Exponential Distribution

Exponential distribution can be viewed as the distribution of waiting time between the occurrence of events described by a Poisson variable. If T is the time elapsed before the occurrence of a Poisson event, it can also be shown that T is the recurrence time between the occurrence of any two events. This analysis will show that the distribution of the variable T is the exponential.

Definition: If X is a continuous random variable with pdf
f(x) = $\lambda e^{-\lambda x}$ for x > 0
= 0 otherwise
then X is said to have an exponential distribution with parameter λ.

The cdf of an exponential variable is F(x) = 1- $e^{-\lambda x}$
The parameter λ of the exponential distribution represents the average number of events per unit time in the corresponding Poisson Process.

Negative exponential distribution is the other name by which exponential distributions are known.

Exponential Distribution Mean

The mean or the expected value of an exponential distribution is given in terms of the parameter λ of the distribution.

$\mu = E(X) = \frac{1}{\lambda}$

Sometimes an exponential distribution is also described using μ, the mean as the parameter as follows

$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$

Variance of Exponential Distribution

The variance of an exponential distribution is also defined in terms of the parameter λ.

$Var(X) = \frac{1}{\lambda^2}$

Standard Deviation of Exponential Distribution

The standard deviation of an exponential Distribution

$\sigma_X = \sqrt{Var(X)} = \frac{1}{\lambda}$

The mean and the standard deviation of an exponential distribution are equal

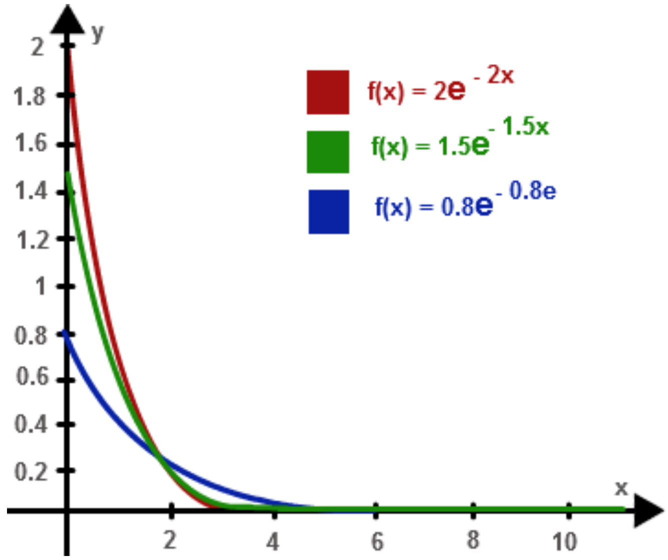
Median of Exponential Distribution

Median of an exponential distribution = $\frac{ln2}{\lambda}$

As ln 2 < 1, Median of an exponential distribution is less than its mean.

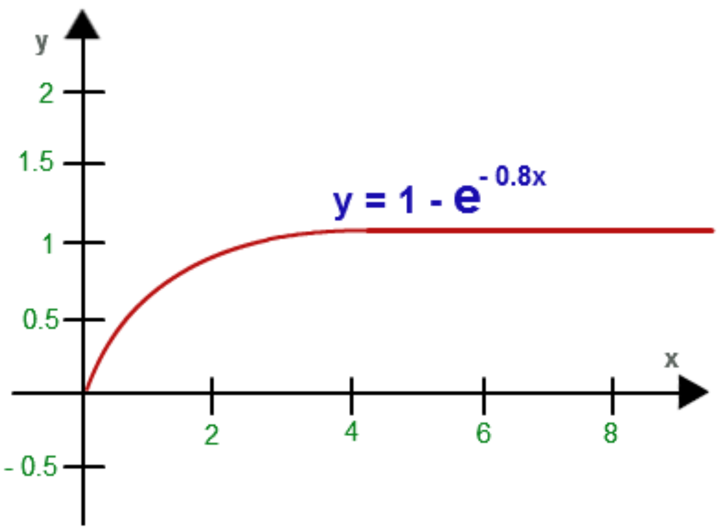
Exponential Distribution Graph

The graphs of probability distribution functions of three exponential distributions with λ = 2, 1.5 and 0.8 are given below.



The shapes of all three graphs are same resembling any exponential graph. Note the exponential function does not have shape parameter, but only a rate parameter. As the value of λ increases, the graph gets closer to the Y axis. The total area under each of the curves shown is equal to 1.

The graph of cumulative probability distribution for the distribution f(x) = $0.8e^{-0.8x}$ is given below.



The cdf function whose graph is shown above is
F(x) = $1 - e^{-0.8x}$.
F(x) has a practical utility in calculating the probabilities of an exponential function.

Exponential Distribution Properties

1. The general statistical properties of an exponential distribution X ~ exponential(λ) are Mean = Standard deviation = $\frac{1}{\lambda}$. Variance = $\frac{1}{\lambda^2}$
2. An important property of exponential distributions is the Memoryless Property. This means if we take the

distribution to represent waiting time for an event, the probability of waiting a given a length of time is not dependent on the time already waited. P(X < a + b | X

3. The sum of n independent exponential random variables all with a common parameter $\lambda > 0$ gives rise to a Gamma distribution defined by the parameters n and λ.
4. If $X_1, X_2, \ldots X_n$ are exponential random variables defined by corresponding parameters $\lambda_1, \lambda_2, \ldots \lambda_n$ then $\min(X_1, X_2, \ldots X_n)$ defines an exponential distribution with parameter $\lambda_1 + \lambda_2 + \cdots + \lambda_n$.
5. If X_1 and X_2 are independent exponential random variables with parameters λ_1 and λ_2 then the probability that the event related to X_1 occurs before the event related to X_2 has occurred is $P(X_1 < X_2) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

Double exponential Distribution

Double exponential distributions are also known as Laplace's Distribution. This can be viewed as the difference distribution of two independent exponential random variables with equal parameters. The pdf of a double exponential distribution is given by

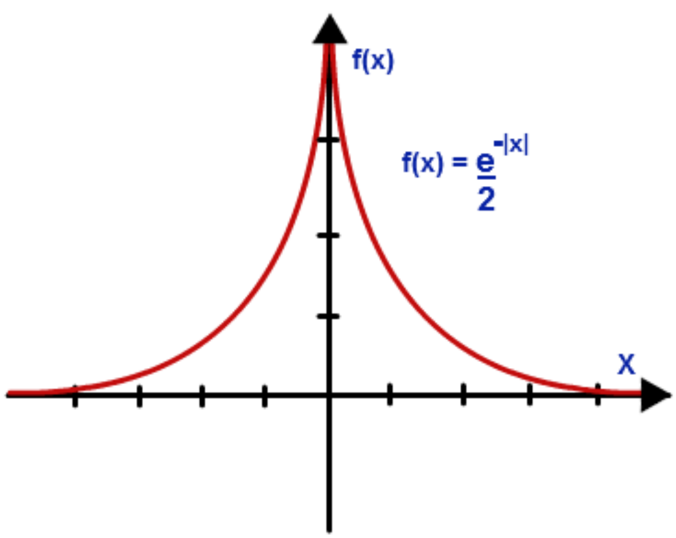
$f(x) = \frac{e^{-\frac{|x-\mu|}{\theta}}}{2\theta}$ where the parameters μ and θ describe the location and scale correspondingly.

The cdf of a double exponential distribution is given by

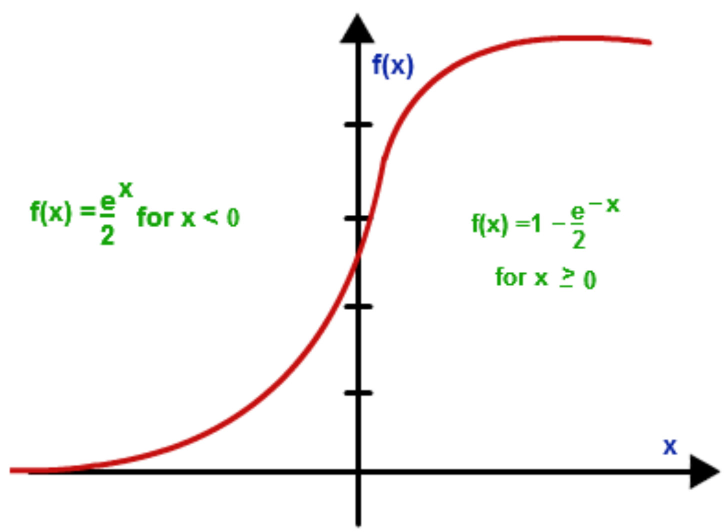
$F(x) = \frac{1}{2}[1 + \sum(x-\mu)(1 - e^{-\frac{|x-\mu|}{\theta}})]$

The graphs of the pdf and cdf of a double exponential function with $\mu = 0$ and $\theta = 1$ are given below.

Double Exponential pdf



Double Exponential cdf



Shifted Exponential Distribution

The definition of an exponential distribution with pdf, f(x) = $\lambda e^{-\lambda x}$ assumes that the distribution starts at x = 0. A shifted exponential distribution starts at x = a where a > 0. The pdf and cdf for a shifted exponential distribution are defined as
f(x) = $\lambda e^{-\lambda(x-a)}$ for x > a
= 0 othewise.

F(x) = 1 - $e^{-\lambda(x-a)}$

Truncated Exponential Distribution

A truncated exponential distribution is formed by considering 0 ≤ x ≤ b, that is restricted or truncated on the right by b.

The truncated version of the exponential distribution X ~ Exp(λ) is denoted by Y ~ TEXP(λ, b). The pdf of the truncated exponential distribution is given by,
f(y : λ) = $\lambda e^{-\lambda y(1-e^{-\lambda b})^{-1}}$ for 0 < y ≤ b
= 0 Otherwise

The mean of such a truncated exponential distribution is given by
μ = $\frac{1}{\lambda} - b(e^{\lambda b} - 1)^{-1}$

Bivariate Exponential Distribution

The development of generalized exponential distributions with an additional shape parameter has motivated the definition of bivariate exponential distributions in terms of rate as well as shape parameters.

Exponential Distribution Examples

Solved Example

Question: The customers using the Bank by Phone during peak hours for a hypothetical bank can be modeled by a Poisson Process with an average of 100 customers using the service per hour. What is the probability that no customer uses this facility between 8:00 AM to 8 : 05 AM assuming the duration falls under a peak hour.

Solution:

Let X represent the time in hours from the start of the interval till the first caller calls. Thus X has an exponential distribution
with λ = 100. We can ignore the calls received prior to 8:00 AM by virtue of Memoryless Property of exponential distributions.

We need to find the probability of X > 5 minutes or X > $\frac{1}{12}$ hr.

We can use the cdf F(x) for this purpose
F(x) = 1- $e^{-\lambda x}$ = 1 - e^{-100x}

P(X > $\frac{1}{12}$) = 1 - P(X ≤ $\frac{1}{12}$) = 1 - F($\frac{1}{12}$)

= 1 - (1 - $e^{-\frac{100}{12}}$)

= $e^{-\frac{100}{12}}$ = 0.00024

Related Concepts

Exponential Probability Distribution (//www.mathcaptain.com/statistics/probability-distribution.html)	Exponentiation (//www.mathcaptain.com/number-sense/exponents.html)	Derivative of Exponential (//www.mathcaptain.com/calculus/deriva of-e.html)
Exponential Functions (//www.mathcaptain.com/algebra/exponential-function.html)	Exponential Graph (//www.mathcaptain.com/algebra/exponential-growth.html)	Exponential Regression (//www.mathcaptain.com/statistics/regre

T Distribution Normal Distribution (//www.mathcaptain.com/statistics/t-distribution.html)

Applications of Exponential and Logarithmic Functions (//www.mathcaptain.com/algebra/exponential-and-logarithmic-functions.html)

conversion of exponential form (//www.mathcaptain.com/number-sense/exponential-form.html)

Define Exponential Decay (//www.mathcaptain.com/algebra/expon decay.html)

derivative of exponential function definition (//www.mathcaptain.com/calculus/deriva exponential-function.html)

Continuous Distributions (//www.mathcaptain.com/probability/continuous-distribution.html)

Related Calculators

Calculate Exponential (//calculator.mathcaptain.com/exponential-function-calculator.html)

Exponential Equation (//calculator.mathcaptain.com/exponential-equation-calculator.html)

Compound Interest Calculator Continuously (//calculator.mathcaptain.com/compoun interest-calculator.html)

Related Formulas

Math Formulas (//formulas.mathcaptain.com/math-formulas.html)

Physics Formulas (//formulas.mathcaptain.com/physics-formulas.html)

Chemistry Formulas (//formulas.mathcaptain.com/chemistry-formulas.html)