Exponential Examples

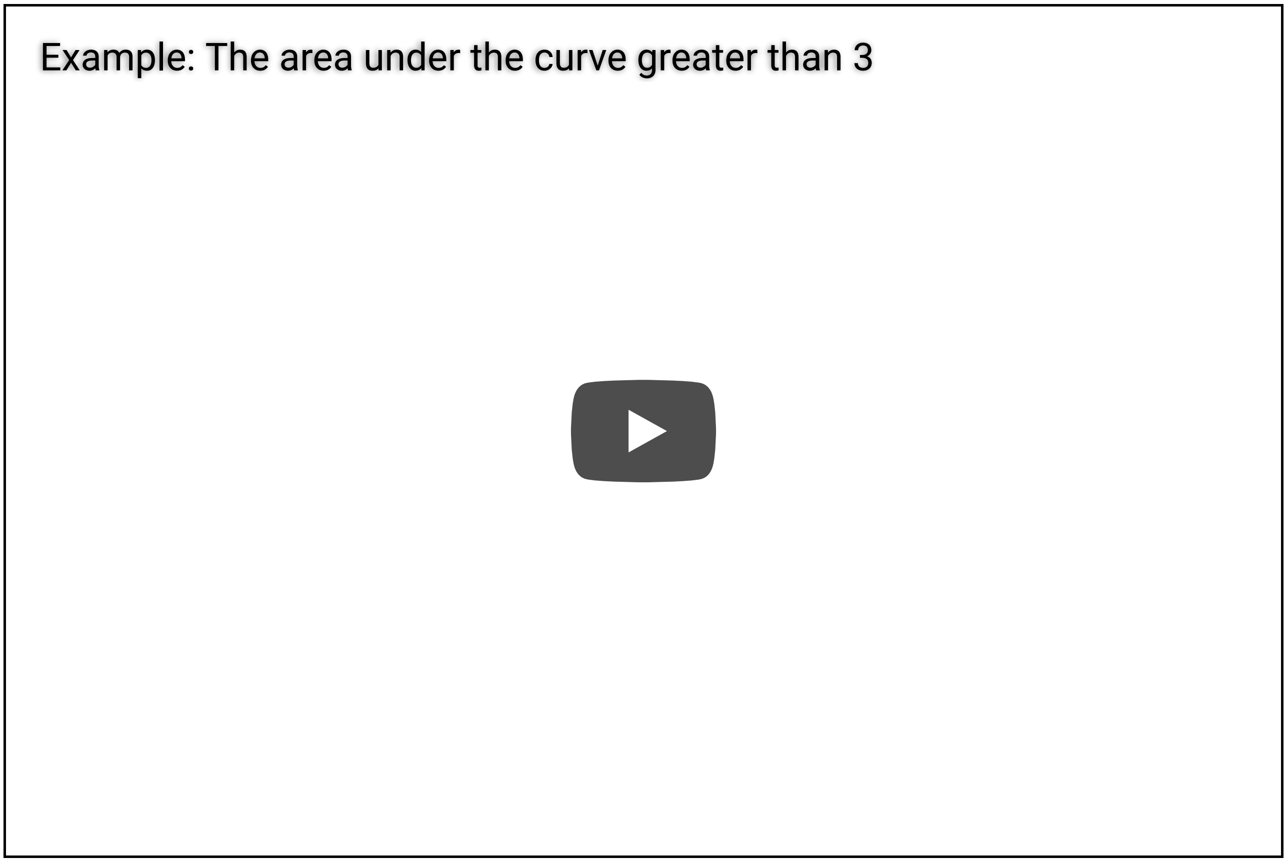
Example

Students arrive at a local bar and restaurant according to an approximate Poisson process at a mean rate of 30 students per hour. What is the probability that the bouncer has to wait more than 3 minutes to card the next student?

Solution. If we let X equal the number of students, then the Poisson mean λ is 30 students per 60 minutes, or 1/2 student per minute! Now, if we let W denote the (waiting) time between students, we can expect that there would be, on average, $\theta = 1/\lambda = 2$ minutes between arriving students. Because W is (assumed to be) exponentially distributed with mean $\theta = 2$, its probability density function is:

$$f(w) = \frac{1}{2}e^{-w/2}$$

for $w \geq 0$. Now, we just need to find the area under the curve, and greater than 3, to find the desired probability:



Example

The number of miles that a particular car can run before its battery wears out is exponentially distributed with an average of 10,000 miles. The owner of the car needs to take a 5000-mile trip. What is the probability that he will be able to complete the trip without having to replace the car battery?

Solution. At first glance, it might seem that a vital piece of information is missing. It seems that we should need to know how many miles the battery in question already has on it before we can answer the question! Hmmm.... or do we? Well, let's let X denote the number of miles that the car can run before its battery wears out. Now, suppose the following is true:

$$P(X > x + y | X > x) = P(X > y)$$

If it is true, it would tell us that the probability that the car battery wears out in more than y = 5000 miles doesn't matter if the car battery was already running for x = 0 miles or x = 1000 miles or x = 15000 miles. Now, we are given that X is exponentially distributed. It turns out that the above statement is true for the exponential distribution (you will be asked to prove it for homework)! It is for this reason that we say that the exponential distribution is "**memoryless**."

It can also be shown (do you want to show that one too?) that if X is exponentially distributed with mean θ, then:

$$P(X > k) = e^{-k/\theta}$$

Therefore, the probability in question is simply:

$$P(X > 5000) = e^{-5000/10000} = e^{-1/2} \approx 0.604$$

We'll leave it to the gentleman in question to decide whether that probability is large enough to give him comfort that he won't be stranded somewhere along a remote desert highway!