \frac{2\pi \times 3600}{60}(x + 1) = V_2 \sin(\alpha_2)$$

$$V_2 \cos(\alpha_2) = 205.8$$

$$\frac{565.5+\frac{2\pi\times3600}{60}(x+1)}{205.8}=\frac{V_2\sin(\alpha_2)}{V_2\cos(\alpha_2)}$$

$$2.75+1.83(x+1)=\tan\alpha_2$$

$$\alpha_2=\tan^{-1}(2.75+1.83(x+1))$$

2.2.4 Calculate V_2

$$V_2=\frac{205.8}{\cos(\tan^{-1}(2.75+1.83(x+1)))}$$

Or

$$V_2=\sqrt{(565.5+\frac{2\pi\times3600}{60}(x+1))^2+205.8^2}$$

2.2.5 Calculate W_3

$$W_3\cos(-1\times(-\frac{20}{L_1}x+\beta_2))=205.8$$

$$W_3=\frac{205.8}{\cos(-1\times(-\frac{20}{L_1}x+\beta_2))}$$

$$W_{u3}=205.8\tan(-1\times(-\frac{20}{L_1}x+\beta_2))$$

2.2.6 Calculate V_3, V_{u3}, V_{a3}, and α_3

$$V_{u3}=W_{u3}+U$$

$$V_{u3}=205.8\tan\left(-1\times\left(-\frac{20}{L_1}x+\beta_2\right)\right)+\frac{2\pi\times3600}{60}(x+1)$$

$$\frac{V_{u3}}{W_a}=\tan(\alpha_3)$$

$$\frac{205.8\tan\left(-1\times\left(-\frac{20}{L_1}x+\beta_2\right)\right)+\frac{2\pi\times3600}{60}(x+1)}{205.8}=\tan(\alpha_3)$$

$$\alpha_3 = \tan^{-1}\left(\tan\left(-1 \times \left(-\frac{20}{L_1}x + \beta_2\right)\right) + 1.83(x+1)\right)$$

2.2.7 Calculate R

$$R = \frac{{W_3}^2 - {W_2}^2}{{W_3}^2 - {W_2}^2 + {V_2}^2 - {V_1}^2}$$

By replacing them by their respective values we get:

$$R = \frac{(205.8\tan(-1 \times (-\frac{20}{0.5}x + 70)))^2 - 601.8^2}{565.5 + \frac{2\pi \times 3600}{60}(x+1))^2 + 205.8^2 - 205.8^2 + (205.8\tan(-1 \times (-\frac{20}{0.5}x + 70)))^2 - 601.8^2}$$

$$R = \frac{(205.8\tan(-1 \times (-\frac{20}{0.5}x + 70)))^2 - 601.8^2}{565.5 + \frac{2\pi \times 3600}{60}(x+1))^2 + (205.8\tan(-1 \times (-\frac{20}{0.5}x + 70)))^2 - 601.8^2}$$

Calculate $\dot{m}$

If $w = 218489\ kJ/kg$ and W=5000 000 kJ/s then, $\dot{m} = \frac{W}{w} = \frac{2000000}{218489} = 9.15\ kg/s$

2.3 Loss analysis parameters

Note: For the loss analysis, T03 is constant throughout the blade, while T3 changes.

2.3.1 Designing the shape of the blade

First the density should be calculated, it is $\rho = \frac{\dot{m}}{\dot{Q}}$ the mass flow rate is constant, while the density changes depending on the speed of the flow and the area (basically the volumetric flowrate).

I will assume $\frac{b}{c_x} = 3$, therefore the axial-chord length is $c_x = \frac{L_1}{3}$,and the chord length taking an average camber angle of 58 degree from different reading is $c = \frac{c_x}{\cos 58}$ and the blade length is 0.5, and therefore the chord is 0.166. Now the pitch will be determined from the blade loading coefficient, which is already determined, but its average will be used to find “s”. $\psi = \frac{\sum \psi_n}{n}$, and therefore, the Zweifel correlation will be used. “s” has been determined to be, 12.5159. it will be the same for the stator and rotor.

2.3.2 Finding zeta and the Reynold numbers

We can now move on to calculate the Reynold’s number at the exit of the stator and the inlet of the rotor. The viscosity will be defined by the temperature of air at point 2, $T_{01} = T_{02}$ and $T_2 = T_{02} - \frac{V_2{}^2}{2Cp}$, as T_2 changes along the blade so will the viscosity, this will complicated things too much and so an average velocity will be taken with an average temperature and an average pressure, at a temperature of around 1250 K or 977 degree C the viscosity is 4.77402×10^{-5}. And therefore, the Reynold’s number for the rotor is 27239 and for the stator it is 23566

Now for calculating ξ of the stator and rotor, it will vary along the blade and therefore will be displayed in the table in the end, for ξ we used the equations available to us in the course, for the $\frac{b}{c_x}$ no adjustments were needed as it was taken to be 3, but for the Re the numbers were different than 10^5 and therefore needed adjustments by using the available equation $\left(\frac{10^5}{R_e}\right)^{\frac{1}{4}}$.

For the pressure losses the equations the Mach number, zeta, gamma, and stagnation pressure are used, but the pressure loss is dependent on the position on the blade and the results will be shown in the final table.

2.4 Losses (profile, tip clearance, and secondary flow losses)

2.4.1 Profile losses

for the profile losses, the flow at the inlet is axial and therefore the equation requires no adjustment, the profile losses at the stator are Y_{pa} and the losses at the rotor are Y_{pe}.

$$\mathrm{Y_{pa}} = \left[-0.627 \times \left(\frac{\alpha_2}{100}\right)^2 + 0.821 \times \left(\frac{\alpha_2}{100}\right) - 0.129\right] \times \left(\frac{s}{c}\right)^2 + \left[1.489 \times \left(\frac{\alpha_2}{100}\right)^2 - 1.676 \times \left(\frac{\alpha_2}{100}\right) + 0.242\right] \times \left(\frac{s}{c}\right) + \left[-0.356 \times \left(\frac{\alpha_2}{100}\right)^2 + 0.399 \times \left(\frac{\alpha_2}{100}\right) + 0.0077\right]$$

$$\mathrm{Y_{pe}} = \left[-1.56 \times \left(\frac{\alpha_2}{100}\right)^2 + 1.55 \times \left(\frac{\alpha_2}{100}\right) - 0.064\right] \times \left(\frac{s}{c}\right)^2 + \left[3.73 \times \left(\frac{\alpha_2}{100}\right)^2 - 3.43 \times \left(\frac{\alpha_2}{100}\right) + 0.290\right] \times \left(\frac{s}{c}\right) + \left[-0.83 \times \left(\frac{\alpha_2}{100}\right)^2 + 0.78 \times \left(\frac{\alpha_2}{100}\right) + 0.078\right]$$

2.4.2 Secondary flow and tip clearance losses

The tip clearance $\frac{k}{c} = 0.02$ and the blades are shrouded and therefore B=0.37

The coefficient C_L will be taken as per the course by neglecting C_D

$$C_{Lstator} = 2 \times \frac{s}{c}(\tan\alpha_2 - \tan\alpha_1) \times \tan\alpha_m$$

$$C_{Lrotor} = 2 \times \frac{s}{c}(\tan\beta_2 - \tan\beta_3) \times \tan\beta_m$$

And

$$(Y_s + Y_k)_{rotor} = \frac{c}{b} \times \left(0.0334 \times \frac{\cos\beta_3}{\cos\beta_2} + B \times \left(\frac{k}{c}\right)^{0.78}\right) \times \frac{C_{Lrotor}}{s/c} \times \frac{\cos^2\beta_3}{\cos^3\beta_m}$$

$$(Y_s)_{stator} = \frac{c}{b} \times \left(0.0334 \times \frac{\cos\alpha_2}{\cos\alpha_1}\right) \times \frac{C_{Lstator}}{\frac{s}{c}} \times \frac{\cos^2\alpha_2}{\cos^3\alpha_m}$$

2.5 Finding the isentropic efficiency

It is straightforward now all the properties have been calculated, now the isentropic efficiency can be calculated to rate the design,

$$r = \frac{P_{01}}{P_{03}}$$

$$\eta = \frac{1 - \left(\frac{1}{r}\right)^{\frac{(\gamma-1)}{\gamma}\eta_p}}{1 - \left(\frac{1}{r}\right)^{\frac{(\gamma-1)}{\gamma}}}$$

2.5 Results and summaries (graphs and tables mainly)

2.5.1 Calculation summary

Table 1: calculation summary

V_1	205.8 m/s
V_{u1}	0
V_{a1}	205.8 m/s
α_1	0
W_1	0
W_{u1}	0
W_{a1}	0
β_1	0
U_1	0
V_2	$\frac{205.8}{\cos(\tan^{-1}(2.75 + 1.83(x+1)))}$
V_{u2}	$1956.2 + \frac{2\pi \times 3600}{60}(x + 1.35)$
V_{a2}	205.8 m/s
α_2	$\tan^{-1}(2.75 + 1.83(x+1))$
W_2	601.8 m/s
W_{u2}	565.5 m/s
W_{a2}	205.8 m/s
β_2	70 degrees

V_3	$\dfrac{205.8}{\cos(\tan^{-1}\left(\tan\left(-1\times\left(-\frac{20}{0.5}x+70\right)\right)+1.83(x+1)\right))}$
V_{u3}	$205.8\tan(-1\times\left(-\frac{20}{0.5}x+70\right))+\frac{2\pi\times 3600}{60}(x+1)$
V_{a3}	205.8
α_3	$\tan^{-1}(-\tan\left(-1\times\left(-\frac{20}{0.5}x+70\right)\right)+1.83(x+1))$
W_3	$\dfrac{205.8}{\cos(-1\times(-\frac{20}{0.5}x+70))}$
W_{u3}	$205.8\tan(-1\times(-\frac{20}{0.5}x+70))$
W_{a3}	205.8 m/s
β_3	$-1\times(-\frac{20}{0.5}x+70)$
U	$\frac{2\pi\times 3600}{60}(x+1)$
R	$\dfrac{(205.8\tan(-1\times(-\frac{20}{0.5}x+70)))^2-601.8^2}{565.5+\frac{2\pi\times 3600}{60}(x+1))^2+(205.8\tan(-1\times(-\frac{20}{0.5}x+70)))^2-601.8^2}$
$\dot{m}$	$9.15\ kg/s$

2.5.2 Graphs (all figures were graphed from MATLAB)

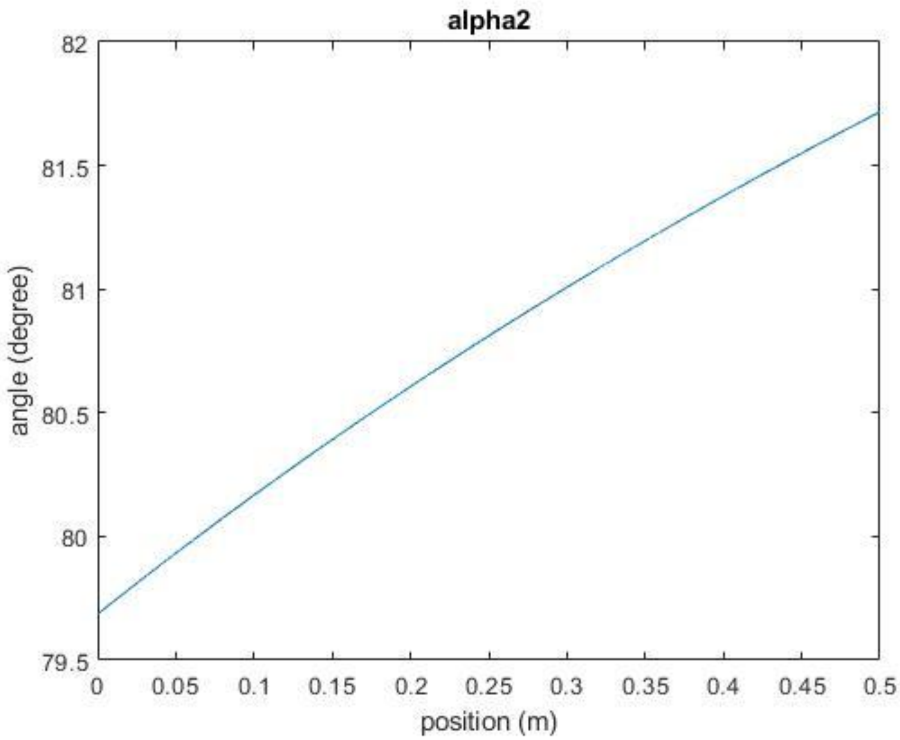

Figure 1: alpha 2 as a function of position

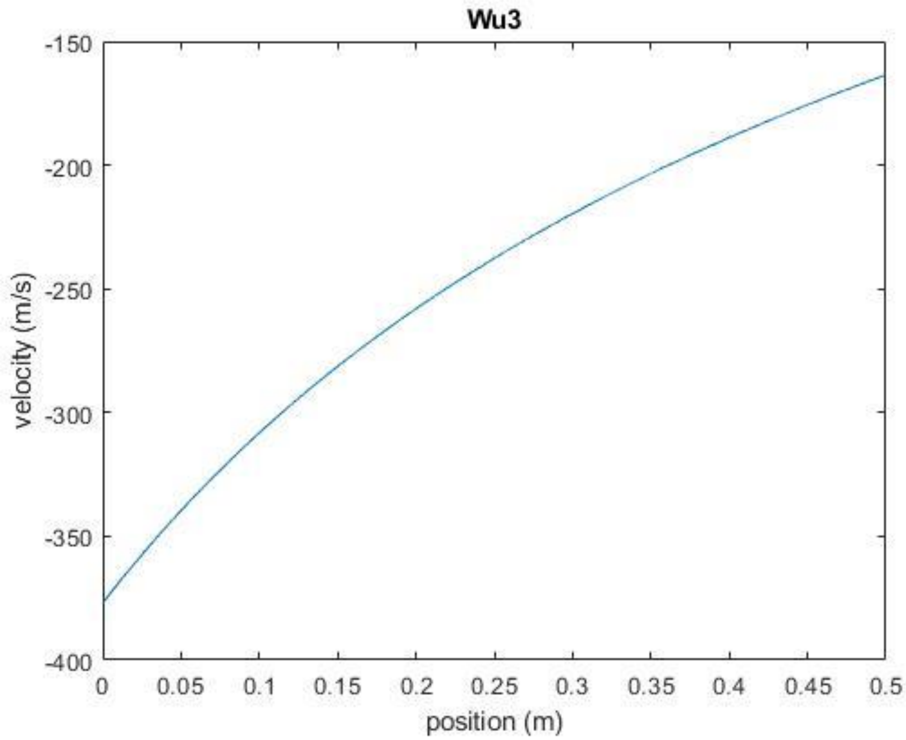

Figure 2: Wu3 as a function of position

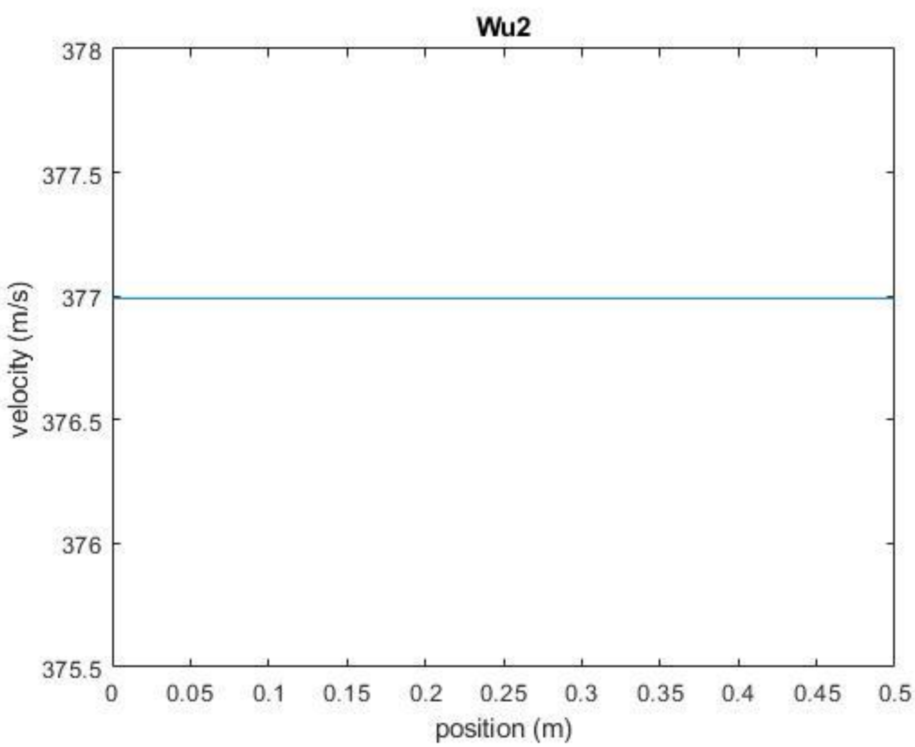

Figure 3: Wu2 as a function of position

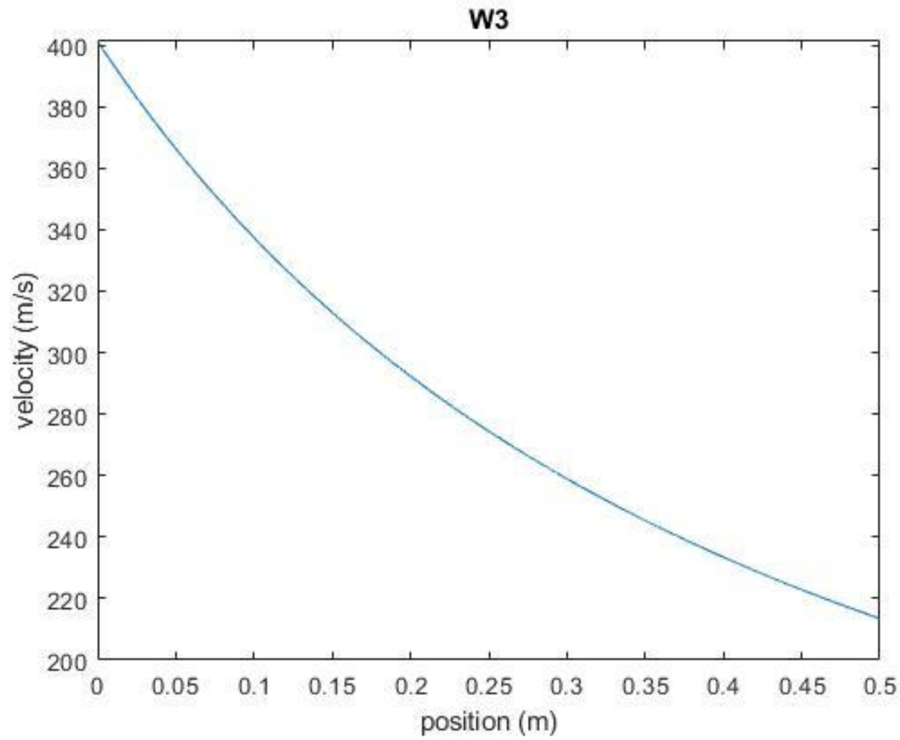

Figure 4: W3 as a function of position

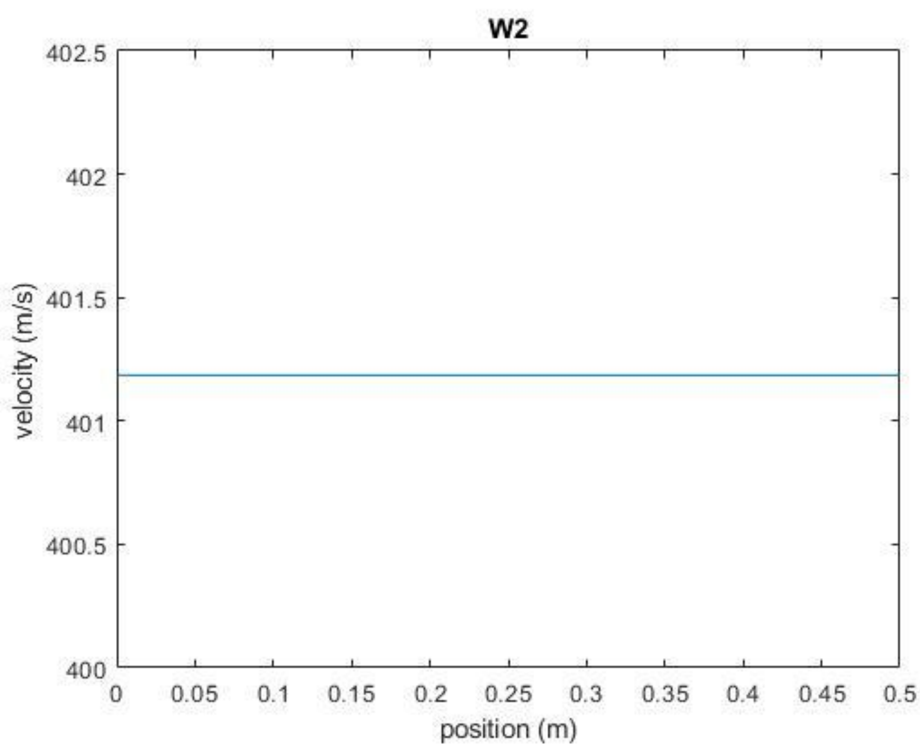

Figure 5: W2 as a function of position

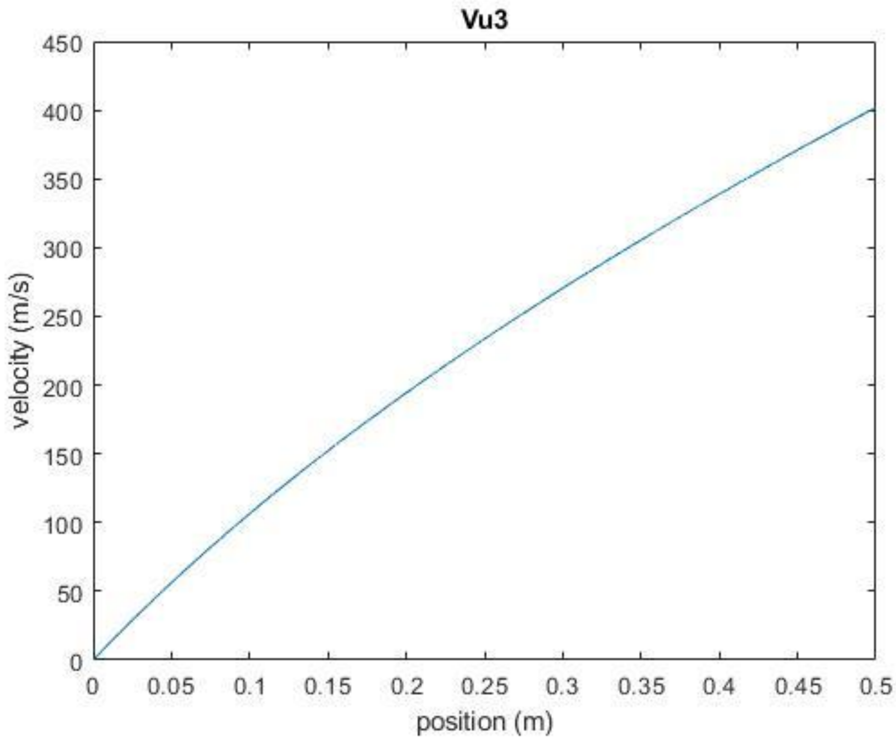

Figure 6: Vu3 as a function of position

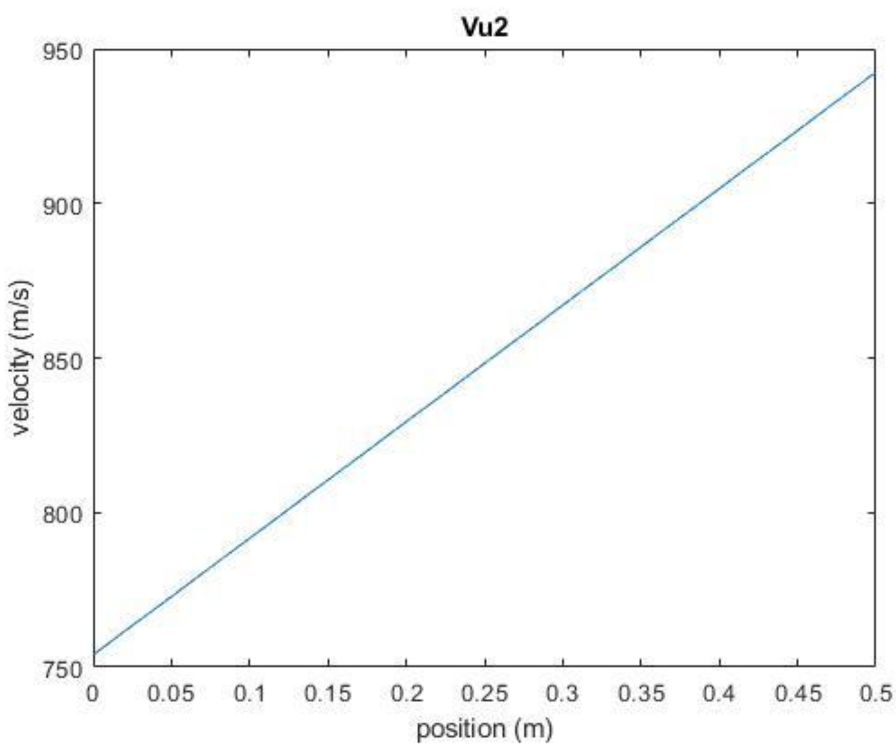

Figure 7: Vu2 as a function of position

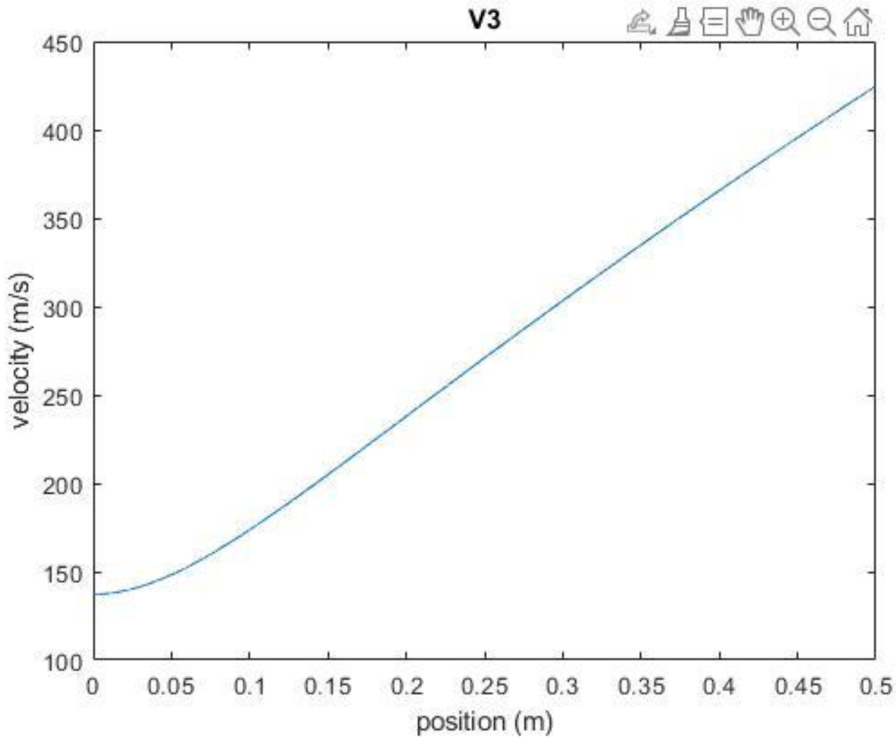

Figure 8: V3 as a function of position

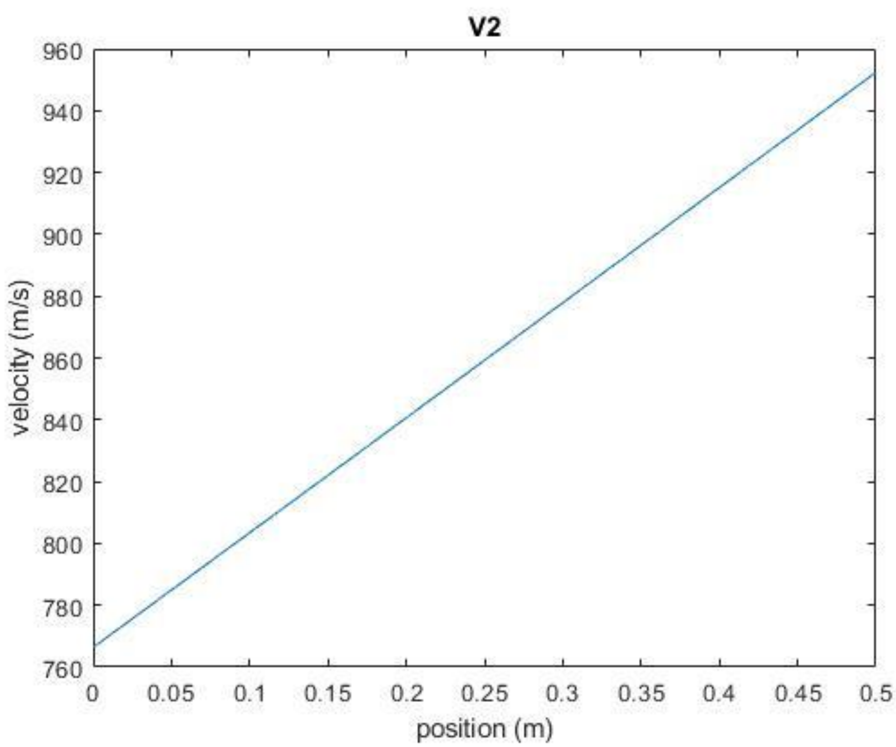

Figure 9: V2 as a function of position

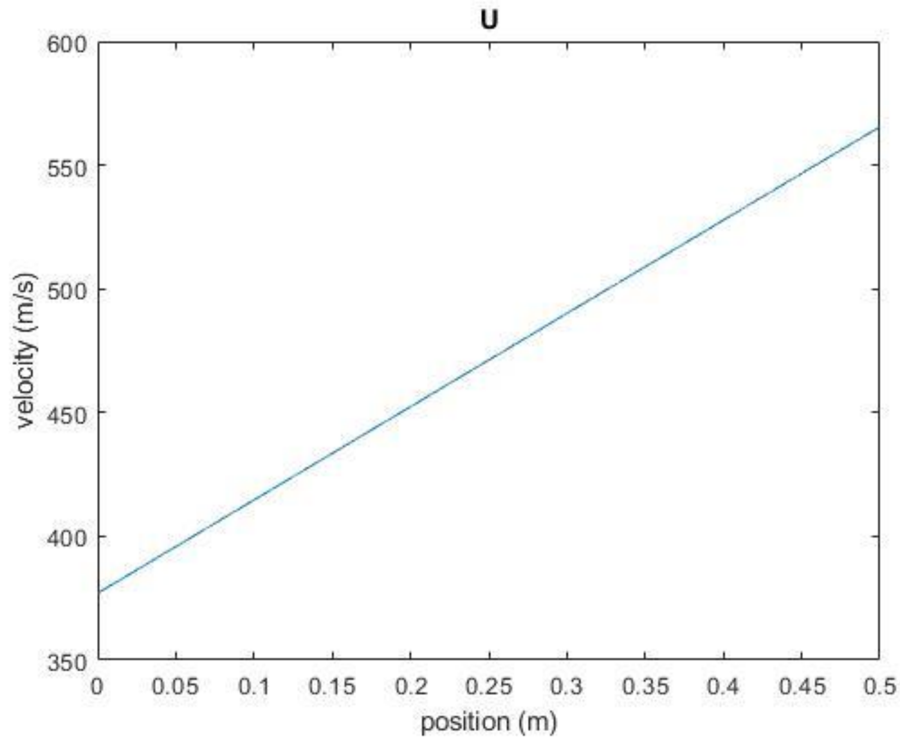

Figure 10: U as a function of position

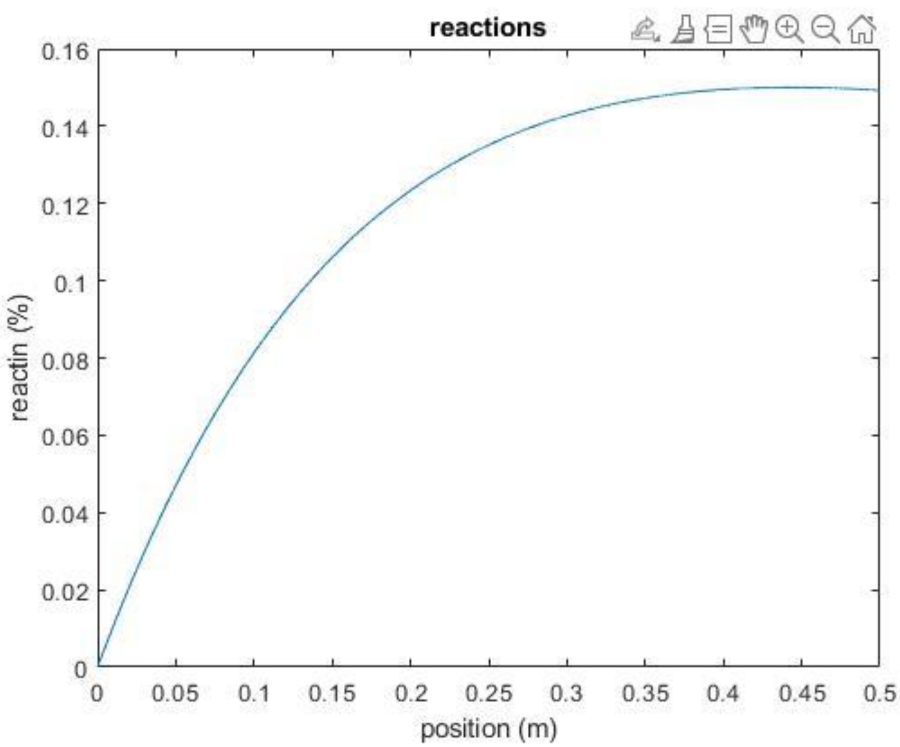

Figure 11: Reactions as a function of position

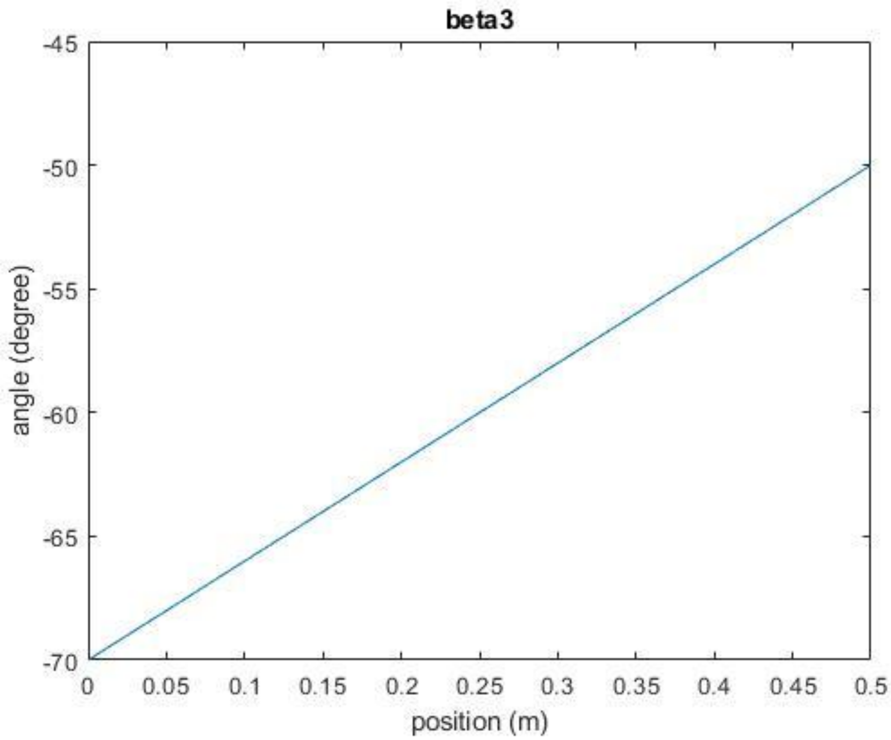

Figure 12: beta 3 as a function of position

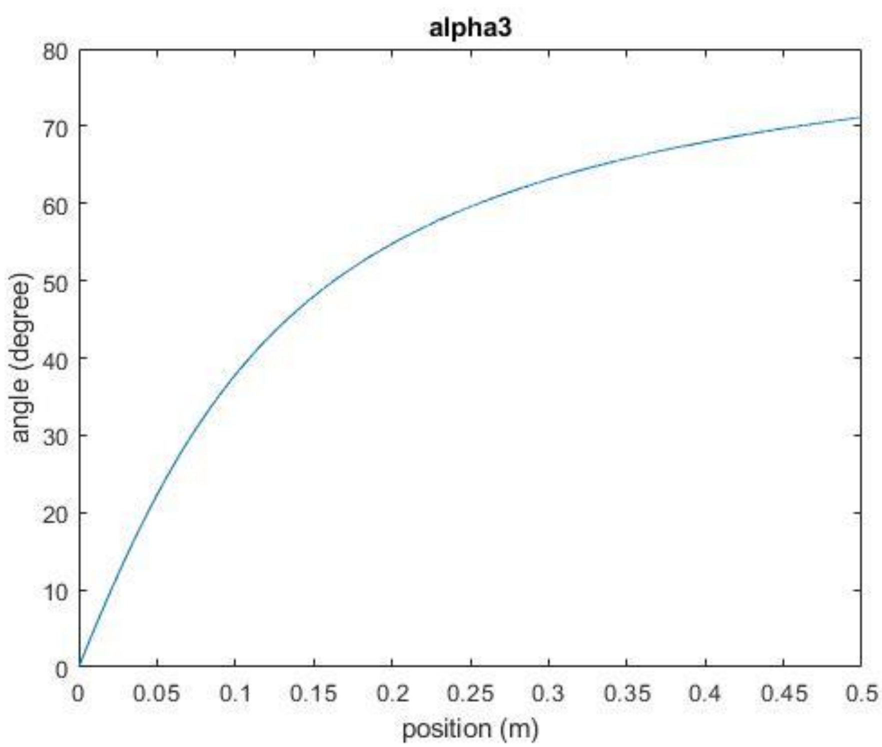

Figure 13: alpha 3 as a function of position

2.5.3 Table of all the coefficients and values

Table 2: results

properties	hub	50% of the blade	casing
y (m)	0	0.25	0.5
mass flow rate (kg/s)	9.9264	9.9264	9.9264
beta3 (rad)	-1.2217	-1.0472	-0.87266
beta2 (rad)	1.2217	1.2217	1.2217
U (m/s)	376.9911	471.2389	565.4867
Wu2 (m/s)	376.9911	376.9911	376.9911
W2 (m/s)	401.1856	401.1856	401.1856
alpha2 (degree)	1.3908	1.4104	1.4262
V2 (m/s)	766.366	859.2565	952.4138
W3 (m/s)	401.1856	274.4271	213.4664
Wu3 (m/s)	-376.991	-237.661	-163.525
alpha3 (degree)	0	-1.0397	-1.2418
V3 (m/s)	137.2135	270.899	424.7363
Vu3 (m/s)	5.68E-14	233.5781	401.9619
Vu2 (m/s)	753.9822	848.23	942.4778
R	0	0.13511	0.14929
T1 (K)	1591.028	1591.028	1591.028
T2 (K)	1320.123	1248.164	1167.739
T3 (K)	1391.028	1365.029	1314.033
P1 (kPa)	84.1191	84.1191	84.1191
P2 (kPa)	11.1451	10.5376	9.85858
P3 (kPa)	122.0639	369.767	855.6266
T01 (K)	1600	1600	1600
T02 (K)	1600	1600	1600
T03 (K)	1400	1400	1400
P01 (kPa)	257.539	257.539	257.539

P02 (kPa)	874.9767	1096.469	1344.019
P03 (kPa)	122.0639	369.767	855.6266
ksi	2	2.7211	3.1231
psi	0.36397	0.29118	0.24265
Fu (N)	7484.33	12728.59	17530.59
blade length (m)	0.5	0.5	0.5
blade chord (m)	0.16667	0.16667	0.16667
pitch	0.13232	0.13232	0.13232
zeta stator	0.12914	0.12984	0.13042
zeta rotor	0.13915	0.13103	0.12351
Mach number at 2	1.0612	1.2236	1.4022
Mach number at 3	0.1851	0.3689	0.5895
pressure loss at the stator (kPa)	87.57864	146.7179	237.2164
pressure loss at the rotor (kPa)	0.400516	4.53798	25.2763
profile losses stator	0.059803	0.060873	0.061764
profile losses rotor	0.17303	0.17702	0.18031
secondary flow and clearance tip losses rotor	0	0.066923	0.25998
secondary flow losses stator	0.003641	0.004612	0.006027
total loss coefficient stator	0.063444	0.065485	0.067791
total loss coefficient stator	0.17303	0.24394	0.4403
pressure ratio	2.1099	0.69649	0.30099
polytropic efficiency	0.89957	0.89957	0.89957

3. Discussion

When comparing an impulse turbine to a twisted blade turbine, we can see that the blade loading in the twisted blade does not increase as much as the impulse turbine and that is because the reaction increases as we get further away from the root, and in the impulse turbine the blade is both impulsive at the base and at the tip. We can see that a high efficiency and how power output are possible, all while keeping low stresses on the blade 7484.33 N at the root and 17530 N at the tip, the tip and pressure losses are also very low, with 87.5 kPa at the root and 237.2 kPa at the tip, the total loss coefficients also increase from root to tip, but are kept low, and finally the pressure ratio, the pressure ratio is high but not as high as it would be for an impulse turbine of the same size and power output. An important note to take is the polytropic efficiency, it would rate the design itself, and as it seems the polytropic efficiency is nearly the same as the stage efficiency, which is a sign of a good design.

4. Conclusion

Twisted blades have been used for a while now in turbines, they are very efficient, and although not discussed in the paper, have a good resistance reduction when rotating, as the turbine rotates the closer to the tip one is the faster the blade is spinning, which when taking into consideration the resistance of the fluid around results in an immense stress on the blade and limits the speed at which it can rotate, with turbine blades this issue is tackled. Now coming back to the paper, twisted blades are a revolutionary innovation when it comes to turbomachinery, their outlet relative angle and reaction change along the blade and from my design it results in a decreased stress on the blades (Fu), high efficiency , decreased total loss coefficient (Ypa and Ype).

5. Recommendations

Using twisted blades for small applications is not a bad idea, due to its special attribute that when it is used off design the effect is not as drastic as non-twisted turbine blades.

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