

Lab1

Linear Programming Solver

Team Members

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Linear Programming Solver

Class Constructor

`__init__` Method

The constructor initializes the solver with:

- `objective`: The coefficients of the objective function.
- `constraints`: The coefficients of constraint equations.
- `rhs`: The right-hand side values of constraints.
- `constraint_types`: The types of constraints (`<=`, `>=`, `=`).
- `var_restrictions`: Restrictions on variables.
- `method`: The solving method (`simplex`, `big-m`, `two-phase`, etc.).
- `type`: The optimization type (`max` or `min`).

The objective function is negated if the problem type is minimization.

Solving Methods

`solve` Method

This method determines which solving algorithm to apply based on the `method` attribute.

Simplex Solver

The Simplex Solver is designed to solve linear programming problems using the Simplex Method. The key components of this implementation are:

1. Initialization

The solver takes as input:

- A matrix of constraint coefficients.
- A vector of constraint right-hand side (RHS) values.
- A vector of objective function coefficients.
- A binary vector indicating unrestricted variables.
- A boolean flag indicating whether the problem is a maximization or minimization problem.

These inputs are converted into a structured format suitable for the Simplex algorithm.

2. Handling Slack and Unrestricted Variables

- Slack variables are added to convert inequalities into equalities.
- Unrestricted variables are split into the difference of two non-negative variables.

3. Simplex Algorithm Execution

- Identify the entering variable based on the most negative coefficient in the objective function.
- Compute the minimum ratio test to determine the leaving variable.
- Perform row operations to update the tableau.
- Repeat the process until an optimal solution is reached or an unbounded solution is detected.

big_m_method

The Big M Method incorporates artificial variables into the objective function with a large penalty coefficient (M):

- Artificial variables are added for equality ($=$) and \geq constraints.
- The objective function is modified by penalizing artificial variables with a large negative or positive M.
- The Simplex algorithm proceeds normally, ensuring artificial variables leave the basis as soon as possible.

If artificial variables remain in the final solution, the problem is infeasible.

two_phase_method

The Two-Phase Method avoids the use of a large M by solving the problem in two steps:

Phase 1: Feasibility Check

- A temporary objective function is defined as the sum of all artificial variables.
- The Simplex algorithm is applied to minimize this function.
- If the optimal value is nonzero, the original problem is infeasible.

Phase 2: Standard Simplex Execution

- The original objective function is restored, and artificial variables are removed.
- The Simplex method continues to find the optimal solution.

Preemptive Goal Programming

Preemptive Goal Programming is an extension of linear programming that prioritizes multiple goals by minimizing deviations from their target values in a hierarchical manner.

1. Initialization

The solver initializes with:

- A matrix of goal coefficients and RHS values.
- Constraint coefficients and values.
- A binary vector for unrestricted variables.
- Goal directions (\geq , \leq , or $=$).

2. Constructing the Initial Tableau

- Deviation variables (positive and negative) are introduced to account for goal violations.
- Structural constraints are added as part of the standard linear programming model.

3. Handling Unrestricted Variables

Similar to the Simplex Solver, unrestricted variables are represented as the difference between two non-negative variables.

4. Objective Function Setup

Each priority level has a separate objective function that minimizes deviations from its corresponding goal. The objectives are processed sequentially, ensuring higher-priority goals are optimized first.

5. Solution Process

- The tableau is updated iteratively.
- Basic variables are adjusted based on pivoting operations.
- The final solution provides the optimized values for the decision variables and deviation measures.

ScreenShots from application

A screenshot of a web application titled "Linear Programming Calculator". The interface is divided into two tabs: "Linear Programming" (active) and "Goal Programming". The "Linear Programming" tab contains the following sections:

- Objective Function:** A text input field with the placeholder "Enter objective function (e.g., $3x_1 + 2x_2$)" and a dropdown menu set to "Maximize".
- Constraints:** A text input field with the placeholder "Constraint 1 (e.g., $2x_1 + 3x_2 \leq 10$)" and an "Add Constraint" button.
- Variable Types:** A section header with no visible input fields.
- Method Selection:** A dropdown menu set to "Big M Method".
- Solve:** A large blue button at the bottom.

A screenshot of a web application titled "Goal Programming Calculator". The interface is divided into two tabs: "Linear Programming" and "Goal Programming" (active). The "Goal Programming" tab contains the following sections:

- Constraints:** A text input field with the placeholder "Constraint 1 (e.g., $2x_1 + 3x_2 \leq 10$)" and an "Add Constraint" button.
- Goals:** A section header with the subtext "Enter goals due to their priority". Below it is a text input field with the placeholder "Goal 1 (e.g., $2x_1 + 3x_2 \leq 10$)" and an "Add Goal" button.
- Solve:** A large blue button at the bottom.

Linear Programming

Goal Programming

Linear Programming Calculator

Objective Function

Maximize

Constraints

✖

✖

Add Constraint

Variable Types

x1

Non-negative

x2

Non-negative

x3

Non-negative

Method Selection

Big M Method

Solve

Optimal Solution

x1	x2	x3	z
6.43	0.57	0.00	7.57

Big m steps

Steps

Step 1

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	-1.00	-2.00	-1.00	0.00	100.00	100.00	0.00
A1	1.00	1.00	1.00	0.00	1.00	0.00	7.00
A2	2.00	-5.00	1.00	-1.00	0.00	1.00	10.00

Step 2

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	-301.00	398.00	-201.00	100.00	0.00	0.00	-1700.00
A1	1.00	1.00	1.00	0.00	1.00	0.00	7.00
A2	2.00	-5.00	1.00	-1.00	0.00	1.00	10.00

Step 3

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	-354.50	-50.50	-50.50	0.00	150.50	-195.00
A1	0.00	3.50	0.50	0.50	1.00	-0.50	2.00
x1	1.00	-2.50	0.50	-0.50	0.00	0.50	5.00

Step 4

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	0.00	0.14	0.14	101.29	99.86	7.57
x2	0.00	1.00	0.14	0.14	0.29	-0.14	0.57
x1	1.00	0.00	0.86	-0.14	0.71	0.14	6.43

Two-phase steps

New Tab x React App x +

localhost:3000

Open in app

	x1	x2	x3	z
	6.43	0.57	0.00	7.57

Steps

Step 1

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	0.00	0.00	0.00	1.00	1.00	0.00
A1	1.00	1.00	1.00	0.00	1.00	0.00	7.00
A2	2.00	-5.00	1.00	-1.00	0.00	1.00	10.00

Step 2

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	-3.00	4.00	-2.00	1.00	0.00	0.00	-17.00
A1	1.00	1.00	1.00	0.00	1.00	0.00	7.00
A2	2.00	-5.00	1.00	-1.00	0.00	1.00	10.00

Step 3

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	-3.50	-0.50	-0.50	0.00	1.50	-2.00
A1	0.00	3.50	0.50	0.50	1.00	-0.50	2.00
x1	1.00	-2.50	0.50	-0.50	0.00	0.50	5.00

Step 4

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	0.00	0.00	0.00	1.00	1.00	0.00
x2	0.00	1.00	0.14	0.14	0.29	-0.14	0.57

New Tab x React App x +

localhost:3000

Open in app

A1	0.00	3.50	0.50	0.50	1.00	-0.50	2.00
x1	1.00	-2.50	0.50	-0.50	0.00	0.50	5.00

Step 4

Basic	x1	x2	x3	s1	A1	A2	RHS
Z	0.00	0.00	0.00	0.00	1.00	1.00	0.00
x2	0.00	1.00	0.14	0.14	0.29	-0.14	0.57
x1	1.00	0.00	0.86	-0.14	0.71	0.14	6.43

Step 5

Basic	x1	x2	x3	s1	RHS
Z	-1.00	-2.00	-1.00	0.00	0.00
x2	0.00	1.00	0.14	0.14	0.57
x1	1.00	0.00	0.86	-0.14	6.43

Step 6

Basic	x1	x2	x3	s1	RHS
Z	-1.00	0.00	-0.71	0.29	1.14
x2	0.00	1.00	0.14	0.14	0.57
x1	1.00	0.00	0.86	-0.14	6.43

Step 7

Basic	x1	x2	x3	s1	RHS
Z	0.00	0.00	0.14	0.14	7.57
x2	0.00	1.00	0.14	0.14	0.57
x1	1.00	0.00	0.86	-0.14	6.43

Goal programming

Linear Programming

Goal Programming

Goal Programming Calculator

Constraints

1500x1+3000x2<=15000

Add Constraint

Goals

Enter goals due to their priority

100x1+400x2>=1200

200x1>=1000

250x2>=800

Add Goal

Solve

Optimal Solution

x1	x2
5.00	2.50

Optimal Solution

	x1	x2
	5.00	2.50

Steps

Step 1

	basic	x1	x2	S0+	S1+	S2+	S0-	S1-	S2-	S3	RHS
Z0	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
Z1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
Z2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00
S0+	100.00	400.00	-1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1200.00
S1+	200.00	0.00	0.00	-1.00	0.00	0.00	1.00	0.00	0.00	0.00	1000.00
S2+	0.00	0.00	250.00	0.00	0.00	-1.00	0.00	0.00	1.00	0.00	800.00
S3	1500.00	3000.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	15000.00

Step 2

	basic	x1	x2	S0+	S1+	S2+	S0-	S1-	S2-	S3	RHS
Z0	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
Z1	200.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	1000.00
Z2	-62.50	0.00	0.63	0.00	-1.00	-0.63	0.00	0.00	0.00	0.00	50.00
x2	0.25	1.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00
S1+	200.00	0.00	0.00	-1.00	0.00	0.00	1.00	0.00	0.00	0.00	1000.00
S2-	-62.50	0.00	0.63	0.00	-1.00	-0.63	0.00	1.00	0.00	0.00	50.00
S3	750.00	0.00	750	0.00	0.00	-750	0.00	0.00	1.00	0.00	6000.00

Step 3

	basic	x1	x2	S0+	S1+	S2+	S0-	S1-	S2-	S3	RHS
Z0	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
Z1	200.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	1000.00
Z2	-62.50	0.00	0.63	0.00	-1.00	-0.63	0.00	0.00	0.00	0.00	50.00
x2	0.25	1.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00
S1+	200.00	0.00	0.00	-1.00	0.00	0.00	1.00	0.00	0.00	0.00	1000.00
S2-	-62.50	0.00	0.63	0.00	-1.00	-0.63	0.00	1.00	0.00	0.00	50.00
S3	750.00	0.00	750	0.00	0.00	-750	0.00	0.00	1.00	0.00	6000.00

Step 3

	basic	x1	x2	S0+	S1+	S2+	S0-	S1-	S2-	S3	RHS
Z0	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
Z1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
Z2	0.00	0.00	0.63	-0.31	-1.00	-0.63	0.31	0.00	0.00	0.00	362.50
x2	0.00	1.00	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.00	1.75
x1	1.00	0.00	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	5.00
S2-	0.00	0.00	0.63	-0.31	-1.00	-0.63	0.31	1.00	0.00	0.00	362.50
S3	0.00	0.00	750	3.75	0.00	-750	-3.75	0.00	1.00	0.00	2250.00

Step 4

	basic	x1	x2	S0+	S1+	S2+	S0-	S1-	S2-	S3	RHS
Z0	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
Z1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
Z2	0.00	0.00	0.00	-0.63	-1.00	0.00	0.63	0.00	-0.08	0.00	175.00
x2	0.00	1.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.00	2.50
x1	1.00	0.00	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	5.00
S2-	0.00	0.00	0.00	-0.63	-1.00	0.00	0.63	1.00	-0.08	0.00	175.00
S0+	0.00	0.00	1.00	0.50	0.00	-1.00	-0.50	0.00	0.13	0.00	300.00