

Make up Examination-Sept.2023  
I/II Semester Diploma Examination  
ENGINEERING MATHEMATICS (20SC01T)  
(Exam Date/Time: 23<sup>rd</sup> Sep.2023/2.00pm-5.00pm)

Time: 3 Hours]

[Max .Marks: 100

Instructions: (1) Answer all questions. (2) Each section carries 20 marks.

Section-I

1. (a) If  $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ , then find  $2A + 3B$ . 4

OR

If  $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$  then find  $A + A^T$  matrix .

(b) Find the characteristics roots of the matrix  $A = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$  6

OR

Find the inverse of the matrix  $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

(c) Find the adjoint of the matrix  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$  5

OR

Solve the equations  $2x+y=1$ ;  $3x+2y=1$  by using Cramer's rule.

(d) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ , find  $AB$  matrix and also find  $(AB)^T$  matrix . 5

OR

If  $\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 0 & x \end{vmatrix}$  find the value of  $x$ .

Section-II

2. (a) Find the equation of a straight line with slope 5 and y-intercept 3. 4

OR

Write the standard form of equation of straight line with

a) One point  $(x_1, y_1)$  having slope  $m$  .

b) Two points  $(x_1, y_1)$  and  $(x_2, y_2)$  .

(b) Find the equation straight line passing through two points  $(3,4)$  and  $(5,6)$  6

OR

Find equation of straight line passing through the point  $(1,2)$  which makes an angle  $45^\circ$

With positive direction of  $x$  - axis.

(c) Find the acute angle between the lines  $x-2y+1=0$  and  $2x+6y-5=0$ . 5

OR

Prove that the given straight lines  $3x-4y-7=0$  and  $9x-12y-11=0$  are parallel.

(d) Find the equation of straight line parallel to  $5x+6y-10=0$  and passing through the Point  $(-3, 3)$  5

OR

Find the equation of the line cutting off equal intercepts and passing through the point  $(-2, 5)$

Section-III

3. (a) Convert  $120^\circ$  into radian and  $\frac{3\pi}{2}$  into degree 4

OR

Prove that  $\sin 2A = 2 \sin A \cos A$

- (b) Prove that  $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$  6

OR

If  $\tan A = \frac{5}{12}$  and  $180^\circ < A < 270^\circ$  then find the value of  $\sin A$  and  $\cos A$

- (c) Simplify  $\frac{\cos(360^\circ - A) \tan(360^\circ + A)}{\cot(270^\circ - A) \sin(90^\circ + A)}$  5

OR

Prove that  $\frac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A} = \tan A$ .

- (d) Show that  $\cos 100^\circ + \cos 80^\circ = 0$  5

OR

Show that  $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$ .

Section-IV

4. (a) If  $y = x^3 + 3\cos x + 4e^x + 2$  then find  $\frac{dy}{dx}$ . 4

OR

If  $y = (x+1)(x-1)$  then find  $\frac{dy}{dx}$ .

- (b) Find the maximum and minimum value of a function  $y = 2x^3 - 15x^2 + 36x + 6$  6

OR

If  $s = t^3 - 2t^2 + 6t + 8$  is the equation of motion of a particle in meters, find the acceleration at the end of 3 secs

- (c) If  $y = a \cos mx + b \sin mx$  then prove that  $\frac{d^2y}{dx^2} + m^2y = 0$ . 5

OR

Find the derivative of a function  $\frac{1 + \sin x}{1 - \sin x}$  w.r.t.x.

- (d) Find the equation of tangent to the curve  $y = 1 - x^3$  at the point  $(2, 3)$  5

OR

If  $y = (1 + x^2) \tan^{-1} x$  then find  $\frac{dy}{dx}$ .

5. (a) Evaluate  $\int \tan^2 x \, dx$  4

OR

Integrate the function  $\sin x + \frac{1}{x} + x^3 - 7$  w.r.t.x.

- (b) Find area bounded by the curve  $y = x^2 + 2$ , the x-axis and the ordinates at  $x=1$  and  $x=2$  6

OR

Find the volume of the solid generated by revolving the line  $y^2=2x+1$  about x-axis between the ordinates  $x=0$  and  $x=2$

- (c) Evaluate the indefinite integral  $\int (x \sin x) dx$  using integration by parts. 5

OR

Evaluate  $\int_0^1 \frac{(\tan^{-1} x)^4}{1+x^2} dx$

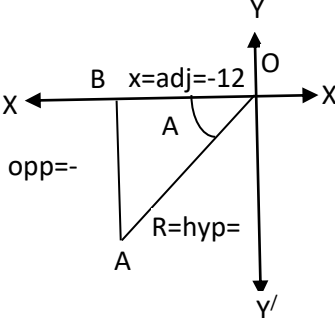
DIPLOMA MAKEUP EXAMINATION SEPT-2023  
MODEL ANSWERS OF ENGINEERING MATHEMATICS [20SC01T]

Section-I		
Q No	Answers	Marks
1 (a)	$2A+3B=2\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}+3\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $=\begin{bmatrix} 4 & 2 \\ 6 & 8 \end{bmatrix}+\begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}=\begin{bmatrix} 7 & 14 \\ 12 & 17 \end{bmatrix}$ <div style="text-align: center;">OR</div> $\text{If } A=\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix} \Rightarrow A^T=\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$ $A+A^T=\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}+\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}=\begin{bmatrix} 6 & 4 \\ 4 & 0 \end{bmatrix}$	4M
1(b)	$A=\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ <p>characteristic equation is given by</p> $\lambda^2-(a+d)\lambda+(ad-bc)=0$ $\lambda^2-(5+3)\lambda+(15-8)=0$ $\lambda^2-8\lambda+7=0$ $\lambda^2-7\lambda-\lambda+7=0$ $\lambda(\lambda-7)-1(\lambda-7)=0$ $(\lambda-7)(\lambda-1)=0$ $\lambda=7 \text{ or } \lambda=1$ <p><math>\therefore \lambda=7,1</math> are required characteristic roots</p> <div style="text-align: center;">OR</div> $A=\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ $ A =\begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}=\cos^2 x+\sin^2 x=1 \neq 0$ <p>Inter change principal diagonal elements and sign change secondary diagonal elements in matrix A to get adj(A)</p> $\text{adj}(A)=\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ $A^{-1}=\frac{1}{ A }\text{adj}(A)=\frac{1}{1}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ $=\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$	6M
1(c)	$\text{If } A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ <p>CF of <math>2= 2 =2</math></p> <p>CF of <math>-1=- -1 =1</math></p> <p>CF of <math>-1=- -1 =1</math></p> <p>CF of <math>2= 2 =2</math></p> <p><math>\therefore \text{adj}(A)=\begin{bmatrix} 2 &amp; 1 \\ 1 &amp; 2 \end{bmatrix}</math></p> <div style="text-align: center;">OR</div> <p>Given equations <math>2x+y=1; 3x+2y=1</math></p> $\Delta=\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}=4-3=1$ $\Delta_x=\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}=2-1=1 \quad \& \quad \Delta_y=\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}=2-3=-1$ $x=\frac{\Delta_x}{\Delta}=\frac{1}{1}=1 \quad \text{and} \quad y=\frac{\Delta_y}{\Delta}=\frac{-1}{1}=-1$ <p><math>\therefore x=1 \&amp; y=-1</math></p>	5M
1(d)	$AB=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $AB=\begin{bmatrix} 2+6 & 1+8 \\ 6+12 & 3+16 \end{bmatrix}=\begin{bmatrix} 8 & 9 \\ 18 & 19 \end{bmatrix}$ $(AB)^T=\left(\begin{bmatrix} 8 & 9 \\ 18 & 19 \end{bmatrix}\right)^T=\begin{bmatrix} 8 & 18 \\ 9 & 19 \end{bmatrix}$ <div style="text-align: center;">OR</div> $\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix}=\begin{vmatrix} 3 & 2 \\ 0 & x \end{vmatrix}$ $4x-6=3x-0$ $4x-3x=6$ $x=6$	5M

Section-II

2(a)	<p>Given, slope = <math>m = 5</math>  Y - intercept = <math>c = 3</math>  Equation of st line is  <math>y = mx + c</math>  <math>y = 5x + 3</math>  <math>5x + 3 = y</math>  <math>5x - y + 3 = 0</math></p>	OR	<p>(a) <math>y - y_1 = m(x - x_1)</math>  (b) <math>y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)</math></p>	4M
2(b)	<p><math>A = (3, 4)</math> and <math>B = (5, 6)</math>  <math>y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)</math>  <math>y - 4 = \frac{6 - 4}{5 - 3}(x - 3)</math>  <math>y - 4 = \frac{2}{2}(x - 3)</math>  <math>y - 4 = x - 3</math>  <math>x - 3 - y + 4 = 0</math>  <math>x - y + 1 = 0</math></p>	OR	<p><math>A(x_1, y_1) = (1, 2)</math> &amp; <math>\theta = 45^\circ</math>  <math>m = \tan \theta = \tan 45^\circ = 1</math>  <math>y - y_1 = m(x - x_1)</math>  <math>y - 2 = 1(x - 1)</math>  <math>y - 2 = x - 1</math>  <math>x - 1 - y + 2 = 0</math>  <math>x - y + 1 = 0</math></p>	6M
2(c)	<p>Given: <math>x - 2y + 1 = 0</math> &amp; <math>2x + 6y - 5 = 0</math>  <math>m_1 = -\frac{a}{b} = -\frac{1}{-2} = \frac{1}{2}</math> &amp; <math>m_2 = -\frac{a}{b} = -\frac{2}{6} = -\frac{1}{3}</math>  <math>\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}</math>  <math>\tan \theta = \frac{\left  \frac{1}{2} - \left(-\frac{1}{3}\right) \right }{\left  1 + \frac{1}{2} \left(-\frac{1}{3}\right) \right } = \frac{\left  \frac{1}{2} + \frac{1}{3} \right }{\left  1 - \frac{1}{6} \right } = \frac{\left  \frac{3+2}{6} \right }{\left  \frac{6-1}{6} \right } = \frac{\left  \frac{5}{6} \right }{\left  \frac{5}{6} \right } = 1</math>  <math>\Rightarrow \tan \theta = \tan 45^\circ</math>  <math>\therefore \theta = 45^\circ</math></p>	OR	<p>Given lines <math>3x - 4y - 7 = 0</math>  <math>9x - 12y - 11 = 0</math>  <math>m_1 = -\frac{a}{b} = -\frac{3}{-4} = \frac{3}{4}</math>  <math>m_2 = -\frac{a}{b} = -\frac{9}{-12} = \frac{3}{4}</math>  <math>\therefore m_1 = m_2 = \frac{3}{4}</math>  Hence the given lines are parallel .</p>	5M
2(d)	<p>Consider given line <math>5x + 6y - 10 = 0</math>  slope of given line = <math>-\frac{a}{b} = -\frac{5}{6}</math>  <math>\therefore</math> Slope of required line = <math>m = -\frac{5}{6}</math>  <math>A(x_1, y_1) = (-3, 3)</math> &amp; <math>m = -\frac{5}{6}</math>  <math>y - y_1 = m(x - x_1)</math>  <math>y - 3 = -\frac{5}{6}(x - (-3))</math>  <math>6(y - 3) = -5(x + 3)</math>  <math>6y - 18 + 5x + 15 = 0</math>  <math>5x + 6y - 3 = 0</math></p>	OR	<p>Let x - intercept = <math>a = k</math>  y - intercept = <math>b = k</math>  <math>\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{k} + \frac{y}{k} = 1 \Rightarrow \frac{x + y}{k} = 1</math>  <math>x + y = k</math> ----- (1)  Since (1) passes through <math>(-2, 5)</math>  <math>-2 + 5 = k</math>  <math>k = 3</math> substitute in equation (1)  <math>x + y = 3</math>  <math>x + y - 3 = 0</math> is the required equation of st line.</p>	5M

Section-III

3(a)	$120^0 = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ $\frac{3\pi}{2} = \frac{3\pi}{2} \times \frac{180}{\pi} = 270^0$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> $\sin(A+B) = \sin A \cos B + \cos A \sin B$ <p>put <math>B = A</math></p> $\sin(A+A) = \sin A \cos A + \cos A \sin A$ $\sin 2A = 2 \sin A \cos A$ <p>Hence proved the result.</p>	4M
3(b)	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $LHS = \tan(45^0 + A)$ $= \frac{\tan 45^0 + \tan A}{1 - \tan 45^0 \tan A}$ $LHS = \frac{1 + \tan A}{1 - \tan A} = RHA$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> <p>Given <math>\tan A = \frac{5}{12} = \frac{opp}{Adj}</math></p> $OA^2 = OB^2 + AB^2$ $r^2 = (-12)^2 + (-5)^2 = 144 + 25 = 169$ $r = \pm\sqrt{169} = \pm 13$ <p><math>r = 13</math> Since Hyp is always positive</p> $\sin A = \frac{Opp}{Hyp} = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$ $\cos A = \frac{Adj}{Hyp} = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$ 	6M
3(c)	$\frac{\cos(360^0 - A) \tan(360^0 + A)}{\cot(270^0 - A) \sin(90^0 + A)}$ $= \frac{\cos A \times \tan A}{\tan A \times \cos A}$ $= 1$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> $LHS = \frac{1 - \cos 2A + \sin 2A}{1 + \cos 2A + \sin 2A}$ $LHS = \frac{2 \sin^2 A + 2 \sin A \cos A}{2 \cos^2 A + 2 \sin A \cos A}$ $LHS = \frac{2 \sin A (\sin A + \cos A)}{2 \cos A (\cos A + \sin A)}$ $LHS = \frac{\sin A}{\cos A} = \tan A.$	5M
3(d)	$LHS = \cos 100^0 + \cos 80^0$ $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$ $LHS = 2 \cos\left(\frac{100+80}{2}\right) \cos\left(\frac{100-80}{2}\right)$ $LHS = 2 \cos 90 \times \cos 10$ $LHS = 2 \times 0 \times \cos 10$ $LHS = 0 = RHS$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> $L.H.S = \cos 20^0 \cos 40^0 \cos 80^0$ $= (\cos 40^0 \cos 20^0) \cos 80^0$ $W.K.T \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$ $L.H.S = \frac{1}{2} [\cos(40^0 + 20^0) + \cos(40^0 - 20^0)] \cos 80^0$ $= \frac{1}{2} (\cos 60^0 + \cos 20^0) \cos 80^0$ $= \frac{1}{2} \left( \frac{1}{2} + \cos 20^0 \right) \cos 80^0$ $= \frac{1}{4} \cos 80^0 + \frac{1}{2} \cos 80^0 \cos 20^0$ $= \frac{1}{4} \cos(180 - 100^0) + \frac{1}{2} \times \frac{1}{2} [\cos(80^0 + 20^0) + \cos(80^0 - 20^0)]$ $= \frac{1}{4} \times -\cos 100^0 + \frac{1}{4} (\cos 100^0 + \cos 60^0)$ $= -\frac{1}{4} \cos 100^0 + \frac{1}{4} \cos 100^0 + \frac{1}{4} \cos 60^0 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = R.H.S$	5M

## Section-IV

4(a)	$y = x^3 + 3\cos x + 4e^x + 2$ $\frac{dy}{dx} = 3x^2 + 3 \times -\sin x + 4e^x + 0$ $\frac{dy}{dx} = 3x^2 - 3\sin x + 4e^x$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> $y = (x+1)(x-1)$ $y = x^2 - 1^2 = x^2 - 1$ $\frac{dy}{dx} = 2x - 0 = 2x$	4M
4(b)	$y = 2x^3 - 15x^2 + 36x + 6$ <i>Differentiate w.r.t.x</i> $\frac{dy}{dx} = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 6)$ $\frac{dy}{dx} = 6x^2 - 30x + 36$ $0 = 6x^2 - 30x + 36$ $6x^2 - 30x + 36 = 0$ <i>Divide both side by 6</i> $x^2 - 5x + 6 = 0$ $x^2 - 5x + 6 = 0$ $x^2 - 3x - 2x + 6 = 0$ $x(x-3) - 2(x-3) = 0$ $(x-3)(x-2) = 0$ $x-3 = 0 \quad \text{or} \quad x-2 = 0$ $x = 3 \quad \text{or} \quad x = 2$ $x = 2, 3$ are stationary point s $\frac{dy}{dx} = 6x^2 - 30x + 36$ <i>Again diff w.r.t.x</i> $\frac{d^2y}{dx^2} = 6 \times 2x - 30 + 0$ $\frac{d^2y}{dx^2} = 12x - 30$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">OR</div> <p>When <math>x = 2</math></p> $\left(\frac{d^2y}{dx^2}\right)_{x=2} = 12(2) - 30 = 24 - 30 = -6 < 0$ The function is maxima at $x = 2$ and its maximum value is given $y = 2x^3 - 15x^2 + 36x + 6$ <i>put <math>x = 2</math></i> $y_{\max} = 2(2)^3 - 15(2)^2 + 36(2) + 6$ $y_{\max} = 2(8) - 15 \times 4 + 72 + 6$ $y_{\max} = 16 - 60 + 72 + 6$ $y_{\max} = 94 - 60 = 34$ <p>When <math>x = 3</math></p> $\left(\frac{d^2y}{dx^2}\right)_{x=3} = 12(3) - 30 = 36 - 30 = 6 > 0$ Therefore the function is minima at $x = 3$ and its minimum value is given $y = 2x^3 - 15x^2 + 36x + 6$ <i>put <math>x = 3</math></i> $y_{\min} = 2(3)^3 - 15(3)^2 + 36(3) + 6$ $y_{\min} = 2(27) - 15 \times 9 + 108 + 6$ $y_{\min} = 54 - 135 + 108 + 6$ $y_{\min} = 168 - 135 = 33$	6M
	$s = t^3 - 2t^2 + 6t + 8$ <i>Differentiate w.r.t.t</i> $\frac{ds}{dt} = \frac{d}{dt}(t^3 - 2t^2 + 6t + 8)$ $\frac{ds}{dt} = 3t^2 - 2 \times 2t + 6 + 0$ $\frac{ds}{dt} = 3t^2 - 4t + 6$	$\frac{d^2s}{dt^2} = 3 \times 2t - 4 + 0$ $\frac{d^2s}{dt^2} = 6t - 4$ Acceleration at time $t = 3\text{sec}$ $a = \frac{d^2s}{dt^2} = 6t - 4$ $a_{t=3\text{sec}} = 6(3) - 4 = 18 - 4 = 14 \text{ meter/sec}$	

<p>4(c)</p>	<p><math>y = a \sin mx + b \sin mx</math>  <i>Differentiate w.r.t.x</i>  <math>\frac{dy}{dx} = \frac{d}{dx}(a \sin mx + b \cos mx)</math>  <math>\frac{dy}{dx} = a \times \cos mx \times m + b \times -\sin mx \times m</math>  <math>\frac{dy}{dx} = am \cos mx - bm \sin mx</math>  <i>Again diff w.r.t.x</i>  <math>\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(am \cos mx - bm \sin mx)</math>  <math>\frac{d^2 y}{dx^2} = am \times -\sin mx \times m - bm \times \cos mx \times m</math>  <math>\frac{d^2 y}{dx^2} = -am^2 \sin mx - bm^2 \cos mx =</math>  <math>\frac{d^2 y}{dx^2} - m^2(a \sin mx + b \cos mx)</math>  <math>\frac{d^2 y}{dx^2} = -m^2 y</math> From equation (1)  <math>\frac{d^2 y}{dx^2} + m^2 y = 0</math></p>	<p style="text-align: center;">OR</p> <p><math>y = \frac{1 + \sin x}{1 - \sin x}</math>  <i>Differentiate w.r.t.x</i>  <math>\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1 + \sin x}{1 - \sin x}\right)</math>  <i>w.k.t</i> <math>\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}</math>  <math>\frac{dy}{dx} = \left( \frac{(1 - \sin x) \frac{d}{dx}(1 + \sin x) - (1 + \sin x) \frac{d}{dx}(1 - \sin x)}{(1 - \sin x)^2} \right)</math>  <math>\frac{dy}{dx} = \frac{(1 - \sin x)(0 + \cos x) - (1 + \sin x)(0 - \cos x)}{(1 - \sin x)^2}</math>  <math>\frac{dy}{dx} = \frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}</math>  <math>\frac{dy}{dx} = \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}</math>  <math>\frac{dy}{dx} = \frac{2 \cos x}{(1 - \sin x)^2}</math></p>	<p>5M</p>
<p>4(d)</p>	<p><math>y = 1 - x^3</math>  <i>Differentiate w.r.t.x</i>  <math>\frac{dy}{dx} = \frac{d}{dx}(1 - x^3)</math>  <math>\frac{dy}{dx} = 0 - 3x^2 = -3x^2</math>  <math>\left(\frac{dy}{dx}\right)_{at A(2,3)} = -3(2)^2 = -3 \times 4 = -12</math>  <i>Now A = (2,3) and m = -12</i>  <math>y - y_1 = m(x - x_1)</math>  <math>y - 3 = -12(x - 2)</math>  <math>y - 3 = -12x + 24</math>  <math>y - 3 + 12x - 24 = 0</math>  <math>12x + y - 27 = 0</math></p>	<p style="text-align: center;">OR</p> <p><math>y = (1 + x^2) \tan^{-1} x</math>  <i>Differentiate w.r.t.x</i>  <math>\frac{dy}{dx} = \frac{d}{dx}[(1 + x^2) \tan^{-1} x]</math>  <i>w.k.t</i> <math>\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}</math>  <math>\frac{dy}{dx} = (1 + x^2) \frac{d}{dx}(\tan^{-1} x) + \tan^{-1} x \frac{d}{dx}(1 + x^2)</math>  <math>\frac{dy}{dx} = (1 + x^2) \times \frac{1}{(1 + x^2)} + \tan^{-1} x \times (0 + 2x)</math>  <math>\frac{dy}{dx} = 1 + 2x \tan^{-1} x</math></p>	<p>5M</p>



## Section-V

5(a)	<p>Let <math>I = \int \tan^2 x \, dx</math></p> <p>W.K.T <math>\tan^2 x = \sec^2 x - 1</math></p> $I = \int (\sec^2 x - 1) dx$ $= \tan x - x + c$	OR	$I = \int \left( \sin x + \frac{1}{x} + x^3 - 7 \right) dx$ $I = -\cos x + \log x + \frac{x^4}{4} - 7x + c$	4M
5(b)	<p>Given: <math>y = (x^2 + 2)</math> and <math>x = 1, x = 2</math></p> <p>W.K.T Area bounded by the curve with xiax is</p> $A = \int_a^b y \, dx$ $A = \int_1^2 (x^2 + 2) dx = \left( \frac{x^3}{3} + 2x \right)_1^2$ $A = \left( \frac{2^3}{3} + 2 \times 2 \right) - \left( \frac{1^3}{3} + 2 \times 1 \right) = \frac{8}{3} + 4 - \frac{1}{3} - 2$ $A = \frac{8 \times 1}{3} - \frac{1 \times 1}{3} + 2 \times 3 = \frac{8-1+6}{3} = \frac{13}{3} \text{ sq units}$	OR	<p>Given: <math>y^2 = 2x + 1</math> and <math>x = 0, x = 2</math></p> <p>W.K.T Volume generated by the curve with xiax is</p> $V = \pi \int_a^b y^2 \, dx$ $V = \pi \int_0^2 (2x + 1) dx = \pi \left( 2 \times \frac{x^2}{2} + x \right)_0^2$ $V = \pi (x^2 + x)_0^2 = \pi \{ (2^2 + 2) - (0^2 + 0) \}$ $V = \pi \{ (4 + 2) - 0 \} = 6\pi \text{ cubic units}$	
5(c)	<p><math>I = \int x \sin x \, dx</math></p> <p>WKT <math>\int uv \, dx = u \int v \, dx - \int \left( \int v \, dx \times \frac{du}{dx} \right) dx</math></p> $I = x \int \sin x \, dx - \int \left( \int \sin x \, dx \times \frac{d(x)}{dx} \right) dx$ $I = x \times -\cos x - \int (-\cos x \times 1) dx$ $I = -x \cos x + \int \cos x \, dx$ $I = -x \cos x + \sin x + c$	OR	<p>Let <math>I = \int_0^1 \frac{(\tan^{-1} x)^4}{1+x^2} dx</math></p> <p>put <math>\tan^{-1} x = t</math></p> $\frac{1}{1+x^2} = \frac{dt}{dx} \Rightarrow \frac{1}{1+x^2} dx = dt$ <p><math>x = 0 \Rightarrow t = \tan^{-1} 0 = 0</math></p> <p><math>\Rightarrow x = 1 \Rightarrow t = \tan^{-1} 1 = \frac{\pi}{4}</math></p> $I = \int_0^{\frac{\pi}{4}} t^4 dt = \left[ \frac{t^5}{5} \right]_0^{\frac{\pi}{4}} = \frac{1}{5} \left[ \left( \frac{\pi}{4} \right)^5 - (0)^5 \right] = \frac{1}{5} \left( \frac{\pi^5}{1024} - 0 \right)$ $I = \frac{\pi^5}{5120}$	
5(d)	<p>Let <math>I = \int \frac{x+1}{x^2+2x+1} dx</math></p> <p>put <math>x^2 + 2x + 1 = t</math></p> <p>Diff w.r.t.x</p> $2x + 2 + 0 = \frac{dt}{dx} \Rightarrow 2(x+1) dx = dt$ $(x+1) dx = \frac{dt}{2}$ $I = \int \frac{1}{t} \times \frac{dt}{2} = \frac{1}{2} \log t + c$ $I = \frac{1}{2} \log (x^2 + 2x + 1) + c$	OR	<p>Let <math>I = \int \sin^3 x \, dx</math></p> <p>W.K.T <math>\sin 3x = 3 \sin x - 4 \sin^3 x</math></p> $\sin^3 x = \frac{1}{4} (3 \sin x - \sin 3x)$ $I = \int \frac{1}{4} (3 \sin x - \sin 3x) dx$ $= \frac{1}{4} \int (3 \sin x - \sin 3x) + c$ $= \frac{1}{4} \left( -3 \cos x - x - \frac{\cos 3x}{3} \right) + c$ $= \frac{1}{4} \left( -3 \cos x + \frac{\cos 3x}{3} \right) + c$	