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[Home](#) > Exponential Properties

# Exponential Properties

Here, we present and prove four key properties of an exponential random variable.

**Theorem.** The exponential probability density function:

$$f(x) = \frac{1}{\theta} e^{-x/\theta}$$

for  $x \geq 0$  and  $\theta > 0$  is a valid probability density function.

**Proof.**

Proof: Is the exponential PDF a valid PDF?



**Theorem.** The moment generating function of an exponential random variable  $X$  with parameter  $\theta$  is:

$$M(t) = \frac{1}{1 - \theta t}$$

for  $t < 1/\theta$ .

**Proof.** The moment generating function is by definition:

$$M(t) = E(e^{tX}) = \int_0^\infty e^{tx} \left( \frac{1}{\theta} \right) e^{-x/\theta} dx$$

Simplifying and rewriting the integral as a limit, we have:

$$M(t) = \frac{1}{\theta} \lim_{b \rightarrow \infty} \int_0^b e^{x(t-1/\theta)} dx$$

Integrating, we have:

$$M(t) = \frac{1}{\theta} \lim_{b \rightarrow \infty} \left[ \frac{1}{t - 1/\theta} e^{x(t-1/\theta)} \right]_{x=0}^{x=b}$$

Evaluating at  $x = 0$  and  $x = b$ , we have:

$$M(t) = \frac{1}{\theta} \lim_{b \rightarrow \infty} \left[ \frac{1}{t - 1/\theta} e^{b(t-1/\theta)} - \frac{1}{t - 1/\theta} \right] = \frac{1}{\theta} \lim_{b \rightarrow \infty} \left\{ \left( \frac{1}{t - 1/\theta} \right) e^{b(t-1/\theta)} \right\} - \frac{1}{t - 1/\theta}$$

Now, the limit approaches 0 provided  $t - 1/\theta < 0$ , that is, provided  $t < 1/\theta$ , and so we have:

$$M(t) = \frac{1}{\theta} \left( 0 - \frac{1}{t - 1/\theta} \right)$$

Simplifying more:

$$M(t) = \frac{1}{\theta} \left( -\frac{1}{\frac{\theta t - 1}{\theta}} \right) = \frac{1}{\theta} \left( -\frac{\theta}{\theta t - 1} \right) = -\frac{1}{\theta t - 1}$$

and finally:

$$M(t) = \frac{1}{1 - \theta t}$$

provided  $t < 1/\theta$ , as was to be proved.

**Theorem.** The mean of an exponential random variable  $X$  with parameter  $\theta$  is:

$$\mu = E(X) = \theta$$

**Proof.**

Proof: The mean of an exponential random variable  $X$



**Theorem.** The variance of an exponential random variable  $X$  with parameter  $\theta$  is:

$$\sigma^2 = Var(X) = \theta^2$$

**Proof.**

Proof: The variance of an exponential random variable  $X$



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