Probability Formula

Binomial Probability Formula

The binomial distribution is one of the most useful probability distribution in statistic. It is a applicable in varieties of situations. Consider we have a sequence of n identical trials, where each trial can be result in one of two possibilities, known as success and failure, the experiment is called Bernoulli trial.

If we toss a coin, outcome will be "head" or "tail". In that one we can choose as "success" and other as "failure".

Binomial Distribution

Binomial distribution is applicable to data where one of two mutually exclusive and independent outcomes are possible as a result of a single experiment. The experiment is a Bernoulli trial, if binomial distribution is applied.

A random variable X is said to follow Binomial distribution with parameters n and p (probability of success) if its probability density function is
f(r) = $^{n}C_{r}\ p^{r}\ q^{n-r}$
where r = 0, 1, 2, ...

1) p always lies between 0 and 1 (0< p<1)
2) p + q = 1 or q = 1 - p

Binomial Probability Formula

The binomial probability formula is the result of getting exactly r successes in n number of trials.

The Binomial probability formula is given as follow:

P(r success in n trials) = ${}^nC_r\ p^r\ q^{n-r}$

Where,

p = the probability of success and

q = the probability of failure (or complement of the event)

n = Total number of trials

r = number of specific events we want to obtain

Also ${}^nC_r$ represents selection of r events from n, it can be written as: ${}^nC_r = \frac{n!}{r!(n-r)!}$

Examples

Lets have a look on some examples based on Binomial Probability:

Example 1: Probability that a batsman scores a century in a cricket match is $\frac{2}{3}$. What is the probability that out of 5 matches, he may score century in:

i) exactly 2 matches

ii) none of the matches

Solution:

Here "success" is denoted by "scoring century"

Probability to scores a century in a cricket match is $\frac{2}{3}$.

Say, p = $\frac{2}{3}$

"Failure" is denoted by "not scoring century". We know that

q = 1 - p = 1 - $\frac{2}{3} = \frac{1}{3}$.

Total number if matches n = 5.

Binomial probability formula is given by ${}^nC_r\ p^r\ q^{n-r}$

i) Find the probability that he scores century in exactly two matches.

That is r = 2.

P(scoring century in exactly 2 matches)

$= {}^5C_2 \ (\frac{2}{3})^2(\frac{1}{3})^{5-2}$

$= {}^5C_2 \ (\frac{2}{3})^2(\frac{1}{3})^3$

$= 10 * (\frac{4}{9})*(\frac{1}{27})$

$= \frac{40}{243}$

P(scoring century in exactly 2 matches) = $\frac{40}{243}$

ii) We have to find the probability that he scores century in none of the matches.
That is r = 0

P(scoring century in none of the matches)

$= {}^5C_0 \ (\frac{2}{3})^0(\frac{1}{3})^{5-0}$

$= {}^5C_0 \ (\frac{2}{3})^0(\frac{1}{3})^5$

$= 1 * 1 *(\frac{1}{243})$

$= \frac{1}{243}$

P(scoring century in none of the matches) = $\frac{1}{243}$

Example 2: A basket contains 70 good apples and 30 are spoiled. Three apples are drawn at random from basket. What is the probability that of the 3 apples,

i) Exactly two good apples

ii) At least one is good

iii) At most two are good

Solution:

Here "success" is denoted by "good apples"

Given there are 70 good apples and 30 bad apples.

That is P(good apples) = p = $\frac{70}{100}$ = 0.70.

"Failure" is denoted by "getting bad apples". We know that

q = 1 - p = 1 - 0.70 = 0.30.

Number of apples selected n = 3.

According to binomial probability formula = ${}^{n}C_{r}\ p^{r}\ q^{n-r}$

i) Probability of getting exactly two good apples.

Here r = 2.

P(getting exactly 2 good apples)

$= {}^{3}C_{2}\ (0.70)^{2}(0.30)^{3-2}$

$= {}^{3}C_{2}\ (0.70)^{2}(0.30)^{1}$

= 3 * (0.49)*(0.30)

= 0.441

P(getting exactly 2 good apples) = 0.441

ii) Probability of getting at least one good apple.

That is r = 1, 2 or 3.

P(getting at least one good apple)

$= {}^{3}C_{1}\ (0.70)^{1}(0.30)^{3-1} + {}^{3}C_{2}\ (0.70)^{2}(0.30)^{3-2} + {}^{3}C_{3}\ (0.70)^{3}(0.30)^{3-3}$

$= {}^{3}C_{1}\ (0.70)^{1}(0.30)^{2} + {}^{3}C_{2}\ (0.70)^{2}(0.30)^{1} + {}^{3}C_{3}\ (0.70)^{3}(0.30)^{0}$

= 3 * (0.70)*(0.09) + 3 * (0.49)*(0.30) + 3 * (0.343)*(1)

= 0.189 + 0.441+ 0.343

P(getting at least one good apple) = 0.973

iii) Probability of getting at most two good apples.
That is r = 0, 1 or 2

P(getting at most two good apples)

$= {}^{3}C_{0}\ (0.70)^{0}(0.30)^{3-0} + {}^{3}C_{1}\ (0.70)^{1}(0.30)^{3-1} + {}^{3}C_{2}\,(0.70)^{2}(0.30)^{3-2}$

$= {}^3C_0\ (0.70)^0(0.30)^3 + {}^3C_1\ (0.70)^1(0.30)^2 + {}^3C_2\ (0.70)^2(0.30)^1$

= 1 * (1)*(0.027) + 3 * (0.70)*(0.09) + 3 * (0.49)*(0.30)

= 0.027 + 0.189 + 0.441

P(getting at most two good apples) = 0.657.

Example 3: Consider families with 4 children. What is the probability of having

i) two boys and two girls
ii) at least one boy
iii) no boys
iv) at most two boys

Solution: Consider p = "success" = "getting boys"

Probability of getting a boy and getting a girls is $\frac{1}{2}$.
i.e. p = $\frac{1}{2}$.

Also q = "Failure" = denoted by "getting girls".
=> q = 1 - p = 1 - $\frac{1}{2}$ = $\frac{1}{2}$.

Number of children n = 4.

Apply binomial probability formula, p(r success in n trials) = ${}^nC_r\ p^r\ q^{n-r}$

i) Probability of getting exactly two boys and two girls.
That is r = 2.

P(getting exactly 2 boys)

$= {}^4C_2\ (\frac{1}{2})^2(\frac{1}{2})^{4-2}$

$= {}^4C_2\ (\frac{1}{2})^2(\frac{1}{2})^2$

$= 6 * \frac{1}{16} = \frac{6}{16} = \frac{3}{8}$

P(getting exactly 2 boys and 2 girls) = $\frac{3}{8}$

ii) Probability of getting no boys.

That is r = 0.

P(getting no boys)

$= {}^4C_0\ (\frac{1}{2})^0(\frac{1}{2})^{4-0}$

$= {}^4C_0\ (\frac{1}{2})^0(\frac{1}{2})^4$

$= 1 * 1 * \frac{1}{16} = \frac{1}{16}$

P(getting no boys) = $\frac{1}{16}$

iii) Probability of getting at most two boys.

That is r = 0, 1 or 2

P(getting at most two boys)

$= {}^4C_0\ (\frac{1}{2})^0(\frac{1}{2})^{4-0} + {}^4C^1\ (\frac{1}{2})^1(\frac{1}{2})^{4-1} + {}^4C_2\ (\frac{1}{2})^2(\frac{1}{2})^{4-2}$

$= {}^4C_0\ (\frac{1}{2})^0(\frac{1}{2})^4 + {}^4C_1\ \ 1.(\frac{1}{2})^1(\frac{1}{2})^3 + {}^4C_2\ (\frac{1}{2})^2(\frac{1}{2})^2$

$= 1 * (1)*(\frac{1}{16}) + 4 * (\frac{1}{2})*(\frac{1}{8}) + 6 * (\frac{1}{4})*(\frac{1}{4})$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16}$

P(getting at most two boys) = $\frac{11}{16}$

Practice Example

Try Yourself:
1) A box contains 100 bulbs, 20 of which are defective. 10 of the bulbs are selected for inspection. Find the probability that

i) all 10 are defective
ii) all 10 are good
iii) at least one is defective
iv) at most three are defective.

2) A coin is tossed 16 times. What is the probability of obtaining
i) exactly five heads
ii) 10 or more tails
Empirical Probability Formula »

Topics in Binomial Probability Formula

Binomial Probability Examples