3619

Code : 20SC01T

Register Number | | | | | | | | | | |

I Semester Diploma Examination, April/May-2021

ENGINEERING MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (i) Answer **one** full question from each section.

(ii) **One** full question carries **20** marks.

SECTION – I

1\. (a) Find the value of x,

if $\begin{vmatrix} 1 & 2 & 9 \\ 2 & x & 10 \\ 3 & 7 & -6 \end{vmatrix} = 0.$ **4**

(b) If $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$, find AB. **5**

(c) Solve the equations $x + y = 0$, $y + z = 1$ and $x + z = 3$ for y by Cramer's rule. **5**

(d) If $A = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$ find A^{-1}. **6**

2\. (a) Evaluate $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ **4**

(b) If $A = \begin{pmatrix} -1 & 0 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ prove that adj (AB) = [adj(B) adj(A)]. **5**

(c) Verify whether AB = BA for the matrices

$A = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 4 \\ 0 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix}$. **5**

(d) Find the characteristic equation and eigen values for the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$. **6**

Turn over

SECTION – II

3. (a) Find the slope of the line passing through the points (2, 4) and (8, 7).

(b) Write the standard form of point-slope form of the straight line and find the equation of the straight line passing through the point (5, 6) and slope of 3 units.

(c) Find the equation of the straight line whose 'x'-intercept and y-intercept are 3 and 4 respectively by writing the standard form of it.

(d) Find the acute angle between the lines $x + 3y + 1 = 0$ and $2x - y + 4 = 0$.

4. (a) (i) Find the slope of the straight line which is making an angle of 30° with the x-axis.

(ii) Find the x-intercept and y-intercept of the line $3x - 2y = 6$.

(b) Find the equation of the straight line which has an angle of inclination 45° with x-axis and y-intercept of 2 units by writing its standard form.

(c) Write the standard form of straight line. Find the equation of the straight line passing through the points (2, –3) and (5, 4).

(d) Find the equation of the straight line passing through the points (–3, 2) and perpendicular to the line $4x - y + 7 = 0$.

SECTION – III

5. (a) (i) Express $\frac{5\pi^C}{2}$ in degrees.

(ii) Express 105° in radians.

(b) Prove that :

$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A.$$

(c) Prove that : $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(d) Prove that : $\cos 20 \cos 40 \cos 60 \cos 80 = \frac{1}{16}$.

6. (a) Find the value of $\cos 75°$. 4

(b) Simplify

$$\frac{\sin(-\theta)}{\sin(\pi-\theta)} - \frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi-\theta)} + \frac{\cos\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{3\pi}{2}-\theta\right)}.$$ 5

(c) If $\tan A = \frac{1}{3}$; $\tan B = \frac{1}{2}$, find $\tan(A+B)$. 5

(d) Without using calculator and table find the value of 6

$\sin 600° \cos 330° + \cos 120° \sin 150°$.

SECTION – IV

7. (a) If $y = 3x^3 + 5\log x - 2e^{3x} + \tan^{-1}x$ find $\frac{dy}{dx}$. 4

(b) If $y = \frac{1-\tan x}{1+\tan x}$ find $\frac{dy}{dx}$ 5

(c) If $y = (e^x - \sin^{-1}x + 4\log x)^{10}$ find $\frac{dy}{dx}$. 5

(d) If $S = t^3 - t^2 + 9t + 8$ where S is the distance travelled by particle in t seconds. Find the velocity and acceleration at $t = 2$ seconds. 6

8. (a) If $y = x^5 - 3e^{[illegible]} + 2\cos x + \sin^{-1}x$ find $\frac{dy}{dx}$. 4

(b) If $y = x^2 \log(e^x)$ find $\frac{dy}{dx}$. 5

(c) If $y = \tan^{-1}x$ show that $(1+x^2)y_2 + 2xy_1 = 0$. 5

(d) Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8x - 6$ at the point $(1, -1)$. 6

[Turn over

SECTION – V

9. (a) Evaluate $\int \left(x^4 + \frac{5}{x} + e^x - 3\,\text{cosec}^2 x\right) dx$.

(b) Evaluate $\int_0^{\pi/2} \sin^2 x \, dx$.

(c) Evaluate $\int x \log x \, dx$.

(d) Find the area bounded by the curve $y = x^2 + 1$, x-axis and the coordinates $x = 1; x = 2$.

10. (a) Evaluate $\int_1^2 x^3 \, dx$.

(b) Evaluate $\int \sin^6 x \cos x \, dx$.

(c) Evaluate $\int x e^x \, dx$.

(d) Find the volume generated by rotating the curve $y = \sqrt{x+2}$ about x-ax between $x = 0$ and $x = 2$.

I Semester Diploma Examination, March/April 2021

Subject: Engineering Mathematics   Subject Code: 20SC01T

SCHEME OF VALUATION

Q. No.	Particular	Marks
	SECTION – 1	
	a) Expansion	2
	Answer	2 = $\boxed{4}$
	b) Multiplication by each row	1 + 1 + 1
	Simplification	2 = $\boxed{5}$
1	c) Finding Δ, Δ_2	2 + 2
	Finding y	1 = $\boxed{5}$
	d) Finding adjoint of A	3
	Finding $\lvert A\rvert$, formula	1 + 1 = $\boxed{6}$
	Simplification	1
	a) Expansion	2
	Simplification of $\lvert A\rvert$	1 = $\boxed{4}$
	Result	1
	b) Finding AB	1
	Finding $adj(A), adj(B), adj(AB)$	1 + 1 + 1 = $\boxed{5}$
2	Proof	1
	c) Finding AB and BA	2 + 2
	Result	1 = $\boxed{5}$
	d) Formula $\lvert A-\lambda I\rvert = 0$	1
	Finding characteristic equation	3 = $\boxed{6}$
	Eigen values (Characteristic Roots)	2
	SECTION – 2	
	a) Slope Formula	1
	Substitution	1 = $\boxed{4}$
	Simplification and Result	2
	b) Point-Slope form	1
	Substitution	1 = $\boxed{5}$
	Simplification and Result	3
3	c) Intercept formula	1
	Substitution	1 = $\boxed{5}$
	Simplification and Result	3
	d) Angle formula	1
	Finding m_1, m_2	1 + 1 = $\boxed{6}$
	Substitution	1
	Simplification and Result	2

Q.	Steps	Marks	Sub-total	Total
4	a) (i) Slope Formula	1	$\rightarrow \boxed{2}$	$= \boxed{4}$
	Substitution and calculation	1		
	(ii) Finding x – intercept	1	$\rightarrow \boxed{2}$	
	Finding y – intercept	1		
	b) Slope-intercept form	1		$= \boxed{5}$
	Finding m	1		
	Substitution	1		
	Calculation and Result	2		
	c) Two-point form formula	1		$= \boxed{5}$
	Substitution	1		
	Calculation and Result	3		
	d) Condition for perpendicular lines	1		$= \boxed{6}$
	Finding slope	1		
	Point slope form	1		
	Simplification and Result	3		

SECTION – 3

Q.	Steps	Marks	Sub-total	Total
5	a) (i) Conversion formula	1	$\rightarrow \boxed{2}$	$= \boxed{4}$
	Calculation	1		
	(ii) Conversion formula	1	$\rightarrow \boxed{2}$	
	Calculation	1		
	b) Each term formula	1+1+1+1		$= \boxed{5}$
	Simplification	1		
	c) Applying $\sin 3\theta = \sin(2\theta + \theta)$	1		$= \boxed{5}$
	Apply $\sin(A+B)$ formula	1		
	$\sin 2\theta$ formula	1		
	Simplification	2		
	d) $\cos 60^\circ$ value	1		$= \boxed{6}$
	$\cos A \cos B$ formula	2		
	Simplification and proof	3		
6	a) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$	1		$= \boxed{4}$
	$\cos(30^\circ + 45^\circ)$ formula	1		
	Simplification	2		
	b) Simplification of each term	3		$= \boxed{5}$
	Calculation and Result	2		
	c) $\tan(A+B)$ formula	1		$= \boxed{5}$
	Substitution each term	1+1		
	Calculation and Result	2		
	d) Simplification of each term	1+1+1+1		$= \boxed{6}$
	Calculation and Result	2		

SECTION – 4

Q. No.	Marking criteria	Marks	Total
7	a) Differentiation of each term	1+1+1+1	$\boxed{4}$
	b) Quotient Rule formula	1	$\boxed{5}$
	Differentiation of each term	2	
	Simplification and result	2	
	c) Chain rule formula	1	$\boxed{5}$
	Differentiation	1	
	Differentiation of each term	1+1+1	
	d) Finding $(v = ds/dt)$	2	$\boxed{6}$
	Finding acceleration $(a = d^2s/dt^2)$	2	
	Finding v	1	
	Finding a	1	
8	a) Differentiation of each term	1+1+1+1	$\boxed{4}$
	b) Product Rule formula	1	$\boxed{5}$
	Differentiation of x^2	1	
	Differentiation $\log(e^x)$	1+1	
	Simplification and result	1	
	c) Finding y_1	1+1	$\boxed{5}$
	Finding y_2	1+1	
	Simplification	1	
	d) Differentiation y	1	$\boxed{6}$
	Finding slope	1	
	Point-Slope form	1	
	Simplification and calculation	3	

SECTION – 5

Q. No.	Marking criteria	Marks	Total
9	a) Integrating each term	1+1+1+1	$\boxed{4}$
	b) Integration of $\sin^2 x$ formula	1	$\boxed{5}$
	Integration of each term	1+1	
	Substitution upper and lower limit	1	
	Simplification and result	1	
	c) Formula of integration by parts	1	$\boxed{5}$
	Substitution	1	
	Simplification	3	
	d) Formula of Area	1	$\boxed{6}$
	Substitution	1	
	Integration of each term	1+1	
	Calculation and result	2	

	a) Integration --	1	$= \boxed{4}$
	Substitution of upper limit and lower limits ----------------------	1	
	Calculation and result --	2	
	b) Substitution --	1	$= \boxed{5}$
	Differentiation --	1	
	Integration & Result --	3	
10	c) Formula of integration by parts --------------------------------	1	$= \boxed{5}$
	Substitution --	1	
	Calculation and result --	3	
	d) Formula of Volume --	1	$= \boxed{6}$
	Substitution --	1	
	Integration ---	1	
	Substitution of upper and lower limits ----------------------------	1	
	Simplification and Result --	2	

"Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and some scheme of valuation prepared by me are correct."

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Department of Technical Education

I Semester Diploma Examination, April/May-2021

ENGINEERING MATHEMATICS: 20SC01T

MODEL ANSWERS

Section: I

1. a. Find the value of x $\begin{vmatrix} 1 & 2 & 9 \\ 2 & x & 0 \\ 3 & 7 & -6 \end{vmatrix} = 0$

Solution:

$$\begin{vmatrix} 1 & 2 & 9 \\ 2 & x & 0 \\ 3 & 7 & -6 \end{vmatrix} = 0$$

$$1(-6x-0)-2(-12-0)+9(14-3x)=0$$

$$-6x+24+126-27x=0$$

$$-33x+150=0$$

$$33x=150$$

$$x=\frac{150}{33}=\frac{50}{11}$$

b. If $A=\begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ *and* $B=\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$ then find AB.

Solution:

$$AB=\begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$$

$$AB=\begin{bmatrix} 10+4 & -2+1 \\ 20+20 & -4+5 \end{bmatrix}$$

$$AB=\begin{bmatrix} 14 & -1 \\ 40 & 1 \end{bmatrix}$$

c. Solve the equations $x + y = 0, y + z = 1 \,\&\, x + z = 3$ **for y by cramer's Rule.**

Solution: Given; $x + y + 0z = 0, 0x + y + z = 1 \,\&\, x + 0y + z = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1) + 0(0-1) = 1 + 1 + 0 = 2$$

$$\Delta = 2$$

$$\Delta_y = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 1(1-3) + 0(0-1) + 0(0-1) = 1(-2) + 0 + 0 = -2$$

$$\Delta_y = -2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-2}{2} = -1$$

d. If $A=\begin{bmatrix}3 & 1 & 2\\ -2 & 1 & 1\\ 3 & 0 & 2\end{bmatrix}$ find A^{-1}

Solution:

$$|A|=\begin{vmatrix}3 & 1 & 2\\ -2 & 1 & 1\\ 3 & 0 & 2\end{vmatrix}=3(2-0)-1(-4-3)+2(0-3)=6+7-6=7\neq 0$$

$$\text{Co}-\text{factor of }3=(2-0)=2$$
$$\text{Co}-\text{factor of }1=-(-4-3)=7$$
$$\text{Co}-\text{factor of }2=(0-3)=-3$$
$$\text{Co}-\text{factor of }-2=-(2-0)=-2$$
$$\text{Co}-\text{factor of }1=(6-6)=0$$
$$\text{Co}-\text{factor of }1=-(0-3)=3$$
$$\text{Co}-\text{factor of }3=(1-2)=-1$$
$$\text{Co}-\text{factor of }0=-(3+4)=-7$$
$$\text{Co}-\text{factor of }2=(3+2)=5$$

$$adjA=\begin{bmatrix}2 & -2 & -1\\ 7 & 0 & -7\\ -3 & 3 & 5\end{bmatrix}\quad \therefore\quad A^{-1}=\frac{adjA}{|A|}=\frac{1}{7}\begin{bmatrix}2 & -2 & -1\\ 7 & 0 & -7\\ -3 & 3 & 5\end{bmatrix}$$

2. a Evaluate $\begin{vmatrix}2 & 3 & -1\\ 3 & -2 & 1\\ 1 & 1 & 2\end{vmatrix}$

Solution:

$$\begin{vmatrix}2 & 3 & -1\\ 3 & -2 & 1\\ 1 & 1 & 2\end{vmatrix}=2(-4-1)-3(6-1)-1(3+2)=2(-5)-3(5)-1(5)$$
$$=-10-15-5=-30$$

b If If $A=\begin{pmatrix}-1 & 0\\ 5 & 3\end{pmatrix}$ *and* $B=\begin{pmatrix}3 & 5\\ 2 & 4\end{pmatrix}$ prove that $\operatorname{adj}(AB)=\operatorname{adj}(B)\operatorname{adj}(A)$

Solution:

$$AB=\begin{pmatrix}-1 & 0\\ 5 & 3\end{pmatrix}\begin{pmatrix}3 & 5\\ 2 & 4\end{pmatrix}=\begin{pmatrix}-3+0 & -5+0\\ 15+6 & 25+12\end{pmatrix}=\begin{pmatrix}-3 & -5\\ 21 & 37\end{pmatrix}$$

$$\operatorname{adj}(AB)=\begin{pmatrix}37 & 5\\ -21 & -3\end{pmatrix}\text{-------------------(1)}$$

$$\text{If } A=\begin{pmatrix}-1 & 0\\ 5 & 3\end{pmatrix} \text{ then } \operatorname{adj}A=\begin{pmatrix}3 & 0\\ -5 & -1\end{pmatrix}$$

$$\text{If } B=\begin{pmatrix}3 & 5\\ 2 & 4\end{pmatrix} \text{ then } \operatorname{adj}B=\begin{pmatrix}4 & -5\\ -2 & 3\end{pmatrix}$$

$$\operatorname{adj}(B)\operatorname{adj}(A)=\begin{pmatrix}4 & -5\\ -2 & 3\end{pmatrix}\begin{pmatrix}3 & 0\\ -5 & -1\end{pmatrix}=\begin{pmatrix}12+25 & 0+5\\ -6-15 & 0-3\end{pmatrix}$$

$$\operatorname{adj}(B)\operatorname{adj}(A)=\begin{pmatrix}37 & 5\\ -21 & -3\end{pmatrix}\text{-----------------(2)}$$

From equations (1) & (2)

$\operatorname{adj}(AB)=\operatorname{adj}(B)\operatorname{adj}(A)$

Hence proved the result

c. Verify whether AB=BA for the matrices $A=\begin{bmatrix}1 & 0 & 5\\ -1 & 2 & 1\\ 5 & 4 & 3\end{bmatrix}$ *and* $B=\begin{bmatrix}3 & -1 & 4\\ 0 & -1 & 1\\ 2 & 4 & -2\end{bmatrix}$

Solution:

$$AB=\begin{bmatrix}1 & 0 & 5\\ -1 & 2 & 1\\ 5 & 4 & 3\end{bmatrix}\begin{bmatrix}3 & -1 & 4\\ 0 & -1 & 1\\ 2 & 4 & -2\end{bmatrix}=\begin{bmatrix}3+0+10 & -1-0+20 & 4+0-10\\ -3+0+2 & 1-2+4 & -4+2-2\\ 15+0+6 & -5-4+12 & 20+4-6\end{bmatrix}=\begin{bmatrix}13 & 19 & -6\\ -1 & 3 & -4\\ 21 & 3 & 18\end{bmatrix}\text{---(1)}$$

$$BA=\begin{bmatrix}3 & -1 & 4\\ 0 & -1 & 1\\ 2 & 4 & -2\end{bmatrix}\begin{bmatrix}1 & 0 & 5\\ -1 & 2 & 1\\ 5 & 4 & 3\end{bmatrix}=\begin{bmatrix}3+1+20 & 0-2+16 & 15-1+12\\ 0+1+5 & 0-2+4 & 0-1+3\\ 2-4-10 & 0+8-8 & 10+4-6\end{bmatrix}=\begin{bmatrix}24 & 14 & 26\\ 6 & 2 & 2\\ -12 & 0 & 8\end{bmatrix}\text{----(2)}$$

From Equations (1) and (2) $\quad AB\neq BA$

d. Find the characteristic equation and Eigen values for the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

Solution:

$$A - \lambda I = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-\lambda & -1 \\ -3 & 1-\lambda \end{bmatrix}$$

AW.k.t Chalractrestic Equation is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 \\ -3 & 1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(1-\lambda) - 3 = 0$$

$$2 - 2\lambda - \lambda + \lambda^2 - 3 = 0$$

$\lambda^2 - 3\lambda - 1 = 0$ charactrestic Equation

$a = 1, b = -3 \,\&\, c = -1$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$\lambda = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

$\therefore \lambda = \frac{3+\sqrt{13}}{2} \;\&\; \lambda = \frac{3-\sqrt{13}}{2}$ are the required charactrestic roots

Section-II

3. a. Find the slope of the line passing through the points (2,4) and (8,7)

Solution:

Given: line joining points (2, 4) and (8,7)

W.K.T Slope of straight line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7-4}{8-2}$$

$$m = \frac{3}{6}$$

$$m = \frac{1}{2}$$

b. Write the standard form of point-slope form of the straight line and find the equation of the straight line passing through the point (5,6) and slope of 3 units.

Solution:

The standard form of Slope-point form of equation of straight line is

$y - y_1 = m(x - x_1)$

Given; point $A(x_1, y_1) = (5,6)$ and slope=m=3

w.k.t Equation of straight line is of the form

$y - y_1 = m(x - x_1)$

$y - 6 = 3(x - 5)$

$y - 6 = 3x - 15$

$3x - 15 = y - 6$

$3x - 15 - y + 6 = 0$

$3x - y - 9 = 0$

c. Find the equation of straight line whose x-intercept and y-intercept are 3 and 4 respectively by writing the standard form of it.

Solution:

Given, x-intercept=a=3 and y-intercept=b=4

w.k.t, Equation of straight line of intercept form

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{4} = 1$

$\Rightarrow \frac{4x + 3y}{12} = 1$

$\Rightarrow 4x + 3y = 12$

$\Rightarrow 4x + 3y - 12 = 0$

d. Find the acute angle between the lines x+3y+1=0 and 2x-y+4=0

Solution:

Given; x+3y+1=0------------------------------- (1)

2x-y+4=0------------------------------- (2)

Slope of line (1) = $m_1 = -\frac{a}{b} = -\frac{1}{3}$

Slope of line (2) = $m_2 = -\frac{a}{b} = -\frac{2}{-1} = 2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-\frac{1}{3} - 2}{1 + \left(-\frac{1}{3}\right) \times 2}\right| = \left|\frac{\frac{-1-6}{3}}{\frac{3-2}{3}}\right| = \left|\frac{\frac{-7}{3}}{\frac{1}{3}}\right| = |-7| = 7$

$\Rightarrow \tan\theta = 7$

$\therefore \theta = \tan^{-1}(7)$

4. a (i) Find the slope of the straight line which is making an angle of 30^0 with the x-axis

Given: Inclination $\theta = 30^0$

W.K.T Slope of straight line is

$$m = \tan\theta$$

$$m = \tan 30^0$$

$$m = \frac{1}{\sqrt{3}}$$

a. (ii) Find the x-intercept and y-intercept of the line 3x-2y=6

Given; a straight line 3x-2y-6=0------------------------ (1)

Comparing with std st line ax+by+c=0---------------------- (2)

$$\Rightarrow a = 3, b = -2 \ \& \ c = -6$$

$$\text{x} - \text{intercept} = -\frac{c}{a} = -\frac{-6}{3} = 2$$

$$\text{y} - \text{intercept} = -\frac{c}{b} = -\frac{-6}{-2} = -3$$

b. Find the equation of the straight line which has an angle of inclination 45^0 with x-axis and y-intercept of 2 units by writing its standard form.

Solution:

Given; Inclination $\theta = 45^0$ and Y-intercept=c=2

w.k.t. slope of line

$$m = \tan\theta$$

$$m = \tan 45^0 = 1$$

W.K.T, Equation of straight line is of the form

$$y = mx + c$$

$$y = 1x + 2$$

$$x + 2 = y$$

$$x + 2 - y = 0$$

$$x - y + 2 = 0$$

c. **Write the standard form of straight line. Find the equation of the straight line passing through the points (2,-3) and (5,4)**

Solution:

Two-point form of equation of straight line is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Given; Here $A(x_1, y_1) = (2,-3) \;\&\; B(x_2, y_2) = (5,4)$

Let P(x,y) be any point on the line.

w.k.t, Equation of line passing through two-points is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 3 = \frac{4+3}{5-2}(x-2)$$

$$y + 3 = \frac{7}{3}(x-2)$$

$$3(y+3) = 7(x-2)$$

$$3y + 9 = 7x - 14$$

$$7x - 14 - 3y - 9 = 0$$

$$7x - 3y - 23 = 0$$

d. **Find the equation of straight line passing through the points (-3,2) and perpendicular to the line 4x-y+7=0**

Solution:

Given line is 4x-y+7=0

It is in the form ax+by+c=0

Here a=4,b=--1

Slope of given line$= -\frac{a}{b} = -\frac{4}{-1} = 4.$

Since the required line is perpendicular to given line,

$\therefore$Slope of required line $= -\frac{1}{\text{slope of given line}} = -\frac{1}{4}.$

The required line passes through the point $A(x_1, y_1) = (-3,2)$ with

slope=m=$-\frac{1}{4}$ and its equation is given by

$\Rightarrow \quad y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 2 = -\frac{1}{4}(x - (-3))$

$\Rightarrow \quad 4(y-2) = -1(x+3)$

$\Rightarrow \quad 4y - 8 = -x - 3$

$\Rightarrow \quad 4y - 8 + x + 3 = 0$

$\Rightarrow \quad x + 4y - 5 = 0$ is the required line.

Alternative method

Consider straight line

$4x - y + 7 = 0 ----------(1)$

$ax + by + c = 0 ----------(2)$

By compairing equations (1) &(2)

$a = 4, b = -1$

Equation of straight line perpendicular to(1) *is given by*

$bx - ay + k = 0$

$-1x - 4y + k = 0$

$x + 4y - k = 0 -----------(3)$

Since required line is passing through $(-3,2)$

$-3 + 4(2) - k = 0$

$k = 8 - 3 \qquad \Rightarrow k = 5$

Subtitute $k = 5$ *in* (3)

$x + 4y - 5 = 0$

Section-III

5. a. (i) Express $\frac{5\pi^c}{2}$ in degrees.

Solution:

$$\frac{5\pi^C}{2} = \frac{5\pi}{2} \times \frac{180^0}{\pi} = 5 \times 90^0 = 450^0.$$

a. (ii) Express 105^0 in radians

Solution:

$$105^0 = 105 \times \frac{\pi}{180} = \frac{7\pi}{12}.$$

b. Prove that $\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A$

Solution:

$$L.H.S = \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$

$$L.H.S = \frac{\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B}{\cos A\cos B - \sin A\sin B + \cos A\cos B + \sin A\sin B}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B} = \frac{\sin A}{\cos A} = \tan A = R.H.S$$

c. Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$

Solution:

$$\sin 3\theta = \sin(2\theta + \theta)$$

$$\sin 3\theta = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$$

$$\sin 3\theta = 2\sin\theta\cos\theta \times \cos\theta + (1 - 2\sin^2\theta)\sin\theta$$

$$\sin 3\theta = 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$$

$$\sin 3\theta = 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$$

$$\sin 3\theta = 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Hence proved the result

d. Prove that $\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \frac{1}{16}$.

Solution:

$$L.H.S = \cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0$$

$$= \cos 60^0 \left(\cos 40^0 \cos 20^0\right) \cos 80^0$$

$$W.K.T \ \cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \ \& \cos 60^0 = \frac{1}{2}$$

$$L.H.S = \frac{1}{2} \times \frac{1}{2}\left[\cos\left(40^0 + 20^0\right) + \cos\left(40^0 - 20^0\right)\right]\cos 80^0$$

$$= \frac{1}{4}\left(\cos 60^0 + \cos 20^0\right)\cos 80^0$$

$$= \frac{1}{4}\left(\frac{1}{2} + \cos 20^0\right)\cos 80^0$$

$$= \frac{1}{8}\cos 80^0 + \frac{1}{4}\cos 80^0 \cos 20^0$$

$$= \frac{1}{8}\cos\left(180 - 100^0\right) + \frac{1}{4} \times \frac{1}{2}\left[\cos\left(80^0 + 20^0\right) + \cos\left(80^0 - 20^0\right)\right]$$

$$= \frac{1}{8} \times -\cos 100^0 + \frac{1}{8}\left(\cos 100^0 + \cos 60^0\right)$$

$$= -\frac{1}{8}\cos 100^0 + \frac{1}{8}\cos 100^0 + \frac{1}{8}\cos 60^0$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = R.H.S$$

6. **a. Find the value of** $\cos 75^0$

Solution:

$$\cos 75^0 = \cos\left(45^0 + 30^0\right)$$

$$W.K.T \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos 75^0 = \cos 45^0 \cos 30^0 - \sin 45^0 \sin 30^0$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 75^0 = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

b. Simplify $\dfrac{\sin(-\theta)}{\sin(\pi-\theta)}-\dfrac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi-\theta)}+\dfrac{\cos\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{3\pi}{2}-\theta\right)}$

Solution:

$$\frac{\sin(-\theta)}{\sin(\pi-\theta)}-\frac{\tan\left(\frac{\pi}{2}-\theta\right)}{\cot(\pi-\theta)}+\frac{\cos\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{3\pi}{2}-\theta\right)}$$

$$=\frac{-\sin\theta}{\sin\theta}-\frac{\cot\theta}{-\cot\theta}+\frac{-\sin\theta}{-\sin\theta}$$

$$=-1+1+1=1$$

c. If $\tan A=\dfrac{1}{3}$ *and* $\tan B=\dfrac{1}{2}$, **find the value of** $\tan(A+B)$.

Solution:

Given $\tan A=\dfrac{1}{3}$ *and* $\tan B=\dfrac{1}{2}$

Consider

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}$$

$$=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{6}}=\frac{\frac{2+3}{6}}{\frac{6-1}{6}}=\frac{\frac{5}{6}}{\frac{5}{6}}$$

$$\tan(A+B)=1$$

d. Without using calculator and table find the value

$$\sin 600^0 \cos 330^0 + \cos 120^0 \sin 150^0$$

Solution:

$$\begin{aligned}
\textit{Consider } & \sin 600^0 \cos 330^0 + \cos\left(120^0\right)\sin 150^0 \\
&= \sin\left(360 + 240^0\right)\cos\left(360 - 30^0\right) + \cos\left(180 - 60^0\right)\sin\left(180 - 30^0\right) \\
&= \sin 240^0 \cos 30^0 + \times - \cos 60^0 \sin 30^0 \\
&= \sin\left(180 + 60^0\right)\cos 30^0 - \cos 60^0 \sin 30^0 \\
&= -\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 \\
&= -\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{1}{2} \\
&= -\frac{3}{4} - \frac{1}{4} = \frac{-3-1}{4} = -\frac{4}{4} \\
&= -1
\end{aligned}$$

Section-IV

7. a. If $y = 3x^3 + 5\log x - 2e^{3x} + \tan^{-1} x$ **find** $\frac{dy}{dx}$.

Solution:

$$y = 3x^3 + 5\log x - 2e^{3x} + \tan^{-1} x$$

Differenti ate w.r.t.x

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^3 + 5\log x - 2e^{3x} + \tan^{-1} x\right)$$

$$\frac{dy}{dx} = 3 \times 3x^2 + 5 \times \frac{1}{x} - 2 \times e^{3x} \times 3 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = 9x^2 + \frac{5}{x} - 6e^{3x} + \frac{1}{1+x^2}$$

b. If $y = \frac{1-\tan x}{1+\tan x}$ **find** $\frac{dy}{dx}$.

Solution:

$$y = \frac{1-\tan x}{1+\tan x}$$

Differentiate w.r.t.x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$w.k.t\ \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \left(\frac{(1+\tan x)\frac{d}{dx}(1-\tan x) - (1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}\right)$$

$$\frac{dy}{dx} = \frac{(1+\tan x)(0-\sec^2 x) - (1-\tan x)(0+\sec^2 x)}{(1+\tan x)^2}$$

$$\frac{dy}{dx} = \frac{(1+\tan x)(-\sec^2 x) - (1-\tan x)(\sec^2 x)}{(1+\tan x)^2}$$

$$\frac{dy}{dx} = \frac{-\sec^2 x - \tan x \sec^2 x - \sec^2 x + \tan x \sec^2 x}{(1+\tan x)^2}$$

$$\frac{dy}{dx} = \frac{-2\sec^2 x}{(1+\tan x)^2}$$

c. If $y = \left(e^x - \sin^{-1} x + 4\log x\right)^{10}$ **find** $\frac{dy}{dx}$

Solution:

$$y = \left(e^x - \sin^{-1} x + 4\log x\right)^{10}$$

$$Diff \ \ w.r.t.x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x - \sin^{-1} x + 4\log x\right)^{10}$$

$$\frac{dy}{dx} = 10\left(e^x - \sin^{-1} x + 4\log x\right)^{9}\left(e^x - \frac{1}{\sqrt{1-x^2}} + \frac{4}{x}\right)$$

d. If $S = t^3 - t^2 + 9t + 8$ **where S is the distance travelled by particle in t seconds.**
Find the velocity and acceleration at t=2 seconds
Solution:

$$S = t^3 - \cancel{9}t^2 + 9t + 8$$

$t^3 - t^2 + 9t + 8$

$$Differentiate \ \ w.r.t.time \ t$$

$$\frac{dS}{dt} = \frac{d}{dt}\left(t^3 - t^2 + 9t + 8\right)$$

$$\frac{dS}{dt} = 3t^2 - 2t + 9 + 0$$

$$\frac{dS}{dt} = 3t^2 - 2t + 9$$

$$Again \ \ diff \ \ w.r.t.x$$

$$\frac{d^2S}{dt^2} = 6t - 2 + 0$$

$$\frac{d^2S}{dt^2} = 6t - 2$$

$$wkt \ Velocity \quad V = \frac{ds}{dt}$$

$$V = 3t^2 - 2t + 9$$

$$V_{at\ t=2} = 3(2)^2 - 2(2) + 9 = 12 - 4 + 9 = 17 unit \backslash \sec$$

$$wkt \ acceleration \quad a = \frac{d^2s}{dt^2}$$

$$a = 6t - 2$$

$$a_{at\ t=2} = 6 \times 2 - 2 = 12 - 2 = 10 \ unit \backslash \sec^2$$

8. a. If $y = x^5 - 3e^{-x} + 2\cos x + \sin^{-1} x$ *find* $\frac{dy}{dx}$.

Solution:

$$y = x^5 - 3e^{-x} + 2\cos x + \sin^{-1} x$$

Diff w.r.t.x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 - 3e^{-x} + 2\cos x + \sin^{-1} x\right)$$

$$\frac{dy}{dx} = 5x^4 + 3e^{-x} - 2\sin x + \frac{1}{\sqrt{1-x^2}}$$

b. If $y = x^2 \log e^x$ *find* $\frac{dy}{dx}$.

Solution:

$$y = x^2 \log e^x$$

Diff w.r.t.x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 \log e^x\right)$$

$$\frac{dy}{dx} = x^2 \times \frac{1}{e^x} \times e^x + \log e^x \times 2x$$

$$\frac{dy}{dx} = x^2 + 2x\log\left(e^x\right)$$

c If $y = \tan^{-1} x$ **then prove that** $\left(1+x^2\right)y_2 + 2xy_1 = 0$.

Solution:

$$y = \tan^{-1} x$$

Differentiate w.r.t.x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\frac{dy}{dx} = \frac{1}{1+x^2} \text{---------------(1)}$$

Again differentiate w.r.t.x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-1}{\left(1+x^2\right)^2} \times (0+2x)$$

$$\frac{d^2y}{dx^2} = \frac{-2x}{\left(1+x^2\right)^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{\left(1+x^2\right)}\frac{-2x}{\left(1+x^2\right)} = \frac{dy}{dx} \times \frac{-2x}{\left(1+x^2\right)} \quad \textit{from (1)}$$

$$\left(1+x^2\right)\frac{d^2y}{dx^2} = -2x\frac{dy}{dx}$$

$$\left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

$$\left(1+x^2\right)y_2 + 2xy_1 = 0$$

d. Find the equation of the tangent to the curve $y=2x^3-5x^2+8x-6$ at the point (1,-1)

Solution:

$$y=2x^3-5x^2+8x-6$$

Differentiate w.r.t.x

$$\frac{dy}{dx}=\frac{d}{dx}\left(2x^3-5x^2+8x-6\right)$$

$$\frac{dy}{dx}=2\times 3x^2-5\times 2x+8-0$$

$$\frac{dy}{dx}=6x^2-10x+8$$

$$\left(\frac{dy}{dx}\right)_{at\ A(1,-1)}=6(1)^2-10\times 1+8=6-10+8=4$$

$$m=4$$

The equation of tangent to the curve at the point A (1, -1) with slope m=4 is

$$y-y_1=m(x-x_1)$$

$$y-(-1)=4(x-1)$$

$$y+1=4x-4$$

$$4x-4-y-1=0$$

$$4x-y-5=0$$

Section-V

9. a. Evaluate $\int\left(x^4+\frac{5}{x}+e^x-3\operatorname{cosec}^2 x\right)dx$

Solution:

$$I=\int\left(x^4+\frac{5}{x}+e^x-3\operatorname{cosec}^2 x\right)dx$$

$$I=\frac{x^5}{5}+5\times\log x+e^x-3\times-\cot x$$

$$I=\frac{x^5}{5}+5\log x+e^x+3\cot x+c$$

b. Evaluate $\int_0^{\frac{\pi}{2}}\sin^2 x\,dx$

Solution:

$$\int_0^{\frac{\pi}{2}}\sin^2 x\,dx=\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2x}{2}\right)dx$$

$$=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin 2\times\frac{\pi}{2}}{2}\right)-\left(0-\frac{\sin 2\times 0}{2}\right)\right]$$

$$=\frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0-\frac{\sin 0}{2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}-0-0+0\right]=\frac{\pi}{4}$$

c. Evaluate $\int(x\log x)dx$

Solution:

$$I=\int(\log x\ x)dx$$

$$\int uv\,dx=u\int v\,dx-\int\left\{\int v\,dx\times\frac{du}{dx}\right\}dx$$

$$I=\log x\times\frac{x^2}{2}-\int\left\{\int(x)dx\times\frac{d(\log x)}{dx}\right\}dx$$

$$I=\frac{x^2}{2}\log x-\int\left(\frac{x^2}{2}\times\frac{1}{x}\right)dx$$

$$I=\frac{x^2}{2}\log x-\frac{1}{2}\int x\,dx$$

$$I=\frac{x^2}{2}\log x-\frac{1}{2}\times\frac{x^2}{2}+c$$

$$I=\frac{x^2}{2}\log x-\frac{x^2}{4}+c$$

d. Find the area bounded by the curve $y = x^2 + 1$, x-axis and the coordinates at x=1 and x=2

Solution:

Given $y = x^2 + 1$ and a=x=1 &b= x=2

w.k.t. Area bounded by the curve is

$$A = \int_a^b y\,dx$$

$$A = \int_1^2 (x^2+1)dx = \left(\frac{x^3}{3}+x\right)_1^2 = \left(\frac{2^3}{3}+2\right)-\left(\frac{1^3}{3}+1\right)$$

$$= \frac{8}{3}+2-\frac{1}{3}-\frac{1}{1}$$

$$= \frac{8}{3}-\frac{1}{3}+\frac{1}{1}$$

$$= \frac{8-1+3}{3} = \frac{11-1}{3} = \frac{10}{3} squnit$$

10. a. Evaluate $\int_1^2 x^3 dx$.

Solution:

$$\int_1^2 x^3 dx. = \left[\frac{x^4}{4}\right]_1^2 = \frac{2^4}{4}-\frac{1^4}{4} = \frac{16}{4}-\frac{1}{4} = \frac{16-1}{4} = \frac{15}{4}$$

b. Evaluate $\int \sin^6 x \cos x dx$.

Solution:

$$I = \int \sin^6 x \cos x dx.$$

$Put \quad \sin x = t$

$\cos x dx = dt$

$$I = \int t^6 dt = \frac{t^7}{7} + c$$

$$I = \frac{(\sin x)^7}{7} + c$$

c. **Evaluate** $\int xe^x \, dx$

Solution:

$$I = \int xe^x \, dx$$

$$\int uv dx = u\int v dx - \int\left\{\int v dx \times \frac{du}{dx}\right\}dx$$

$$I = x \times \int e^x dx - \int\left(\int e^x dx \times \frac{d(x)}{dx}\right)dx$$

$$I = xe^x - \int\left(e^x \times 1\right)dx$$

$$I = xe^x - \int\left(e^x\right)dx$$

$$I = xe^x - e^x + c$$

d. Find the volume generated by rotating the curve $y=\sqrt{x+2}$ about x-axis between x=0 and x=2

Solution:

Given; $y=\sqrt{x+2}$ and x=a=0 & x=b=2

w.k.t, Volume of solid generated by rotating the curve is

$$V=\pi\int_a^b y^2 dx$$

$$V=\pi\int_0^2 \left(\sqrt{x+2}\right)^2 dx$$

$$V=\pi\int_0^2 (x+2)dx$$

$$V=\pi\left[\frac{x^2}{2}+2x\right]_0^2=\pi\left\{\left(\frac{2^2}{2}+2\times2\right)-\left(\frac{0^2}{2}+2\times0\right)\right\}=\pi(2+4-0)$$

$V=6\pi$ *cubic units*

Certified that model answers prepared by me for code 20SC01T are from prescribed text book and. Scheme of valuation, model answer prepared by me are correct and equal weightage should be given to alternative methods also.

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