Unit One

(1) $\lim\limits_{h\to 0}\left(\dfrac{\cot(x+h)-\cot x}{h}\right) = \ldots\ldots$

(a) $\csc^2 x$ (b) $\sec^2 x$ (c) $-\csc^2 x$ (d) $\cot x$

---

(2) $\lim\limits_{h\to 0}\dfrac{\sec(\frac{\pi}{4}+h)-\sec(\frac{\pi}{4})}{h} = \ldots\ldots$

(a) $\sqrt{2}$ (b) zero (c) $\dfrac{1}{\sqrt{2}}$ (d) Undefined

---

(3) If $f(x)$ is a polynomial function, then $\lim\limits_{h\to 0}\dfrac{f'(x+h)-f'(x)}{h} = \ldots\ldots$

(a) $f'(x)$ (b) $f''(x)$ (c) $f'(h)$ (d) $f''(h)$

---

(4) If $\lim\limits_{h\to 0}\dfrac{f'(1)-f'(1+h)}{2h} = 21$ where $f(x) = 2x^4 - ax^3$, then $a = \ldots\ldots$

(a) 11 (b) 12 (c) 13 (d) 14

---

(5) If $y = f(x)$ and $f(x+h) - f(x) = 5x^2h + h^2$, then $\dfrac{d^3y}{dx^3} = \ldots\ldots$

(a) 6 (b) 10 (c) 14 (d) 18

---

(6) If $y = \sin x - \dfrac{1}{3}\sin^3 x$, then $\dfrac{dy}{dx} = (\ldots\ldots)^3$

(a) $-\cos x$ (b) $\tan x$ (c) $\cos x$ (d) $\sin x$

---

(7) $\dfrac{d}{dx}[x^2 + \dfrac{d}{dx}(x + \sec x)] = \ldots\ldots$

(a) $2x + 1 + \sec x \tan x$ (b) $2x + 2\sec^3 x - \sec x$

(c) $2 + \sec^3 x - \sec x$ (d) $2x + \sec^2 x \tan^2 x$

---

(8) If $f(x) = 8x \sin x \cos x \cos 2x$, then $f'(\dfrac{\pi}{8}) = \ldots\ldots$

(a) -2 (b) zero (c) 1 (d) 2

---

(9) If $y = \cot ax$ and $\dfrac{dy}{dx} + 4(1 + y^2) = 0$, then $a = \ldots\ldots$

(a) 1 (b) -2 (c) 4 (d) -4

---

(10) If $y = \sin x \sec\left(\frac{\pi}{2} - x\right)$ where x is an acute angle, then $\frac{dy}{dx}$ = ……….

(a) zero (b) 1

(c) $\cos x \csc x + \sin x \sec x$ (d) $\sin x \cos x + \sec x \cot x$

(11) If $0 < x < \frac{\pi}{2}$ and $f(\tan x) = \sin x$, then $f'(1)$ = ……

(a) 1 (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{2}}{4}$ (d) $\frac{\sqrt{2}}{6}$

(12) If $f(x) = x^2$, $g(x) = \cot x$ and $h(x) = (f \circ g)(x)$, then $h'\left(\frac{\pi}{4}\right)$ = …….

(a) -4 (b) 4 (c) zero (d) -1

(13) If $0<x<\frac{\pi}{2}$ and $f(\sin x) = a \cos x$ where a is a constant and $f'\left(\frac{3}{5}\right) = -6$, then a =…

(a) 2 (b) 4 (c) 6 (d) 8

(14) If $f(x) = \cos 3x \cos x - \sin 3x \sin x$, then $f''\left(\frac{\pi}{4}\right)$= ………

(a) 2 (b) 4 (c) 8 (d) 16

(15) If $f(\sin x) = \sin^2 x$, then $f''(1)$ = …….

(a) 1 (b) 2 (c) π (d) $\frac{\pi}{2}$

(16) If $f(x) = 2 \sin \frac{x}{2} \cos \frac{x}{2}$, then the 1000^{th} derivative of this function equals …..

(a) $\sin x$ (b) $-\sin x$ (c) $-\cos x$ (d) $\sin^{-1} x$

(17) If $x^3 y^2 = 1$, then $\left[\frac{dy}{dx}\right]_{y=1}$ = ………..

(a) 1.5 (b) 2 (c) -1.5 (d) 1

(18) The rate of change of the slope of the tangent to the function $f : f(x) = 2x^3$ at $x = 3$ equals …….

(a) 36 (b) 54 (c) 6 (d) 12

(19) If $x^2 + y^2 = 4$, prove that: $y^3 \frac{d^2y}{dx^2}$ = ……

(a) -4 (b) 4 (c) 2 (d) -2

(20) If (x , y) is any point on the unit circle , then ……

(a) $y\,y''-2(y')^2+1=0$ (b) $yy''-(y')^2+1=0$ (c) $y\,y''-(y')^2-1=0$ (d) $yy''+2(y')^2+1=0$

(21) If $x^2+y^2=2\,x\,y$, then $y'=$ …….

(a) -1 (b) 0 (c) 1 (d) 2

(22) If $y^2-2\sqrt{x}=0$, then $\frac{dy}{dx}=$ ......

(a) $\frac{2y}{\sqrt{x}}$ (b) $\sqrt{x}$ (c) $\frac{x}{y^2}$ (d) $\frac{1}{y^3}$

(23) If $x=(1-y)(1+y)(1+y^2)(1+y^4)$, then $\frac{d^2y}{dx^2}=$ ……..

(a) $\frac{-1}{8}y^{-7}$ (b) $-56\,y^6$ (c) $\frac{-7}{64}y^{-15}$ (d) $\frac{7}{8}y^6$

(24) If $x^3+3x^2y+3x\,y^2+y^3=15$, then $\frac{dy}{dx}=$ ....

(a) 1 (b) -1 (c) 3 (d) -3

(25) If $f'(x)=x\,f(x)$, $f(3)=-5$, then $f''(3)=$ ……

(a) -50 (b) 50 (c) -40 (d) 15

(26) If $h(x)=|x|$, then $h'(-5)=$ …………

(a) 5 (b) 1 (c) -5 (d) -1

(27) If $y=2t^3+7$ and $z=t^2-4$, then the rate of change of y with respect to z equals:

(a) 2t (b) 3t (c) 6 (d) 12

(28) If $y=\frac{z+1}{z-1}$, $x=\frac{z-1}{z+1}$, then $\frac{dy}{dx}=$ ……. at $x=2$

(a) $\frac{-1}{8}$ (b) $\frac{-1}{4}$ (c) $\frac{1}{4}$ (d) -4

(29) If $x=2z^3-5$, $y=6z^2+1$ find $\frac{d^2y}{dx^2}=$ ……… at $z=1$

(a) $\frac{-1}{3}$ (b) $\frac{-1}{9}$ (c) $\frac{-2}{9}$ (d) $\frac{-2}{3}$

(30) The rate of change of $\sin x^3$ with respect to $\cos x^3$ equals ……

(a) $-\cot x^3$ (b) $\cot x^3$ (c) $\tan x^3$ (d) $-\tan x^3$

(31) If $x = \sin^3\theta, y = \cos^3\theta$, then $\frac{d^2y}{dx^2}$ = ……. at $\theta = \frac{\pi}{4}$

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$ (c) $\frac{4\sqrt{3}}{3}$ (d) $\frac{4\sqrt{2}}{3}$

(32) If $y = \sqrt{f(x)}$, and $f'(2) = 4$, $f(2) = 9$, then $\frac{dy}{dx}$ = …... when $x = 2$

(a) $\frac{4}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

(33) If $f(5x) = x^2 + x$, then $f'(2)$ = …..

(a) 5 (b) 1 (c) $\frac{3}{25}$ (d) $\frac{9}{25}$

(34) If $f(x) = \frac{2}{x+1}$, $g(x) = 3x$, then $\frac{d}{dx}[(f \circ g)(x)]$ = …… at $x = -2$

(a) $-\frac{2}{25}$ (b) 6 (c) $\frac{1}{25}$ (d) $-\frac{6}{25}$

(35) If $f(x) = 3x^2 - 2$, then $(f \circ f)'(-1)$ = …….

(a) - 36 (b) - 18 (c) zero (d) 18

(36) If $f(2x+1) = x \cdot h(2x-5)$ and $h(1) = 2$, $h'(1) = 4$, then $f'(7)$ = …..

(a) 4 (b) 7 (c) 11 (d) 13

(37) If $f(x) = g(x^2)$, then $\frac{f'(2)}{g'(4)}$ = …….

(a) 1 (b) 2 (c) 3 (d) 4

(38) If the function g is the inverse of the function f where f and g are differentiable functions on R and $f'(a) = 2$, $f(a) = b$, then $g'(b)$ = ……

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) 1

(39) If $f(x) = x^2$, then $(f \circ f')'(1)$ = …….

(a) 2 (b) 4 (c) 8 (d) 9

(40) If $y = x^n$ where n is a natural number and $\frac{d^4y}{dx^4} = 360\,x^{n-4}$, then the value of n = …..

(a) 7 (b) 13 (c) 5 (d) 6

(41) If $f(x) = 20x^{n-1}$ and $f'''(x) = c$ where $c \in R$, $n \in z^+$, then n + c ……

(a) 104 (b) 123 (c) 124 (d) 125

(42) If the function $f : f(x) = \begin{cases} ax^2 + bx + c & x < 0 \\ 2x + 5 & x \geq 0 \end{cases}$ is differentiable twice at x = 0 , then $a + b - c = $ …..

(a) 7 (b) 5 (c) 3 (d) - 3

(43) If $y = 4 + \cot x - \sec^2 x$, then the equation of normal when $x = \frac{\pi}{4}$ is …….

(a) $x - 6y + 18 - \frac{\pi}{4} = 0$ (c) $x + 6y + 18 - \frac{\pi}{4} = 0$

(c) $x - 6y + 18 + \frac{\pi}{4} = 0$ (d) $x - 6y - 18 - \frac{\pi}{4} = 0$

(44) If two parametric equations to a curve are $x = t^2 - 6t$, $y = 8\sqrt{t - 2}$, then the equation of the tangent when t = 6 is ………

(a) $x - 3y - 48 = 0$ (b) $x - 3y + 48 = 0$ (c) $x + 3y - 48 = 0$ (d) $x + 3y + 48 = 0$

(45) The area of the triangle bounded by x-axis , tangent and normal to the curve $3x^2 + y^2 = 12$ at point (-1,3) = …… unit area

(a) 9 (b) 8 (c) 10 (d) 7

(46) If the tangent and the normal to the curve $x^2 + 3xy + y^2 + 1 = 0$ at point A(-1,1) intersect x-axis at the two points B , C , then the area of the triangle ABC = …… unit area

(a) 1.5 (b) 1 (c) 0.5 (d) 2

(47) The equation the normal to the curve $x = t^2 + 2$, $y = t^3 + 1$ when t = 1 is ……

(a) $3x - 2y - 5 = 0$ (b) $2x - 3y - 12 = 0$

(c) $2x + 3y - 12 = 0$ (d) $3x + 2y - 5 = 0$

(48) The straight line $y + x - 1 = 0$ touches the curve of the function : $f(x) = x^2 - 3x + a$ then a = …….

(a) 1 (b) 2 (c) 3 (d) 4

(49) The tangent to the curve $y = 3x^2 - 5$ at point (1, - 2) passing through point …..

(a) (5,-2) (b) (3,1) (c) (2,-4) (d) (0,-8)

(50) If the equation of the normal to the curve $y = f(x)$ at point $(1,1)$ is $x + 4y = 5$, then $f'(1) =$

(a) -3 (b) $-\frac{1}{4}$ (c) 4 (d) -4

(51) The equation of tangent to the curve of the function $f(x) = (2x+3)^3$ at the point $(-1,1)$ is ....

(a) $6y + x = 7$ (b) $y = 6x + 7$ (c) $2y - 3x = 1$ (d) $y + 2x = 5$

(52) The slope of the normal to the curve of the function $y = \left|x^3\right|$ at the point $(-2, 8)$ is……..

(a) -12 (b) 12 (c) $\frac{1}{12}$ (d) $-\frac{1}{2}$

(53) The tangent to the curve $x^2 - xy + y^2 = 27$ which is drawn at the point $(6, 3)$ makes an angle of measure …….° with the positive direction of x-axis

(a) $90°$ (b) zero (c) $45°$ (d) $180°$

(54) The two curves of the functions $f(x)$ and $g(x)$ are tangential at the point $(2, 4)$ and $f'(2) = 3$, then $g'(2) =$ …..

(a) 2 (b) 3 (c) 4 (d) 5

(55) If the tangent to the curve of the function $y = x^2 + a$ at the point $(1, b)$ crosses the x-axis at $x = -1$, then $a \times b =$ ……

(a) 3 (b) 4 (c) 12 (d) -12

(56) The angle measure that the tangent to the curve $\sin 2x = \tan y$ makes with the positive direction of x-axis at the point $\left(\frac{3\pi}{4}, \frac{3\pi}{4}\right)$ equals …….

(a) zero (b) $135°$ (c) $45°$ (d) $26°34'$

(57) If the tangent to the curve $y = x^3 - 3x^2$ makes an obtuse angle with the positive direction of the x-axis then $x \in$…….

(a) $[0, 2]$ (b) $]0, 2[$ (c) $R - [0, 2]$ (d) $R -]0, 2[$

(58) The rate of increase of side length of an equilateral triangle is 2 cm./sec. , then the rate of increase of its perimeter is …… cm./sec.

(a) 2 (b) 8 (c) 4 (d) 6

(59) *If $y = x^2 - 3x$, then $\frac{dy}{dt} = \frac{dx}{dt}$ at x = …….*

(a) 1 (b) 2 (c) 3 (d) 4

(60) *If a particle moves on the curve : $y^2 + x^2 = 10$ such that $\frac{dy}{dt} = 4$, then $\frac{dx}{dt}$ = ….. at the point ($\sqrt{5}$, - $\sqrt{5}$)*

(a) 2 (b) $2\sqrt{5}$ (c) 4 (d) $4\sqrt{5}$

(61) *A cube melt preserving its shape by rate 1 cm^3./sec. , then the rate of change of its edge length when its volume 8 cm^3. is ……… cm./sec.*

(a) $\frac{1}{12}$ (b) $\frac{1}{192}$ (c) $\frac{-1}{24}$ (d) $\frac{-1}{12}$

(62) *If the side length of an equilateral triangle = a and it increases at a rate (k) , then the rate of increase of the triangle area equals ……..*

(a) $\frac{2}{\sqrt{3}}ak$ (b) $\sqrt{3}\,ak$ (c) $\frac{\sqrt{3}}{2}ak$ (d) $\frac{\sqrt{2}}{\sqrt{3}}ak$

(63) *The length of a rectangle is twice its width , if the rate of change of its width is 3 cm./sec. , then the rate of change of its diagonal length = ……. cm./sec.*

(a) 6 (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $\sqrt{15}$

(64) *The rate of increase of surface area of a sphere is 6 cm^2./sec. at this moment its radius equals 30 cm. , then the rate of increase of its volume = ……. cm^3./sec.*

(a) 180 (b) 40 (c) 90 (d) 90π

(65) *An isosceles triangle , the length of each of the two equal sides is 6 cm. and the measure of their included angle equals (x) , if the rate of change of (x) is $(\frac{\pi}{90})^{rad}$ per minute , then the rate of change of its area at x = 30° is ……..*

(a) $\frac{\sqrt{3}}{10}\pi$ (b) $\frac{\pi}{10}$ (c) $9\sqrt{3}$ (d) 9

(66) *The slope of the tangent to the curve y =f (x) at a point = $\frac{1}{2}$ and the x-coordinate of this point decreases at a rate 3 unit/sec. then the rate of change of its y-coordinate equals …….. unit/sec.*

(a) $-\frac{1}{6}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{6}$ (d) $\frac{3}{2}$

(67) *The height of right circular cone equals to its base diameter the rate of change of its base radius = $\frac{1}{\pi}$ cm./sec. then the rate of change of its volume = …… cm^3./sec. when its base radius length = 5 cm.*

(a) 50π (b) $\frac{250}{3}\pi$ (c) 150 (d) 50

(68) *A man observes a plane flies horizontally at 3 km. high above him and with speed 480 km./h. , then the rate of change of the distance between the man and the plane after 30 sec. later = …….*

(a) $\frac{320}{3}$ km./h. (b) 384km./sec. (c) 384 m./sec. (d) $\frac{320}{3}$ m./sec.

(69) *10 meter ladder is leaning against a vertical wall and its lower end on a horizontal ground , if the lower end slides 2 m./min. then the rate of change of inclination angle with the horizontal at the moment the lower end at a distance 8 m. equals ……. rad.min.*

(a) 3 (b) - 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(70) *A circular segment , the radius length of its circle is 10 cm. and measure of its central angle (x°) changes in the rate $3^{rad.}$ per minute , then the rate of increasing in the area of the circular segment at $x = 60^\circ$ is …… cm^2./min.*

(a) 125 (b) 75 (c) 150 (d) 300