7651

Code : 20SC01T

Register Number | | | | | | | | | | |

I/II Semester Diploma Examination, February/March-2023

ENGINEERING MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (i) Answer **one** full question from each section.

(ii) Each section carries **20** marks.

SECTION – I

1. (a) Define square matrix with an example. **4**

OR

If $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$, then find $3A - 2B$.

(b) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$, then find the inverse of matrix A if it exists. **6**

OR

Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

(c) Solve the system of linear equations by applying Cramer's rule : **5**

$x + y = 1$

$2x + y = 2$

OR

The Maruti Motor Company Ltd. has 2 outlets, one in Delhi and one in Mumbai, among other things, it sells Baleno, Ertiga and Brezza cars. The monthly sales of these cars at the two stores for two months are given in the following tables :

January Sales

	Delhi	**Mumbai**
Baleno	45	30
Ertiga	35	25
Brezza	20	18

February Sales

	Delhi	**Mumbai**
Baleno	42	28
Ertiga	36	20
Brezza	22	16

Use matrix subtraction to calculate the change in sales of each product in each store from January to February.

(d) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, find AB matrix and also find $(AB)^T$ matrix. **5**

OR

For the matrix $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, verify $|A| \cdot I = A \cdot (\text{adj } A)$ where $|A|$ stands for determinant of A and I is a unit matrix of order 2×2.

SECTION – II

2. (a) Find slope and x-intercept of the line $3x + 4y + 7 = 0$. **4**

OR

Write the standard form of equation of straight line in (i) general form and (ii) one-point form.

(b) Find the equation of line passing through the point (1, 2) and parallel to the line $2x - 3y + 1 = 0$. **6**

OR

Find the equation of straight line passing through the points (2, 3) and (4, 6).

(c) Find the equation of straight line using intercepts form whose x-intercept is 3 and y-intercept is 2. **5**

OR

Show that the angle between the lines
$x - y + 4 = 0$ and $2x - y + 5 = 0$ is $\tan^{-1}(1/3)$.

(d) If the line inclined at angle of 45° with +ve direction of x-axis and having y-intercept '5' unit, then find its equation using slope-intercept form. **5**

OR

Write the condition of slopes for 2 lines to be parallel and show that the lines $2x + y - 4 = 0$ and $6x + 3y + 10 = 0$ are parallel.

SECTION – III

3. (a) Convert 40° into radians and $\frac{8\pi}{7}$ into degree. **4**

OR

Prove that $\tan(45° - A) = \frac{1 - \tan A}{1 + \tan A}$.

(b) Simplify $\frac{\sin(360° + A) \cdot \tan(180° + A)}{\cos(90° - A) \cdot \cot(270° - A)}$. **6**

OR

If $\tan\theta = \frac{3}{4}$ where θ is in I quadrant, show that the value of $5\sin\theta + 5\cos\theta = 7$.

(c) Write the formula for $\cos(A + B)$ then find the value of $\cos 75°$. 5

OR

Prove that $\frac{\sin 2A}{\sin A} - \frac{\cos 2A}{\cos A} = \sec A$.

(d) Prove that $\frac{\sin 6\theta + \sin 2\theta}{\cos 6\theta + \cos 2\theta} = \tan 4\theta$. 5

OR

Prove that $\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \tan\theta$

SECTION – IV

4. (a) If $y = \tan x + 4e^x - 6 + \sqrt{x}$, then find $\frac{dy}{dx}$. 4

OR

Differentiate $x^2 \cdot e^x$ w.r.t. x.

(b) Find the derivative of $y = \frac{1 + \tan x}{1 - \tan x}$ w.r.t. x. 6

OR

If $y = 2x^4 - 3x^3 - 2x^2 + x - 1$, find $\frac{d^2y}{dx^2}$ at $x = 0$.

(c) Distance travelled by a particle in 't' second is given by $S = 2t^3 - t^2 + 5t - 6$. Find the velocity and acceleration of particle at $t = 2$ second. 5

OR

Find the maximum and minimum values of function $y = 2x^3 - 3x^2 - 36x + 10$.

(d) If $y = x^2$, show that $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$. 5

OR

Find the equation of tangent to the curve $y = x^2 + x - 1$ at the point (1, 1).

SECTION – V

5. (a) Integrate $e^x + \frac{1}{1 + x^2} - \sin x + x^3$ w.r.t. x. 4

OR

Evaluate $\int x^2 \cdot (1 + x) \cdot dx$.

(b) Evaluate $\int \cos^2 x \cdot dx$. **6**

OR

Show that $\int_0^{\pi/4} \tan^2 x \cdot \sec^2 x \cdot dx = 1/3$.

(c) With the use of definite integrals find the area bounded by the curve $y = x^3 - 2$, x-axis and $x = 0, x = 1$. **5**

OR

The curve $y^2 = x + 2$ is rotated about x-axis. Find the volume of solid generated by revolving the curve between $x = 2$ and $x = 5$.

(d) Evaluate the indefinite integral $\int x \cdot e^x \cdot dx$ using integration by parts. **5**

OR

Evaluate $\int_0^1 \frac{e^x}{1 + e^x} \cdot dx$.

I/II Semester Diploma Examination, February-2023

Course Name: Engineering Mathematics　　　　Course Code: 20SC01T

MODEL ANSWERS

<table>
<tr><th>Q.No.</th><th>Answers</th><th>Q.No.</th><th>Answers</th></tr>
<tr>
<td rowspan="2">1(a)</td>
<td rowspan="2">Square matrix: - It is a matrix with equal number of Rows and Columns

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

OR

Given $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$

3A-2B=3 $\begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$

=$\begin{bmatrix} 12 & 15 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$

=$\begin{bmatrix} 6 & 7 \\ 5 & 4 \end{bmatrix}$</td>
<td>1(b)</td>
<td>(1-λ) (3-λ)-8=0
$\lambda^2 - 4\lambda - 5 = 0$

$\lambda^2 - 5\lambda + \lambda - 5 = 0$

$\lambda(\lambda - 5) + 1(\lambda - 5) = 0$

λ=5 & λ=-1</td>
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<tr>
<td>1(c)</td>
<td>x+y=1 & 2x+y=2

Δ=$\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$

$\Delta_x = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$

$\Delta_y = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$

$x = \frac{\Delta_x}{\Delta} = \frac{=1}{-1} = 1 \& y = \frac{\Delta_y}{\Delta} = \frac{0}{-1} = 0$</td>
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<tr>
<td rowspan="2">1(b)</td>
<td rowspan="2">Given A=$\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ $then$ detA=$\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 6 - 2 = 4$

adjA=$\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$

$A^{-I} = \frac{adjA}{detA} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$

OR

A=$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

Characteristic equation is $|A - \lambda I| = 0$

$\left|\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right| = 0$

$\begin{vmatrix} 1 - \lambda & 2 \\ 4 & 3 - \lambda \end{vmatrix} = 0$</td>
<td>1(c)</td>
<td>OR

$J = \begin{pmatrix} 45 & 30 \\ 35 & 25 \\ 20 & 18 \end{pmatrix}, F = \begin{pmatrix} 42 & 28 \\ 36 & 20 \\ 22 & 16 \end{pmatrix}$

$D = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, M = \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix}$</td>
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1(d)	$A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B = \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$ $AB = \begin{bmatrix}2+6 & 1+8\\6+12 & 3+16\end{bmatrix} = \begin{bmatrix}8 & 9\\18 & 19\end{bmatrix}$ $(AB)^T = \begin{bmatrix}8 & 18\\9 & 19\end{bmatrix}$	2(b)	OR Given points are (2,3) & (4,6) Two-point form is $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (or) $\frac{y-3}{x-2} = \frac{3}{2}$ $2y-6 = 3x-6(\ or)\ 3x-2y = 0$
1(d)	OR $A = \begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$ $\lvert A \rvert = \begin{vmatrix}3 & 1\\5 & 2\end{vmatrix} = 6-5 = 1$ $\lvert A \rvert.I = 1.\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = L.H.S$ $A.adjA = \begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\-5 & 3\end{bmatrix}$ $= \begin{bmatrix}6-5 & -3+3\\10-10 & -5+6\end{bmatrix}$ $= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = R.H.S$	2(c)	$\text{Intercepts form is} \frac{x}{a} + \frac{y}{b} = 1$ Given $x - int. = 3$ & $y - int. = 2$ Then $\frac{x}{3} + \frac{y}{2} = 1$ Therefore, the equation is $2x + 3y - 6 = 0$ OR Slope of x-y+4=0 is $m_1 = 1$, $Slope$ of 2x-y+5=0 is $m_2 = 2$, Formula Θ=$\tan^{-1}\left(\frac{m_2-m_1}{1+m_2.m_1}\right)$ $= \tan^{-1}\left(\frac{2-1}{1+2.1}\right) = \tan^{-1}\left(\frac{1}{3}\right)$
2(a)	$Equation\ of\ line\ is\ 3x + 4y + 7 = 0$ $Slope = \frac{-a}{b} = \frac{-3}{4}$ $X - int. = \frac{-c}{a} = \frac{-7}{3}$ OR General form of straight line is $ax + by + c = 0$ One-point form is $(y - y_1) = m(x - x_1)$	2(d)	Θ=45^0, $m = \tan 45^0$, $y - int. = C = 5$ Equation of line is y=mx+c i.e., y=1.x+5 or x-y+5=0 **(OR)** Parallel condition is $m_1 = m_2$ Slope of 2x+y-4=0 is $m_1 = \frac{-2}{1} = -2$ Slope of 6x+3y+10=0 is m_2=$\frac{-6}{3} = -2$, $There\ fore\ m_1 = m_2$
2(b)	2x-3y+1=0, Slope $m_1 = \frac{-a}{b}$=$\frac{-2}{-3} = \frac{2}{3}$, $m_2 = \frac{2}{3}$(II) One-point form is $(y - y_1) = m(x - x_1)$ $(y-2) = \frac{2}{3}(x-1)$ & $2x - 3y + 4 = 0$	3(a)	40^0=40X1^0 =40X$\frac{\pi^c}{180}$=$\frac{2\pi^c}{9} = 0.6981^c$ and $\frac{8\pi^c}{7}$=$\frac{8\pi}{7}$X$\frac{180^0}{\pi}$ $= (205.42)^0$ $= 205^0 42^I 51^{II}$

3(a)	OR $$\tan(45^0 - A) = \frac{\tan 45^0 - \tan A}{1 + \tan 45^0 . \tan A}$$ $= \frac{1-\tan A}{1+\tan A}$	3(d)	Given $\frac{\sin 6\theta+\sin 2\theta}{\cos 6\theta+\cos 2\theta}=\frac{2\sin\left(\frac{6\theta+2\theta}{2}\right)\cos\left(\frac{6\theta-2\theta}{2}\right)}{2\cos\left(\frac{6\theta+2\theta}{2}\right)\cos\left(\frac{6\theta-2\theta}{2}\right)}$ $=\frac{\sin 4\theta}{\cos 4\theta} = \tan 4\theta$ OR $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}$ $=\frac{1-(1-2sin^2\theta)+2\sin\theta\cos\theta}{1+2cos^2\theta-1+2\sin\theta\cos\theta}=\frac{2sin^2\theta+2\sin\theta\cos\theta}{2cos^2\theta+2\sin\theta\cos\theta}$ $= \frac{2\sin\theta(\sin\theta+\cos\theta)}{2cos\theta(\cos\theta+\sin\theta)}$ =$\tan\theta$
3(b)	Simplify $\frac{\sin(360^0+A)\tan(180^0+A)}{\cos(90^0-A)\cot((270^0-A))}=\frac{\sin A.\tan A}{\sin A.\tan A}=1$ OR Given $\tan\theta = \frac{3}{4}$, $Then\ hypotenuse = 5$ Therefore $\sin\theta = \frac{3}{5}$, $\cos\frac{4}{5}$ $5\sin\theta + 5\cos\theta = 5\left(\frac{3}{5}\right) + 5\left(\frac{4}{5}\right)=7$	4(a)	Given $y = \tan x + 4e^x - 6 + \sqrt{x}$ $\frac{dy}{dx} = sec^2x + 4e^x - 0 + \frac{1}{2\sqrt{x}}$ OR $$\frac{d(x^2.e^x)}{dx} = x^2e^x + e^x.2x$$
3(c)	$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $$\cos(75^0) = \cos(45^0 + 30^0)$$ $$= \cos 45^0\cos 30^0 - \sin 45^0 \sin 30^0$$ $= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$ OR Let$\frac{sin2A}{sinA} - \frac{\cos 2A}{\cos A}$ $$= \frac{\sin 2A\cos A - \cos 2A\sin A}{\sin A\cos A}$$ $= \frac{\sin(2A-A)}{\sin A\cos A} = \frac{\sin A}{\sin A\cos A} = \frac{1}{\cos A} = \sec A$	4(b)	$$y = \frac{(1+\tan x)}{(1-\tan x)}$$ $\frac{dy}{dx}$ $= \frac{(1-\tan x)sec^2x - (1+\tan x)(-sec^2x)}{(1-\tan x)^2}$ $=\frac{sec^2x-\tan x sec^2x+sec^2x+\tan x sec^2x}{(1-\tan x)^2}=\frac{2sec^2x}{(1-\tan x)^2}$ OR $$y = 2x^4 - 3x^2 - 2x^2 + x - 1$$ $$\frac{dy}{dx} = 8x^3 - 9x^2 - 4x + 1$$ $\frac{d^2y}{dx^2} = 24x^2 - 18x - 4, at\ x = 0, \frac{d^2y}{dx^2} = -4$

4(c)	S=$2t^3 - t^2 + 5t - 6$ V=$\frac{ds}{dt} = 6t^2 - 2t + 5$ a=$\frac{dv}{dt} = 12t - 2$ At t=2 , V=25 At t=2 , a=22 OR $Y = 2x^3 - 3x^2 - 36x + 10$, $\frac{dy}{dx} = 6x^2 - 6x - 36$, $\frac{d^2y}{dx^2} = 12x - 6$ Let $x^2 - x - 6 = 0$, $x^2 - 3x + 2x - 6 = 0$, $(x-3)(x+2) = 0$, x=3 or x= -2 Put x=3 in $\frac{d^2y}{dx^2}$ and its value is 30(+ve) Function has minimum at x=3 Put x=-2 in $\frac{d^2y}{dx^2}$ and its value is -30(-ve) Function has maximum at x=-2 Maximum value is (x=-2 in given eqn.) 54 Minimum value is (x=3 in given eqn.) -71	5(a)	$\int e^x + \frac{1}{1+x^2} - \sin x + x^3$.dx $= e^x + \tan^{-1} x + \cos x + \frac{x^4}{4}$+C OR $\int x^2 \, (1+x).dx$ $= \int x^2 + x^3.dx$ $= \frac{x^3}{3} + \frac{x^4}{4} + c$
4(d)	If $y = x^2$, $\frac{dy}{dx} = 2x$, $\frac{d^2y}{dx^2} = 2$ Let $x \, \frac{d^2y}{dx^2} - \frac{dy}{dx}$ $x(2) - 2x$ =0 OR $y = x^2 + x - 1$, One-point form is $\frac{dy}{dx} = 2x + 1$, $(y - y_1) = m(x - x_1)$ Slope m(at x=1)=3 $(y - y_1) = m(x - x_1)$ $(y-1) = 3(x-1)$ i.e. 3x-y-2=0	5(b)	$\int cos^2x.dx = \int \frac{1+\cos 2x}{2}.dx$ = $\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$ OR $\int_0^{\frac{\pi}{4}} tan^2x sec^2x.dx$ Put tanx = t $\int_0^{\frac{\pi}{4}} t^2.dt = \left[\frac{t^3}{3}\right]$ = $\frac{tan^3x}{3}$ sec^2x .dx=dt where limits are 0 to $\frac{\pi}{4}$ then answer is $\frac{1}{3}$

5(c)	Area=$\int_a^b y.dx = \int_0^1 x^3 - 2.dx = \left[\frac{x^4}{4} - 2x\right]$ $with\ limits\ 0\ to\ 1$ $= \left(\left(\frac{1^4}{4} - 2(1)\right)\right) - 0$ Answer is $\frac{-7}{4}$ OR Volume=$\pi\int_a^b y^2\,.dx = \pi\int_2^5 x + 2.dx$ $= \pi\left[\frac{x^2}{2} + 2x\right]$with limits 2 to 5 Answer is $\frac{33\pi}{2}$		
5(d)	$\int x.e^x.dx = x\int e^x.dx - \int \frac{d}{dx}x.\int e^x.dx.dx$ $= x\int e^x.dx - \int 1.e^x.dx$ $= xe^x - e^x + c$ OR $\int_0^1 \frac{e^x}{1+e^x}.dx = \int_0^1 \frac{dt}{t}$=$[logt] = [\log(1+e^x)]$ With limits 0 to 1 Put 1+$e^x = t$ $log(1+e) - \log(1+1)$ $e^x.dx = dt$ $= log\frac{(1+e)}{2}$		
Note: - Give equal weightage for alternate answers			

Certified that the model answers prepared by me for code 20SC01T are from prescribed textbook and model answers and scheme of valuation prepared by me are correct.