Lesson 5.1-5.2

Quiz, 9 questions

5/9 points (55%)

Try again once you are ready.

Required to pass: 75% or higher

You can retake this quiz up to 3 times every 8 hours.

Back to Week 2

Retake

1 / 1 points

1.
For Questions 1-5, consider the following scenario:

You are trying to ascertain your American colleague's political preferences. To do so, you design a questionnaire with five yes/no questions relating to current issues. The questions are all worded so that a "yes" response indicates a conservative viewpoint.

Let θ be the unknown political viewpoint of your colleague, which we will assume can only take values $\theta = \text{conservative}$ or $\theta = \text{liberal}$. You have no reason to believe that your colleague leans one way or the other, so you assign the prior $P(\theta = \text{conservative}) = 0.5$.

Assume the five questions are independent and let Y count the number of "yes" responses. If your colleague is conservative, then the probability of a "yes" response on any given question is 0.8. If your colleague is liberal, the probability of a "no" response on any given question is 0.7.

- What is an appropriate likelihood for this scenario?

$f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y}$

$f(y \mid \theta) = \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{liberal}\}}$

Correct
If your colleague is conservative, the number of "yes" responses will follow a Binomial(5, 0.8). If liberal, the number of "yes" responses will follow a Binomial(5, 0.3).

$f(y \mid \theta) = \binom{5}{y} 0.3^y 0.7^{5-y} I_{\{\theta=\text{conservative}\}} + \binom{5}{y} 0.8^y 0.2^{5-y} I_{\{\theta=\text{liberal}\}}$

$f(y \mid \theta) = \binom{5}{y} 0.2^y 0.8^{5-y}$

$f(y \mid \theta) = \theta^y e^{-\theta} / y!$

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0 / 1 points

2.
Political preferences:

- Suppose you ask your colleague the five questions and he answers "no" to all of them. What is the MLE for θ?

$\hat{\theta} = \text{conservative}$

This should not be selected
Plug $y = 0$ into the likelihood and determine which θ returns the highest value.

Note also that if your colleague answered "no" to all conservative-leaning questions, he is likely not conservative.

$\hat{\theta} = \text{liberal}$

None of the above. The MLE is a number.

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1 / 1 points

3.
Political preferences:

- Recall that Bayes' theorem gives $f(\theta \mid y) = \frac{f(y|\theta)f(\theta)}{\sum_\theta f(y|\theta)f(\theta)}$. What is the corresponding expression for this problem?

○ $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y0.2^{5-y}(0.5)^2}{\binom{5}{y}0.8^y0.2^{5-y}(0.5)+\binom{5}{y}0.3^y0.7^{5-y}(0.5)}$

○ $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y0.2^{5-y}(0.2)I_{\{\theta=\text{conservative}\}}+\binom{5}{y}0.3^y0.7^{5-y}(0.7)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^y0.2^{5-y}(0.2)+\binom{5}{y}0.3^y0.7^{5-y}(0.7)}$

○ $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y0.2^{5-y}(0.5)+\binom{5}{y}0.3^y0.7^{5-y}(0.5)}{\binom{5}{y}0.8^y0.2^{5-y}(0.5)+\binom{5}{y}0.3^y0.7^{5-y}(0.5)}$

○ $f(\theta \mid y) = \frac{\binom{5}{y}0.8^y0.2^{5-y}(0.5)I_{\{\theta=\text{conservative}\}}+\binom{5}{y}0.3^y0.7^{5-y}(0.5)I_{\{\theta=\text{liberal}\}}}{\binom{5}{y}0.8^y0.2^{5-y}(0.5)+\binom{5}{y}0.3^y0.7^{5-y}(0.5)}$

Correct
The prior probability was 0.5 for both values of θ.

Notice we have added over all possibilities of θ to get the denominator. Hence, the denominator contains no θ and evaluates to a number when we plug in y. This is the marginal probability of observing y, which gives us the normalizing constant.

○ $f(\theta \mid y) = \frac{\theta^y e^{-\theta}(0.5)/y!}{0.8^y e^{-.8}(0.5)/y!+0.3^y e^{-.3}(0.5)/y!}$

1 / 1 points

4.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is conservative, given that he responded "no" to all of the questions. Round your answer to three decimal places.

.001

Correct Response
This is $P(\theta = \text{conservative} \mid Y = 0) = \frac{.2^5(.5)}{.2^5(.5)+.7^5(.5)} = \frac{.2^5}{.2^5+.7^5}$.

1 / 1 points

5.
Political preferences:

- Evaluate the expression in Question 3 for $y = 0$ and report the posterior probability that your colleague is liberal, given that he responded "no" to all of the questions. Round your answer to three decimal places.

.998

Correct Response
This is $P(\theta = \text{liberal} \mid Y = 0) = \frac{.7^5(.5)}{.2^5(.5)+.7^5(.5)} = \frac{.7^5}{.2^5+.7^5}$.

Note that once we have calculated the posterior probability for this hypothesis, we can easily obtain the posterior probability for the opposite (complementary) hypothesis with $P(\theta = \text{conservative} \mid Y = 0) = 1 - P(\theta = \text{liberal} \mid Y = 0)$.

0 / 1
points

6.
For Questions 6-9, consider again the loaded coin example from the lesson.

Recall that your brother has a fair coin which comes up heads 50% of the time and a loaded coin which comes up heads 70% of the time.

Suppose now that he has a third coin which comes up tails 70% of the time. Again, you don't know which coin your brother has brought you, so you are going to test it by flipping it 4 times, where X counts the number of heads. Let θ identify the coin so that there are three possibilities $\theta = \text{fair}$, $\theta = \text{loaded favoring heads}$, and $\theta = \text{loaded favoring tails}$.

Suppose the prior is now $P(\theta = \text{fair}) = 0.4$, $P(\theta = \text{loaded heads}) = 0.3$, and $P(\theta = \text{loaded tails}) = 0.3$. Our prior probability that the coin is loaded is still 0.6, but we do not know which loaded coin it is, so we split the probability evenly between the two options.

- What is the form of the likelihood now that we have three options?

- $f(x \mid \theta) = \binom{4}{x}\left[0.5^4(0.4)I_{\{\theta=\text{fair}\}} + 0.3^x 0.7^{4-x}(0.3)I_{\{\theta=\text{loaded heads}\}} + 0.7^x 0.3^{4-x}(0.3)I_{\{\theta=\text{loaded tails}\}}\right]$
- $f(x \mid \theta) = \binom{4}{x}\left[0.5^4(0.4)I_{\{\theta=\text{fair}\}} + 0.7^x 0.3^{4-x}(0.3)I_{\{\theta=\text{loaded heads}\}} + 0.3^x 0.7^{4-x}(0.3)I_{\{\theta=\text{loaded tails}\}}\right]$

This should not be selected
Notice that the likelihood has been multiplied by the prior; this is the numerator for the posterior.

- $f(x \mid \theta) = \binom{4}{x}0.5^x 0.5^{4-x}I_{\{\theta=\text{fair}\}} + \binom{4}{x}0.3^x 0.7^{4-x}I_{\{\theta=\text{loaded heads}\}} + \binom{4}{x}0.7^x 0.3^{4-x}I_{\{\theta=\text{loaded tails}\}}$
- $f(x \mid \theta) = \binom{4}{x}0.5^x 0.5^{4-x}I_{\{\theta=\text{fair}\}} + \binom{4}{x}0.7^x 0.3^{4-x}I_{\{\theta=\text{loaded heads}\}} + \binom{4}{x}0.3^x 0.7^{4-x}I_{\{\theta=\text{loaded tails}\}}$

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1 / 1
points

7.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the MLE for θ?

- $\hat{\theta} = \text{fair}$

Correct
$\binom{4}{2}0.5^4$ is the highest value among the three when $X = 2$.

- $\hat{\theta} = \text{loaded heads}$
- $\hat{\theta} = \text{loaded tails}$
- None of the above. The MLE is a number.

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0 / 1
points

8.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is the fair coin? Round your answer to two decimal places.

.22

Incorrect Response
Use Bayes theorem as in the lesson and in question 3. That is, find
$P(\theta = \text{fair} \mid X = 2) = \frac{P(X=2|\theta=\text{fair})P(\theta=\text{fair})}{P(X=2|\theta=\text{fair})P(\theta=\text{fair})+P(X=2|\theta=\text{loaded heads})P(\theta=\text{loaded heads})+P(X=2|\theta=\text{loaded tails})P(\theta=\text{loaded tails})}$

0 / 1
points

9.
Loaded coins:

- Suppose you flip the coin four times and it comes up heads twice. What is the posterior probability that this is a loaded coin (favoring either heads or tails)? Round your answer to two decimal places.

Hint: $P(\theta = \text{fair} \mid X = 2) = 1 - P(\theta = \text{loaded} \mid X = 2)$, so you can use your answer from the previous question rather than repeat the calculation from Bayes' theorem (both approaches yield the same answer).

.38

Incorrect Response
Use the hint, or calculate $P(\theta = \text{loaded} \mid X = 2) = P(\theta = \text{loaded heads} \mid X = 2) + P(\theta = \text{loaded tails} \mid X = 2)$ using Bayes' theorem.