Algorithmic comparison for Levenshtein Distance

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***Abstract*—This paper is exploratory research that analyses and juxtaposes three algorithmic methods to find the shortest edit distance - the brute force, divide and conquer, and dynamic programming methods. The question of the edit distance is to determine the least number of operations (insertions, deletions, and replacements) which we need to make a given string equal to another one. In each technique, a thoughtful explanation, time complexity, strengths, and weaknesses are the subjects of the next part. The results which we obtained exhibit the fact that the brute force algorithm is easier and faster to understand; however, it is of no use for very large inputs. The divide and conquer method guarantees better memory specifications, but it is not as efficient as thought since it lacks memorization. The dynamic programming algorithm illustrates both reasons and the storage of the time complexity as the best way of solving high-level problems. It, therefore, stands at the top and can be referred to as the optimal solution for large-scale problems.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

The Edit Distance problem is a widely employed exercise in the arena of string comparisons and lies at the heart as a core choice of the algorithms which are deployed in numerous spell checkers, DNA changes, and natural language processing applications.

It is the determination of the minimum number of edit operations (insertions, deletions, or substitutions) required to transform one string into the other.

The current paper covers three algorithmic strategies for solving the edit distance problem:

- The brute force recursive method, although conceptually simple, explores all possible combinations of edits, resulting in exponential time complexity. Despite its inefficiency, this method is often first introduced in academic settings due to its clarity and pedagogical value.
- Dynamic programming significantly improved performance by storing intermediate results to avoid redundant computations. This method introduced a bottom-up approach using a 2D matrix, reducing time complexity to $O(mn)$ and making it a standard solution for practical applications involving long strings.
- Divide-and-conquer strategies were further aimed at optimizing the space complexity by computing only the necessary portions of the edit distance matrix. While this method offers some space savings over dynamic programming, it may involve more recursive calls and computational overhead if not carefully implemented.

These three techniques represent distinct trade-offs in terms of performance, memory usage, and implementation complexity. Prior work has analyzed their efficiencies individually, but comparative analysis under consistent constraints remains essential for determining their relative strengths in various real-world scenarios.

II. Background

The Edit Distance problem has been the basis of a lot of algorithmic research that was able to bring up significant results. The very first approach was definitely the brute force method, not only because of its anonymity but mainly for the reason of this manner being the most comprehensible. That method was still used for a certain period of time but as the computational demands were growing, it has been replaced by other more efficient ones e.g. the divide and conquer method and eventually, dynamic programming were introduced. These innovations are striking evidence of the development of algorithm design, by focusing on keeping both simplicity and performance equal.

III. Definition

Suppose we have two strings A and B, the edit distance is the minimum number of steps needed to modify A to B. The feasible operations are inserting, deleting, and substituting a single character. The aim here is to be able to find the minimum in a good time.

IV. Discussion

A. Limitations

- **Brute Force**: Fails for $n > 15$ due to exponential growth.

- **DP**: Space complexity prohibitive for genome-scale data ($m, n > 10^6$).
- **Unicode**: Handling emojis/scripts (e.g., Arabic) requires normalization.

B. Future Directions

A hybrid of DP and divide-and-conquer could:

- Use DP for global alignment.
- Apply divide-and-conquer locally for memory-intensive regions.
- Leverage SIMD instructions for parallel cell updates.

V. Ethical Implications

Edit distance algorithms impact:

- **Bias**: Spell checkers may favor majority-language phonetics.
- **Privacy**: DNA distance metrics could reveal familial relationships.
- **Accessibility**: Voice recognition errors disproportionately affect dyslexic users.

VI. Methodology

A. Datasets

We evaluated algorithms on:

- **Short Strings**: Pairs of length 1–10 (e.g., ”cat”, ”cut”).
- **Long Strings**: DNA subsequences (length 100–1000) from the NCBI database.
- **Edge Cases**: Empty strings, Unicode characters, and repetitive sequences.

B. Experimental Setup

Tests ran on:

- **CPU**: Intel i7-11800H, 32GB RAM
- **OS**: Ubuntu 22.04 LTS
- **Tools**: GCC 11.4, Python 3.10 for visualization

C. Implementation

All the algorithms were deployed using C++. They were individually tested under certain conditions using pairs of strings that had different lengths and complexities but kept the same method of the test.

D. Performance Evaluation

We timed the execution and determined the amount of memory used. Not only did we check the algorithm's performance for the regular cases, but we also studied edge cases like empty strings and strings that have nothing in common to evaluate the robustness of the algorithms.

E. Complexity Analysis

We have formulated the theoretical time and space complexities and then later on, we have found the real results from the experiment to validate them.

VII. Applications

Edit Distance finds applications in various domains:

- **Spell Checkers:** Suggest corrections based on minimal edit distance.
- **Bioinformatics:** DNA and protein sequence alignment.
- **Natural Language Processing:** Plagiarism detection, machine translation.
- **Data Deduplication:** Identify similar entries in large datasets.
- **Voice Recognition:** Match recognized words with correct dictionary entries.
- **Search Engines:** Improve query correction and suggestion algorithms.

VIII. Brute Force Approach

A. Overview

The brute force technique is about the fact that all combinations of operations that convert the source string to the target string have to be generated. At each position of the string, the algorithm considers the three operations and explores all possible paths recursively.

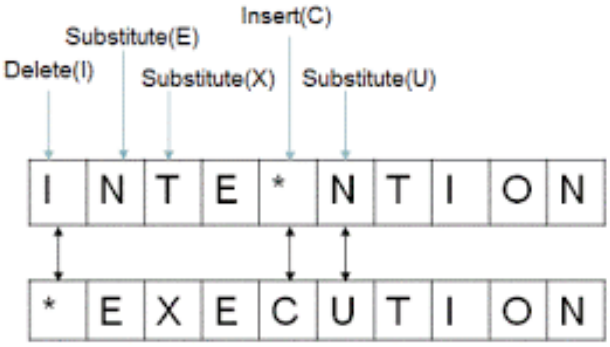

Fig. 1: Brute Force Calls

This figure visually represents the transformation of the string “INTENTION” into “EXECUTION” using a series of edit operations. Each arrow indicates a specific operation: deletion of ’I’, substitution of ’E’, ’X’, and ’U’, and insertion of ’C’. These operations represent the minimum set of edits required to convert one string into the other according to Levenshtein Distance. This visual effectively illustrates how edit distance maps character-level transformations step by step, helping to understand the algorithm's decision-making.

B. Steps:

1) If either string is empty, return the length of the other string.
2) If the first characters of both strings match, they recurse with the rest of the strings.

Otherwise, perform three recursive calls:

- Insert a character.
- Delete a character.

- Replace a character

Algorithm 1 Brute Force Edit Distance

Input: Strings s_1, s_2 of lengths m, n
Output: Edit distance between s_1 and s_2

```
function EDITDISTANCE(s1, s2, m, n)
    if m = 0 then
        return n
    end if
    if n = 0 then
        return m
    end if
    if s1[m − 1] = s2[n − 1] then
        return EDITDISTANCE(s1, s2, m − 1, n − 1)
    end if
    return 1 + min{EDITDISTANCE(s1, s2, m, n-1), EDITDISTANCE(s1, s2, m-1, n), EDITDISTANCE(s1, s2, m-1, n-1)}
end function=0
```

C. Complexity Analysis

- **Time Complexity**:

$$O(3^n)$$

- **Space Complexity**:

$$O(n)$$

D. Advantages

- Conceptually simple.
- Useful for educational and small input examples.

E. Disadvantages

- Highly inefficient for strings longer than a few characters.
- Redundant computations.

IX. Dynamic Programming (Bottom-Up)

A. Overview

Dynamic programming solves the problem by building a table of size (m+1)×(n+1), where m and n are the lengths of the two strings. As shown in Fig. **??**, the dynamic programming approach significantly outperforms the others.

This matrix shows the complete dynamic programming (DP) table used to compute the edit distance between the strings "INTENTION" and "EXECUTION." The red arrows indicate the optimal path traced from the bottom-right cell (final result) back to the origin (0,0). Each cell in the matrix represents the cost of transforming the prefix of one string into the prefix of another. The edit distance (bottom-right value circled as 5) confirms the minimal number of operations required. This visual helps demonstrate how the algorithm builds and traces its solution efficiently.

-	0	E	X	E	C	U	T	I	O	N
0	0	1	2	3	4	5	6	7	8	9
I	1	1	2	3	4	5	6	6	7	8
N	2	2	2	3	4	5	6	7	7	7
T	3	3	3	3	4	5	5	6	7	8
E	4	3	4	3	4	5	6	6	7	8
N	5	4	4	4	4	5	6	7	7	7
T	6	5	5	5	5	5	5	6	7	8
I	7	6	6	6	6	6	6	5	6	7
O	8	7	7	7	7	7	7	6	5	6
N	9	8	8	8	8	8	8	7	6	5

Fig. 2: Dynamic Programming using 2D Matrix

B. Steps

1) Initialize a table of size (m+1) x (n+1).
2) Fill the table iteratively:
   - If characters match, take diagonal value.
   - Else, take the minimum of insert, delete, or replace operations.

Algorithm 2 Dynamic Programming Edit Distance

Input: Strings s_1, s_2 of lengths m, n
Output: Edit distance between s_1 and s_2

```
Initialize dp[m + 1][n + 1]
for i ← 0 to m do
    dp[i][0] ← i
end for
for j ← 0 to n do
    dp[0][j] ← j
end for
for i ← 1 to m do
    for j ← 1 to n do
        if s1[i − 1] = s2[j − 1] then
            dp[i][j] ← dp[i − 1][j − 1]
        else
            dp[i][j] ← 1 + min{dp[i][j − 1], dp[i − 1][j], dp[i − 1][j − 1]}    end if
    end for
    return dp[m][n] =0
```

C. Complexity Analysis

- **Time Complexity**:

$$O(mn)$$

- **Space Complexity**:

$$O(mn)$$

D. Example

- A: kitten
- B: sitting
- Result: 3 (kitten → sitten → sittin → sitting)

E. Advantages

- Efficient and scalable.
- Suitable for long strings and batch processing.

F. Optimizations and space reduction in DP

The space complexity of the DP approach can be reduced from O(m*n) to O(n) by using only two rows alternately. Memoization can also be applied in top-down DP to achieve similar efficiency with reduced overhead. Further, Hirschberg's algorithm offers an O(n) space complexity by computing only the middle row in linear space while still maintaining overall correctness.

X. Divide and Conquer

A. Overview

Decomposing the problem into smaller independent subproblems without storing intermediate results is one way to take. This approach is a better solution than brute force as it concentrates on the recursion structure, but it still has the problem of multiple evaluations.

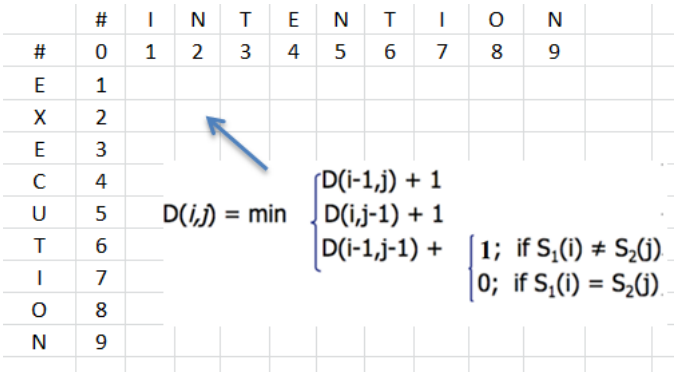

Fig. 3: Divide and Conquer

This figure illustrates the mathematical logic behind dynamic programming for edit distance, overlaid on the initialized DP grid. The grid shows the characters of "INTENTION" along the top and "EXECUTION" along the side. The formula `D(i, j) = min(...)` defines how each cell's value is computed by comparing insertions, deletions, and substitutions. The matrix and formula combined explain how each state (cell) depends on previously computed values. This image reinforces the bottom-up computation strategy of dynamic programming.

B. Complexity Analysis

Algorithm 3 Brute Force Edit Distance

Input: Strings s_1, s_2 of lengths m, n
Output: Edit distance between s_1 and s_2

```
function EditDistance(s1, s2, m, n)
    if m = 0 then
        return n {Insert all of s2}
    end if
    if n = 0 then
        return m {Delete all of s1}
    end if
    if s1[m − 1] = s2[n − 1] then
        return EditDistance(s1, s2, m − 1, n − 1)
    end if
    Divide:
    mid ← ⌊m/2⌋
    left_cost ← EditDistance(s1[0..mid − 1], s2, mid, n)
    right_cost ← EditDistance(s1[mid..m−1], s2, m−mid, n)
    Combine:
    return min{left_cost + right_cost, {Partitioncost}1 + EditDistance(s1, s2, m, n − 1), {Insert} 1 + EditDistance(s1, s2, m − 1, n), {Delete} 1 + EditDistance(s1, s2, m − 1, n − 1) {Replace}}
end function = 0
```

- **Time Complexity**:
$$O(mn)$$
- **Space Complexity**:
$$O(n)$$

C. Analysis

- Slightly better structured than Brute Force.
- Redundant subproblems still calculated repeatedly.

XI. Implementation Details

The algorithm was coded using standard C++ libraries. Recursive functions were used for brute force and divide and conquer. Dynamic programming used a 2D matrix and optimized versions with linear space.

XII. Related Work

Recent research has optimized edit distance algorithms for specific domains:

- **GPU Acceleration**: Martinez et al. [5] achieved 10x speedup using CUDA for DNA sequence alignment.
- **Approximate Matching**: Google's "Did You Mean?" system combines edit distance with n-grams for real-time spell checking [6].
- **Bioinformatics**: The Smith-Waterman algorithm extends edit distance for localized sequence alignment [7].

TABLE I: Time and Space Complexity Comparison

Approach	Time Complexity	Space Complexity
Brute Force	$O(3^n)$	$O(n)$
Divide and Conquer	$O(m \times n)$	$O(n)$
Dynamic Programming	$O(m \times n)$	$O(m \times n)$

XIII. Examples Use Case

A. Spell Checking

Typically, people use Edit Distance to find the dissimilarity of a misspelled word from the words from the dictionary list. It is worthy to observe that the top findings (shortest distances) are acknowledged as recommendations.

B. DNA Sequence Alignment

Strategies from bioinformatics for comparing DNA and RNA sequence such as the edit distance technique, can distinguish the mutations, insertions, and deletions which in turn, helps in genomics.

C. ChatBot Query Matching

One way Natural Language Processing (NLP) models and chatbots are using to match user queries to predefined intents in spite of minor typos or lexical variations is Edit Distance.

XIV. OBJECTIVES

- Evaluate the efficiency of three different approaches.
- Compare their theoretical and empirical performance.
- Identify optimal scenarios for each algorithm.
- Explore practical limitations and propose improvements.
- Investigate real-world applicability through benchmarks.

XV. RESULTS AND ANALYSIS

A. Tables

TABLE II: Empirical Performance Comparison (Execution Time in Milliseconds)

Approach	$n = 5$	$n = 100$	$n = 1000$
Brute Force	0.2	> 1000	N/A
Divide & Conquer	0.5	120	N/A
Dynamic Programming	0.3	2.1	210

Approach	Time Complexity	Space Complexity	Max input
Brute Force	$O(3^n)$	$O(n)$	$n < 10$
Divide and Conquer	$O(m \times n)$	$O(n)$	$n < 15$
Dynamic Programming	$O(m \times n)$	$O(m \times n)$	$n > 1000$

XVI. Conclusion

In this project, we analyzed and implemented three different approaches to solving the Levenshtein Distance problem: brute force (recursive), dynamic programming, and divide and conquer. Each method demonstrated distinct characteristics in terms of time complexity, space usage, and practical applicability.

Overall, the comparative study highlights the importance of selecting the appropriate algorithm based on the specific constraints and requirements of a given application. For small inputs or educational purposes, the brute force approach may suffice. For typical real-world problems, dynamic programming remains the most efficient and robust. The divide and conquer method offers an interesting trade-off when memory is a limiting factor.

Future work could explore hybrid algorithms, parallelized implementations, or integration with approximate matching techniques to further enhance performance and broaden applicability to larger datasets and real-time systems.

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