Code : 20SC01T

1477

Register Number | | | | | | | | | | |

I/II Semester Diploma Examination, August/September-2022

ENGINEERING MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (1) Answer **one** full question from each section.
(2) Each section carries **20** marks.

SECTION – I

1. (a) Define matrix. List any four types of matrices. **4**

OR

If $X = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 5 & 0 \\ 3 & 2 \end{bmatrix}$, then find $4X + 3Y$.

(b) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then find the inverse of matrix A if it exists. **6**

OR

Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

(c) Applying Cramer's rule solve the system of linear equations

$3x + 2y = 8$

$2x + 5y = 9$

OR

The revenue generated by sales of vehicles in two branches of motor vehicle dealer in the month of January 2022 was as follows : **5**

Vehicle Type	**Revenue in US-Dollars**	
	Branch – 1	**Branch – 2**
2-wheeler	140	100
3-wheeler	30	40
4-wheeler	50	20

If 1 US-Dollar = Rs. 75 at the time then compute the revenue generated by these branches in rupees using scalar multiplication.

(d) If $A = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$, find AB and BA. Also check whether the commutative law AB = BA holds good or not. **5**

OR

For the matrix $A = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$, verify that $A(\text{adj } A) = |A|.\ I$ where I is an identity matrix of order 2.

SECTION – II

2. (a) Find x and y intercepts of line $2x + 4y + 5 = 0$. **4**

OR

Write the standard form of equation of straight line in (i) intercept form and (ii) two-point form.

(b) Find the equation of line passing through the point (2, 3) and parallel to the line $3x + 2y - 1 = 0$. **6**

OR

Show that the angle between the lines $3x + y + 5 = 0$ and $2x + 4y - 7 = 0$ is 45°.

(c) Find the equation of diameter of circle passing through the points (–3, 4) and (1, 2). **5**

OR

If A(2, 3), B(6, 5) and C(4, –3) are vertices of a triangle, then find equation of its side AB.

(d) Find slopes of straight lines $x + 2y - 5 = 0$ and $3x + 6y + 1 = 0$. Are these lines parallel to each other ? Give reason. **5**

OR

If a line is inclined at angle 135° with positive direction of x-axis and passes through the point (3, 4), then find its equation.

SECTION – III

3. (a) Convert 60° into radians and $\frac{5\pi}{6}$ into degree. **4**

OR

If $\tan A = \frac{1}{3}$, $\tan B = \frac{1}{2}$, find $\tan(A + B)$.

(b) Prove that $\frac{1 + \cos 2\theta + \sin 2\theta}{1 - \cos 2\theta + \sin 2\theta} = \cot\theta$. 6

OR

Prove that $\frac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°} = -1$.

(c) Using the trigonometric ratios of compound angle, determine the values sin 15° and cos 15°. 5

OR

Find the value of sin 120° cos 330° – sin 240° cos 390° using ratios of allied angles.

(d) Prove that sin 2A = 2 sin A cos A. Also verify the result for A = 30°. 5

OR

Prove that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$.

SECTION – IV

4. (a) Differentiate $x^3 \sin x$ w.r.t. x. 4

OR

If $y = \sin 2x + 5e^x - 3\log x + 7$, then find $\frac{dy}{dx}$.

(b) Find the derivative of $y = \frac{1 - \sin x}{1 + \sin x}$ w.r.t. x. 6

OR

If $y = 3\sin x + 4\cos x$ find $\frac{d^2y}{dx^2}$ at $x = 0$.

(c) Distance travelled by a particle in 't' seconds is given by $s = t^3 - t^2 + 9t + 8$. Find the velocity and acceleration of particle at t = 2 seconds. 5

OR

Find the maximum profit that a company make if the profit function $y(x)$ is given by $y = 41 - 72x - 18x^2$.

(d) Check whether the lines $2x - y + 4 = 0$ and $x + 2y - 13 = 0$ respectively represents the tangent and normal to the curve $y = x^2 + 5$ at (1, 6) on it. 5

OR

If $y = \tan^{-1} x$, prove that $(1 + x^2)\, y_2 + 2xy_1 = 0$.

SECTION – V

5. (a) Integrate $\frac{1}{x} + \cos x - e^{3x} + \frac{1}{\sqrt{1-x^2}}$ w.r.t. x.

OR

Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$. 4

(b) Show that $\int_0^{\pi/2} \sin^3 x\,dx = \frac{2}{3}$. 6

OR

Evaluate $\int_0^{\pi/4} \tan x \sec^2 x\,dx$.

(c) With the use of definite integrals find the area bounded by the curve $y = x^2 + 3$, x-axis and $x = 0$, $x = 2$. 5

OR

The curve $y = \sqrt{x^2 + 5x}$ is rotated about x-axis. Find the volume of solid generated by revolving the curve between $x = 1$ and $x = 2$.

(d) Evaluate the indefinite integral $\int x \cos x\,dx$ using integration by parts. 5

OR

Evaluate $I = \int (3x^2 + e^x)\,dx$. Find the derivative of obtained integral to verify that integration is inverse process of differentiation.

I/II Semester Diploma Examination, August/September – 2022

Course Name: Engineering Mathematics **Course Code: 20SC01T**

SCHEME OF VALUATION

Q. No.	Particular	Marks
SECTION – I		
1 (a)	Definition Types (Each ½ Marks)	2 2 } = $\boxed{4}$
	OR	
	Finding $4X$ and $3Y$ Finding $4X+3Y$	2 2 } = $\boxed{4}$
1 (b)	Finding $\lvert A\rvert$ Finding $adjA$ Formula Simplification	1 3 1 1 } = $\boxed{6}$
	OR	
	Writing $\lvert A-\lambda I\rvert=0$ Finding Characteristic Equation Finding Characteristic roots	1 3 2 } = $\boxed{6}$
1 (c)	Finding Δ, Δ_1, Δ_2 Finding x and y	$1+1+1$ $1+1$ } = $\boxed{5}$
	OR	
	Writing Given Data in Matrix Form Scalar Multiplication Answer	2 2 1 } = $\boxed{5}$
1 (d)	Finding AB Finding BA Writing $AB \neq BA$	2 2 1 } = $\boxed{5}$
	OR	
	Finding $adjA$ Finding $A(adjA)$ Finding $\lvert A\rvert$ Finding $\lvert A\rvert I$ and Rest	1 2 1 1 } = $\boxed{5}$

Q. No.	Particular	Marks
SECTION – II		
2 (a)	Writing x – intercept Writing y – intercept	2 2 } = $\boxed{4}$
	OR	
	Writing intercept form Writing two-point form	2 2 } = $\boxed{4}$
2 (b)	Writing $3x+2y+k=0$ Getting value of k Result	2 2 2 } = $\boxed{6}$
	OR	
	Finding m_1 and m_2 Writing $\tan\theta$ Getting $\tan\theta=1$ Finding $\theta=45^\circ$	2 1 2 1 } = $\boxed{6}$
2 (c)	Writing two-point form Substituting values Simplification	2 1 2 } = $\boxed{5}$
	OR	
	Writing two-point form Substituting values Simplification	2 1 2 } = $\boxed{5}$
2 (d)	Finding m_1 Finding m_2 Result with reason	2 2 1 } = $\boxed{5}$
	OR	
	Finding slope Writing Slope point form Substituting values Simplification	2 1 1 1 } = $\boxed{5}$

Q. No.	Particular	Marks
	SECTION – III	
3 (a)	Converting 60° to radian Converting $5\pi/6$ to degree	2 2 } = $\boxed{4}$
	OR	
	$\tan(A+B)$ formula Simplification Result	1 2 } = $\boxed{4}$ 1
3 (b)	Formula for $1+\cos 2\theta$; $1-\cos 2\theta$; $\sin 2\theta$ Simplification Result	3 2 } = $\boxed{6}$ 1
	OR	
	Using $\tan(A+B)$ formula writing $\tan 135^\circ$ Simplification Result	3 2 } = $\boxed{6}$ 1
3 (c)	Expansion using $\sin(A-B)$ and $\cos(A-B)$ Simplification Results	2 2 } = $\boxed{5}$ 1
	OR	
	Finding $\sin 120^\circ, \cos 330^\circ$ Finding $\sin 240^\circ, \cos 390^\circ$ Result	$1+1$ $1+1$ } = $\boxed{5}$ 1
3 (d)	Writing $\sin 2A = \sin(A+A)$ Simplification and result Substituting $A = 30^\circ$ Rest	1 2 1 } = $\boxed{5}$ 1
	OR	
	Simplifying $\cos 40^\circ \cos 20^\circ$ Simplifying $\cos 80^\circ \cos 20^\circ$ Rest	2 2 } = $\boxed{5}$ 1

Q. No.	Particular	Marks
	SECTION – IV	
4 (a)	Using product rule Derivative of $\sin x$ & x^3	2 $1+1$ } = $\boxed{4}$
	OR	
	Derivative of each term 1 Mark (1+1+1+1)	} = $\boxed{4}$
4 (b)	Using Quotient rule Derivative of $1+\sin x$ Derivative of $1-\sin x$ Simplification	2 1 1 } = $\boxed{6}$ 2
	OR	
	Finding $\frac{dy}{dx}$ Finding $\frac{d^2y}{dx^2}$ Finding $\frac{d^2y}{dx^2}$ at $x = 0$	2 2 } = $\boxed{6}$ 2
4 (c)	Finding $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ Finding Velocity Finding Acceleration	$2+1$ 1 } = $\boxed{5}$ 1
	OR	
	Finding $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ Getting value of x Checking of $\frac{d^2y}{dx^2} < 0$ Result	$1+1$ 1 } = $\boxed{5}$ 1 1
4 (d)	Finding dy/dx and Slope Finding tangent Finding normal Result	$1+1$ 1 1 } = $\boxed{5}$ 1
	OR	
	Getting $y_1 = \frac{1}{1+x^2}$ Second order derivative Simplification	2 2 } = $\boxed{5}$ 1

Q. No.	Particular	Marks
	SECTION – V	
5 (a)	Integration of each term 1 mark (1+1+1+1)	$\} = \boxed{4}$
	OR	
	Integration Substituting limits Result	2 1 1 $\} = \boxed{4}$
5 (b)	Using $\sin 3x$ formula Integration Substituting limit values Simplification	2 1 1 2 $\} = \boxed{6}$
	OR	
	Substituting for $\tan x$ Finding new limits Integration Simplification	1 1 2 2 $\} = \boxed{6}$

Q. No.	Particular	Marks
	SECTION – V	
5 (c)	Writing Area Formula Integration Simplification and result	1 2 2 $\} = \boxed{5}$
	OR	
	Writing Volume Formula Integration Simplification and result	1 2 2 $\} = \boxed{5}$
5 (d)	Formula for integration by parts Substitution Calculation and result	1 1 3 $\} = \boxed{5}$
	OR	
	Integration Differentiation Result	2 2 1 $\} = \boxed{5}$

"Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and scheme of valuation prepared by me are correct".

Award full marks for alternative methods of answers.

I/II Semester Diploma Examination, August/September – 2022

Course Name: Engineering Mathematics **Course Code: 20SC01T**

MODEL ANSWERS

SECTION – I

1 (a)

Matrix: A rectangular arrangement of numbers in rows and columns enclosed within brackets is called as a matrix.

Types of Matrices: (Any four)

(i) Zero Matrix (ii) Row Matrix (iii) Column Matrix (iv) Rectangular Matrix (v) Square Matrix (vi) Diagonal Matrix (vii) Scalar Matrix (viii) Identity Matrix

OR

Given $X = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 5 & 0 \\ 3 & 2 \end{bmatrix}$

Consider $4X + 3Y = 4\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + 3\begin{bmatrix} 5 & 0 \\ 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 12 \\ 4 & 16 \end{bmatrix} + \begin{bmatrix} 15 & 0 \\ 9 & 6 \end{bmatrix}$$

$$4X + 3Y = \begin{bmatrix} 23 & 12 \\ 13 & 22 \end{bmatrix}$$

1 (b)

Given $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Consider $|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$

$$= 1(16-9) - 3(4-3) + 3(3-4)$$

$$= 7 - 3 - 3$$

$$|A| = 1 \neq 0$$

Therefore, A^{-1} exists.

To find adjoint of A, first to find cofactors of its elements

$A_{11} = +(16-9) = 7$; $A_{12} = -(4-3) = -1$; $A_{13} = +(3-4) = -1$

$A_{21} = -(12-9) = -3$; $A_{22} = +(4-3) = 1$; $A_{23} = -(3-3) = 0$

$A_{31} = +(9-12) = -3$; $A_{32} = -(3-3) = 0$; $A_{33} = +(4-3) = 1$

We have, $adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \Rightarrow adjA = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Using, $A^{-1} = \dfrac{1}{|A|} adjA$

$$\Rightarrow \quad A^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

	OR
	Given $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ The characteristic equation of A is $\lvert A-\lambda I\rvert=0$ $\Rightarrow \quad \begin{vmatrix}1-\lambda & 4\\ 2 & 3-\lambda\end{vmatrix}=0$ $\Rightarrow \quad (1-\lambda)(3-\lambda)-8=0$ $\Rightarrow \quad 3-\lambda-3\lambda+\lambda^2-8=0$ $\Rightarrow \quad \lambda^2-4\lambda-5=0$ $\Rightarrow \quad \lambda^2-5\lambda+\lambda-5=0$ $\Rightarrow \quad \lambda(\lambda-5)+1(\lambda-5)=0$ $\Rightarrow \quad (\lambda-5)(\lambda+1)=0$ $\Rightarrow \quad (\lambda-5)=0$ or $(\lambda+1)=0$ $\Rightarrow \quad \lambda=5$ or $\lambda=-1$ Therefore, the characteristic roots of the given matrix are $\lambda=5$ and $\lambda=-1$.
1 (c)	Given System of linear equations is $3x+2y=8$ --- (1) $2x+5y=9$ --- (2) Consider, $\Delta=\begin{vmatrix}3 & 2\\ 2 & 5\end{vmatrix}=15-4=11$ $\Delta_1=\begin{vmatrix}8 & 2\\ 9 & 5\end{vmatrix}=40-18=22$ $\Delta_2=\begin{vmatrix}3 & 8\\ 2 & 9\end{vmatrix}=27-16=11$ Using, $x=\dfrac{\Delta_1}{\Delta}$ and $y=\dfrac{\Delta_2}{\Delta}$ $\Rightarrow \quad x=\dfrac{22}{11}$ and $y=\dfrac{11}{11}$ $\Rightarrow \quad x=2$ and $y=1$
	OR
	The revenue (in US-Dollars) generated by sales of vehicles in two branches of motor vehicle dealer in the month of January 2022 is represented in matrix form as $D=\begin{bmatrix}140 & 100\\ 30 & 40\\ 50 & 20\end{bmatrix}$ Given $1\,\text{US-Dollar}=\text{Rs.}75$, then the revenue generated (in rupees) in matrix form is Revenue in rupees, $R=75\begin{bmatrix}140 & 100\\ 30 & 40\\ 50 & 20\end{bmatrix}=\begin{bmatrix}10500 & 7500\\ 2250 & 3000\\ 3750 & 1500\end{bmatrix}$ (i) Revenue generated by Branch-1 $=10500+2250+3750=16500$ rupees (ii) Revenue generated by Branch-2 $=7500+3000+1500=12000$ rupees

1 (d)	Given $A=\begin{bmatrix}2 & 3\\2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\0 & 4\end{bmatrix}$ Consider $AB=\begin{bmatrix}2 & 3\\2 & 5\end{bmatrix}\begin{bmatrix}3 & -1\\0 & 4\end{bmatrix}$ $=\begin{bmatrix}6+0 & -2+12\\6+0 & -2+20\end{bmatrix}$ $AB=\begin{bmatrix}6 & 10\\6 & 18\end{bmatrix}$ --- (1) and $BA=\begin{bmatrix}3 & -1\\0 & 4\end{bmatrix}\begin{bmatrix}2 & 3\\2 & 5\end{bmatrix}$ $=\begin{bmatrix}6-2 & 9-5\\0+8 & 0+20\end{bmatrix}$ $BA=\begin{bmatrix}4 & 4\\8 & 20\end{bmatrix}$ --- (2) From (1) and (2), $AB \neq BA$ i.e., commutative law does not holds good.
	OR
	Given $A=\begin{bmatrix}5 & -2\\3 & 1\end{bmatrix}$ $\Rightarrow \quad adjA=\begin{bmatrix}1 & 2\\-3 & 5\end{bmatrix}$ Consider, $A(adjA)=\begin{bmatrix}5 & -2\\3 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & 5\end{bmatrix}$ $=\begin{bmatrix}5+6 & 10-10\\3-3 & 6+5\end{bmatrix}$ $A(adjA)=\begin{bmatrix}11 & 0\\0 & 11\end{bmatrix}$ --- (1) Consider, $\lvert A\rvert=\begin{vmatrix}5 & -2\\3 & 1\end{vmatrix}$ $=5+6$ $\lvert A\rvert=11$ Consider, $\lvert A\rvert I=11\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\lvert A\rvert I=\begin{bmatrix}11 & 0\\0 & 11\end{bmatrix}$ --- (2) From (1) and (2), $A(adjA)=\lvert A\rvert I$

SECTION – II	
2 (a)	Given line is $2x+4y+5=0$ $\equiv \quad ax+by+c=0$ $\Rightarrow \quad a=2;\ b=4;\ c=5$ We have, $x-\text{intercept}=-\dfrac{c}{a} \quad \Rightarrow \quad x-\text{intercept}=-\dfrac{5}{2}$ $y-\text{intercept}=-\dfrac{c}{b} \quad \Rightarrow \quad y-\text{intercept}=-\dfrac{5}{4}$ **Alternate Method-1:** Given line is $2x+4y+5=0$ --- (1) To get x-intercept substituting $y=0$ in (1), we get $2x+4(0)+5=0$ $2x+5=0$ $x=-\dfrac{5}{2}$ i.e., $x-\text{intercept}=-\dfrac{5}{2}$ To get y-intercept substituting $x=0$ in (1), we get $2(0)+4y+5=0$ $4y+5=0$ $y=-\dfrac{5}{4}$ i.e., $y-\text{intercept}=-\dfrac{5}{4}$ **Alternate Method-2:** Given line is $2x+4y+5=0$ $\Rightarrow \quad 2x+4y=-5$ Dividing by (-5), $\dfrac{2x}{-5}+\dfrac{4y}{-5}=\dfrac{-5}{-5}$ $\Rightarrow \quad \dfrac{x}{(-5/2)}+\dfrac{y}{(-5/4)}=1$ Comparing with $\dfrac{x}{a}+\dfrac{y}{b}=1$ we get, $a=-\dfrac{5}{2}$ i.e., $x-\text{intercept}=-\dfrac{5}{2}$ and $b=-\dfrac{5}{4}$ i.e., $y-\text{intercept}=-\dfrac{5}{4}$
	OR
	(i) Standard form of equation of straight line in intercept form is $\dfrac{x}{a}+\dfrac{y}{b}=1$ where 'a' is x-intercept and 'b' is y-intercept (ii) Standard form of equation of straight line in two-point form is $\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}$ OR $(y-y_1)=\left(\dfrac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$ where (x_1,y_1) and (x_2,y_2) are two points through which the line passes.

2 (b)	Given line is $3x+2y-1=0$ --- (1) Any line parallel to (1) is $3x+2y+k=0$ --- (2) Since the required line (2) passes through the point $(2,3)$, $(2) \Rightarrow \quad 3(2)+2(3)+k=0$ $\Rightarrow \quad 6+6+k=0$ $\Rightarrow \quad k=-12$ Substituting the value of k in (2) we get, $3x+2y-12=0$ which is the required line. **Alternate Method:** Given line is $3x+2y-1=0$ --- (1) $\equiv \quad ax+by+c=0$ $\Rightarrow \quad a=3;\ b=2;\ c=-1$ Slope of line (1), $m=-\frac{a}{b} \quad \Rightarrow \quad m=-\frac{3}{2}$ Since the required line is parallel to given line (1), $(\text{Slope of required line}) = (\text{Slope of given line})$ $\Rightarrow \quad \text{Slope of required line} = -\frac{3}{2}$ As the required line passes through the point $(2,3)$ having $\text{Slope} = -\frac{3}{2}$ its equation is given by $(y-y_1)=m(x-x_1)$ Here $m=-\frac{3}{2}$ and $(x_1,y_1)=(2,3)$ $(y-3)=-\frac{3}{2}(x-2)$ $\Rightarrow \quad 2(y-3)=-3(x-2)$ $\Rightarrow \quad 2y-6=-3x+6$ $\Rightarrow \quad 2y-6+3x-6=0$ $\Rightarrow \quad 3x+2y-12=0$
	OR
	Given lines are $3x+y+5=0$ --- (1) $2x+4y-7=0$ --- (2) Using slope of line $=-\frac{a}{b}$, Slope of line (1) is $m_1=-\frac{3}{1} \quad \Rightarrow \quad m_1=-3$ Slope of line (2) is $m_2=-\frac{2}{4} \quad \Rightarrow \quad m_2=-\frac{1}{2}$

	Let θ be the acute angle between lines (1) and (2) then $$\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$$ $$\Rightarrow \quad \tan\theta = \left\|\frac{(-3) - \left(-\frac{1}{2}\right)}{1 + (-3)\left(-\frac{1}{2}\right)}\right\|$$ $$\Rightarrow \quad \tan\theta = \left\|\frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}}\right\|$$ $$\Rightarrow \quad \tan\theta = \left\|\frac{(-5/2)}{(5/2)}\right\|$$ $$\Rightarrow \quad \tan\theta = \|-1\|$$ $$\Rightarrow \quad \tan\theta = 1$$ $$\Rightarrow \quad \tan\theta = \tan 45^\circ$$ $$\Rightarrow \quad \theta = 45^\circ$$
2 (c)	The diameter of circle passing through the points $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (1, 2)$ is given by $$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$ $$\Rightarrow \quad (y - 4) = \left(\frac{2 - 4}{1 + 3}\right)(x + 3)$$ $$\Rightarrow \quad (y - 4) = \left(\frac{-2}{4}\right)(x + 3)$$ $$\Rightarrow \quad (y - 4) = \frac{-1}{2}(x + 3)$$ $$\Rightarrow \quad 2(y - 4) = -1(x + 3)$$ $$\Rightarrow \quad 2y - 8 = -x - 3$$ $$\Rightarrow \quad 2y - 8 + x + 3 = 0$$ $$\Rightarrow \quad x + 2y - 5 = 0$$
	OR
	Given $A = (2,3)$, $B = (6,5)$ and $C = (4,-3)$. The side AB of triangle is the line joining the points $A = (x_1, y_1) = (2,3)$ and $B = (x_2, y_2) = (6,5)$, its equation is given by $$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$ $$\Rightarrow \quad (y - 3) = \left(\frac{5 - 3}{6 - 2}\right)(x - 2)$$ $$\Rightarrow \quad (y - 3) = \left(\frac{2}{4}\right)(x - 2)$$

	$\Rightarrow \quad (y-3)=\frac{1}{2}(x-2)$ $\Rightarrow \quad 2(y-3)=1(x-2)$ $\Rightarrow \quad 2y-6=x-2$ $\Rightarrow \quad x-2-2y+6=0$ $\Rightarrow \quad x-2y+4=0$
2 (d)	Given lines are $x+2y-5=0$ --- (1) $3x+6y+1=0$ --- (2) Using slope of line $=-\frac{a}{b}$, Slope of line (1) is $m_1=-\frac{1}{2}$ Slope of line (2) is $m_2=-\frac{3}{6} \quad \Rightarrow \quad m_2=-\frac{1}{2}$ Here $m_1=m_2$ i.e., the slopes of given lines are equal. Therefore the given lines are parallel to each other.
	OR
	Given, Inclination $(\theta)=135^\circ$ and point $(x_1,y_1)=(3,4)$ We have, Slope of line $(m)=\tan\theta$ $\Rightarrow \quad m=\tan 135^\circ$ $=\tan(90^\circ+45^\circ)$ $=-\cot 45^\circ$ $m=-1$ The required equation of line passing through the point $(x_1,y_1)=(3,4)$ and having slope $m=-1$ is given by $(y-y_1)=m(x-x_1)$ $\Rightarrow \quad (y-4)=-1(x-3)$ $\Rightarrow \quad y-4=-x+3$ $\Rightarrow \quad y-4+x-3=0$ $\Rightarrow \quad x+y-7=0$
SECTION – III	
3 (a)	(i) Consider $60^\circ=60\times\frac{\pi}{180}$ radians $60^\circ=\frac{\pi}{3}$ radians (ii) Consider $\frac{5\pi}{6}=\frac{5\pi}{6}\times\frac{180}{\pi}$ degrees $\frac{5\pi}{6}=150^\circ$

<table>
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Given $\tan A = \frac{1}{3}$ and $\tan B = \frac{1}{2}$

Using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\tan(A+B) = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}$$

$$\Rightarrow \quad \tan(A+B) = \frac{\left(\frac{2+3}{6}\right)}{\left(\frac{6-1}{6}\right)}$$

$$\Rightarrow \quad \tan(A+B) = \frac{(5/6)}{(5/6)}$$

$$\Rightarrow \quad \tan(A+B) = 1$$

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<tr><td>3 (b)</td><td>

Consider $\text{LHS} = \frac{1+\cos 2\theta + \sin 2\theta}{1-\cos 2\theta + \sin 2\theta}$

Using $1+\cos 2\theta = 2\cos^2\theta$; $1-\cos 2\theta = 2\sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

$$\text{LHS} = \frac{2\cos^2\theta + 2\sin\theta\cos\theta}{2\sin^2\theta + 2\sin\theta\cos\theta}$$

$$= \frac{2\cos\theta(\cos\theta + \sin\theta)}{2\sin\theta(\sin\theta + \cos\theta)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

LHS = RHS

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<tr><td></td><td>OR</td></tr>
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Consider $\text{LHS} = \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$

Using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\text{LHS} = \tan(69^\circ + 66^\circ)$$

$$= \tan 135^\circ$$

$$= \tan(90^\circ + 45^\circ)$$

$= -\cot 45^\circ$ $\quad \because \tan(90^\circ + \theta) = -\cot\theta$

$= -1$ $\quad \because \cot 45^\circ = 1$

LHS = RHS

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3 (c)	Consider $\sin 15^\circ = \sin(45^\circ - 30^\circ)$ Using $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\sin 15^\circ = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$ Consider $\cos 15^\circ = \cos(45^\circ - 30^\circ)$ Using $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $\cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$ $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$ **Alternate Method:** Consider $\sin 15^\circ = \sin(60^\circ - 45^\circ)$ Using $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\sin 15^\circ = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$ Consider $\cos 15^\circ = \cos(60^\circ - 45^\circ)$ Using $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $\cos 15^\circ = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$ $= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$ $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
	OR
	(i) $\sin 120^\circ = \sin(90^\circ + 30^\circ)$ $= \cos 30^\circ$ $\sin 120^\circ = \frac{\sqrt{3}}{2}$ (ii) $\cos 330^\circ = \cos(270^\circ + 60^\circ)$ $= \sin 60^\circ$ $\cos 330^\circ = \frac{\sqrt{3}}{2}$

	(iii) $\sin 240^\circ = \sin(180^\circ + 60^\circ)$ $= -\sin 60^\circ$ $\sin 240^\circ = -\dfrac{\sqrt{3}}{2}$ (iv) $\cos 390^\circ = \cos(360^\circ + 30^\circ)$ $= \cos 30^\circ$ $\cos 390^\circ = \dfrac{\sqrt{3}}{2}$ Consider $\sin 120^\circ \cos 330^\circ - \sin 240^\circ \cos 390^\circ = \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(-\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{3}{4} + \dfrac{3}{4}$ $\sin 120^\circ \cos 330^\circ - \sin 240^\circ \cos 390^\circ = \dfrac{3}{2}$
3 (d)	Consider $\sin 2A = \sin(A + A)$ $\sin 2A = \sin A \cos A + \cos A \sin A$ $\quad \because \sin(A+B) = \sin A \cos B + \cos A \sin B$ $\Rightarrow \quad \sin 2A = 2 \sin A \cos A$ --- (1) To verify the result (1) for $A = 30^\circ$, substituting $A = 30^\circ$ in (1) we get $\sin 2(30^\circ) = 2 \sin 30^\circ \cos 30^\circ$ $\sin 60^\circ = 2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$ $\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2}$ which implies the result (1) holds good for $A = 30^\circ$
	OR
	Consider $\text{LHS} = \cos 20^\circ \cos 40^\circ \cos 80^\circ$ $= (\cos 40^\circ \cos 20^\circ) \cos 80^\circ$ $\quad \because \cos A \cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$ $= \dfrac{1}{2}\left[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)\right] \cos 80^\circ$ $= \dfrac{1}{2}\left[\cos 60^\circ + \cos 20^\circ\right] \cos 80^\circ$ $= \dfrac{1}{2}\left[\dfrac{1}{2} + \cos 20^\circ\right] \cos 80^\circ$ $\quad \because \cos 60^\circ = \dfrac{1}{2}$ $= \dfrac{1}{4} \cos 80^\circ + \dfrac{1}{2} \cos 80^\circ \cos 20^\circ$ $= \dfrac{1}{4} \cos 80^\circ + \dfrac{1}{2} \times \dfrac{1}{2}\left[\cos 100^\circ + \cos 60^\circ\right]$ $= \dfrac{1}{4} \cos 80^\circ + \dfrac{1}{4} \cos 100^\circ + \dfrac{1}{4} \cos 60^\circ$ $= \dfrac{1}{4} \cos 80^\circ + \dfrac{1}{4} \cos(180^\circ - 80^\circ) + \dfrac{1}{4} \times \dfrac{1}{2}$ $\quad \because \cos 60^\circ = \dfrac{1}{2}$ $= \dfrac{1}{4} \cos 80^\circ - \dfrac{1}{4} \cos 80^\circ + \dfrac{1}{8}$ $\quad \because \cos(180^\circ - \theta) = -\cos\theta$ $= \dfrac{1}{8}$ $\text{LHS} = \text{RHS}$

SECTION – IV	
4 (a)	Let $y = x^3 \sin x$ Using product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ Differentiating w.r.t. x, $\frac{dy}{dx} = x^3 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^3)$ $= x^3(\cos x) + \sin x(3x^2)$ $\frac{dy}{dx} = x^3 \cos x + 3x^2 \sin x$
	OR
	Given $y = \sin 2x + 5e^x - 3\log x + 7$ Differentiating w.r.t. x, $\frac{dy}{dx} = (\cos 2x \times 2) + 5(e^x) - 3\left(\frac{1}{x}\right) + 0$ $\frac{dy}{dx} = 2\cos 2x + 5e^x - \frac{3}{x}$
4 (b)	Let $y = \frac{1-\sin x}{1+\sin x}$ Using product rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Differentiating w.r.t. x, $\frac{dy}{dx} = \frac{(1+\sin x)(0-\cos x)-(1-\sin x)(0+\cos x)}{(1+\sin x)^2}$ $= \frac{-\cos x - \cos x \sin x - \cos x + \cos x \sin x}{(1+\sin x)^2}$ $\frac{dy}{dx} = \frac{-2\cos x}{(1+\sin x)^2}$
	OR
	Given $y = 3\sin x + 4\cos x$ Differentiating w.r.t. x, $\frac{dy}{dx} = 3(\cos x) + 4(-\sin x)$ $\frac{dy}{dx} = 3\cos x - 4\sin x$ Again differentiating w.r.t. x, $\frac{d^2y}{dx^2} = 3(-\sin x) - 4(\cos x)$ $\frac{d^2y}{dx^2} = -3\sin x - 4\cos x$ $\Rightarrow \left[\frac{d^2y}{dx^2}\right]_{x=0} = -3(\sin 0) - 4(\cos 0)$ $= -3(0) - 4(1)$ $\left[\frac{d^2y}{dx^2}\right]_{x=0} = -4$

4 (c)	Given $s = t^3 - t^2 + 9t + 8$ Differentiating w.r.t. t, $\frac{ds}{dt} = 3t^2 - 2t + 9(1) + 0$ $\frac{ds}{dt} = 3t^2 - 2t + 9$ Again differentiating w.r.t. t, $\frac{d^2s}{dt^2} = 3(2t) - 2(1) + 0$ $\frac{d^2s}{dt^2} = 6t - 2$ (i) Velocity at $t = 2$ seconds is $v = \left(\frac{ds}{dt}\right)_{t=2}$ $= 3(2)^2 - 2(2) + 9$ $= 12 - 4 + 9$ $v = 17$ unit/sec (ii) Acceleration at $t = 2$ seconds is $a = \left(\frac{d^2s}{dt^2}\right)_{t=2}$ $= 6(2) - 2$ $= 12 - 2$ $v = 10$ unit/sec^2
	OR
	Given profit function is $y = 41 - 72x - 18x^2$ Differentiating w.r.t. x, $\frac{dy}{dx} = 0 - 72(1) - 18(2x)$ $\frac{dy}{dx} = -72 - 36x$ Again differentiating w.r.t. x, $\frac{d^2y}{dx^2} = -0 - 36(1)$ $\frac{d^2y}{dx^2} = -36$ Profit is maximum if $\frac{dy}{dx} = 0$ $\Rightarrow \quad -72 - 36x = 0$ $\Rightarrow \quad 36x = -72$ $\Rightarrow \quad x = -\frac{72}{36}$ $\Rightarrow \quad x = -2$ At $x = -2$, $\left[\frac{d^2y}{dx^2}\right]_{x=-2} = -36 < 0$ Therefore profit function is maximum at $x = -2$ and the maximum profit is given as Maximum profit $= [y]_{x=-2}$ $= 41 - 72(-2) - 18(-2)^2$ $= 41 + 144 - 72$ Maximum profit $= 113$

4 (d)	Given curve is $y = x^2 + 5$ Differentiating w.r.t. x, $\frac{dy}{dx} = 2x + 0$ $\frac{dy}{dx} = 2x$ Slope of tangent at $(1,6)$ is $m = \left[\frac{dy}{dx}\right]_{(1,6)}$ $= 2(1)$ $m = 2$ = (i) Equation of tangent to the given curve at $(x_1, y_1) = (1,6)$ is $(y - y_1) = m(x - x_1)$ $\Rightarrow \quad (y - 6) = 2(x - 1)$ $\Rightarrow \quad y - 6 = 2x - 2$ $\Rightarrow \quad 2x - 2 - y + 6 = 0$ $\Rightarrow \quad 2x - y + 4 = 0$ --- (1) (ii) Equation of normal to the given curve through $(x_1, y_1) = (1,6)$ is $(y - y_1) = \frac{-1}{m}(x - x_1)$ $\Rightarrow \quad (y - 6) = \frac{-1}{2}(x - 1)$ $\Rightarrow \quad 2y - 12 = -x + 1$ $\Rightarrow \quad 2y - 12 + x - 1 = 0$ $\Rightarrow \quad x + 2y - 13 = 0$ --- (2) From (1) and (2), the given lines $2x - y + 4 = 0$ and $x + 2y - 13 = 0$ respectively represents the tangent and normal to the given curve $y = x^2 + 5$ at $(1,6)$.
	OR
	Given curve is $y = \tan^{-1} x$ Differentiating w.r.t. x, $y_1 = \frac{1}{1 + x^2}$ $(1 + x^2) y_1 = 1$ Again differentiating w.r.t. x, $(1 + x^2) y_2 + y_1 (0 + 2x) = 0$ $\quad \because \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $(1 + x^2) y_2 + 2x y_1 = 0$
SECTION – V	
5 (a)	Let $I = \int \left(\frac{1}{x} + \cos x - e^{3x} + \frac{1}{\sqrt{1 - x^2}} \right) dx$ $I = \int \frac{1}{x} dx + \int \cos x dx - \int e^{3x} dx + \int \frac{1}{\sqrt{1 - x^2}} dx$ $I = \log x + \sin x - \frac{e^{3x}}{3} + \sin^{-1} x + c$
	OR

	Let $I=\int_0^1 \frac{1}{1+x^2}dx$ $=\left[\tan^{-1}x\right]_0^1$ $=\tan^{-1}(1)-\tan^{-1}(0)$ $=\frac{\pi}{4}-0$ $I=\frac{\pi}{4}$
5 (b)	Let $I=\int_0^{\pi/2}\sin^3 xdx$ Using $\sin 3x=3\sin x-4\sin^3 x \quad \Rightarrow \quad 4\sin^3 x=3\sin x-\sin 3x$ $\Rightarrow \quad \sin^3 x=\frac{1}{4}\left[3\sin x-\sin 3x\right]$ $I=\int_0^{\pi/2}\frac{1}{4}\left[3\sin x-\sin 3x\right]dx$ $=\frac{1}{4}\left[3(-\cos x)-\left(\frac{-\cos 3x}{3}\right)\right]_0^{\pi/2}$ $=\frac{-3}{4}\left[\cos x\right]_0^{\pi/2}+\frac{1}{12}\left[\cos 3x\right]_0^{\pi/2}$ $=\frac{-3}{4}\left[\cos\frac{\pi}{2}-\cos 0\right]+\frac{1}{12}\left[\cos\frac{3\pi}{2}-\cos 0\right]$ $=\frac{-3}{4}\left[0-1\right]+\frac{1}{12}\left[0-1\right]$ $I=\frac{2}{3}$ **Alternate Method:** Let $I=\int_0^{\pi/2}\sin^3 xdx$ $=\int_0^{\pi/2}\sin^2 x\sin xdx$ $I=\int_0^{\pi/2}(1-\cos^2 x)\sin xdx$ Put $\cos x=t$ Differentiating, $-\sin xdx=dt \quad \Rightarrow \quad \sin xdx=-dt$ Lower limit: When $x=0$, $t=\cos 0=1$ Upper limit: When $x=\pi/2$, $t=\cos(\pi/2)=0$ $\therefore \; I=\int_1^0(1-t^2)(-dt) \quad \Rightarrow \quad I=\int_1^0(t^2-1)dt$ $\Rightarrow \quad I=\left[\frac{t^3}{3}-t\right]_1^0$ $\Rightarrow \quad I=\frac{1}{3}\left(0-1\right)-(0-1)$ $\Rightarrow \quad I=\frac{2}{3}$

	OR
	Let $I = \int_0^{\pi/4} \tan x \sec^2 x dx$ Put $\tan x = t$ Differentiating, $\sec^2 x dx = dt$ Lower limit: When $x = 0$, $t = \tan 0 = 0$ Upper limit: When $x = \pi/4$, $t = \tan(\pi/4) = 1$ $\therefore \ I = \int_0^1 t dt$ $= \left[\frac{t^2}{2}\right]_0^1$ $= \frac{1}{2}\left[1^2 - 0^2\right]$ $I = \frac{1}{2}$
5 (c)	Given curve is $y = x^2 + 3$ Required area, $A = \int_a^b y dx$ $A = \int_0^2 (x^2 + 3) dx$ $= \left[\frac{x^3}{3} + 3x\right]_1^2$ $= \frac{1}{3}\left[2^3 - 0^3\right] + 3\left[2 - 0\right]$ $= \frac{8}{3} + 6$ $A = \frac{26}{3}$ Sq.Units
	OR
	Given curve is $y = \sqrt{x^2 + 5x}$ Required volume, $V = \pi \int_a^b y^2 dx$ $V = \pi \int_1^2 (x^2 + 5x) dx$ $= \pi \left[\frac{x^3}{3} + \frac{5x^2}{2}\right]_1^2$ $= \pi \left[\frac{1}{3}\left(2^3 - 1^3\right) + \frac{5}{2}(2^2 - 1^2)\right]_1^2$ $= \pi \left[\frac{7}{3} + \frac{15}{2}\right]$ $V = \frac{59\pi}{6}$ Cubic.Units

5 (d)	Let $I = \int x \cos x dx$ Integrating by parts using $\int uv dx = u\int v dx - \int\left[\frac{du}{dx} \cdot \int v dx\right]dx$, we get $I = x(\sin x) - \int (1 \times \sin x) dx$ $= x\sin x - \int \sin x dx$ $= x\sin x - (-\cos x) + c$ $I = x\sin x + \cos x + c$	
	OR	
	Given $I = \int (3x^2 + e^x) dx$ --- (1) $= 3\left(\frac{x^3}{3}\right) + e^x + c$ $I = x^3 + e^x + c$ Differentiating w.r.t x, $\frac{dI}{dx} = 3x^2 + e^x + 0$ $\frac{dI}{dx} = 3x^2 + e^x$ --- (2) From (1) and (2), it is observed that integration is inverse process of differentiation.	

"Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and scheme of valuation prepared by me are correct".

Award the full marks for alternative methods of answers.