Lesson 5.1-5.2

Quiz, 9 questions

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1.
For Questions 1-5, consider the following scenario:

You are trying to ascertain your American colleague's political preferences. To do so, you design a questionnaire with five yes/no questions relating to current issues. The questions are all worded so that a "yes" response indicates a conservative viewpoint.

Let θ be the unknown political viewpoint of your colleague, which we will assume can only take values $\theta = \text{conservative}$ or $\theta = \text{liberal}$. You have no reason to believe that your colleague leans one way or the other, so you assign the prior $P(\theta = \text{conservative}) = 0.5$.

Assume the five questions are independent and let Y count the number of "yes" responses. If your colleague is conservative, then the probability of a "yes" response on any given question is 0.8. If your colleague is liberal, the probability of a "no" response on any given question is 0.7.

- What is an appropriate likelihood for this scenario?

- $f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y}$
- $f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{liberal}\}}$
- $f(y \mid \theta) = \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{liberal}\}}$
- $f(y \mid \theta) = \binom{5}{y} 0.2^y 0.8^{5-y}$
- $f(y \mid \theta) = \theta^y e^{-\theta} / y!$

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2.
Political preferences:

- Suppose you ask your colleague the five questions and he answers "no" to all of them. What is the MLE for θ?

- $\hat{\theta} = \text{conservative}$
- $\hat{\theta} = \text{liberal}$
- None of the above. The MLE is a number.

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3.
Political preferences:

- Recall that Bayes' theorem gives $f(\theta \mid y) = \frac{f(y|\theta)f(\theta)}{\sum_\theta f(y|\theta)f(\theta)}$. What is the corresponding expression for this problem?

- ( ) $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)^2}{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}$
- ( ) $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y 0.2^{5-y}(0.2)I_{\{\theta=\text{conservative}\}}+\binom{5}{y}0.3^y 0.7^{5-y}(0.7)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^y 0.2^{5-y}(0.2)+\binom{5}{y}0.3^y 0.7^{5-y}(0.7)}$
- ( ) $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}$
- ( ) $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)I_{\{\theta=\text{conservative}\}}+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^y 0.2^{5-y}(0.5)+\binom{5}{y}0.3^y 0.7^{5-y}(0.5)}$
- ( ) $f(\theta \mid y) = \frac{\theta^y e^{-\theta}(0.5)/y!}{0.8^y e^{-.8}(0.5)/y!+0.3^y e^{-.3}(0.5)/y!}$

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4.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is conservative, given that he responded "no" to all of the questions. Round your answer to three decimal places.

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5.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is liberal, given that he responded "no" to all of the questions. Round your answer to three decimal places.

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6.
For Questions 6-9, consider again the loaded coin example from the lesson.

Recall that your brother has a fair coin which comes up heads 50% of the time and a loaded coin which comes up heads 70% of the time.

Suppose now that he has a third coin which comes up tails 70% of the time. Again, you don't know which coin your brother has brought you, so you are going to test it by flipping it 4 times, where X counts the number of heads. Let θ identify the coin so that there are three possibilities $\theta = \text{fair}$, $\theta = \text{loaded favoring heads}$, and $\theta = \text{loaded favoring tails}$.

Suppose the prior is now $P(\theta = \text{fair}) = 0.4$, $P(\theta = \text{loaded heads}) = 0.3$, and $P(\theta = \text{loaded tails}) = 0.3$. Our prior probability that the coin is loaded is still 0.6, but we do not know which loaded coin it is, so we split the probability evenly between the two options.

- What is the form of the likelihood now that we have three options?

  - $f(x \mid \theta) = \binom{4}{x} \left[0.5^4(0.4)I_{\{\theta=\text{fair}\}} + 0.3^x 0.7^{4-x}(0.3)I_{\{\theta=\text{loaded heads}\}} + 0.7^x 0.3^{4-x}(0.3)I_{\{\theta=\text{loaded tails}\}}\right]$
  - $f(x \mid \theta) = \binom{4}{x} \left[0.5^4(0.4)I_{\{\theta=\text{fair}\}} + 0.7^x 0.3^{4-x}(0.3)I_{\{\theta=\text{loaded heads}\}} + 0.3^x 0.7^{4-x}(0.3)I_{\{\theta=\text{loaded tails}\}}\right]$
  - $f(x \mid \theta) = \binom{4}{x}0.5^x 0.5^{4-x} I_{\{\theta=\text{fair}\}} + \binom{4}{x}0.3^x 0.7^{4-x} I_{\{\theta=\text{loaded heads}\}} + \binom{4}{x}0.7^x 0.3^{4-x} I_{\{\theta=\text{loaded tails}\}}$
  - $f(x \mid \theta) = \binom{4}{x}0.5^x 0.5^{4-x} I_{\{\theta=\text{fair}\}} + \binom{4}{x}0.7^x 0.3^{4-x} I_{\{\theta=\text{loaded heads}\}} + \binom{4}{x}0.3^x 0.7^{4-x} I_{\{\theta=\text{loaded tails}\}}$

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7.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the MLE for θ?

  - $\hat{\theta} = \text{fair}$
  - $\hat{\theta} = \text{loaded heads}$
  - $\hat{\theta} = \text{loaded tails}$
  - None of the above. The MLE is a number.

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8.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is the fair coin? Round your answer to two decimal places.

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9.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is a loaded coin (favoring either heads or tails)? Round your answer to two decimal places.

Hint: $P(\theta = \text{fair} \mid X = 2) = 1 - P(\theta = \text{loaded} \mid X = 2)$, so you can use your answer from the previous question rather than repeat the calculation from Bayes' theorem (both approaches yield the same answer).

Enter answer here

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