Probability Formula

Poisson Distribution

In probability theory, the Poisson distribution is a very common discrete probability distribution. A Poisson distribution helps in describing the chances of occurrence of a number of events in some given time interval or given space conditionally that the value of average number of occurrence of the event is known. This is a major and only condition of Poisson distribution.

The distribution that arises from the Poisson experiment is termed as Poisson distribution. The number of successes that are resulting from a Poisson experiment is termed as a Poisson random variable.

An experiment in statistics is termed as Poisson experiment when it possesses the following probabilities:

1) The outcomes of the experiment can be easily classified as either success or failure.

2) The average of the number of successes within a region that is specified is known.

3) The probability of occurrence of a success is always proportional to the size of the specified region.

4) The probability of occurrence of success in a very small region is zero virtually.

It is to be noted that the region that is specified can take different forms like area, length, time period etc.

Formula

The following formula can be used for computation of Poisson probability:

$$P(x, \mu) = \frac{(e^{-\mu})(\mu^x)}{x!}$$

Here, 'x' represents the actual number of occurring successes that are resulting from the Poisson experiment, the value is 'e' is 2.71828 approximately, 'μ' is the average of the number of successes that are within a specified region.
More elaborately, the following notation is useful in case of Poisson distribution:

a) 'e' : this is a constant whose value is equal to 2.71828 approximately. Basically it is the base value of the system of natural logarithm.

b) μ: it is the mean value of the number of successes that are occurring in the region specified.

c) x: it is the actual number of the successes that are occurring in the region specified.

d) P (x, μ): it is the probability or we can say the Poisson probability of the condition of occurrence of exactly 'x' number of successes in the conducted Poisson experiment, when it is given that the mean value of the number of occurring successes is μ.A Poisson distribution possesses the properties below:

1) The expected value of an distribution s the overall average of the distribution. We use notation to represents expected value is μ_x or E(x). In case of Poisson distribution, μ_x = E(x) = λ. ('μ' is equal to the mean of the distribution).

2) In Poisson distribution we see a very close relationship between the expected mean and the variance. (sometimes, 'μ' is equal to the variance of the distribution).

Examples

Let us few examples on Poisson distribution to make clear this topic more.

Example 1: A man was able to complete 3 files a day on an average. Find the probability that he can complete 5 files the next day.

Solution: Here we know this is a Poisson experiment with following values given:
μ = 3, average number of files completed a day

x = 5, the number of files required to be completed next day

And e = 2.71828 being a constant

On substituting the values in the Poisson distribution formula mentioned above we get the Poisson probability in this case.

We get,

$$P(x,\mu) = \frac{(e^{-\mu})(\mu^{x})}{x!}$$

$$\rightarrow P(5, 3) = \frac{(2.71828)^{-3}(3^{5})}{5!}$$

= 0.1008 approximately.

Hence the probability for the person to complete 5 files the next day is 0.1008 approximately.

Example 2: If customers come into a bank with variance 36/hour. Find the standard deviation of customer visit per hour using Poisson distribution.

Solution: According to a Poisson distribution, the expected value is μ_x = variance = λ = 36 customers per hour.

Now the standard deviation = $\sigma = \sqrt{\lambda}$ ($\because$ standard deviation = square root of variance)

$\sigma = \sqrt{36}$ = 6 customers per hour.

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Expected Value »

Topics in Poisson Distribution

Poisson Distribution Examples