Project 1 Report: Martingale

Youssef Sultan
ysultan@gatech.edu

Question 1

In experiment 1 we wanted to track the winnings of Professor Balch's original betting strategy by running the simulator through 10 episodes with an infinite amount of capital (each episode consisting of 1000 bets). Based on the results it was found that for 10 episodes, it took anywhere from 148-180 sequential bets to win $80.

We use the binomial distribution model to calculate the probability of winning $80 within 1000 sequential bets since there are two possible outcomes when betting.

$$P(K\ successes\ in\ n\ trials) = \binom{n}{k} p^k q^{n-k}$$

The average number of successful bets or successes it takes to make $80 based on the 10 episodes described above was approximately 163, however, there could be other cases where this average is different. To get a more robust average, the simulator was run for 1,000,000 episodes; the average successes it took to win $80 was 168 spins which can be used to justify K in the equation. The following model can be then seen as the following:

$$P(168\ successes\ in\ 1000\ trials) = \binom{1000}{168} .4736^{168} .5263^{1000-168}$$

Note: This equation refers to P(K ≥ 168)

Where n is equal to the number of sequential bets, K is the average amount of spins it takes on average to reach $80 based on 1,000,000 episodes, p being the probability of winning for one spin and q being the probability of losing for one spin. Our final probability that is calculated is .9999999999.

Question 2

In experiment 1 the estimated expected value of winnings for each bet can be seen as the following:

$$E[X] = 1(.4736)$$

Note: This equation refers to one single bet

Where the value x ($1) is multiplied by its probability of occurring (.4736). This means that for each bet the expected value of winnings is $0.4736. If this is multiplied by 1000 sequential bets, the expected value would become $473.60 this is of course if betting were to continue after winning $80.

Question 3

In experiment 1 the upper and lower standard deviation lines have very high variation and do not stabilize throughout the graph until they reach convergence at the $80 limit set in the simulator. This occurs because the probability of having positive or negative spikes in total winnings increases not only as the number of episodes in the experiment increases but also because the amount of money is infinite. The standard deviation lines do converge abruptly at the end at $80 since this is the limit set in the simulator; at that point, the standard deviation is 0 for all episodes since $80 continues to be populated in the NumPy array. However, they do not converge only because the number of sequential bets increases.

Question 4

In experiment 2 since the bankroll is limited to $256, the probability of winning $80 becomes significantly less than when having an infinite bankroll. To get a more robust probability of winning $80 within 1000 consecutive bets given the finite bankroll, rather than using 1,000 episodes to compute the probability, the simulator was run for 1,000,000 episodes to compute more accurately how many times $80 in winnings was reached out of all episodes. It was found that $80 in winnings was reached in 642,938 episodes out of the 1,000,000 which gives a probability of 642938/1000000 or .643. This makes sense as when having a limit in capital, the chances are significantly lower to reach a specified amount in winnings.

Question 5

Given the probability from question 4, this can be seen as the following:

$$E[X] = 80(.643) + (-256(1 - .643))$$

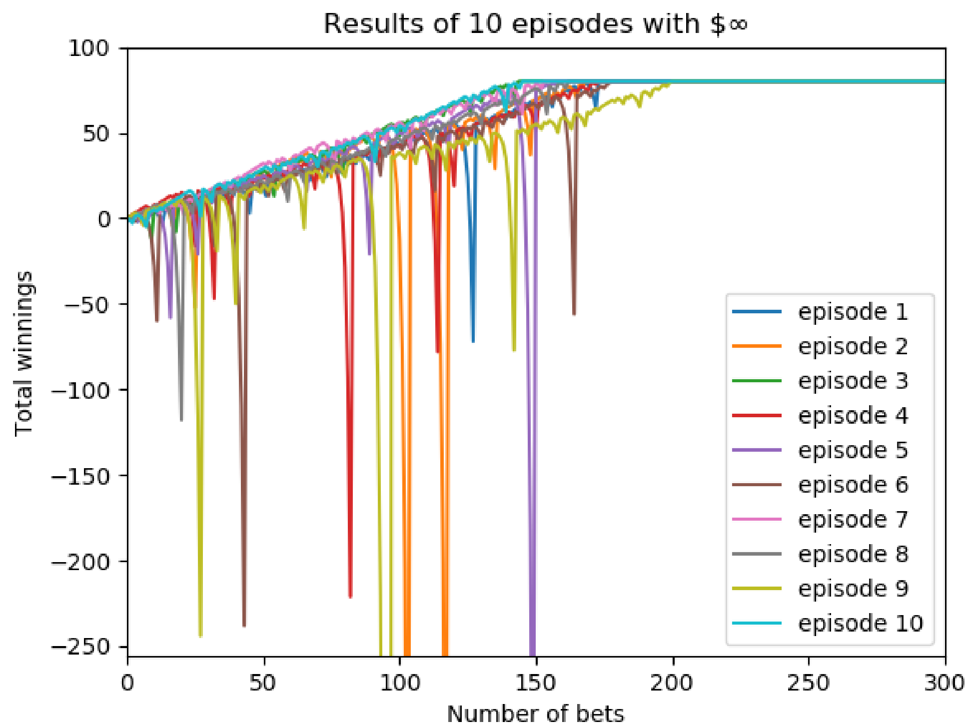
Where the value x_1 (\$80) is multiplied by its probability of occurring (.643) + the value x_2 (-\$256) is multiplied by its probability of occurring (1-.643). The resulting answer is -\$39.952 after 1000 sequential bets.

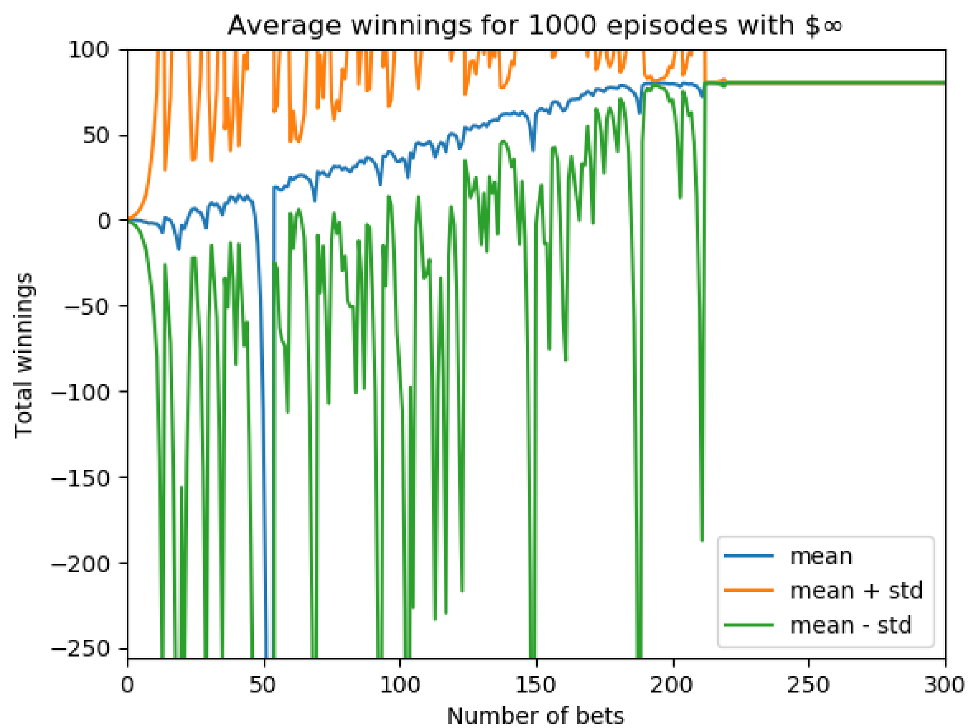
Question 6

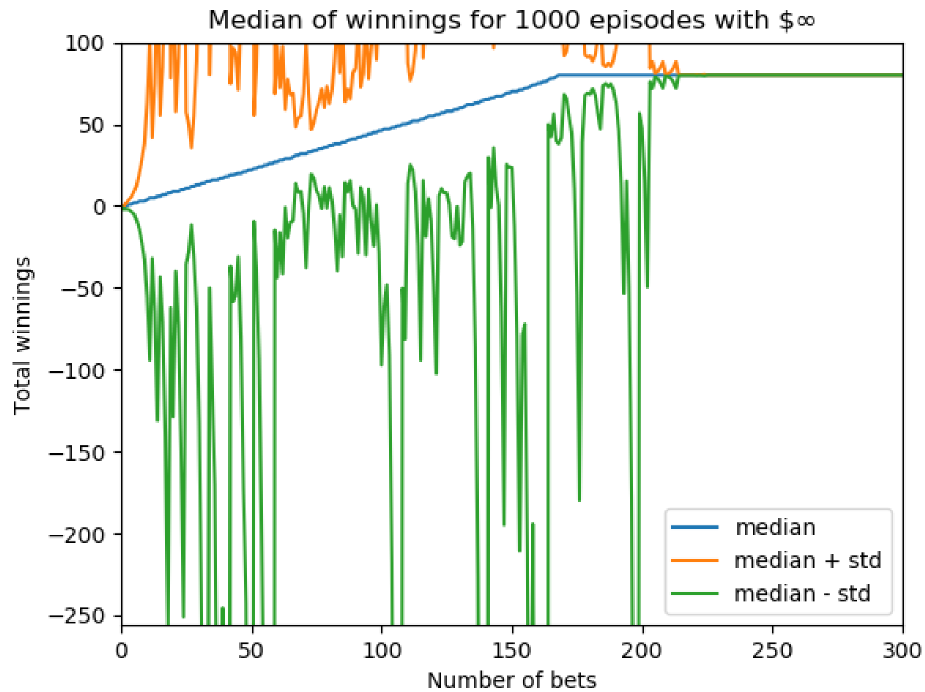
In experiment 2 the upper and lower standard deviation lines have very low variation and do stabilize throughout the graph. This occurs because the probability of having positive or negative spikes in total winnings decreases not only as the number of episodes in the experiment increases but also because the amount of money is finite. The standard deviation lines become parallel with the mean line when the \$80 limit is reached for the average total winnings. They do not converge or meet together at the same point as in experiment 1 because of the probability or potential of winnings to be -\$256 given the finite bankroll.

Question 7

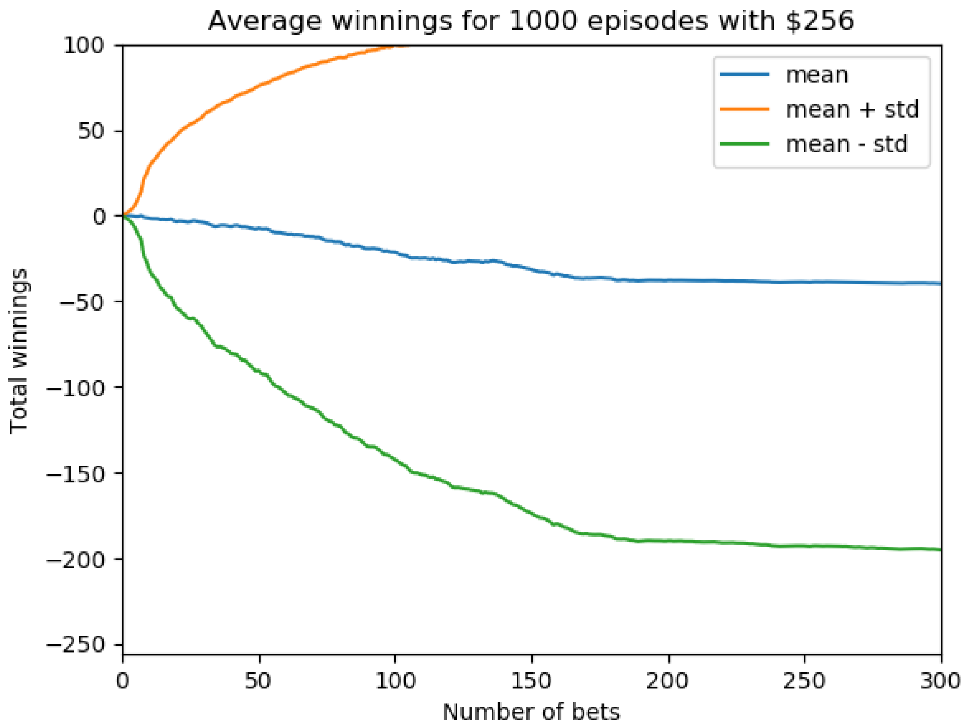
Using expected values is beneficial as it provides an average that accounts for the probability of an outcome occurring. In this case, its usage gives an insightful way to decide whether there is a good reason to utilize this gambling methodology. Rather than using one specific random episode which only accounts for one potential possibility, the expected value takes every possibility of outcome into account.

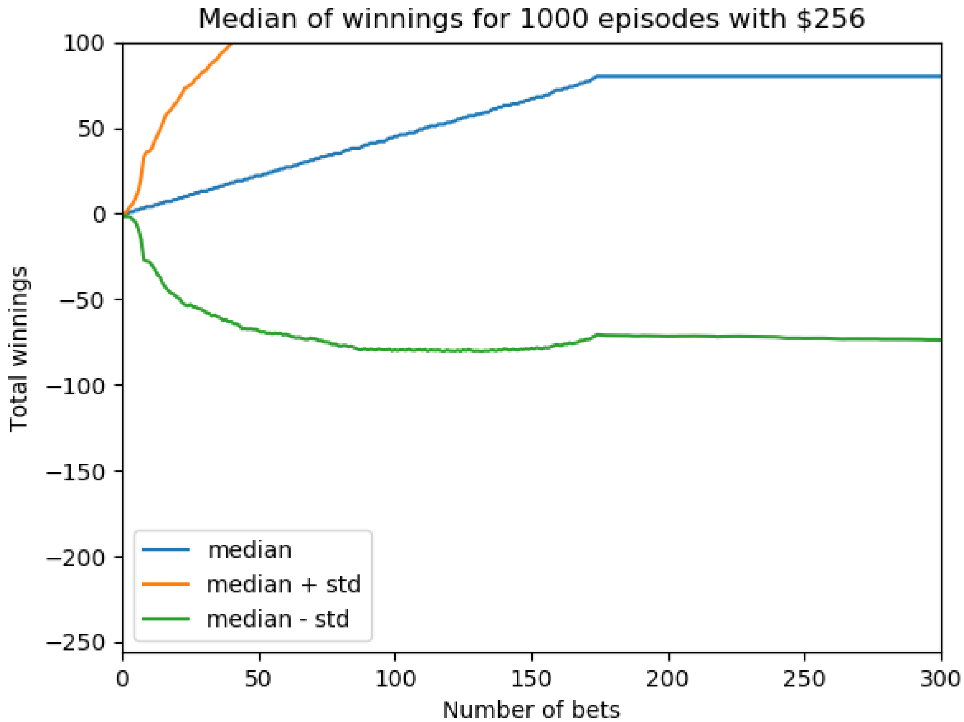

Results of 10 episodes with $∞
Total winnings
Number of bets
episode 1
episode 2
episode 3
episode 4
episode 5
episode 6
episode 7
episode 8
episode 9
episode 10
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300


Average winnings for 1000 episodes with $∞
Total winnings
Number of bets
mean
mean + std
mean - std
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300



Median of winnings for 1000 episodes with $∞
Total winnings
Number of bets
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300
median
median + std
median - std


Average winnings for 1000 episodes with $256
mean
mean + std
mean - std
Total winnings
Number of bets
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300


Median of winnings for 1000 episodes with $256
Total winnings
Number of bets
median
median + std
median - std
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300