Simulating Portfolio Growth: Integrating Historical Inflation for Enhanced Financial Forecasting

Simulation ISYE 6664

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Abstract:

Conventional retirement savings calculators often offer simplistic, static parameters that fail to capture the dynamic nature of financial markets. Many individuals, particularly those unfamiliar with investment intricacies, overlook the profound influence of variable parameters, intricately tied to both local and global macroeconomic landscapes. This project adopts a comprehensive approach, empowering users to tailor inputs according to their unique perspectives while acknowledging the uncertainties of inflation inherent in financial projections. By accommodating diverse economic scenarios, our calculator facilitates a nuanced analysis of how fluctuations in key parameters could shape an individual's retirement savings trajectory with inflation taken into account and simulated.

Background And Problem Description

As young professionals who started their careers during and after the COVID pandemic, we saw how the pandemic impacted the world and especially the financial markets. We saw company stocks hit new lows, people close to retirement lost a lot of their retirement savings, and interest rates almost 0%. We then saw company stocks recover and inflation starting to pick up to reach a high of more than 8% since 2000 and that is excluding the food and gas prices which inevitably skyrocketed in the aftermath of geopolitical events. Currently, and while inflation is decreasing, the financial market awaits the central bank's effective decisions on the current interest rates which are impacting how people are deciding to invest their money in fixed income assets or in stocks as well as impacting companies who raised capital through debt. These events shaped our worldview and prompted our group to create our own retirement savings calculator that is more nuanced than what is currently available. We want to understand how we can best protect ourselves in the future, from a monetary perspective, subject to the geopolitical and socioeconomic changes in the world.

As such, we would like to understand the future value (FV) of a retirement portfolio given certain macroeconomic conditions. By varying inputs such as starting contribution, employee match, pre-tax 401k or after-tax 401k and financial indicators within specified intervals, we aim to integrate the probability density function (PDF) over the inflation rate to estimate the portfolio's future value. We decided to integrate over inflation because we found that it is one of the important factors that impacts how the interest rates develop and the economy grows and consequently how the financial markets and investments are impacted.

This simulation is designed to aid individuals in interpreting the potential future value of their retirement portfolio given their intuition of predictions of specific economic conditions. For instance, if a user anticipates an inflationary environment, they can use this tool to explore how such a scenario might affect the growth of their savings. Often, the widely-used calculators assume a target rate of inflation of 2% which is deemed “healthy” and optimal. However, this static value does not encompass the potential loss or gain in the savings portfolio when the economic regime is dealing with higher inflation rates for longer periods especially when the user is approaching retirement. In this case, the real value of the portfolio is less than anticipated because their dollars are not worth the same as when they were first invested.

To this purpose, we build a tutorial that exploits Monte Carlo integration methods to help us achieve the goal of estimating savings more dynamically and evaluate the confidence intervals of the estimations derived from the simulations.

Main Findings (Programming-Oriented Problem)

For this project, we chose to do a tutorial on simulating a retirement account's savings using Monte Carlo integration methods in Python. The main focus is to understand the portfolio value given different taxation methods of each account type, factored with the blend of returns from small, mid and large cap fidelity funds and whether one can be in the positive if the penalty is applied (liquidation before 59.5). The original question trying to be solved is given an employee match of X, can one be able to liquidate their portfolio in Y years with the penalty and still be in the green? We then add the simulation factor on this to understand the derivations with inflation taken into account for a more robust estimate. Previous calculators have been simplified with annual returns fixed, this is more nuanced allowing for the user to find an optimal set of parameters for their retirement account with their employer.

The parameters we initially defined as inputs in a function are as follows as the proof of concept (not the main function):

- Individual annual contributions
- Employer contribution match
- Annual rate of return: this is initially fixed but later in the simulation as we develop our calculator, we use assets allocations to be able to determine the rate of return

- The number of years until retirement or liquidation of assets
- The expected tax bracket at retirement or liquidation of assets
- The type of retirement account: pre-tax 401k, post-tax 401k, roth 401k
- Penalty in case of early withdrawal or liquidation of assets: if the investor chooses to liquidate before 59.5, a penalty is applied to the future value after taxes if the taxes apply to the retirement account

In the first part, our FV calculator returns a cost basis, the future value before taxes or penalty applied and the future value after taxes and penalties depending on the retirement account.

The annual total contributions are the total of both individual and employer match contributions. What we consider a cost basis is the amount contributed individually without employer match. Earnings are defined as the difference between the future value and the total contributions.

If the account is a pre-tax 401k, then we apply an income tax on the future value of the portfolio in addition to a penalty in case a penalty is warranted.

If the account is after-tax 401k, taxes are only applied to the earnings based on the tax bracket in addition to a penalty should it be warranted.

If the account is a roth-401k, the taxes are already paid at the time of contributions, as such the take-home of the user is the actual future value, in addition to a penalty should it be warranted.

In the second revision of the function denoted as “Step 1” in the jupyter notebook attached, we allow the user to input their desired asset allocations from a mix of a small-capital, mid-capital and a large-capital stock fund which must equal 1.0. Investing in a small capital fund offers room for growth but also brings a greater risk and volatility. Large capital funds offer stability and dividends but do not grow as fast. Mid-capital stock funds represent the best of both worlds as they tend to be more stable than small capital yet offer more growth potential than large capital. We propose these funds to the user to help them decide on their risk appetite and whether they'd like to balance out the investment growth and risk thus returns. The dataset relevant to these funds' prices date from 2011.

In addition we account for the taxes and penalty, taxation will be different based on the retirement account selected, pre-tax 401k, after-tax 401k or roth-401k. This is done because it is generally a hard decision to select which retirement account yields the most benefit when employee matches vary. With a 10% penalty parameter in addition the user can be able to tell if they have a higher probability of being within the positive after the selected years.

After taxation in the function we account for the inflation. We use Monte Carlo integration to estimate the expected future value of savings by simulating multiple scenarios of inflation of each investment period. As such, we allow the user to decide on how they'd like to consider the input of inflation rate: if they prefer to input an interval with a lower bound and an upper bound (Uniform RV from interval), or if they would like to let the calculator randomly choose on their behalf (Uniform Random Choice). Hence, we offer the option of inputs: inflation_rate_interval and use_historical_inflation

In the case of being provided an interval, i.e. the user chooses to input their desired range through inflation_rate_interval parameter, we know where the minimum and maximum values occur over time. As such, we use the random.uniform() function available in Python. This function helps us generate a random floating-point number within a specified interval with the assumption of independence between every random value generated.

In the case of allowing the calculator to choose a random value, i.e. use_historical_inflation=True, we use random.choice() that allows sampling with replacement from historical inflation data. To maintain consistency with our assets data, we obtain the historical and monthly inflation data from 2011 to early 2024 where 2011 is the start date of all the fidelity funds.

Based on these inputs, in an experiment we conducted 20,000 simulations and estimated the FV under 20,000 different scenarios. We chose a high number of simulations equal to 20,000 because as the number of Monte Carlo simulations increase, the accuracy of the expected future value of the savings improves thanks to the Law of Large Numbers. The estimate becomes more reliable and converges towards the true expected value of savings. Then we take the mean of all of the simulations and plot it on the graph for each timepoint to understand where the middle bound is.

Below is an example of the historical inflation rates considered in the simulation (Figure 1). Its probability distribution is skewed with outliers at the tail from the most recent years, as evidenced in Figure 2. Figure 2 also shows the evolution of the returns of the assets over the years and we can notice a sharp drop in returns in 2020. We can also notice that the volatility of the returns is higher after 2019 than before 2019. Post 2019 also corresponds to a time of high inflation rate that was causing uncertainty in the financial markets about whether the US would be able to avoid a recession and have a “soft landing” and how soon the central bank would be able to cut the rates.

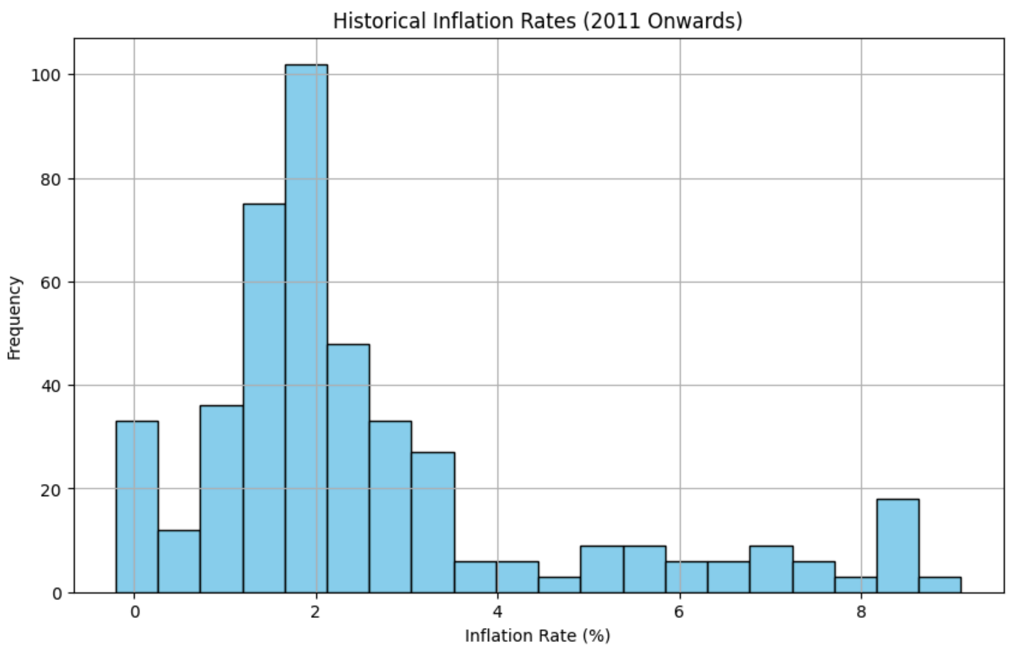

Figure1 : Histogram of historical inflation rates from 2011 to 2024

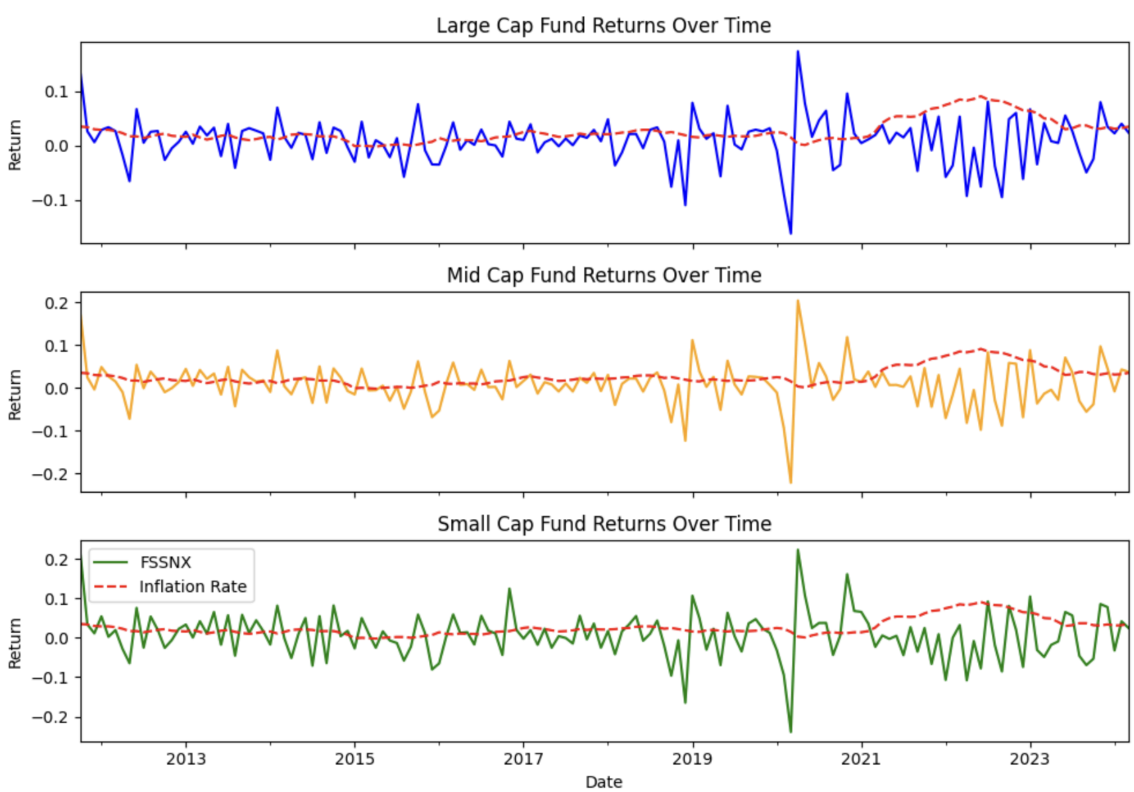

Figure 2: Asset returns over time with inflation rate over-layed on each fund

As an example for the use of the calculator, and to see how the inflation would impact the savings, we decide to test with the following input parameters:

- Pre-tax salary = 110,000
- Annual contribution = 6%
- Employee match = matches 100% of the first 3% and 50% of the next 3% contributed
- Small cap allocation = 20%
- Mid cap allocation = 20%
- Large capital allocation = 60%
- Use_historical_inflation = True
- Number of simulation = 20,000
- Years = 30
- Income Tax bracket = 26%
- Account type: pre-tax 401k
- No penalty

The calculator returns the following:

> *0% penalty applied | assumes 9.69% yearly returns | annual contribution $6600.0 | employee match $4950.0*
>
> ***Calculations after 30 years on a pre-tax 401k and salary 110000:***
>
> - *Cost Basis with Employee Match Over Time: $346,500.0*
> - *Cost Basis without Employee Match Over Time: $198,000.0*
> - *Future Value before tax or inflation: $1,965,416.95*
> - *Future Value after tax without inflation (26.0%): $1,454,408.54*
> - *Future Value after tax and inflation simulation applied averaging all runs (26.0%): $1,417,827.07*
> - *Net Gain after tax before inflation: $1,256,408.54*
> - *Net Gain after tax after inflation simulation: $1,219,827.07*

Inevitably, the inflation simulation decreases the expected value of the savings by 3% on average which is higher than the inflation rate most generic retirement savings calculators assume.

In Figure 4, we can see the evolution over time of the future value before tax and penalty, contributions, the future value after tax, penalty and inflation.

The orange line represents the average of the simulations at each point in time. The gray shaded area represents the area between the lower and upper bound estimates of the future value of savings. The gray area is essentially the 20,000 lines or simulations of the portfolio value with different inflation rates sampled. It looks like one big line because they are all close to each other. This plot gives us a great understanding of where our portfolio value can be with upper and lower bounds. Caveats include that this is under the assumption of returns that follow historical returns of the three blended allocations.

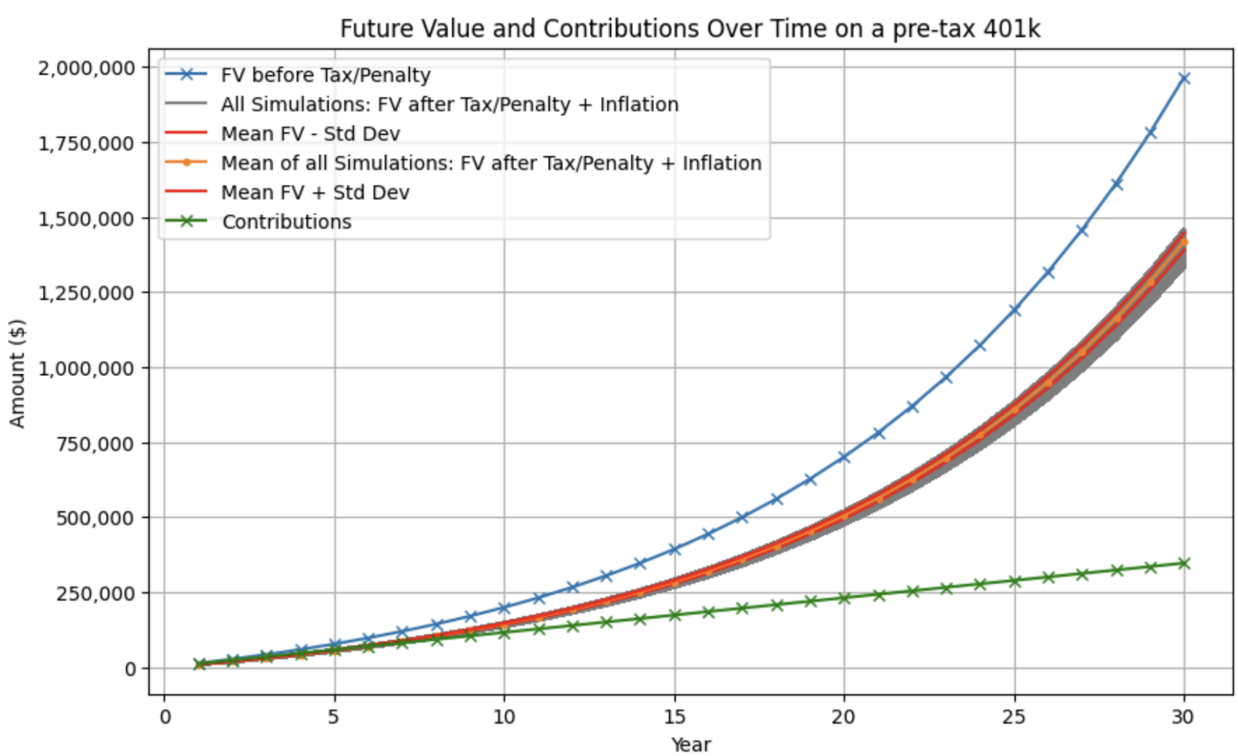

Figure 4: Future value of savings simulating inflation from historical rates

Conclusion:

In this project, we developed a nuanced simulation tool for estimating the future value of portfolios, accounting for different inflationary environments. Unlike conventional retirement savings calculators, our simulator integrates historical inflation data, offering a more tailored approach to forecasting portfolio growth. However, several caveats emerged from our analysis.

Firstly, the inflation data's timespan aligns with the timespan of the stock funds, commencing in 2011. While this limitation restricts the historical context, it ensures relevance to our selected timeframe. Secondly, our reliance on these historical inflation rates assumes future inflation is sampled from this distribution in this time period which includes major world events such as geopolitical shifts and COVID. This can induce bias in our estimation.

To address these limitations and enhance our simulator's performance, future studies should focus on predictive modeling for inflation. By training and testing such models, we can better anticipate inflationary trends, thus improving the accuracy of portfolio projections. Furthermore, integrating additional variables like interest rates and volatility in joint simulations offers a more comprehensive understanding of portfolio dynamics.

Moreover, incorporating a volatility model enables dynamic asset allocation, optimizing portfolio weights without user intervention. This adaptive approach ensures portfolios remain resilient amidst changing market conditions.

Additionally, expanding the asset pool to include specific stocks and sectors demands increased computational power.

In conclusion, while our simulation tool provides valuable insights into portfolio growth under varying inflationary scenarios, there remains scope for refinement and expansion. By leveraging predictive modeling and incorporating dynamic asset allocation strategies, future iterations of our simulator can offer enhanced accuracy and utility in financial planning. This calculator provides a more granular view of boundaries in which a portfolio may reside in given inflation taken into account; this is under the assumption that future inflation rates follow the distribution of the historical ones.

References

https://www.investopedia.com/terms/s/small-cap.asp
https://fundresearch.fidelity.com/mutual-funds/summary/316146182
https://fundresearch.fidelity.com/mutual-funds/summary/316146265
https://fundresearch.fidelity.com/mutual-funds/summary/31635V729
https://www.usinflationcalculator.com/inflation/historical-inflation-rates/
https://www.statisticshowto.com/probability-and-statistics/skewed-distribution/