Binomial Distribution

Binomial distribution is the discrete probability distribution with parameters n and p. This is the basis for the popular binomial test of statistical significance. In a binomial distribution there are always two mutually exclusive events (as binomial means two).

They are termed as fail and success. It is very much clear that the sum of fail and success probabilities will be 1. This further indicates that if the probability for fail is ‘f’, then the probability for success will be 1 - f.

Formula

The formula that we commonly used for determining the distribution of binomial types is more generally known also as the formula for “Bernoulli trials” (when n = 1).

Let us assume that in an experiment done, ‘n’ is representing the number of trials attempted, and that ‘k’ is the count of successes that is to be attained in those ‘n’ trials. This implies that number of failures clearly will be ‘n - k’.

Assuming, ‘s’ to be the probability of succeeding in a trial, we get that the probability of failure is ‘1 - s’.

Then the formula for calculating the achievement of ‘k’ successes in ‘n’ trials is given below:

P (‘k’ successes in ‘n’ trials) = $C(n, k)s^k(1-s)^(n-k)$

C (n, k) is called the coefficient for binomial distribution or binomial coefficient. It is on this coefficient that the distribution is named.

The factorial of any number ‘m’ is the product of all natural numbers starting from m, (m - 1) to 1.
Hence, C (n, k) is evaluated as below:

C (n, k) = $\frac{n!}{(k!(n-k)!)}$

With n! = n * (n - 1) * … * 2 * 1

If k > $\frac{n}{2}$ then the following is applicable,

f (k, n, s) = f (n - k, n, 1 - s)

This holds true for every k > $\frac{n}{2}$.

Examples

Couple of binomial distribution problems are given below:

Example 1: A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution:

Here, n = 20, n - k = 5, k = 20 - 5 = 15

Here the probability of success = probability of giving a right answer = s = $\frac{1}{4}$

Hence, the probability of failure = probability of giving a wrong answer = 1 - s
= 1 - $\frac{1}{4}$ = $\frac{3}{4}$

When we substitute these values in the formula for Binomial distribution we get,

So, P (exactly 5 out of 20 answers incorrect) = C (20, 5) * $\left(\frac{1}{4}\right)^{15} * \left(\frac{3}{4}\right)^{5}$

$\rightarrow$ P (5 out of 20) = $\frac{(20*19*18*17*16)}{(5*4*3*2*1)} * \left(\frac{1}{4}\right)^{15} * \left(\frac{3}{4}\right)^{5}$

= 0.0000034 (approximately)

Thus the required probability is 0.0000034 approximately.

Example 2: A die marked A to E is rolled 50 times. Find the probability of getting a “D” exactly 5 times.

Solution:

Here, n = 50, k = 5, n - k = 45.

The probability of success = probability of getting a “D”= s = $\frac{1}{5}$

Hence, the probability of failure = probability of not getting a “D” = 1 - s = $\frac{4}{5}$.

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Topics in Binomial Distribution

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