1601

Code : 20SC01T

Register Number | | | | | | | | | | |

I/II Semester Diploma Examination, Oct./Nov.-2021

ENGINEERING MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Special Note : Students can answer for max. of **100** marks, selecting any sub-section from any main section.

SECTION – I

1. (a) If the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 3 & 2 & x \end{bmatrix}$ is singular, then find 'x'. **4**

(b) If $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, then find $2A + 3B$. **5**

(c) Solve the system of equations $2x + 3y = 5$ and $x + 4y = 5$ by Cramer's Rule. **5**

(d) Find the characteristic roots for the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$. **6**

2. (a) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}$, then find $(A + B)^2$. **4**

(b) If $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 1 & 3 & -2 \end{bmatrix}$, then find $(AB)^T$. **5**

(c) Find the characteristic equation and Eigen roots of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. **5**

(d) Find the Inverse of the matrix $A = \begin{bmatrix} 5 & 1 & 3 \\ 1 & 4 & -1 \\ 2 & 0 & 4 \end{bmatrix}$. **6**

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SECTION – II

3. (a) Find the slope of the straight line passing through the points (1, 2) and (3, 4). **4**

(b) If the x-intercept of the line is 3 units and y-intercept is 2 times x-intercept, then find the equation of the line. **5**

(c) If a straight line makes an angle of inclination of 60° with respect to positive x-axis and passes through the point (1, –1), then find the equation of the straight line. **5**

(d) Find the equation of the straight line parallel to the line joining the points (3, –1) and (4, –2), passing through the point (2, 2). **6**

4. (a) Write the slope and x-intercept of the line $2x + 4y + 5 = 0$. **4**

(b) Find the equation of the straight line passing through the points (4, 2) and (1, 3). **5**

(c) Find the equation of the straight line perpendicular to the line $4x - 2y + 3 = 0$ and passing through the points (1, 2). **5**

(d) Find the acute angle between the lines $7x - 4y = 0$ and $3x - 11y + 5 = 0$. **6**

SECTION – III

5. (a) Express 75° in radian measure and $\frac{7\pi}{2}$ in degree. **4**

(b) Simplify : $\frac{\sec(360 - A)\cot(90 - A)}{\tan(360 + A)\,\mathrm{cosec}(90 + A)}$ **5**

(c) Prove that : $\cos 2\theta = \cos^2\theta - \sin^2\theta$ **5**

(d) Prove that : $\sin 20 \cdot \sin 40 \cdot \sin 80 = \frac{\sqrt{3}}{8}$ **6**

6. (a) Find the value of $\cos (15°)$ **4**

(b) Prove that : $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ **5**

(c) Prove that : $\sin 600° \cos 390° + \cos 480° \sin 150° = -1$ **5**

(d) Show that : $\cos 55 + \cos 65 + \cos 175 = 0$ **6**

SECTION – IV

7. (a) Find the derivative of $y = x^3 + e^{3x} + \sin 3x - 4 \log x$ w.r.t. 'x'. **4**

(b) Find $\frac{dy}{dx}$ for $y = \frac{1+\cos x}{1-\cos x}$ **5**

(c) Find $\frac{dy}{dx}$ for $y = e^{\sin x} + \sin (x^3) + \tan x$ **5**

(d) If $S = t^2 + 6t + 5$ represents the displacement of the particle in motion at time 't', then find the velocity of the particle and acceleration at $t = 3$ secs. **6**

8. (a) If $y = e^x \sin x$, then find $\frac{dy}{dx}$. **4**

(b) If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, then find $\frac{dy}{dx}$. **5**

(c) Find the equation of the tangent to the curve $y = x^3$ at the point (1, 2). **5**

(d) If $y = \tan^{-1} x$, show that $(1 + x^2)y_2 + 2xy_1 = 0$. 6

SECTION – V

9. (a) Evaluate : $\int\left(x^2+\sin x+\frac{1}{x}+e^{2x}\right)dx$ **4**

(b) Evaluate : $\int_0^{\pi/2}\cos^2 x\,dx$ **5**

(c) Evaluate : $\int xe^x\,dx$ **5**

(d) Find the area bounded by the curve $y = x^2 + 3$, x-axis and co-ordinates $x = 1$ & $x = 2$. **6**

10. (a) Evaluate : $\int_1^2 \frac{1}{x}\,dx$ **4**

(b) Evaluate : $\int \frac{\tan^{-1} x}{1+x^2}\,dx$ **5**

(c) Evaluate : $\int x\sec^2 x\,dx$ **5**

(d) Find the volume of the solid generated by revolving the curve $y = \sqrt{x^2+1}$ between $x = 0$ and $x = 2$. **6**

GOVERNMENT OF KARNATAKA
DEPARTMENT OF COLLEGIATE AND TECHNICAL EDUCATION
I/II SEMESTER DIPLOMA EXAMINATIONS, OCT/NOV-2021
Sub: Engineering Mathematics **Code: 20SC01T**

SCHEME AND SOLUTION

SECTION 1		
1	a) Expansion, Simplification, Answer	2+1+1
	b) Finding 2A and 3B, 2A+3B, Sum	2+2+1
	c) Finding Δ, Δ_1, Δ_2, x and y	1+1+1+1+1
	d) General CE, Particular CE, Roots	1+3+2
2	a) A+B, Writing $(A+B)^2$, Product	1+1+2
	b) Writing AB, Product, Transpose	1+3+1
	c) General CE, Particular CE, Roots	1+2+2
	d) Δ, Adjoint, Formula, Inverse	1+3+1+1
SECTION 2		
3	a) Formula, Substitution, Slope	2+1+1
	b) Writing intercepts, General form, Equation	1+2+2
	c) Slope formula, slope, line formula, Equation of the line	1+1+2+1
	d) Finding slope of lines, General equation of the line, equation of the line.	2+2+2
4	a) Slope formula, intercept formula, slope and x-intercept	2+2
	b) Slope formula, slope, line formula, Equation of the line	1+1+2+1
	c)Finding slopes of lines, General equation of the line, Equation of the line.	2+2+1
	d) Finding slopes, formula, Angle	2+2+2
SECTION 3		
5	a) Conversion radian to degree, vice-versa	2+2
	b) Finding each term, Simplification	4+1
	c) cos(A+B) ,Writing 2A=A+A , simplification	2+1+2
	d) sinAsinB formula, Simplification, Applying sinAcosB formula,	2+2+2
6	a) Writing cos15= cos(45-30),cos(A-B) formula, simplification, Answer	1+1+1+1
	b) Writing $\sin 3\theta = \sin(2\theta + \theta)$, $\sin 2\theta$ formula, Simplification	2+2+1
	c) Finding each term, Simplification	4+1

	d) Applying cosC+cosD formula Writing cos120, Simplification and solution.	3+1+2
SECTION 4		
7	a) Each term derivative	1+1+1+1
	b) Applying quotient rule, Each Simplification, Answer	2+1+2
	c) Finding derivative of $\sin(x^3)$ Finding derivative of e^{sinx} Finding derivative of tanx	2+2+1
	d) Differentiating w.r.t, t, Finding velocity, Finding 2nd derivative , Finding Acceleration.	2+1+2+1
8	a) Product rule, Each derivative	2+1+1
	b) Apply chain rule, Apply quotient rule, Simplification and Answer	2+2+1
	c) Finding derivative, Finding slope, Equation of tangent slope point form, Simplification	1+1+2+1
	d) Finding y_1, second derivative, Proof	2+2+2
SECTION 5		
9	a) Evaluation of each integrand	1+1+1+1
	b) Integrand simplification, Evaluation.	2+3
	c) Choosing first and second function Applying by part rule Integral of e^x ,Derivative of x	2+2+1
	d) Applying data in Area formula, Integration, tending the limits.	2+2+2
10	a) Integration, Applying limit, Solution.	2+1+1
	b) Substitution, Integration, Evaluation by tending limits	2+2+1
	c) Rule of parts, integrand Evaluation	2+3
	d) Volume formula, Substituting, Integrating each term, Applying limit	2+1+2+1

GOVERNMENT OF KARNATAKA

DEPARTMENT OF COLLEGIATE AND TECHNICAL EDUCATION

I/II SEMESTER DIPLOMA EXAMINATIONS, OCT/NOV-2021

Sub: Engineering Mathematics **Code: 20SC01T**

SOLUTION

Qno	SECTION 1	MARK
1a)	**If $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 3 & 2 & x \end{bmatrix}$ is Singular Matrix, Find x** **Solution:** Given A is singular i.e $\|A\| = 0$ $\|A\| = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 3 & 2 & x \end{vmatrix} = 0$ $1(2x - 8) - 3(0x - 12) + (-1)(0 - 6) = 0$ $x = -17$	1 1 1 1
1b)	**If $A=\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ $B=\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ find 2A+3B** **Solution:** $2A= \begin{bmatrix} 2X2 & 2X3 \\ 2X1 & 2X(-1) \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & -2 \end{bmatrix}$ $3B = \begin{bmatrix} 3X3 & 3X4 \\ 3X1 & 3X2 \end{bmatrix} = \begin{bmatrix} 9 & 12 \\ 3 & 6 \end{bmatrix}$ $2A+3B = \begin{bmatrix} 4 & 6 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 13 & 18 \\ 5 & 4 \end{bmatrix}$	2 2 1
1c)	**Solve 2x +3y = 5, x +4 y = 5 by Cramer's rule.** Solution:Given system of the equations is 2x +3y = 5 & x +4 y = 5 let $\Delta= \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 8 - 3 = 5$ $\Delta_1= \begin{vmatrix} 5 & 3 \\ 5 & 4 \end{vmatrix} = 20 - 15 = 5$ $\Delta_2= \begin{vmatrix} 2 & 5 \\ 1 & 5 \end{vmatrix} = 10 - 5 = 5;$ $\therefore x = 1$ and $y = 1$	1 1 1 1+1
1d)	**Find the characteristic roots for the matrix$A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$** **Solution:**Given $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$ C.E is given by $\|A - \lambda I\| = 0$ $\left\| \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\| = 0$ $\begin{vmatrix} 2-\lambda & 3 \\ 0 & 4-\lambda \end{vmatrix} = 0$ $(2-\lambda)(4-\lambda) = 0$ $\lambda = 2$ or $\lambda = 4$ are characteristic roots.	1 1 1 1 2

Qno	SECTION 1	MARK
2a)	**If A=$\begin{bmatrix}2 & 1\\0 & 4\end{bmatrix}$ B=$\begin{bmatrix}1 & -1\\3 & 6\end{bmatrix}$ find $(A+B)^2$** **Solution:** $A+B=\begin{bmatrix}2 & 1\\0 & 4\end{bmatrix}+\begin{bmatrix}1 & -1\\3 & 6\end{bmatrix}=\begin{bmatrix}2+1 & 1-1\\0+3 & 4+6\end{bmatrix}=\begin{bmatrix}3 & 0\\3 & 10\end{bmatrix}$ $(A+B)^2=\begin{bmatrix}3 & 0\\3 & 10\end{bmatrix}\begin{bmatrix}3 & 0\\3 & 10\end{bmatrix}=\begin{bmatrix}3x3+0x3 & 3x0+0x10\\3x3+10x3 & 3x0+10x10\end{bmatrix}$ $=\begin{bmatrix}9+0 & 0+0\\9+30 & 0+100\end{bmatrix}=\begin{bmatrix}9 & 0\\39 & 100\end{bmatrix}$	1 1+1 1
2b)	**Find $(AB)^T$ if A = $\begin{bmatrix}1 & 3 & -1\\3 & 2 & 0\\4 & 1 & 3\end{bmatrix}$ B = $\begin{bmatrix}2 & 1 & 3\\4 & 2 & 1\\1 & 3 & -2\end{bmatrix}$** **Solution:** AB=$\begin{bmatrix}1 & 3 & -1\\3 & 2 & 0\\4 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1 & 3\\4 & 2 & 1\\1 & 3 & -2\end{bmatrix}$ $=\begin{bmatrix}2+12-1 & 1+6-3 & 3+3+2\\6+8+0 & 3+4+0 & 9+2+0\\8+4+3 & 4+2+9 & 12+1-6\end{bmatrix}$ $=\begin{bmatrix}13 & 4 & 8\\14 & 7 & 11\\15 & 15 & 7\end{bmatrix}$ $(AB)^T=\begin{bmatrix}13 & 14 & 15\\4 & 7 & 15\\8 & 11 & 7\end{bmatrix}$	1 1 2 1
2c)	**Find the characteristic equation and Eigen roots for the matrix A = $\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$** **Solution:** Given A = $\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$ C.E is given by $\lvert A-\lambda I\rvert=0$ $\left\lvert\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\rvert=0$ $\begin{vmatrix}1-\lambda & 2\\3 & 1-\lambda\end{vmatrix}=0$ $\lambda^2-2\lambda-5=0$ $\lambda=1\pm\sqrt{6}$	1 1 1 1+1
2d)	**Find the inverse of the matrix A = $\begin{bmatrix}5 & 1 & 3\\1 & 4 & -1\\2 & 0 & 4\end{bmatrix}$** **Solution:** Given A = $\begin{bmatrix}5 & 1 & 3\\1 & 4 & -1\\2 & 0 & 4\end{bmatrix}$; $\lvert A\rvert=\begin{vmatrix}5 & 1 & 3\\1 & 4 & -1\\2 & 0 & 4\end{vmatrix}$=50 adjA = $\begin{bmatrix}16 & -4 & -13\\-6 & 14 & 8\\-8 & 2 & 19\end{bmatrix}$, $A^{-1}=Adj(A)/\lvert A\rvert$ $A^{-1}=\frac{1}{50}\begin{bmatrix}16 & -4 & -13\\-6 & 14 & 8\\-8 & 2 & 19\end{bmatrix}$	1 3+1 1

Qno	SECTION 2	MARK
3a)	**Find the slope of a straight line passing through the points $(1, 2)$ & $(3, 4)$** **Solution:** Let $A = (x_1, y_1) = (1, 2)$ and $B = (x_2, y_2) = (3, 4)$ $\therefore$ Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$ $\Longrightarrow m = \frac{4-2}{3-1}$ $=1$	 2 1 1
3b)	**If x- intercept of the line is 3 units and y-intercept is 2 times x-intercept ,the find the equation of the line.** **Solution:** Given that: $a = 3$ and $b = 2a = 2X3 = 6$ Equation of a straight line is $\frac{x}{a} + \frac{y}{b} = 1$ $\Longrightarrow \frac{x}{3} + \frac{y}{6} = 1$ $\Longrightarrow \frac{2x + y}{6} = 1$ $\Longrightarrow 2x + y - 6 = 0$	 1 2 1 1
3c)	**If a straight line makes an angle of 60^o with the positive direction of the x-axis and passes through the point (1, -1), then find the equation of the straight line.** **Solution:** Given $\theta = 60^o$ slope is $m = \tan\theta$ then, $m = \tan\theta = \tan 60^o = \sqrt{3}$ $y - y_1 = m(x - x_1)$ $\sqrt{3}x - y - 1 - \sqrt{3} = 0$	 1 1 2 1
3d)	**Find the equation of the straight line parallel to the line joining the points (3, -1) and (4, -2) and passing through the point (2,2).** **Solution:** $\mathbf{m = -1}$ Since the given line is parallel to required line $m_1 = m_2 = -1$ $y - y_1 = m(x - x_1)$ $y - 2 = -x + 2$ $x + y - 4 = 0$ is the required equation of line.	 1+1 2 1 1

<table>
<tr><th>Qno</th><th>SECTION 2</th><th>MARK</th></tr>
<tr><td>4a)</td><td>Write the slope and x-intercept of line $2x + 4y + 5 = 0$.
Solution: Here $a = 2$, $b = 4$ and $c = 5$
Slope, $m = -\frac{a}{b} = -\frac{2}{4} = -\frac{1}{2}$
$x - \text{intercept} = -\frac{c}{a} = -\frac{5}{2}$</td><td>1+1
1+1</td></tr>
<tr><td>4b)</td><td>Find the equation of the line passing through the points $(4, 2)$ and $(1, 3)$.
Solution: m$=\left(\frac{y_2-y_1}{x_2-x_1}\right)$=-1/3
$y - y_1 = (x - x_1)$
$y - 2 = \left(\frac{3-2}{1-4}\right)(x-4)$
$-x - 3y + 6 + 4 = 0$
$x + y - 10 = 0$ is the required equation of the line.</td><td>1+1

2

1</td></tr>
<tr><td>4c)</td><td>Find the equation of line perpendicular to the line 4x-2y+3=0 and passing through the point (1,2).
Solution: $m= \frac{-4}{-2} = \frac{2}{4} = 2$; $m_1 \times m_2 = -1$; $m_2 = \frac{-1}{2}$
$y - y_1 = m(x - x_1)$
$y - 2 = \frac{-1}{2}(x - 1)$
$x + 2y - 5 = 0$ is the required equation of line.</td><td>1+1
2

1</td></tr>
<tr><td>4d)</td><td>Find the acute angle between the two lines $7x - 4y = 0$ and $3x - 11y + 5=0$

Solution: Given line l_1 $7x - 4y = 0$ and l_2 $3x - 11y + 5 = 0$

Let the slope of l_1 be $m_1 = -\frac{a}{b} = -\frac{7}{-4} = \frac{7}{4}$

Slope of l_2 be $m_2 = -\frac{a}{b} = -\frac{-3}{-11} = \frac{3}{11}$

$\tan\theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

$\tan\theta = \left(\frac{(\frac{7}{4}) - (\frac{3}{11})}{1 + (\frac{7}{4})(\frac{3}{11})}\right) = \left(\frac{77-12}{44+21}\right) = \left(\frac{65}{65}\right) = 1$

$\theta = \tan^{-1}(1) = 45^0$</td><td>1

1

2

1

1</td></tr>
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Qno	SECTION 3	MARK
5a)	**Convert 75° in radians measure and $\frac{7\pi}{2}$ in degree.** **Solution:** $x \text{ degree} = \frac{\pi}{180} \times x \text{ radians}$; $75° = \frac{5\pi}{12}$ radians $x \text{ radians} = \frac{180°}{\pi} \times x \text{ degree}$; $\frac{7\pi}{2}$ radians $= 630$ degree	1+1 1+1
5b)	**Simplify:** $\frac{\sec(360-A)\cot(90-A)}{\tan(360+A)\operatorname{cosec}(90+A)}$ **Solution:** Consider $\frac{\sec(360-A)\cot(90-A)}{\tan(360+A)\operatorname{cosec}(90+A)}$ $= \frac{\sec A \times \tan A}{\tan A \times \sec A} = 1$	1+1+1+1 1
5c)	**Prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.** **Proof:** We have $\cos(A+B) = \cos A\cos B - \sin A\sin B$ put $B = A = \theta$ $\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$.	2 1 1 1
5d)	**Show that $\sin 20^0 \sin 40^0 \sin 80^0 = \frac{\sqrt{3}}{8}$.** **Solution:** $\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ $= \frac{1}{2}[\cos(40^0 - 20^0) - \cos(40^0 + 20^0)] \sin 80^0$ $= \frac{1}{2}[\cos 20^0 - \cos 60^0] \sin 80^0$ $= \frac{1}{2}\left(\cos 20^0 - \frac{1}{2}\right) \sin 80^0$ $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ $= \frac{1}{4}[\sin(80^0 + 20^0) + \sin(80^0 - 20^0)] - \frac{1}{4}\sin(180 - 100^0) = \frac{1}{4}\sin 60^0 = \frac{\sqrt{3}}{8}$	2 1 1 1 1

Qno	SECTION 3	MARK
6a)	**Find the value of $\cos 15^o$** **Solution:** Given $\cos 15^o = \cos(45^o - 30^o)$ We know that $\cos(A - B) = \cos A \cos B + \sin A \sin B \rightarrow (1)$ Substitute $A = 45^o, B = 30^o$ in equation (1), then we get $\cos 15^o = \cos(45^o - 30^o) = \cos 45^o \cos 30^o + \sin 45^o \sin 30^o$ $\cos 15^o = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$	1 1 1 1
6b)	**Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.** Proof: $\sin 3\theta = \sin(2\theta + \theta)$ $\sin 3\theta = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $\sin 3\theta = 2\sin\theta\cos\theta \times \cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $\sin 3\theta = 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$ $\sin 3\theta = 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $\sin 3\theta = 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$	1 1 1 1 1
6c)	**Show that $\sin 600^o \cos 390^o + \cos 480^o \sin 150^o = -1$** **Solution:** $\sin 600^o = \frac{-\sqrt{3}}{2}$; $\cos 390^o = \frac{\sqrt{3}}{2}$; $\cos 480^o = -\frac{1}{2}$; $\sin 150^o = \frac{1}{2}$ Now, Consider LHS $= \sin 120^o \cos 330^o + \cos 420^o \sin 30^o$ $= \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{2}\right) = \frac{-3}{4} - \frac{1}{4} = \frac{-4}{4} = -1 = \text{RHS}$	1+1+1+1 1
6d)	**Prove that $\cos 55 + \cos 65 + \cos 175 = 0$.** **Solution:** $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ $= \cos 55 + \cos 175 + \cos 65$ $= \cos 55 + 2\cos\left(\frac{175 + 65}{2}\right)\cos\left(\frac{175 - 65}{2}\right)$ $= \cos 55 + 2\cos 120 \cos 55$ $= \cos 55 + 2\cos(180 - 60)\cos 55$ $= \cos 55 - \cos 55 = 0$	1 1 1 1 1 1

Qno	SECTION 4	MARK
7a)	**If $y = x^3 + e^{3x} + \sin 3x - 4\log x$ then find $\frac{dy}{dx}$.** **Solution:** $y = x^3 + e^{3x} + \sin 3x - 4\log x$; $\frac{dy}{dx} = 3x^2 + 3e^{3x} + 3\cos 3x - 4(\frac{1}{x})$	1+1+1+1
7b)	**Find $\frac{dy}{dx}$ where $y = \frac{1+\cos x}{1-\cos x}$** **Solution:** $\frac{dy}{dx} = \frac{(1-\cos x)\frac{d}{dx}(1+\cos x)-(1+\cos x)\frac{d}{dx}(1-\cos x)}{(1-\cos x)^2}$ $= \frac{(1-\cos x)(0-\sin x)-(1+\cos x)(0+\sin x)}{(1-\cos x)^2}$ $= \frac{-\sin x(1-\cos x)-\sin x(1+\cos x)}{(1-\cos x)^2}$ $= \frac{-\sin x+\sin x\cos x-\sin x-\sin x\cos x}{(1-\cos x)^2}$ $\frac{dy}{dx} = \frac{-2\sin x}{(1-cosx)^2}$	1 1 1 1 1
7c)	**Find $\frac{dy}{dx}$ for $y = e^{sinx} + \sin(x^3) + \tan x$** **Solution:** $y = \sin(x^3) + e^{sinx} + \tan x$ $\frac{dy}{dx} = \frac{d(e^{sinx}+\sin(x^3)+\tan x)}{dx}$ $=e^{sinx}(\cos x) + \cos(x^3)(3x^2) + sec^2x$	2+2+1
7d)	**The displacement of a particle from one point to another is given by $s = t^2 + 6t + 5$, find the velocity and acceleration at the end of t=3 sec** **Solution:** $\frac{ds}{dt} = 2t + 6 + 0$; $\frac{ds}{dt} = 2t + 6$; $\frac{d^2s}{dt^2} = 2 + 0$; $\frac{d^2s}{dt^2} = 2$ Velocity =12 unit/sec; Acceleration = 2 unit/s^2	2+2 1+1

Qno	SECTION 4	MARK
8a)	**If $y = e^x \sin x$ then find $\frac{dy}{dx}$.** **Solution:** $y = e^x sinx$ $\quad (uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\frac{dy}{dx} = e^x \frac{d}{dx}(sinx) + sinx\frac{d}{dx}(e^x)$ $\frac{dy}{dx} = e^x \cos x + \sin x\, e^x$	2 1+1
8b)	**If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ then find $\frac{dy}{dx}$.** **Solution:** $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ $\frac{dy}{dx} = \frac{d}{dx}\left(\tan\left(\frac{1+x}{1-x}\right)\right) = \frac{1}{1+(\frac{1+x}{1-x})^2} \times \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$ w.k.t $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$; $\frac{dy}{dx}\ \frac{(1-x)^2}{(1-x)^2+(1+x)^2} X \frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}$ $\frac{dy}{dx} = \frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2+(1+x)^2}$ $\frac{dy}{dx} = \frac{2}{2(1+x^2)} = \frac{1}{(1+x^2)}$	2 1 1 1
8c)	**Obtain the equation of tangent to the curve $y = x^3$ at the point $(1, 2)$** **Solution:** $y = x^3$; $\frac{dy}{dx} = 3x^2$; $\left(\frac{dy}{dx}\right)_{at(1,2)} = 3(1) = 3$ $m = 3$ The equation of tangent to the curve at the point (1, 2) with slope m=3 is $Y - y_1 = m(x - x_1)$ $y - 2 = 3(x - 1)$ $y - 2 = 3x - 3$ $\mathbf{3x - y - 1 = 0}$	1+1 2 1
8d)	**If $y = \tan^{-1} x$ then prove that $(1 + x^2)y_2 - 2xy_1 = 0$** **Solution:** $y = \tan^{-1} x$ $y_1 = \frac{d}{dx}(\tan^{-1} x)$ $y_1 = \frac{1}{1+x^2}$ $(1 + x^2)y_1 = 1$ Again differentiate w.r.t.x $(1 + x^2)\frac{d}{dx}(y_1) + y_1\frac{d}{dx}(1 + x^2) = \frac{d}{dx}(1)$ $(1 + x^2)y_2 + y_1(0 + 2x) = 0$ $(1 + x^2)y_2 + 2xy_1 = 0$	1 1 1+1 1 1

Qno	SECTION 5	MARK
9a)	Evaluate $\int\left(x^2+\sin x+\frac{1}{x}+e^{2x}\right)dx$ **Solution:** $I=\frac{x^3}{3}-\cos x+\log x+\frac{e^{2x}}{2}+c$	1+1+1+1
9b)	***Evaluate*** $\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$ **Solution:** $\int_0^{\frac{\pi}{2}}\cos^2 x\,dx=\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2x}{2}\right)dx=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$ $=\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{\sin 2\frac{\pi}{2}}{2}\right)-\left(0+\frac{\sin 2(0)}{2}\right)\right]$ $=\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{0}{2}\right)-\left(0+\frac{0}{2}\right)\right]=\frac{\pi}{4}$	1+1 1+1 1
9c)	**Evaluate:** $\int x\,e^x\,dx$ **Solution:** Here x is Algebraic function and e^x is Exponential function. According to the ILATE rule of choosing the first function, u = I fn = x and v = II fn = e^x $\int(\text{I fn}\times\text{II fn})dx=\text{I fn}\int(\text{II fn})dx-\int\left(\int(\text{II fn})\,dx\right)\frac{d}{dx}(\text{I fn})\,dx$ $I=\int x\,e^x\,dx$ $= x\int e^x\,dx-\int\left(\int e^x\,dx\right)\frac{d}{dx}(x)\,dx$ $= x\,e^x-\int e^x\times 1dx$ $= x\,e^x-\int e^x\,dx$ $= x\,e^x-e^x+c$	1+1 1 1 1
9d)	Find the area bounded by the curve $y=x^2+3$, x – axis and the ordinates $x=1$ & $x=2$. **Solution:** Area enclosed by a curve is $A=\int_a^b y\,dx\rightarrow(1)$ $A=\int_a^b y\,dx=\int_1^2(x^2+3)dx$ $=\left[\frac{x^3}{3}+3x\right]_1^2$ $=\left[\frac{2^3}{3}+3\times 2\right]-\left[\frac{(1)^3}{3}+3(1)\right]$ $=\left[\frac{8}{3}+6\right]-\left[\frac{1}{3}+3\right]$ $=\left[\frac{26}{3}\right]-\left[\frac{10}{3}\right]=\frac{16}{3}\;\therefore A=\frac{16}{3}$ Square units	1 1 1+1 1 1

Qno	SECTION 5	MARK
10a)	**Evaluate:** $\int_1^2 \frac{1}{x}\,dx$ **Solution:** $\int_1^2 \frac{1}{x}\,dx = logx\|_1^2 = [\log 2 - \log 1] = [\log 2 - 0]$ =log2	1+1+1+1
10b)	**Evaluate:** $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ **Solution:** Put $\tan^{-1}x$= t : $\frac{1}{1+x^2}$ dx = dt Let I = $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ = $\int \tan^{-1}x \frac{1}{1+x^2}\,dx$ = $\int t\ .dt$ = $\frac{t^{1+1}}{1+1}$ +c = $\frac{(\tan^{-1}x)^2}{2}$ +c== $\frac{t^2}{2}$ + c	1+1 1 1 1
10c)	**Evaluate:** $\int x\, sec^2x\,dx$ **Solution:** Let I = $\int x\, sec^2x\ dx$ Here x is Algebraic function and $\sec^2x$ is Trigonometric function. According to the ILATE rule of choosing the first function, u = I fn = x and v = II fn = $\sec^2x$ $\int$ (I fn × II fn)dx = I fn $\int$(II fn)dx − $\int(\int$(II fn) dx) $\frac{d}{dx}$(I fn) dx I = $\int x\, sec^2x\ dx$ = x $\int sec^2x$ dx - $\int (\int sec^2x\ dx) \frac{d}{dx}(x)$ dx = x (tan x) - $\int$(tanx) × 1dx = x tan x - $\int$ tanx dx = x tan x -log (sec x) +c	1+1 1 1 1
10d)	Find the volume of the solid generated by revolving the curve $y = \sqrt{x^2+1}$ between x=0 and x=2 **Solution:** We know that, the volume of the solid formed by revolving the curve y=f(x) and the x−axis between x=a and x=b about the x−axis is $V = \pi\int_a^b y^2\,dx$ $= \pi\int_0^2 (\sqrt{x^2+1})^2\,dx$ $= \pi\int_0^2 x^2+1\,dx$ $= \pi\left[\frac{x^3}{3}+x\right]_0^2$ $= \pi\left[(\frac{2^3}{3}+2)-0\right]$ ∴ $V = \frac{14}{3}\pi$ cubic units	1 1 1 1+1 1