

Makeup Examination Nov/Dec - 2022

I / II Semester Diploma Examination

ENGINEERING MATHEMATICS (20SC01T)

Time: 3 Hours ]

[ Max. Marks: 100

**Instruction:** i) Answer ONE full question from each section.

ii) One full question carries 20 marks.

## SECTION – I

1. (a) Write four type of matrices with one example for each. (4)

OR

If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  then Find  $3A + 2B$

(b) If  $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$  then find  $\text{adj}A$  (6)

OR

Find the characteristic roots of the matrix  $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

- (c) Applying Cramer's rule solve the system of linear equations (5)

$$3x + y = 4 \text{ and } x + 3y = 4$$

OR

The Tata motors company Ltd., has two outlets, one in Bengaluru and one in Belgaum, among other things, it sells Tata Nexon, Tata Tiago, and Tata Punch cars. The monthly sales of these cars at the two stores for two months are Given in the following tables:

October sells

	Bengaluru	Belgaum
Tata Nexon	25	35
Tata Tiago	15	20
Tata Punch	18	05

November sells

	Bengaluru	Belgaum
Tata Nexon	35	45
Tata Tiago	30	50
Tata Punch	26	25

Use matrix arithmetic to calculate the change in sales of each product in each Store from October to November.

(d) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  then find the value of  $AB$  and  $(AB)^T$  (5)

OR

For the given matrix  $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  verify  $A \cdot \text{adj}A = |A|I$ , where 'I' is the unit matrix of the order 2.

## SECTION – II

2. (a) Find the slope and y-intercept of the line  $5x - 3y + 9 = 0$  (4)

OR

Find the equation of straight line of slope 3 units and y-intercept 4.

- (b) Find the equation of straight line passing through the point (3,4) and perpendicular to  $4x+2y+3=0$ . (6)

OR

Find the equation of straight line passing through the point (2,-5) and (3,7)

- (c) Using slope point form of straight line find the equation of line passing through the point (1,2), inclined at  $45^\circ$  to the x - axis (5)

OR

Find the equation of straight line whose 'x'- intercept and y-intercept are 5 and 6 respectively. Write the standard form of it.

- (d) Find the equation of straight line passing through the point (2,3) and parallel to  $5x - 4y + 4 = 0$  (5)

OR

If  $\frac{3}{5}$  and  $\frac{7}{3}$  are the slopes of two lines, find the angle between the two lines

### SECTION – III

- 3.(a) Convert  $30^\circ$  into radians and  $\frac{\pi}{12}$  into degree. (4)

OR

Prove that  $\tan(45^\circ + A) = \frac{1+\tan A}{1-\tan A}$

- (b) Prove that  $\sin\theta \cos(90 - \theta) + \cos\theta \sin(90 - \theta) = 1$  (6)

OR

Prove that  $\frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \tan A$ .

- (c) Write the formula for  $\sin(A + B)$  then find the value of  $\sin 75^\circ$  (5)

OR

Simplify  $\frac{\cos(360^\circ - A) \tan(360^\circ + A)}{\cot(270^\circ - A) \sin(90^\circ + A)}$

- (d) Prove that  $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$  (5)

OR

Prove that  $\cos 2\theta = 2 \cos^2 \theta - 1$

#### SECTION - IV

4. (a) Differentiate  $x^3 + x^2 + 3x + 9$  with respect to  $x$ . (4)

OR

If  $y = (x+1)(x-1)$  then find  $\frac{dy}{dx}$ .

- (b) Find the maximum and minimum value of a function (6)

$$y = x^3 - 12x^2 - 27x + 16$$

OR

A ball will throw vertically upwards and reaches maximum height  $s$  in feet.

The height reached is given by  $s = -16t^2 + 64t$ . How much time it takes to reach maximum height? Find also the maximum height reached by the ball.

(c) If  $y = Ae^{mx} + Be^{-mx}$  then prove that  $\frac{d^2y}{dx^2} - m^2y = 0$  (5)

OR

Find the derivative of a function  $\frac{1 + \sin x}{1 - \sin x}$  w.r.t.  $x$ .

(d) Find the equation of normal to the curve  $y = 1 - x^3$  at the point  $(2, 3)$  (5)

OR

If  $y = (1 + x^2)\tan^{-1} x$  then find  $\frac{dy}{dx}$ .

#### SECTION - V

5. a) Integrate  $2x^3 - \frac{3}{x} + 4\cos x + \frac{1}{1+x^2}$  w.r.t.  $x$  (4)

OR

Integrate  $(x - 2)(x + 3)$  w.r.t.  $x$ .

b) Evaluate  $\int \sin^2 x \, dx$  (6)

OR

Evaluate  $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

- c) Using definite integrals, find the area bounded by the curve  $y = 4x - x^2 - 3$ ,  
x-axis and the ordinates  $x = 1$  and  $x = 3$ . (5)

OR

The curve  $y^2 = x^2 + 5x$  is rotated about  $x$ -axis. Find the volume of the solid  
generated by revolving the curve between the ordinates  $x = 1, x = 2$  about  $x$ -axis

- d) Using integration by parts evaluate the indefinite integral  $\int x \sin x \, dx$ . (5)

OR

Evaluate  $\int_0^1 \frac{(\tan^{-1} x)^3}{1+x^2} \, dx$

**SCHEME OF VALUATION**

Q No	Particulars	Marks
Section - I		
1 (a)	Types Example (each $\frac{1}{2}$ marks)	<div> 1 1 1 1 </div> } <div>= 4</div>
OR		
	Finding 3A and 2B	2
	Finding 3A+2B	2
1(b)	Finding minors	4
	Writing cofactor	1
	Writing adjoint	1
OR		
	Writing $ A - \lambda I  = 0$	1
	Finding Characteristic Equation	3
	Characteristic roots	2
1(c)	Finding $\Delta$ , $\Delta_x$ , $\Delta_y$	3
	Finding x and y	2
OR		
	Writing Given Data in Matrix Form	2
	Finding difference	2
	Answer	1
1(d)	Finding AB	3
	Finding $(AB)^T$	2
OR		
	Finding $A \cdot adj A$	2
	Finding $ A I$	2
	Conclusion	1

Q.N	PARTICULAR	MARKS
SECTION - II		
2(a)	Finding slope	2
	Finding y-intercept	2
	OR	
	Writing m=3 and c =4 Formula	1 1

	Substituting values	1
	Final equation	1
2(b)	Writing $4x+2y+K=0$	2
	Getting value of k	2
	result	2
	OR	
	Formula	2
	Substituting values	2
	Simplification and result	2
2(c)	Finding $m = 1$	2
	Formula	1
	Substituting values	1
	Final equation	1
	OR	
	Writing standard form	2
	Substituting values	1
	Simplification	1
	Final equation	1
2(d)	Writing $5x-4y+K=0$	2
	Getting value of k	2
	result	1
	OR	
	Formula	2
	Substituting values	1
	Simplification	1
	Result	1

Q No	Particulars	Marks
Section - III		
3 (a)	Converting $30^0$ to radian	2
	Converting $\frac{\pi}{12}$ to degree	2
	OR	
	$\tan (A+B)$ formula	1
	substituting $45^0$	2
	Result	1
(b)	Allied angle change	2
	Substitution	2
	Trigonometric identity	1
	Result	1
	OR	
	$\sin (A+B)$ and $\cos (A+B)$ (2 mark each)	4
	Simplification	1
	Proving RHS	1
(c)	$\sin (A+B)$ formula	2
	each T – value $\frac{1}{2}$ mark	2
	result	1
	OR	



	Each allied angle (1 mark each) Simplification and result	4 1 } = 5
1(d)	Simplifying $\sin 20^\circ \sin 40^\circ$ Simplifying $\cos 20^\circ \sin 80^\circ$	2 2 1 } = 5
	OR	
	$\cos 2\theta$ formula $\sin^2 \theta$ formula Rest	2 2 1 } = 5

Q.NO	Particular	Marks
SECTION-IV		
4(a)	Differentiation of each term	1+1+1+1 = 4
	OR	
	Applying algebraic formula or multiplication Differentiation and Answer	1 2+1 } = 4
4(b)	Finding first and second derivative of a function Getting value of x Finding maximum and Minimum value	1+1 2 1+1 } = 6
	OR	
	Finding $\frac{ds}{dt}$ Taking $\frac{ds}{dt} = 0$ Getting time value Finding displacement value	2 1 1 2 } = 6
4(c)	Finding $\frac{dy}{dx}$ Finding $\frac{d^2y}{dx^2}$ Simplification and Answer	2 2 1 } = 5
	OR	
	Using Quotient rule Derivative of $1+\sin x$ Derivative of $1-\sin x$ Simplification and answer	2 1 1 1 } = 5
4(d)	Differentiation and getting slope Equation of normal formula Substitution and simplification	2 1 2 } = 5
	OR	
	Using product rule Differentiation each term Answer	2 1+1 1 } = 5

Q.N0	Particular	Marks
<b>SECTION V</b>		
5(a)	Integration of each 1 mark (1+1+1+1)	4M
	<b>OR</b>	
	Simplification Integration	1 M 3M
5(b)	Writing $\sin^2 x$ formula Substitution Integration	2M 1 M 1M
	<b>OR</b>	
	Writing $\tan^2 x$ formula Integration Substituting limit values Simplification	2M 1M 1M 2M
5(c)	Writing Area Formula Integration Simplification and Result	1M 2M 2M
	<b>OR</b>	
	Writing Volume Formula Integration Simplification and Result	1M 2M 2M
5(d)	Formula for integration by parts Substitution Calculation and Result	1M 1M 3M
	<b>OR</b>	
	Substitution Differentiation Finding new limits Simplification	1M 1M 1M 2M

## MODEL ANSWERS

SECTION I		
1(a)	1. Row matrix $A = [1 \ 2 \ 3]$ 2. Column matrix $A = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ 3. Square matrix $S = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 4. Diagonal matrix $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$	
	OR	
	If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then Find $3A + 2B$  $3A = 3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$ $2B = 2 \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$ $3A + 2B = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} + \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$ $= \begin{bmatrix} 9 & 10 \\ 11 & 20 \end{bmatrix}$	
1(b)	To find the adjoint we need to find the cofactors first	
	$C_3 = + \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$ $= +(2 - 3) = -1$ $C_1 = - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$ $= -(1 - 2) = 1$ $C_2 = + \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$ $= +(3 - 4) = -1$	$C_1 = - \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ $= -(1 - 6) = 5$ $C_2 = + \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$ $= +(3 - 4) = -1$ $C_3 = - \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$ $= -(9 - 2) = -7$
	$C_2 = + \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$ $= +(3 - 4) = -1$ $C_3 = - \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$ $= -(3 - 2) = -1$ $C_1 = + \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$ $= +(6 - 1) = +5$	
	Cofactor matrix $C = \begin{bmatrix} -1 & 1 & -1 \\ 5 & -1 & -7 \\ -3 & -1 & 5 \end{bmatrix}$  $\text{Adj}A = [\text{cofactor matrix}]^T$  $\text{adj}A = \begin{bmatrix} -1 & 5 & -3 \\ 1 & -1 & -1 \\ -1 & -7 & 5 \end{bmatrix}$	
	OR	
	Given $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$  Characteristic equation is 'A' defined as $ A - \lambda I  = 0$ $\Rightarrow \begin{vmatrix} 3 - \lambda & 1 \\ 2 & 4 - \lambda \end{vmatrix} = 0$  $\Rightarrow (3 - \lambda)(4 - \lambda) - 2 = 0$ $\Rightarrow 12 - 3\lambda - 4\lambda + \lambda^2 - 2 = 0$	

	$\Rightarrow 12 - 7\lambda + \lambda^2 - 2 = 0$ $\Rightarrow \lambda^2 - 7\lambda + 10 = 0$ $\Rightarrow \lambda^2 - 5\lambda - 2\lambda + 10 = 0$ $\Rightarrow \lambda(\lambda - 5) - 2(\lambda - 5) = 0$ $\Rightarrow (\lambda - 5)(\lambda - 2) = 0$ $\Rightarrow \lambda - 5 = 0 \text{ or } \lambda - 2 = 0$ $\Rightarrow \lambda = 5 \text{ or } \lambda = 2$ <p>Therefore the characteristics roots of the given matrix are <math>\lambda = 5</math> and <math>\lambda = 2</math></p>
1(c)	<p>Given system of linear equations</p> $3x + y = 4$ $x + 3y = 4$ <p>Consider <math>\Delta = \begin{vmatrix} 3 &amp; 1 \\ 1 &amp; 3 \end{vmatrix} = 9 - 1 = 8</math></p> $\Delta_x = \begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix} = 12 - 4 = 8$ $\Delta_y = \begin{vmatrix} 3 & 4 \\ 1 & 4 \end{vmatrix} = 12 - 4 = 8$ $x = \frac{\Delta_x}{\Delta} = \frac{8}{8} = 1, y = \frac{\Delta_y}{\Delta} = \frac{8}{8} = 1$ <p><math>x = 1</math> and <math>y = 1</math></p>
OR	
	<p>By transforming given data to matrix form</p> <p>Let October sells be <span style="float: right;">Let November sells</span></p> $O = \begin{bmatrix} 25 & 35 \\ 15 & 20 \\ 18 & 5 \end{bmatrix} N = \begin{bmatrix} 35 & 45 \\ 30 & 50 \\ 26 & 25 \end{bmatrix}$ <p>Change in sells is given by</p> $O - N = \begin{bmatrix} 25 & 35 \\ 15 & 20 \\ 18 & 5 \end{bmatrix} - \begin{bmatrix} 35 & 45 \\ 30 & 50 \\ 26 & 25 \end{bmatrix}$ $= \begin{bmatrix} -10 & -10 \\ -15 & -30 \\ -8 & -20 \end{bmatrix} \text{ or } \begin{bmatrix} 10 & 10 \\ 15 & 30 \\ 8 & 20 \end{bmatrix}$
1(d)	<p>Given <math>A = \begin{bmatrix} 1 &amp; 2 \\ 3 &amp; 4 \end{bmatrix}</math> and <math>B = \begin{bmatrix} 3 &amp; 2 \\ 1 &amp; 4 \end{bmatrix}</math></p> $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ $= \begin{bmatrix} (1 \times 3) + (2 \times 1) & (1 \times 2) + (2 \times 4) \\ (3 \times 3) + (4 \times 1) & (3 \times 2) + (4 \times 4) \end{bmatrix}$ $= \begin{bmatrix} 3 + 2 & 2 + 8 \\ 9 + 4 & 6 + 16 \end{bmatrix}$ $= \begin{bmatrix} 5 & 10 \\ 13 & 22 \end{bmatrix}$

	$(AB)^T = \begin{bmatrix} 5 & 13 \\ 10 & 22 \end{bmatrix}$
	OR
	<p>Given, <math>A = \begin{bmatrix} 3 &amp; 2 \\ 1 &amp; 4 \end{bmatrix}</math></p> <p><math>\Rightarrow adjA = \begin{bmatrix} 4 &amp; -2 \\ -1 &amp; 3 \end{bmatrix}</math></p> <p>Consider <math>A \cdot adjA = \begin{bmatrix} 3 &amp; 2 \\ 1 &amp; 4 \end{bmatrix} \begin{bmatrix} 4 &amp; -2 \\ -1 &amp; 3 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} (3 \times 4) + (2 \times -1) &amp; (3 \times -2) + (2 \times 3) \\ (1 \times 4) + (4 \times -1) &amp; (1 \times -2) + (4 \times 3) \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 12 - 2 &amp; -6 + 6 \\ 4 - 4 &amp; -2 + 12 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 10 &amp; 0 \\ 0 &amp; 10 \end{bmatrix}</math>----- (1)</p> <p>Consider <math> A I = \begin{vmatrix} 3 &amp; 2 \\ 1 &amp; 4 \end{vmatrix} \begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix}</math></p> <p><math>= (12 - 2) \begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix}</math></p> <p><math>= 10 \begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix} = \begin{bmatrix} 10 &amp; 0 \\ 0 &amp; 10 \end{bmatrix}</math>----- (2)</p> <p>From (1) and (2)</p> <p><math>A \cdot adjA =  A I</math></p>

“Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and scheme of valuation prepared by me are correct”.

“Award full marks for alternative methods of answers”

  
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SECTION - II	
2(a)	<p>Find the slope and y-intercept of the line <math>5x - 3y + 9 = 0</math></p> <p>Given: <math>5x - 3y + 9 = 0</math></p> <p><math>a = 5 \quad b = -3 \quad c = 9</math></p> <p><math>Slope = \frac{-a}{b} = \frac{-5}{-3} = \frac{5}{3}</math></p> <p><math>y - intercept = \frac{-c}{b} = \frac{-9}{-3} = \frac{9}{3} = 3</math></p>

	<p style="text-align: center;"><b>OR</b></p> <p>Find the equation of straight line of slope 3 units and y-intercept 4.  Given: <math>\text{slope } m = 3 \quad c = 4</math>  <math>y = mx + c</math>  <math>y = 3x + 4</math>  <math>3x - y + 4 = 0</math></p>
2(b)	<p>Find the equation of straight line passing through the point (3,4) and perpendicular to <math>4x + 2y + 3 = 0</math>  Given: <math>4x + 2y + 3 = 0 \quad 4x + 2y + 3 = 0</math>  <math>a = 4 \quad b = 2 \quad c = 3 \quad 2x - 4y + k = 0 \dots\dots\dots(1)</math>  <math>\text{Slope} = \frac{-a}{b} = \frac{-4}{2} = -2 \quad (3) - 4(4) + k = 0</math>  <math>(y - y_1) = \frac{-1}{m} (x - x_1) \quad \text{OR} \quad 6 - 16 + k = 0</math>  <math>(y - 4) = \frac{-1}{-2} (x - 3) - 10 + k = 0</math>  <math>(y - 4) = \frac{1}{2} (x - 3) \quad k = 10 \quad (\text{substitute in equ - 1})</math>  <math>2(y - 4) = 1(x - 3) \quad 2x - 4y + 10 = 0</math>  <math>2y - 8 = x - 3</math>  <math>x - 3 - 2y + 8 = 0 \quad \text{Or } x - 2y + 5 = 0</math>  <math>x - 2y + 5 = 0</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Find the equation of straight line passing through the point (2, -5) and (3,7)  Given: <math>(x_1, y_1) = (2, -5)</math>  <math>(x_2, y_2) = (3, 7)</math>  <math>(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)</math>  <math>(y + 5) = \frac{(7 + 5)}{(3 - 2)} (x - 2)</math>  <math>(y + 5) = \frac{12}{1} (x - 2)</math>  <math>1(y + 5) = 12(x - 2)</math>  <math>y + 5 = 12x - 24</math>  <math>12x - y - 29 = 0</math></p>
2(c)	<p>Using slope point form of straight line find the equation of line passing through the point (1,2), inclined at <math>45^\circ</math> to the x-axis  Given: <math>(x_1, y_1) = (1, 2)</math>  <math>\theta = 45^\circ</math>  <math>m = \tan \theta</math>  <math>m = \tan 45</math>  <math>m = 1</math>  <math>(y - y_1) = m(x - x_1)</math>  <math>(y - 2) = 1(x - 1)</math>  <math>y - 2 = x - 1</math>  <math>x - y + 1 = 0</math></p>
	<p style="text-align: center;"><b>OR</b></p> <p>Find the equation of straight line whose 'x'-intercept and y-intercept are 5 and 6 respectively. Write the standard form of it.  Given: <math>a = 5</math>  <math>b = 6</math>  Standard form of equation is <math>\frac{x}{a} + \frac{y}{b} = 1</math></p>

	$\frac{x}{a} + \frac{y}{b} = 1$ $\frac{x}{5} + \frac{y}{6} = 1$ $\frac{(6x+5y)}{30} = 1$ $6x + 5y = 30$ $6x + 5y - 30 = 0$
2(d)	<p>(d) Find the equation of straight line passing through the point (2,3) and parallel to <math>5x - 4y + 4 = 0</math></p> <p>Given: <math>5x - 4y + 4 = 0</math></p> $a = 5 \quad b = -4 \quad c = 4$ $\text{Slope} = \frac{-a}{b} = \frac{-5}{-4} = \frac{5}{4}$ $(y - y_1) = m(x - x_1)$ $(y - 3) = \frac{5}{4}(x - 2)$ $4(y - 3) = 5(x - 2)$ $4y - 12 = 5x - 10$ $5x - 10 - 4y + 12 = 0$ $5x - 4y + 2 = 0$ <p>Alternative method</p> $5x - 4y + 4 = 0$ $5x - 4y + k = 0 \dots\dots\dots(1)$ $5(2) - 4(3) + k = 0$ $10 - 12 + k = 0$ $-2 + k = 0$ $k = 2$ <p>(substitute in equ -1)</p> $5x - 4y + 2 = 0$ <p style="text-align: center;"><b>OR</b></p> <p>If <math>\frac{3}{5}</math> and <math>\frac{7}{3}</math> are the slopes of two lines, find the angle between the two lines</p> <p>Given: <math>m_1 = \frac{3}{5} \tan \theta = \left  \frac{m_1 - m_2}{1 + m_1 m_2} \right  \tan \theta = \left  \frac{\frac{3}{5} - \frac{7}{3}}{1 + \frac{53}{37}} \right </math></p> $m_2 = \frac{7}{3} \tan \theta = \left  \frac{\frac{9-35}{15}}{1 + \frac{21}{15}} \right  \tan \theta = \left  \frac{\frac{-26}{15}}{\frac{15+21}{15}} \right $ $\tan \theta = \left  \frac{-26}{36} \right  \tan \theta = \left  \frac{-13}{18} \right $ $\tan \theta = \frac{13}{18} \theta = \tan^{-1} \left( \frac{13}{18} \right)$

<b>SECTION III</b>	
3(a)	<p>Consider, <math>30^\circ = 30^\circ \times \frac{\pi}{180^\circ}</math></p> $= \frac{\pi}{6} \text{ radians}$ <p>Consider</p> $\frac{\pi}{12} \text{ radians} = \frac{\pi}{12} \times \frac{180^\circ}{\pi} = \frac{180^\circ}{12}$ $\frac{\pi}{12} = 15^\circ$

	OR
	<p>We know that</p> $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ <p>Consider</p> $\text{LHS} = \tan(45^\circ + A)$ $\tan(45^\circ + A) = \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \tan A}$ $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A} = \text{RHS}$
3(b)	<p>Consider</p> $\cos(90 - \theta) = \sin \theta$ $\sin(90 - \theta) = \cos \theta$ $\text{LHS} = \sin \theta \cos(90 - \theta) + \cos \theta \sin(90 - \theta)$ $= \sin \theta \sin \theta + \cos \theta \cos \theta$ $= \sin^2 \theta + \cos^2 \theta = 1 = \text{RHS} \quad [\text{according to first trigonometric identity}]$
	OR
	$\text{LHS} = \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$ $= \frac{(\sin A \cos B + \cos A \sin B) + (\sin A \cos B - \cos A \sin B)}{(\cos A \cos B - \sin A \sin B) + (\cos A \cos B + \sin A \sin B)}$ $= \frac{2 \sin A \cos B}{2 \cos A \cos B}$ $= \frac{\sin A}{\cos A}$ $= \tan A$ $= \text{RHS}$
3(c)	<p>Consider</p> $\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\sin 75^\circ = \sin(30^\circ + 45^\circ)$ $\sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$ $\sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$
	OR
	<p>Consider</p> $\cos(360^\circ - A) = \cos(A)$ $\tan(360^\circ + A) = \tan(A)$ $\cot(270^\circ - A) = \tan(A)$ $\sin(90^\circ + A) = \cos(A)$ <p>Consider <math>\frac{\cos(360^\circ - A) \tan(360^\circ + A)}{\cot(270^\circ - A) \sin(90^\circ + A)}</math></p> $= \frac{\cos A \tan A}{\tan A \cos A} = \frac{1}{1} = 1$
3(d)	$\text{LHS} = \sin 20^\circ \sin 40^\circ \sin 80^\circ$ $= \sin 80^\circ \sin 40^\circ \sin 20^\circ$



	$= -\frac{1}{2}(\cos(120^\circ) - \cos 40^\circ)\sin 20^\circ$ $= -\frac{1}{2}\left(-\frac{1}{2} - \cos 40^\circ\right)\sin 20^\circ$ $= -\frac{1}{2}\left(-\frac{1}{2}\sin 20^\circ - \cos 40^\circ \sin 20^\circ\right)$ $= -\frac{1}{2}\left(-\frac{1}{2}\sin 20^\circ - \frac{1}{2}(\sin(60^\circ) - \sin 20^\circ)\right)$ $= -\frac{1}{2}\left(-\frac{1}{2}\sin 20^\circ - \frac{1}{2}\sin 60^\circ + \frac{1}{2}\sin 20^\circ\right)$ $= -\frac{1}{2}\left(-\frac{1}{2}\sin 60^\circ\right)$ $= -\frac{1}{2}\left(-\frac{1}{2} \times \frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{8}$ $= \text{RHS}$
	OR
	<p style="text-align: center;">We know that</p> $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\cos 2\theta = \cos^2 \theta - (1 - \cos^2 \theta)$ $\cos 2\theta = \cos^2 \theta - 1 + \cos^2 \theta$ $\cos 2\theta = 2 \cos^2 \theta - 1$

“Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and scheme of valuation prepared by me are correct”.

**“Award full marks for alternative methods of answers”**

  
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SECTION-IV	
4 (a)	$\frac{d}{dx}(x^3 + x^2 + 3x + 9) = 3x^2 + 2x + 3 + 0$ $= 3x^2 + 2x + 3$
	OR

	$y = (x+1)(x-1)$ $y = x^2 - 1$ <p>Diff w.r.t.x</p> $\frac{dy}{dx} = \frac{d}{dx}(x^2 - 1) = 2x - 0 = 2x$
4 (b)	$y = x^3 - 12x^2 - 27x + 16$ <p><i>Differentiate w.r.t.x</i></p> $\frac{dy}{dx} = \frac{d}{dx}(x^3 - 12x^2 - 27x + 16)$ $\frac{dy}{dx} = 3x^2 - 12 \times 2x - 27 + 0$ $\frac{dy}{dx} = 3x^2 - 24x - 27$ <p>For function to be maxima or minima put <math>\frac{dy}{dx} = 0</math></p> $0 = 3x^2 - 24x - 27 \quad \Rightarrow \quad 3x^2 - 24x - 27 = 0$ <p>Divide both side by 3</p> $x^2 - 8x - 9 = 0$ <p>Solve the above quadratic equation by factorization method</p> $x^2 - 8x - 9 = 0 \quad -9x^2 = -9x + x$ $x^2 - 9x + x - 9 = 0$ $x(x-9) + 1(x-9) = 0$ $(x-9)(x+1) = 0$ $x-9 = 0 \quad \text{or} \quad x+1 = 0$ $x = 9 \quad \text{or} \quad x = -1$ <p><math>x = 9, -1</math> are the stationary points</p> $\frac{dy}{dx} = 3x^2 - 24x - 27$ <p>Again diff w.r.t.x</p> $\frac{d^2y}{dx^2} = 3 \times 2x - 24 + 0$ $\frac{d^2y}{dx^2} = 6x - 24$

When  $x = -1$

$$\left(\frac{d^2y}{dx^2}\right)_{x=-1} = 6(-1) - 24 = -6 - 24 = -30 < 0$$

Therefore the function is maxima at  $x = -1$  and its maximum value is given

$$y = x^3 - 12x^2 - 27x + 16$$

put  $x = -1$

$$y_{\max} = (-1)^3 - 12(-1)^2 - 27(-1) + 16$$

$$y_{\max} = -1 - 12 \times 1 + 27 + 16$$

$$y_{\max} = -1 - 12 + 27 + 16$$

$$y_{\max} = 43 - 13 = 30$$

When  $x = 9$

$$\left(\frac{d^2y}{dx^2}\right)_{x=9} = 6(9) - 24 = 54 - 24 = 30 > 0$$

Therefore the function is minima at  $x = 9$  and its minimum value is given

$$y = x^3 - 12x^2 - 27x + 16$$

put  $x = 9$

$$y_{\min} = (9)^3 - 12(9)^2 - 27(9) + 16$$

$$y_{\min} = 729 - 12(81) - 243 + 16$$

$$y_{\min} = 729 - 972 - 243 + 16$$

$$y_{\min} = 745 - 1215 = -470$$

OR

$$s = -16t^2 + 64t. \text{----- (1)}$$

Differentiate w.r.t.  $x$

$$\frac{ds}{dt} = \frac{d}{dt}(-16t^2 + 64t) \Rightarrow \frac{ds}{dt} = -16 \times 2t + 64$$

$$\frac{ds}{dt} = -32t + 64$$

when ball reaches maximum height at that time velocity  $v = 0$

$$0 = -32t + 64$$

$$32t = 64$$

$$t = \frac{64}{32} = 2$$

$$t = 2 \text{ sec}$$

Therefore the ball taken time to reach maximum height is 2 sec.

Now find maximum height put  $t = 2 \text{ sec}$  in (1)

$$s = -16t^2 + 64t$$

$$s = -16(2)^2 + 64(2)$$

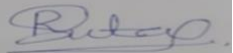
$$s = -16 \times 4 + 128 = -64 + 128 = 64 \text{ feet}$$

4 (c)	<p> <math>y = Ae^{mx} + Be^{-mx}</math> ----- (1)         </p> <p> <i>Differentiate w.r.t.x</i> </p> $\frac{dy}{dx} = \frac{d}{dx}(Ae^{mx} + Be^{-mx})$ $\frac{dy}{dx} = A \times e^{mx} \times m + B \times e^{-mx} \times -m$ $\frac{dy}{dx} = Ame^{mx} - Bme^{-mx}$ <p> <i>Again differentiate w.r.t.x</i> </p> $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(Ame^{mx} - Bme^{-mx})$ $\frac{d^2y}{dx^2} = Am \times e^{mx} \times m - Bm \times e^{-mx} \times -m$ $\frac{d^2y}{dx^2} = Am^2e^{mx} + Bm^2e^{-mx} = m^2(Ae^{mx} + Be^{-mx})$ $\frac{d^2y}{dx^2} = m^2y \text{ from equation (1)}$ $\frac{d^2y}{dx^2} - m^2y = 0$
	OR
	<p>           Let <math>y = \frac{1 + \sin x}{1 - \sin x}</math> </p> <p> <i>Differentiate w.r.t.x</i> </p> $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1 + \sin x}{1 - \sin x}\right) \quad \text{w.k.t } \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} = \left( \frac{(1 - \sin x) \frac{d}{dx}(1 + \sin x) - (1 + \sin x) \frac{d}{dx}(1 - \sin x)}{(1 - \sin x)^2} \right)$ $\frac{dy}{dx} = \frac{(1 - \sin x)(0 + \cos x) - (1 + \sin x)(0 - \cos x)}{(1 - \sin x)^2}$ $\frac{dy}{dx} = \frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$ $\frac{dy}{dx} = \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}$ $\frac{dy}{dx} = \frac{2 \cos x}{(1 - \sin x)^2}$
4 (d)	<p> <math>y = 1 - x^3</math> </p> <p> <i>Differentiate w.r.t.x</i> </p> $\frac{dy}{dx} = \frac{d}{dx}(1 - x^3) \Rightarrow \frac{dy}{dx} = 0 - 3x^2$ $\frac{dy}{dx} = -3x^2$ $\left(\frac{dy}{dx}\right)_{at A(2,3)} = -3(2)^2 = -12$

	$m = -12$ The equation of normal to the curve at the point (2, 3) with slope $m = -12$ is $y - y_1 = \frac{-1}{m}(x - x_1) \Rightarrow y - 3 = \frac{-1}{-12}(x - 2)$ $y - 3 = \frac{1}{12}(x - 2) \Rightarrow 12(y - 3) = 1(x - 2)$ $12y - 36 = x - 2 \Rightarrow x - 2 - 12y + 36 = 0$ $x - 12y + 34 = 0$
	<div style="text-align: center; background-color: #f0f0f0; padding: 5px;"><b>OR</b></div> $y = (1 + x^2)\tan^{-1} x$ <p><i>Differentiate w.r.t. x</i></p> $\frac{dy}{dx} = \frac{d}{dx} [(1 + x^2)\tan^{-1} x]$ <p>w.k.t <math>\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}</math></p> $\frac{dy}{dx} = (1 + x^2) \frac{d}{dx} (\tan^{-1} x) + \tan^{-1} x \frac{d}{dx} (1 + x^2)$ $\frac{dy}{dx} = (1 + x^2) \times \frac{1}{(1 + x^2)} + \tan^{-1} x \times (0 + 2x)$ $\frac{dy}{dx} = 1 + 2x \tan^{-1} x$

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	<b>SECTION V</b>
5 a)	Integrate $2x^3 - \frac{3}{x} + 4\cos x + \frac{1}{1+x^2}$ w.r.t. $x$ $\text{Let } I = \int 2x^3 dx - \int \frac{3}{x} dx + \int 4\cos x dx + \int \frac{1}{1+x^2} dx$ $I = \int 2x^3 - \frac{3}{x} + 4\cos x + \frac{1}{1+x^2} dx$ $I = \frac{x^4}{2} - 3\log x + 4\sin x + \tan^{-1} x + c$
	<b>OR</b>
	Let $I = \int (x - 2)(x + 3) dx$ $I = \int x^2 + x - 6 dx$ $I = \frac{x^3}{3} + \frac{x^2}{2} - 6x + c$

b)	<p>Using <math>\sin^2 x = \frac{1 - \cos 2x}{2}</math></p> <p>Then <math>I = \int \sin^2 x \, dx</math></p> $I = \int \frac{1 - \cos 2x}{2} \, dx$ $I = \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx$ $I = \frac{1}{2}x - \frac{\sin 2x}{4} + c$
	<b>OR</b>
	<p>Using <math>\tan^2 x = \sec^2 x - 1</math></p> $I = \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$ $I = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \, dx$ $I = [\tan x - x]_0^{\frac{\pi}{4}}$ $I = \left( \tan \frac{\pi}{4} - \frac{\pi}{4} \right) - (\tan 0 - 0)$ $I = 1 - \frac{\pi}{4}$
c)	<p>Required Area <math>A = \int_a^b y \, dx</math></p> $A = \int_1^3 4x - x^2 - 3 \, dx$ $A = \frac{4x^2}{2} - \frac{x^3}{3} - 3x$ $A = \left[ \frac{4x^2}{2} - \frac{x^3}{3} - 3x \right]_1^3$ $A = \left[ 2(3)^2 - \frac{3^3}{3} - 3(3) \right] - \left[ 2(1)^2 - \frac{1^3}{3} - 3(1) \right]$ $A = [18 - 9 - 9] - \left[ 2 - \frac{1}{3} - 3 \right] = \frac{4}{3} \text{ sq. units}$
	<b>OR</b>
	<p>Required Volume, <math>V = \int_a^b y^2 \, dx</math></p> $V = \int_1^2 x^2 + 5x \, dx$ $V = \left[ \frac{x^3}{3} + \frac{5x^2}{2} \right]_1^2$ $V = \left[ \frac{(2)^3}{3} + \frac{5(2)^2}{2} \right] - \left[ \frac{(1)^3}{3} + \frac{5(1)^2}{2} \right]$ $V = \frac{59}{6} \text{ cubic units}$
d)	<p>Let <math>I = \int x \sin x \, dx</math></p> <p>Integrating by parts using <math>\int uv \, dx = u \int v \, dx - \int \left[ \frac{du}{dx} \cdot \int v \, dx \right] dx</math>, we get</p> $I = x \int \sin x \, dx - \int (-\cos x) 1 \, dx$ $I = x(-\cos x) + \int \cos x \, dx$

	$I = -x \cos x + \sin x + c$
	<b>OR</b>
	<p>Let <math>I = \int_0^1 \frac{(\tan^{-1}x)^3}{1+x^2} dx</math></p> <p>Substituting <math>\tan^{-1}x = t</math></p> <p>Differentiating w.r.t <math>t</math>,</p> $\frac{1}{1+x^2} = \frac{dt}{dx} \Rightarrow \frac{dx}{1+x^2} = dt$ <p>Lower limit <math>x = 0, t = 0</math>; Upper limit <math>x = 1, t = \frac{\pi}{4}</math></p> $I = \int_0^{\frac{\pi}{4}} t^3 dt = \left[ \frac{t^4}{4} \right]_0^{\frac{\pi}{4}} = \frac{\pi^4}{1024}$