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Number*

**I & II Semester Diploma Examination, June/July-2023**

## **ENGINEERING MATHEMATICS**

Time : 3 Hours ]

[ Max. Marks : 100 ]

**Instructions :** (i) Answer **one** full question from each section.  
(ii) One full question carries **20** marks.

## **SECTION - I**

1. (a) Solve for  $x$ ,  $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 3 \end{vmatrix} = 0$  4

OR

If  $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ , Find  $A + A^T$ .

- (b) Using Cramer's rule, find the solution of the system of equations  $2y - z = 0$ ,  
and  $x + 3y = -4$ ,  $3x + 4y = 4$  6

OR

Which of the matrix has no inverse ?

$$A = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 \\ 12 & 8 \end{bmatrix}$$

- (c) Find the characteristic equation and characteristic roots value for the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

OR

If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ , then verify that  $(A + B)^T = A^T + B^T$

- (d) Consider the matrix

If  $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ , find  $A^{-1}$ .

-  
OR

$$\text{If } A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \text{ & } B = \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}, \text{ find } AB.$$



**SECTION – II**  
**(Match the following)**

**P**

2. (a) (A) Equation of a straight line passing through a given point  $(x, y)$  and having slope  $m$  is  
 (B) Equation of a straight line passing through two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  
 (C) The equation of a straight line whose  $x$  and  $y$ -intercepts are  $a, b$  respectively is  
 (D) If two lines are perpendicular then product of their slopes is equal to

**Q**

- (1)  $\frac{x}{a} + \frac{y}{b} = 1$   
 (2)  $y - y_1 = m(x - x_1)$   
 (3)  $-1$   
 (4)  $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

4

**Answers :**

P	Q
A	
B	
C	
D	

**OR**

**(Match the following)**

**P**

- (A) If two lines with slopes  $m_1$  and  $m_2$  are parallel then ' $\theta$ ' is  
 (B) Equation of a straight line whose slope is  $m$  and  $y$  intercept is  $C$ .  
 (C) Slope of line  $ax + by + c = 0$   
 (D) Slope of a line joining two points  $(x_1, y_1)$  and  $(x_2, y_2)$ .

**Q**

- (1)  $y = mx + c$   
 (2) 0 (zero)  
 (3)  $\frac{y_2 - y_1}{x_2 - x_1}$   
 (4)  $-\frac{a}{b}$

**Answers :**

P	Q
A	
B	
C	
D	

- (b) Find the equation to the perpendicular to the line  $6x - 5y - 2 = 0$  and passing through  $(2, -3)$ . 6

**OR**

Are the lines  $4x + 6y + 7 = 0$  and  $2x + 3y - 1 = 0$  parallel to each other? Justify.

- (c) Find the equation of straight line parallel to  $5x + 6y - 10 = 0$  and passing through the point  $(-3, 3)$ . 5

**OR**

Are the lines  $3x + 4y + 7 = 0$  and  $28x - 21y + 50 = 0$  are perpendicular to each other? Justify.

- (d) Find the angle between the lines  $x + 3y + 5 = 0$  and  $4x + 2y - 7 = 0$  5

**OR**

Find the equation of straight line which passes through the points  $(-2, 3)$  and  $(-5, 6)$ .

**SECTION – III**

3. (a) Determine the value of  $\cos(570^\circ)$  and  $\sin(330^\circ)$ . 4

**OR**

Convert 45 degree into radian and  $\frac{11\pi}{5}$  radian into degree.

- (b) If  $A + B = \frac{\pi}{4}$  prove that  $(1 + \tan A)(1 + \tan B) = 2$  6

**OR**

Prove that  $\sin 3A = 3\sin A - 4 \sin^3 A$

- (c) Given  $\tan A = \frac{18}{17}$  and  $\tan B = \frac{1}{35}$  show that  $A - B = \frac{\pi}{4}$  5

**OR**

Show that :  $\frac{\cos(360^\circ - A) \cdot \tan(360^\circ + A)}{\cot(270^\circ - A) \cdot \sin(90^\circ + A)} = 1$

- (d) Prove that  $\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0$  5

**OR**

Show that  $\frac{\sin 40^\circ + \sin 20^\circ}{\cos 40^\circ + \cos 20^\circ} = \frac{1}{\sqrt{3}}$

**SECTION – IV**

4. (a) If  $y = \sin x + \log x + e^x + \tan x$ , then find  $\frac{dy}{dx} = ?$  4

**OR**

If  $\frac{dy}{dx} = 4x^3 + 3x^2$ , then find  $\frac{d^2y}{dx^2}$  at  $(1, 2)$

- (b) Using chain rule of differentiation, find the derivative of the function  
 $y = (3x + 8)^5$  6

**OR**

Using composite rule find the derivative of the function  $y = \log(\sin(\log x))$

- (c) The distance covered by a body in  $t$  seconds is given by  $S = 4t - 5t^2 + 2t^3$ , find  
 the velocity and acceleration when  $t = 2$  sec. 5

**OR**

Distance travelled by a car is given by  $S = 160t - 16t^2$  metre and time in  
 seconds. When does the car stop?

- (d) Find the maximum and minimum values of the function  $x^3 - x^2 - x = 0$ . 5

**OR**

Find the equation of the tangent to the curve  $y = 2 - 3x + x^2$  at  $(1, 2)$

**SECTION - V**

5. (a) Integrate :  $\cos x + e^x + \frac{1}{x} + x^2$ , w.r. to  $x$ . 4

**OR**

The area under the curve  $y = x^2$  between  $x = 1$ , and  $x = 2$  is equal to ...

- (b) Using the rule of integration by parts evaluate the integral  $\int x \sin 2x \, dx$  6

**OR**

Evaluate  $\int \sin 2x \cos 3x \, dx$

- (c) Find  $\int_0^{\pi/2} \sin^2 x \, dx$  5

**OR**

Evaluate  $\int \sin^5 x \cos x \, dx$

- (d) The area enclosed by the curve  $y = x^2 + 1$ ,  $x$ -axis between  $x = 1$ ,  $x = 3$ ,  
 calculate the area enclosed. 5

**OR**

Find the volume generated by rotating the curve  $y = \sqrt{x+2}$  about  $x$ -axis  
 between  $x = 0$  and  $x = 2$ .

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**SCHEME OF VALUATION****I & II SEMESTER DIPLOMA EXAMINATION**

Sub: Engg Mathematics

June/ July 2023

Code: 20SCOIT

Q.NO	Matter	Marks
1 (a)	Expansion of determinant Simplifying and result	2 2
	<b>OR</b>	
	Finding $A^T$	1
	Finding $A + A^T$	2
	Result	1
1 (b)	Find $\Delta_x, \Delta_y, \Delta_z$ Finding $x, y, z$	3 3
	<b>OR</b>	
	Find $ A  = -1, A^{-1}$ exists	2
	Find $ B  = -1, B^{-1}$ exists	2
	Find $ C  = -1, C^{-1}$ exists	2
1.c	$ A - \lambda I  = 0$ Finding characteristic equation Finding roots	1 2 2
	<b>OR</b>	
	$(A+B)$	1
	$(A+B)^T$	2
	$A^T, B^T$	1
	$A^T + B^T$	
1.d	$ A =7 \neq 0$ Formula $A^{-1} = \frac{\text{adj } A}{ A }$ Find adj A Answer	1 1 2 1
	<b>OR</b>	
	$AB = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}$	1
	Performing AB Answer	3 1

Q.NO	Matter	Marks
2(a)	Matching A-2 B-4 C-1 D-3	1 1 1 1
	<b>OR</b>	
	Matching A-2 B-1 C-4 D-3	1 1 1 1
2 (b)	Finding $m = \frac{6}{5}$ Formula $(y - y_1) = m(x - x_1)$ Substitution and calculation Answer	1 1 3 1
	<b>OR</b>	
	Finding $m_1 = \frac{-2}{3}$ $m_2 = \frac{-2}{3}$ Using condition Answer	2 2 1 1
2.c	Finding $m = \frac{-5}{6}$ Formula $(y - y_1) = m(x - x_1)$ Substitution and calculation Answer	1 1 2 1
	<b>OR</b>	
	Finding $m_1 = \frac{-3}{4}, m_2 = \frac{4}{3}$ Using condition $m_1 \times m_2 = -1$ Answer	2 2 1
2.d	Finding $m_1 = \frac{-1}{3}, m_1 = -2$ Formula $\tan \theta = \left  \frac{m_1 - m_2}{1 + m_1 m_2} \right $ Calculation Answer	2 1 1 1
	<b>OR</b>	
	$\frac{(y - y_1)}{(x - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ Substitution and calculation Answer	1 3 1



Q.NO	Matter	Marks
5.a	For each integration  OR $A = \int_a^b y dx$  Substitution and calculation  Answer	1 marks each=4m  1  2  1
5.b	Integration by part formula Substitution Calculation Answer  OR Integration by part formula Formula $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$ Substitution and calculation Answer	1 1 3 1  1  1 1 3 1
5.c	$\int_0^a f(x) dx = \int_0^a f(a-x) dx$ And calculation $\sin^2 x + \cos^2 x = 1$ $I=\pi/4$  OR $t = \sin x, dt = \cos x dx$ Calculation Answer	1 2 1 1  2 2 1
5.d	$A = \int_a^b y dx$ Substitution and calculation Answer $A=32/3$  OR $V = \pi \int_a^b [y]^2 dx$ Substitution Calculation $V=6 \pi$ Cubic unit	1 3 1  1 1 2 1

$$\begin{aligned}
 & \text{OR } A = \int_a^b y dx - 1 \\
 & = \int_a^{\pi/2} \sin^2 x dx = \int_a^{\pi/2} \frac{1-\cos 2x}{2} dx \rightarrow 1+1 \\
 & = \frac{1}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^{\pi/2} - 1 \text{ . Sub. & Ans } \rightarrow 1+1
 \end{aligned}$$

**I AND II SEMESTER EXAMINATION JUNE/JULY 2023**

**ENGINEERING MATHEMATICS**

CODE: 20SC01T

1. (a)  $1(3x - 12) - 2(6 - 9) + 3(8 - 3x) = 0$

$$3x - 24 + 42 - 9x = 0$$

$$18 - 6x = 0 \Rightarrow x = 3$$

**OR**

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1+1 & 5+2 \\ 2+5 & 3+3 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 7 \\ 7 & 6 \end{bmatrix}$$

1. (b)  $0x+2y-z=0 \quad X+3y+0z=-4 \quad 3x+4y+0z=4$

$$\Delta = \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0(0-0) - 2(0-0) - 1(4-9) = 0 - 0 - 1(-5) = 5$$

$$\Delta_x = \begin{vmatrix} 0 & 2 & -1 \\ -4 & 3 & 0 \\ 4 & 4 & 0 \end{vmatrix} = 0(0-0) - 2(0-0) - 1(-16-12) = 0 - 0 - 1(-28) = 28$$

$$\Delta_y = \begin{vmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0(0-0) - 0(0-0) - 1(4+12) = 0 - 0 - 1(16) = -16$$

$$\Delta_z = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 4 \end{vmatrix} = 0(12+16) - 2(4+12) + 0(4-9) = 0 - 2(16) - 0(-5) = -32$$

$$x = \frac{\Delta_x}{\Delta} = \frac{28}{5} \quad y = \frac{\Delta_y}{\Delta} = \frac{-16}{5}, \quad z = \frac{\Delta_z}{\Delta} = \frac{-32}{5}$$

**OR**

$$|A| = \begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1 - 0 = -1 \quad |B| = \begin{vmatrix} 2 & 6 \\ -1 & -3 \end{vmatrix} = -6 + 6 = 0$$

$$|C| = \begin{vmatrix} 3 & 2 \\ 12 & 8 \end{vmatrix} = -24 - 24 = 0 \quad \text{Matrix B and C does not have inverse.}$$

1. (C) Given  $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Characteristic equation is  $|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2-0 \\ 3-0 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 1-\lambda \end{bmatrix}$$

$$\left| \begin{array}{cc} 1-\lambda & 2 \\ 3 & 1-\lambda \end{array} \right| = 0 \Rightarrow (1-\lambda)(1-\lambda) - 6 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 5 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-5)}}{2 \times 1}$$

$$\Rightarrow \lambda = 1 + \sqrt{6} \text{ and } \lambda = 1 - \sqrt{6}$$

OR

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+2 & 2+0 \\ 3+4 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 7 & 7 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 3 & 7 \\ 2 & 7 \end{bmatrix} \dots \dots \dots (1)$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad B^T = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 7 \end{bmatrix} \dots \dots \dots (2)$$

From equation (1) & (2)  $(A+B)^T = A^T + B^T$

1. (d)  $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = 10 - 3 = 7 \neq 0 \therefore A^{-1} \text{ exists.}$

$$\text{adj } A = \begin{bmatrix} 5 & -1 \\ -3 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{7} \begin{bmatrix} 5 & -1 \\ -3 & 2 \end{bmatrix}$$

OR

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 18-3 & 4+5 \\ -27-15 & -6+25 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -42 & 19 \end{bmatrix}$$

2. (a)

2a

(A) Equation of Straight line Passing through a given point $(x_1, y_1)$ and having slope m is	(2) $(y - y_1) = m(x - x_1)$
(B) equation of straight line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$ is	(4) $\frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}$
(C) The equation of straight line whose x and y intercepts are a, b respectively is	(1) $\frac{x}{a} + \frac{y}{b} = 1$
(D) If two lines are perpendicular to each other then Product of their slope is equal to	(3) -1

OR

(A) if two lines with slopes $m_1$ and $m_2$ are parallel then $\theta$ is	(2) 0 (zero)
(B) equation of straight line whose slope is m and y intercept is C	(1) $y = mx + C$
(C) Slope of line is $ax + by + C = 0$	(4) $\frac{-a}{b}$
(D) slope of line joining two points $(x_1, y_1)$ and $(x_2, y_2)$	(3) $\frac{(y_2-y_1)}{(x_2-x_1)}$

2. (b)

Given equation  $6x - 5y - 2 = 0$ 

$$\text{Slope of given line } m = \frac{-a}{b} = \frac{-6}{-5} = \frac{6}{5}$$

$$\text{Therefore slope of required line is } m = \frac{-5}{6}$$

$$\text{Equation of line passing through } (2, -3) \text{ & having slope } m = \frac{-5}{6}$$

$$(y - y_1) = m(x - x_1), (x_1, y_1) = (2, -3), m = \frac{-5}{6}$$

$$[y - (-3)] = \frac{-5}{6}(x - 2) \Rightarrow 5x + 6y + 8 = 0$$

OR

$$\text{Consider } 4x + 6y + 7 = 0 \dots\dots\dots(1) \quad 2x + 3y - 1 = 0 \dots\dots\dots(2)$$

$$\text{Slope of line (1)} m_1 = \frac{-a}{b} = \frac{-4}{6} = \frac{-2}{3}$$

$$\text{Slope of line (2)} m_2 = \frac{-a}{b} = \frac{-2}{3}$$

since  $m_1 = m_2 = \frac{-2}{3}$  so the lines are parallel to each other.

2. (C) Given line  $5x + 6y - 10 = 0$

$$\text{Slope of given line } m = \frac{-a}{b} = \frac{-5}{6}$$

Since required line is parallel to  $5x + 6y - 10 = 0$

$$\text{So slope of required line is } m = \frac{-5}{6}$$

$$(x_1, y_1) = (-3, 3) \quad m = \frac{-5}{6}$$

$$(y - y_1) = m(x - x_1)$$

$$(y - 3) = \frac{-5}{6}(x + 3) \Rightarrow 6(y - 3) = -5(x + 3)$$

$$6y - 18 = -5x - 15 \Rightarrow 5x + 6y - 3 = 0 \quad \text{required line.}$$

OR

$$\text{Consider } 3x + 4y + 7 = 0 \quad \dots \quad (1) \quad 28x - 21y + 50 = 0 \quad \dots \quad (2)$$

$$\text{Slope of equation (1)} \quad m_1 = \frac{-a}{b} = \frac{-3}{4} \quad \text{Slope of equation (2)} \quad m_2 = \frac{-a}{b} = \frac{-28}{-21} = \frac{4}{3}$$

Since product of slope of two perpendicular lines is equal to  $-1$

$$m_1 \times m_2 = \frac{-3}{4} \times \frac{4}{3} = -1$$

$\therefore$  given lines are perpendicular to each other.

$$2. (d) \text{ let } x + 3y + 5 = 0 \quad \dots \quad (1) \quad 4x + 2y - 7 = 0 \quad \dots \quad (2)$$

$$m_1 = \frac{-a}{b} = \frac{-1}{3}$$

$$m_2 = \frac{-a}{b} = \frac{-4}{2} = -2$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\frac{-1}{3} + 2}{1 + \left(\frac{-1}{3}\right)(-2)} \right| = \left| \frac{\frac{-1+6}{3}}{1 + \frac{2}{3}} \right| = \left| \frac{\frac{5}{3}}{\frac{5}{3}} \right| = 1$$

$$\tan \theta = 1 \Rightarrow \theta = \tan^{-1} 1 = 45^\circ$$

OR

Equation of straight line which passes through  $(x_1, y_1)$  &  $(x_2, y_2)$  is

$$\frac{(y - y_1)}{(x - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)} \quad \text{and} \quad (x_1, y_1) = (-2, 3) \quad (x_2, y_2) = (-5, 6)$$

$$\frac{y - 3}{x + 2} = \frac{6 - 3}{-5 + 2} = -1 \quad \frac{y - 3}{x + 2} = -1$$

$$y - 3 = -1(x + 2) = -x - 2$$

$$y - 3 + x + 2 = 0 \Rightarrow x + y - 1 = 0$$

3. (a)

$$\cos(570^\circ) = \cos(6 \times 90^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\sin(330^\circ) = \sin(4 \times 90^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

OR

$$45^\circ = \frac{\pi}{180} \times 45 = \frac{\pi}{4} = 0.7855 \text{ rad}$$

$$11\frac{\pi}{5} = \frac{180}{\pi} \times 11\frac{\pi}{5} = 36 \times 11 = 396^\circ$$

3. (b)

$$A + B = \frac{\pi}{4} \Rightarrow \tan(A + B) = \tan \frac{\pi}{4} = 1$$

$$\tan(A + B) = 1$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1, \quad \tan A + \tan B = 1 - \tan A \tan B$$

$$\tan A + \tan B + \tan A \tan B = 1 \quad \text{add 1 both the side.}$$

$$1 + \tan A + \tan B + \tan A \tan B = 1 + 1$$

$$1 + \tan A + \tan B (1 + \tan A) = 2$$

$$(1 + \tan A)(1 + \tan B) = 2$$

OR

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$$

$$= 2 \sin A \cos A \cos A + (1 - 2 \sin^2 A) \sin A$$

$$= 2 \sin A \cos^2 A + \sin A - 2 \sin^3 A$$

$$= 2 \sin A (1 - \sin^2 A) + \sin A - 2 \sin^3 A$$

$$= 2 \sin A - 2 \sin^3 A + \sin A - 2 \sin^3 A$$

$$\therefore \sin 3A = 3 \sin A - 4 \sin^3 A$$

3. (c)  $\tan A = \frac{18}{17} \quad \tan B = \frac{1}{35}$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A - B) = \frac{\frac{18}{17} - \frac{1}{35}}{1 + \frac{18}{17} \times \frac{1}{35}} = \frac{630 - 17}{595 + 18} = \frac{613}{613} = 1$$

$$\tan(A - B) = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow (A - B) = \frac{\pi}{4}$$

OR

$$\frac{\cos(360-A) \tan(360+A)}{\cot(270-A) \sin(90+A)} = \frac{\cos(A) \tan(A)}{\tan(A) \cos(A)} = 1$$

3. (d)

$$\begin{aligned} \text{L.H.S } \cos 55^\circ + \cos 65^\circ + \cos 175^\circ &= \cos 55^\circ + 2 \cos\left(\frac{65+175}{2}\right) \cos\left(\frac{65-175}{2}\right) \\ &= \cos 55^\circ + 2 \cos(120^\circ) \cos(-55^\circ) \\ &= \cos 55^\circ + 2 \times \cos(90^\circ + 30^\circ) \cos(55^\circ) \\ &= \cos 55^\circ + 2 \times (-\sin 30^\circ) \cos(55^\circ) \\ &= \cos 55^\circ - 2 \times \frac{1}{2} \cos(55^\circ) \\ &= \cos 55^\circ - \cos 55^\circ = 0 \quad \text{R.H.S } \checkmark \end{aligned}$$

OR

$$\begin{aligned} \frac{\sin 40^\circ + \sin 20^\circ}{\cos 40^\circ + \cos 20^\circ} &= \frac{2 \sin\left(\frac{40+20}{2}\right) \cos\left(\frac{40-20}{2}\right)}{2 \cos\left(\frac{40+20}{2}\right) \cos\left(\frac{40-20}{2}\right)} \\ &= \frac{\sin\left(\frac{40+20}{2}\right)}{\cos\left(\frac{40+20}{2}\right)} = \frac{\sin(30)}{\cos(30)} = \tan 30 = \frac{1}{\sqrt{3}} \checkmark \end{aligned}$$

4. (a)  $y = \sin x + \log x + e^x + \tan x$

$$\frac{dy}{dx} = \frac{d(\sin x)}{dx} + \frac{d \log x}{dx} + \frac{de^x}{dx} + \frac{d \tan x}{dx}$$

$$\frac{dy}{dx} = \cos x + \frac{1}{x} + e^x + \sec^2 x$$

OR

$$y = 4x^3 + 3x^2 \quad \frac{dy}{dx} = 4 \frac{dx^3}{dx} + 3 \frac{dx^2}{dx}$$

$$\frac{dy}{dx} = 4 \times 3x^2 + 3 \times 2x = 12x^2 + 6x \quad \checkmark$$

$$\frac{d^2y}{dx^2} = 12 \frac{d(x^2)}{dx} + 6 \frac{d(x)}{dx} = 12 \times 2x + 6 \times 1 = 24x + 6 \quad \checkmark$$

$$\left( \frac{d^2y}{dx^2} \right)_{(1,2)} = 24(1) + 6 = 30 \quad \checkmark$$

4. (b)

$$y = (3x + 8)^5$$

$$\begin{aligned} \frac{dy}{dx} &= 5(3x + 8)^{5-1} \times \frac{d(3x+8)}{dx} \\ &= 5(3x + 8)^4 \times \left( 3 \frac{d(x)}{dx} + \frac{d(8)}{dx} \right) \\ &= 5(3x + 8)^4 \times (3 \times 1 + 0) \\ &= 15(3x + 8)^4 \quad \checkmark \end{aligned}$$

OR

$$y = \log [\sin (\log x)]$$

$$\frac{dy}{dx} = \frac{d(\log [\sin (\log x)])}{dx} = \frac{1}{\sin(\log x)} \frac{d(\sin(\log x))}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sin(\log x)} (\cos(\log x)) \frac{d(\log x)}{dx}$$

$$\frac{dy}{dx} = \frac{\cos(\log x)}{\sin(\log x)} \left( \frac{1}{x} \right) = \frac{\cot(\log x)}{x}$$

$$4. (c) S = 4t - 5t^2 + 2t^3$$

$$\frac{d(s)}{dt} = \frac{d(4t-5t^2+2t^3)}{dt}$$

$$\frac{d(s)}{dt} = 4 \frac{d(t)}{dt} - 5 \frac{d(t^2)}{dt} + 2 \frac{d(t^3)}{dt}$$

$$\frac{d(s)}{dt} = 4 \times 1 - 5 \times 2t + 2 \times 3t^2 \quad \checkmark$$

$$V = \frac{d(s)}{dt} = 4 - 10t + 6t^2 \quad \checkmark$$

$$V = \left( \frac{d(s)}{dt} \right)_{t=2} = 4 - 10 \times 2 + 6 \times 2^2 = 4 - 20 + 24 = 8$$

$$a = \frac{d(4-10t+6t^2)}{dt} = 0 - 10 \frac{dt}{dt} + 6 \frac{dt^2}{dt} = -10 \times 1 + 6 \times 2t = -10 + 12t$$

$$(a)_{t=2} = -10 + 12(2) = -10 + 24 = 14 \text{ ms}^{-2}$$

**OR**

$$S = 160t - 16t^2$$

$$\frac{d(s)}{dt} = V = \frac{d(160t - 16t^2)}{dt} = 160 \frac{d(t)}{dt} - 16 \frac{d(t^2)}{dt}$$

$$\frac{d(s)}{dt} = 160 \times 1 - 16 \times 2t = 160 - 32t \quad \text{When the car stop}$$

$$V = 0 = 160 - 32t \quad \text{velocity becomes zero. } V=0$$

$$t = \frac{160}{32} = 5 \text{ sec}$$

$$4. (d) \quad y = x^3 - x^2 - x \quad , \quad \frac{dy}{dx} = 3x^2 - 2x - 1$$

$y$  is maximum or minimum, if  $\frac{dy}{dx} = 0$

$$\text{i.e } 0 = 3x^2 - 2x - 1 \text{ or } (3x + 1)(x - 1) = 0$$

$$\therefore x = -\frac{1}{3}, \text{ and } x = 1$$

$$\frac{d^2y}{dx^2} = 6x - 2, \text{ now if } x = -\frac{1}{3} \quad \frac{d^2y}{dx^2} = 6\left(\frac{-1}{3}\right) - 2$$

$$\frac{d^2y}{dx^2} = -4$$

$\therefore y$  is maximum at  $x = -\frac{1}{3}$

$$y = x^3 - x^2 - x$$

$$y = \left(\frac{-1}{3}\right)^3 - \left(\frac{-1}{3}\right)^2 - \left(\frac{-1}{3}\right) = \left(\frac{-1}{27}\right) - \left(\frac{1}{9}\right) + \left(\frac{1}{3}\right) = \left(\frac{5}{27}\right)$$

Maximum value is  $y = \left(\frac{5}{27}\right)$

$$\text{For } x = 1 \quad \frac{d^2y}{dx^2} = 6(1) - 2 = 4 = +ve \text{ value}$$

$\therefore y$  is minimum at  $x = 1$

$$y = x^3 - x^2 - x \quad , \quad y = 1^3 - 1^2 - 1 = -1$$

Minimum value is  $y = -1$  ✓

OR

$$y = 2 - 3x + x^2 \quad \text{diff w.r.t } x$$

$$\frac{dy}{dx} = -3 + 2x \quad , \quad m = \left( \frac{dy}{dx} \right)_{(1,2)} = -3 + 2(1) = -1 \quad \checkmark$$

$$\text{Equation of tangent } y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 1) \quad y - 2 = -x + 1$$

$$y - 2 + x - 1 = 0 \quad x + y - 3 = 0 \quad \checkmark$$

5. (a)  $y = \cos x + e^x + \frac{1}{x} + x^2$

$$\int y dx = \int \cos x dx + \int e^x dx + \int \frac{1}{x} dx + \int x^2 dx$$

$$= \sin x + e^x + \log x + \frac{x^3}{3} + C \quad \checkmark$$

OR

$$\text{Area } A = \int_a^b y dx = \int_1^2 x^2 dx = \left[ \frac{x^3}{3} \right]_1^2 = \left[ \frac{2^3}{3} - \frac{1^3}{3} \right] = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \quad \checkmark$$

$$\therefore A = \frac{7}{3} \text{ square units} \quad \checkmark$$

5. (b)  $\int uv dx = u \int v dx - \int \left[ \frac{du}{dx} \times \int v dx \right] dx$

$$\int x \sin 2x dx = x \int \sin 2x dx - \int \left[ \frac{dx}{dx} \times \int \sin 2x dx \right] dx$$

$$= x \left( \frac{-\cos 2x}{2} \right) - \int \left( 1 \times \left( \frac{-\cos 2x}{2} \right) \right) dx$$

$$= \cancel{x} \left( \frac{-\cos 2x}{2} \right) + \frac{1}{2} \int \cos 2x \, dx$$

$$= -\frac{x \cos(2x)}{2} + \frac{\sin 2x}{4} + C$$

OR

$$\int uv \, dx = u \int v \, dx - \int \left[ \frac{du}{dx} \times \int v \, dx \right] dx$$

$$\int \sin 2x \cos 3x \, dx = \int \frac{1}{2} [\sin(2x + 3x) + \sin(2x - 3x)] \, dx$$

Here using formula  $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$

$$\int \sin 2x \cos 3x \, dx = \frac{1}{2} \int \sin(5x) \, dx + \frac{1}{2} \int \sin(-x) \, dx$$

$$= \frac{1}{2} \left( -\cos \frac{5x}{5} \right) + \frac{1}{2} \int (-\sin x) dx$$

$$= \frac{-\cos 5x}{10} - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 5x}{10} + \frac{\cos x}{2} + C$$

$$5. \text{ (C)} \quad \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad \dots \dots \dots (2)$$

Adding (1) & (2)

$$I + I = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx + \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) dx = \int_0^{\frac{\pi}{2}} 1 dx = [x]_0^{\frac{\pi}{2}} = \left[ \frac{\pi}{2} - 0 \right]$$

$$\Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow \quad I = \frac{\pi}{4} \quad \checkmark$$

OR

$$\int \sin^5 x \cos x \, dx = \int t^5 \, dt \quad \text{put } t = \sin x$$

$$= \frac{t^{5+1}}{5+1} + C$$

$$\frac{dt}{dx} = \cos x$$

$$= \frac{t^6}{6} + C$$

$$dt = \cos x \, dx$$

$$= \frac{(\sin x)^6}{6} + C$$

$$5. (d) \quad \text{Area} = A = \int_a^b y \, dx = \int_1^3 (x^2 + 1) \, dx = \left[ \frac{x^3}{3} + x \right]_1^3 = \frac{1}{3} [x^3]_1^3 + [x]_1^3$$

$$\begin{aligned}
 &= \frac{1}{3} [(3)^3 - (1)^3] + [3 - 1] \\
 &= \frac{1}{3} [27 - 1] + [2] \\
 &= \frac{26}{3} + 2 = \frac{32}{3}
 \end{aligned}$$

OR

$$y = \sqrt{x+2}$$

$$\begin{aligned}
 V &= \pi \int_a^b [y]^2 dx = \pi \int_a^b [f(x)]^2 dx = \pi \int_0^2 (\sqrt{x+2})^2 dx \\
 &= \pi \int_0^2 (x+2) dx = \pi \int_0^2 (x) dx + 2\pi \int_0^2 1 dx \\
 &= \pi \left[ \frac{x^2}{2} \right]_0^2 + 2\pi [x]_0^2 \\
 &= \frac{\pi}{2} [2^2 - 0^2] + 2\pi [2 - 0] \\
 &= \frac{\pi}{2} [4] + 2\pi [2] = 2\pi + 4\pi = 6\pi \checkmark
 \end{aligned}$$

$$V = 6\pi \text{ cubic unit}$$

Note: Give full marks for any other alternate method.