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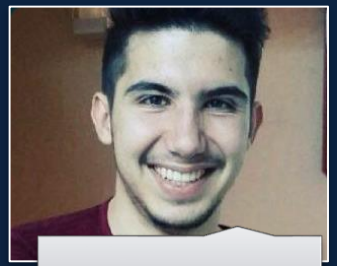
AI
PROJECT P2

Venceslas KOUASSI



Lucas BONMARIN

TOPIC

Unsupervised Learning

Nonnegative Matrix Factorization (NMF)

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PLAN

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NMF : WHAT IS IT?

- NMF (Nonnegative Matrix Factorization) is a matrix factorization method where we constrain the matrices to be nonnegative.

- Suppose we factorize a matrix X into two matrices W and H so that $X \cong WH$
- Each row in X can be considered a data point.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_k \end{bmatrix} \quad W = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_k \end{bmatrix} \quad H = \begin{bmatrix} h_1 \\ h_2 \\ \dots \\ h_k \end{bmatrix}$$

Source : [1]

NMF : WHAT IS IT?

- The i^{th} row in X, x_i, can be written as

components

$$x_i = [w_{i1}\ w_{i2}\ \dots\ w_{ik}] \times \begin{bmatrix} h_1 \\ h_2 \\ \dots \\ h_k \end{bmatrix} = \sum_{j=1}^{k} w_{ij} \times h_i$$

w_i: weights

Source : [1]

- we can interpret x_i to be a weighted sum of some components (or bases), where each row in H is a component, and each row in W contains the weights of each component.
- NMF decomposes each data point into an overlay of certain components

NMF : WHAT IS IT?

- Formalization of an objective function and iterative optimization.
- NMF is an NP-hard problem in general ⟶ local minima.

- Although there are some variants, a generally used measures of distance is the frobenius norm (the sum of element-wise squared errors).

$$minimize \ \|X - WH\|_F^2 \ w.r.t. W, H \ s.t. W, H \geq 0$$

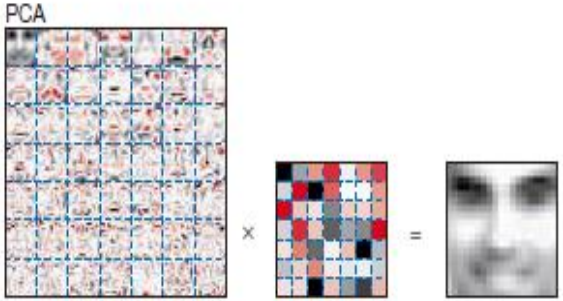
DIFFICULTIES IN NMF

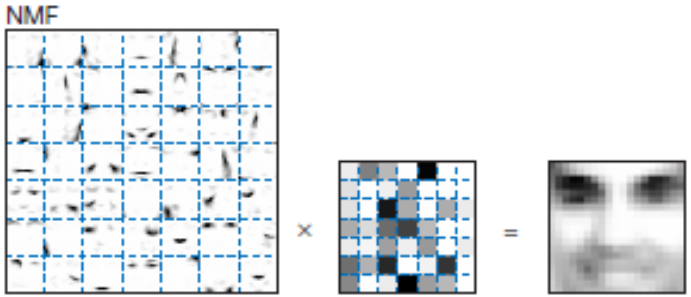
- **Choice of Suitable Model order K :** number of rank-1 matrices within the approximation.
- **Data fitting / Model complexity tradeoff :** A greater K leads to a better data approximation /A smaller K leads to a less complex model (easier to estimate, less
- parameters, etc ...)

- **Ill-posed Problem :** The solution is not unique

NMF : SOME APPLICATIONS

- **Face decomposition : Comparison with PCA**
- **PCA (Principle Components Analysis) is a factorization method that creates both positive and negative factors**





- **Topic Modeling : yield components that could be considered "topics", and decompose term-document matrix into a weighted sum of topics**

CONCLUSION

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