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Beta Distribution

Beta binomial distribution is mostly used to model the number of success in n binomial trials, when the probability of success which is p is a B(α, β) random variable. The flexibility of the Beta distribution function represents a very fair representation of the randomness of the probability of success p. The probability of success can vary randomly, but in any one situation that probability will apply to all trials.

The general form of writing the probability density function of Beta distribution is

$$f(x) = \frac{(x-a)^{p-1}(b-x)^{q-1}}{B(p,q)(b-a)^{p-q-1}} \quad a \leq x \leq b,\ p > 0,\ q > 0.$$

where p and q define the shape parameters, a and b define the lower and upper bounds, respectively, of the distribution and $B\ (p, q)$ is the beta function.

The beta function can be expressed as

$$B(\alpha,\ \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt$$

The standard beta distribution is obtained when $a = 0$ and $b = 1$ in the general form. Hence we get $0 \leq x < 1, p > 0,\ q > 0.$

Beta Binomial Distribution

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A variable with a beta binomial is equivalent to the binomial distribution in terms of distribution with parameter p, which is the distribution with a beta distribution that has parameters α and β. Its probability density function is

$$P(x) = \frac{B(x+\alpha, n-x-\beta)\binom{n}{r}}{B(\alpha,\beta)}$$

where, $B(\alpha,\ \beta)$ is the beta function and $\binom{n}{r}$ is the binomial coefficient.

The value of a beta binomial distribution always returns a discrete value in between 0 and n. When comparing with the best fitting Binomial distribution, the Beta Binomial distribution always has more **variance** as Beta distribution helps to get some more randomness. This is the reason for selecting Beta Binomial against Beta distribution in some conditions.

For example, let us consider using the Beta-Binomial distribution in order to model:

• The death of a number of life insurance policy holders in one year, where some external variable will moderate the probability of their deaths to some degree.

Related Worksheets

Binomial Distribution Problems

Binomial Distribution Worksheet

Binomial Probability Distribution Worksheet

Distributive Property Word Problems

• In a car race of n cars, the number of cars that crash, where the important factor is, not how skillful the individual driver is, but the weather condition on the day.

• The number of juice bottles from a company which are bad, where the predominant factor is not how each bottle is treated, but something to do with the batch as a whole.

Beta Probability Distribution

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Beta distributions are a type of probability distribution that is commonly used to describe uncertainty about the true value of a proportion. The Beta distribution can be defined by the two parameters, alpha and beta with $\alpha = x + 1$ and $\beta = n - x + 1$, where x is the number of positive events out of n trials. The general formula for the probability density function of the beta distribution is

$$f(x) = \frac{(x-a)^{p-1}(b-x)^{q-1}}{B(p,q)(b-a)^{p+q-1}},\ a \le x \le b;\ p,\ q > 0$$

where p and q are the shape parameters, a and b are the lower and upper bounds, respectively, of the distribution, and $B(p, q)$ is the beta function.

Beta Distribution of the First Kind

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The continuous random variable X with probability density function

$$f(x) = \frac{1}{B(p,q)}\ x^{p-1}\ (1 - x)^{q-1} \text{ with } p > 0,\ q > 0,\ 0 \le x \le 1$$

is said to follow the Beta distribution of the first kind with parameters p and q.

where, B (p, q) is called the Beta function and it may be remembered that B(p,q) = $\frac{\Gamma p \Gamma q}{\Gamma p+q}$

where, Γ denotes the Gamma function.

Here, by calculation it can be seen that

Mean of Beta Distribution = $\frac{p}{p+q}$

Variance of Beta Distribution = $\frac{pq}{(p+q)^2(p+q+1)}$

Harmonic mean of Beta Distribution = $\frac{p-1}{p+q-1}$

Beta Distribution of the Second Kind

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The continuous random variable X with probability density function f(x) is said to follow the Beta distribution of the second kind.

$$f(x) = \frac{1}{B(p,q)}\ \frac{x^{p-1}}{(1+x)^{p+q}} \text{ with } p > 0,\ q > 0,\ 0 < x < \infty$$

After the calculation, it can be seen that

Mean of Beta Distribution = $\frac{p}{q-1}$

Variance of Beta Distribution = $\frac{p(p+q-1)}{(p-1)^2(q-2)}$

Distribution Function

In general, it has no closed form. If any of α or β is taken as a positive integer, we can obtain F(x) by using a binomial expansion, which will be a polynomial defined in x and also in general, the power of x will be positive real numbers which will range from 0 to $\alpha + \beta + 1$

Beta Distribution Example

Given below are some of the examples on Beta Distribution.

Solved Example

Question: Solve y = $\frac{x}{1-x}$, p > 0, q > 0, 0 $\leq$ x $\leq$ 1.

Solution:

Consider the function y = $\frac{x}{1-x}$

Lets assume it follows the Beta distribution of the first kind with parameter p and q

So, with p > 0, q > 0, 0 $\leq$ x $\leq$ 1

Now let g(y) be the probability density of Y

So y = $\frac{x}{1-x}$ $\Rightarrow$ x = $\frac{y}{1+y}$

Its derivative

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{(1-x)^2}$$

So g(y) = f(x) $\left|\frac{\mathrm{dx}}{\mathrm{dy}}\right|$

$$= \frac{1}{B(p,q)} \left(\frac{y}{1+y}\right)^{p-1} \left(1 - \frac{y}{1+y}\right)^{q-1} \left(1 - \frac{y}{1+y}\right)^2$$

f(x) = $\frac{1}{B(p,q)} \frac{y^{p-1}}{(1+y)^{p+q}}$ 0 $\leq$ y < ∞

This is the probability distribution function of the Beta distribution of the second kind. So Y follows Beta distribution of the second kind.

Beta Distribution Uses

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The Beta distribution is the conjugate prior which means it has the same functional form to the Binomial likelihood function in Bayesian inference and, so, it is mostly used to describe the uncertainty about a binomial probability, given that a number of n trials have been made with a number of recorded successes s. In these situations, a is set to the value (s + x) and b is set to (n - s + y), where Beta(x, y) is the prior.

The two main uses of Beta distribution are as follows:

- It can be used as the description of uncertainty or it can be taken as random variation of a probability, fraction (or prevalence).
- It can also be taken as a useful distribution which can be changed to create distributions with a wide range of shapes within any finite range.

Generalized Beta Distribution

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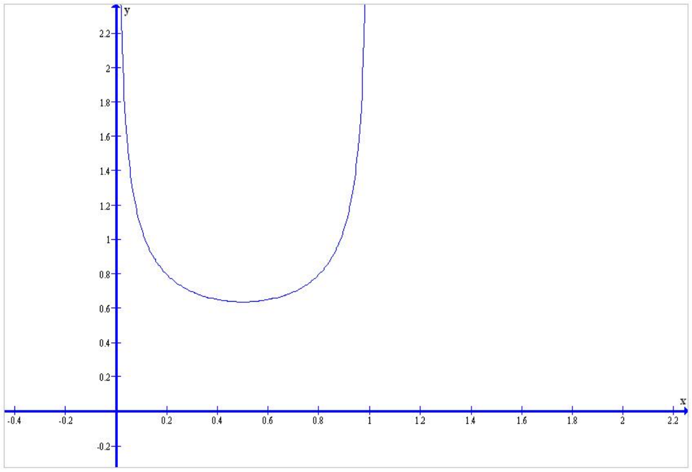
The generalized beta generated distributions are defined similar to that of the definition of the beta generated distributions. To precise the classic beta distribution which is the kernal of the beta generated distribution is replaced by generalized beta models. The pdf of generalized beta distribution is defined by

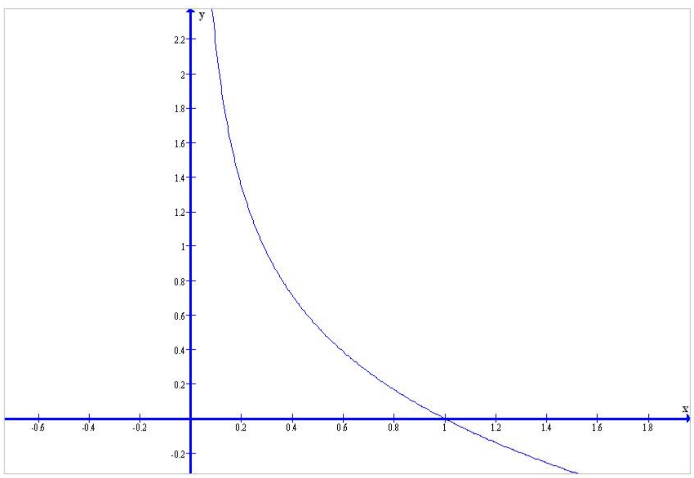
f(z) = $\frac{|a|Z^{ap-1}}{b^{ap}B(p,q)(1+Z/b)^{a})^{p+q}}$, Z $\geq$ 0.

Lets take some beta distribution example for some different values of the parameter. This can be illustrated by the **beta distribution graph**.

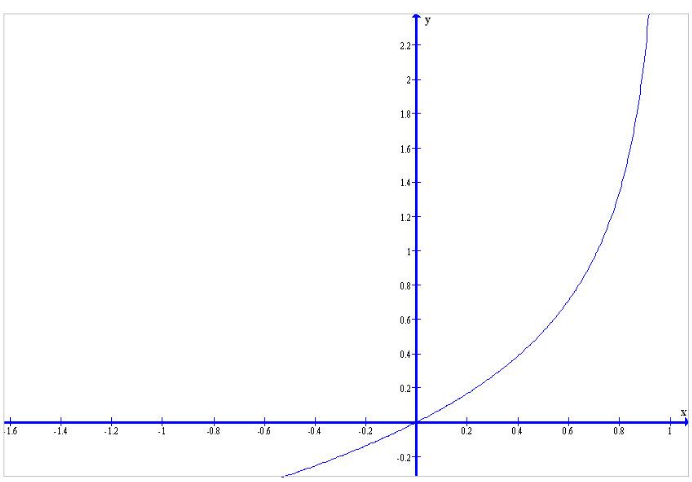
When p = 0.5 and q = 0.5, the graph looks like



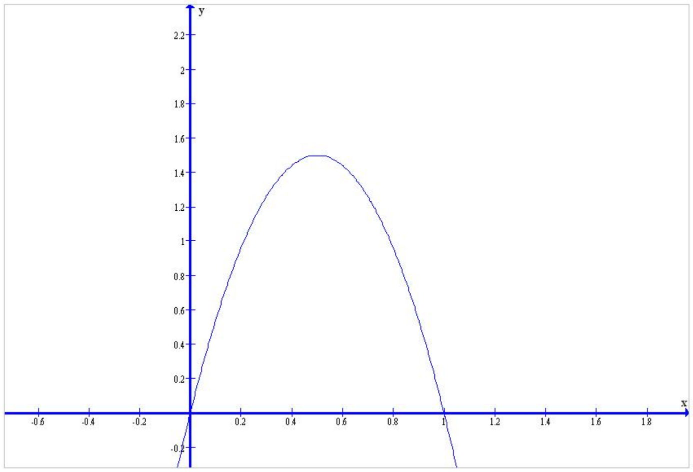
When p = 0.5 and q = 2, the graph looks like



When p = 2 and q = 0.5, the graph looks like



When p = 2 and q = 2, the graph looks like



Bivariate Beta Distribution

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In certain cases beta distribution, for a single conserved scalar, represent the actual PDF more closely than the SML when only second order moment information is considered. This indicates that the higher order moments implicit in the shape of the beta distribution may better represent the mixing process. A joint distribution with a similar property resulting in a bivarata beta distribution.

Multivariate Beta Distribution

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The **multivariate beta distributions** are able to provide appropriate stochastic models for a number of processes, especially for those that involve random proportions. Hence it may become necessary to estimate the parameters of such distributions with known parameter values.

The **Dirichlet distribution** is an example of the multivariate generalization of the beta distribution. The multivariate Beta distribution will play a particular role in the analysis of random moment sequences of matrix measures on the interval [0, 1]. This distribution can be defined by its density. Since the density depends on X only through the determinant of X, the distribution of a multivariate Beta distribution random variable X is invariant under the transformation $X \to O X O^T$ for an orthogonal matrix $O \in O(p)$

where, $O(p) = \{O \in R^{pxp} | OO^T = I_p\}$ denotes the orthogonal group.

The eigenvalues of a multivariate Beta distributed random variable follows the law of the Jacobi ensemble.

Gamma Distribution

Exponential Distribution

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